

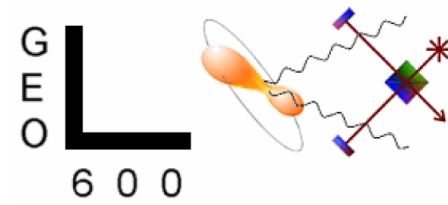
# First LIGO/GEO Upper Limits on Pulsar Gravitational Emissions

**Teviet Creighton**

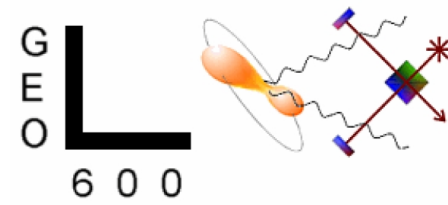
For the Pulsar Upper Limits Working Group  
of the LIGO Scientific Collaboration

**CaJAGWR Seminar  
April 15, 2003**

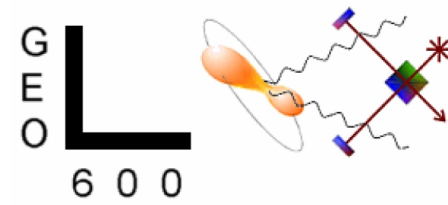
**LIGO-G030189-00-Z**



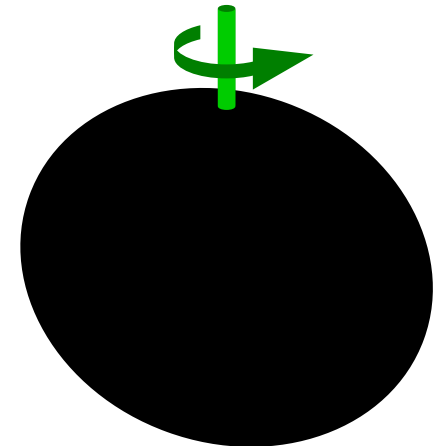
- S1 science run took 3 weeks of data (Aug. 23 – Sep. 9, 2003) on 4 detectors (LIGO L1, H1, H2, and GEO600).
- Data analyzed for signal from PSR J1939+2134, using two methods:
  - ★ Frequency-domain frequentist analysis  $\Rightarrow h_0 < (2.8 \pm 0.3) \times 10^{-22}$
  - ★ Time-domain Bayesian analysis  $\Rightarrow h_0 < (1.0 \pm 0.1) \times 10^{-22}$
- Upper limits were set in each case
- For this pulsar,  $h_0 < 1.0 \times 10^{-22}$  corresponds to ellipticity ratio (non-axisymmetry)  $\epsilon < 7.5 \times 10^{-5}$ .

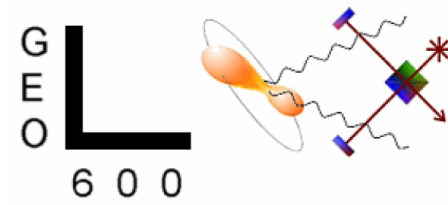


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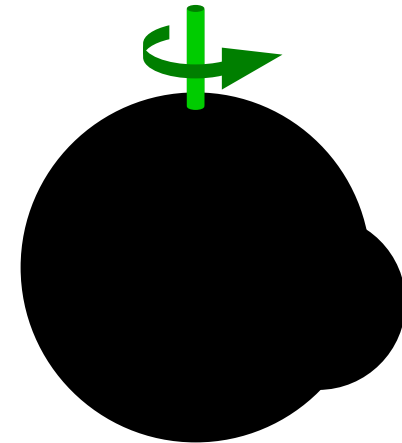


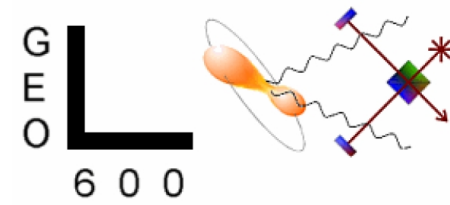
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- Emit gravitational waves if they are *non-axisymmetric*
- Possible mechanisms:



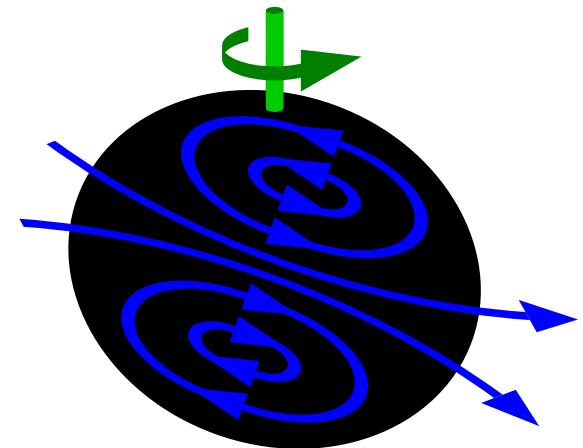


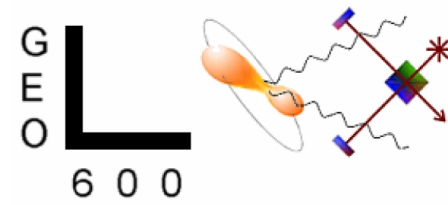
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  - ★ “Mountains” on solid crust



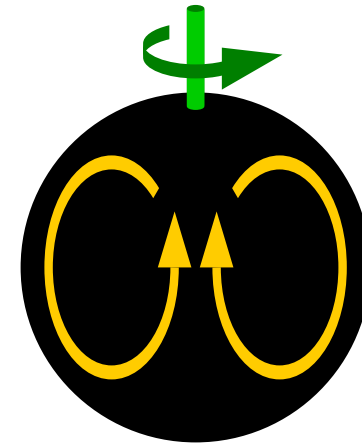


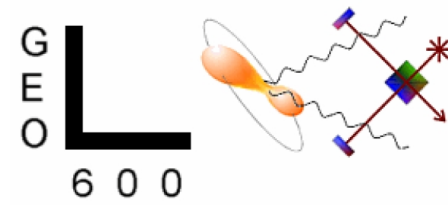
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  - ★ “Trapped” magnetic fields



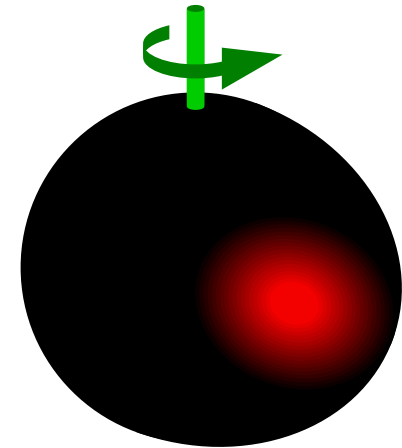


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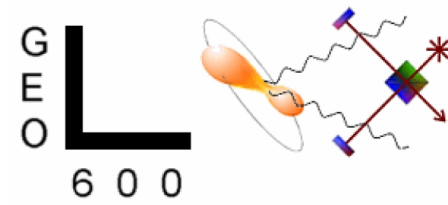




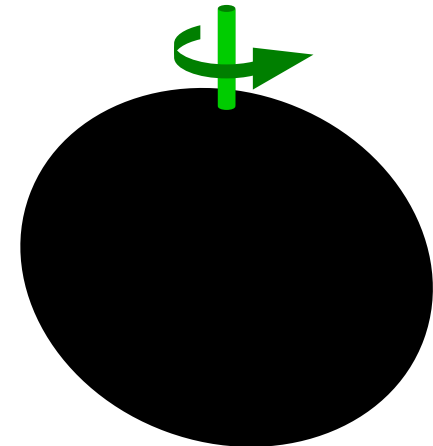
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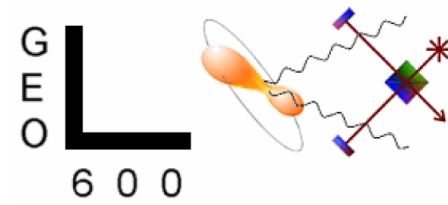






- Pulsars = spinning neutron stars
  - Emit gravitational waves if they are *non-axisymmetric*
  - Possible mechanisms:
    - ★ “Mountains” on solid crust
    - ★ “Trapped” magnetic fields
    - ★ Unstable fluid modes
    - ★ Compositional/thermal inhomogeneities
- ⇒ Most likely for known pulsars
- ★ Emit primarily at GW frequency =  $2 \times$  spin frequency





- Intrinsic amplitude:

$$h_0 = (1.06 \times 10^{-23}) \left( \frac{I}{10^{45} \text{g cm}^2} \right) \left( \frac{1 \text{ kpc}}{r} \right) \left( \frac{f_{\text{gw}}}{1 \text{ kHz}} \right)^2 \left( \frac{\epsilon}{10^{-5}} \right)$$

- Signal in detector is:

$$h(t) = h_0 \left\{ F_+(t, \psi) \frac{1 + \cos^2 \iota}{2} \cos[\Phi(t) + \phi_0] + F_\times(t, \psi) \cos \iota \sin[\Phi(t) + \phi_0] \right\}$$

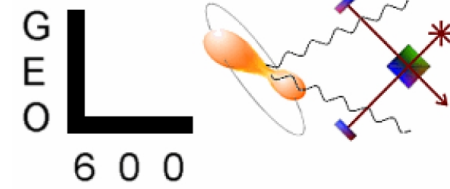
$F_+, F_\times$  = polarization beam patterns (known)  
 $\Phi$  = observed rotation phase (known)

$h_0$  = intrinsic amplitude (above)

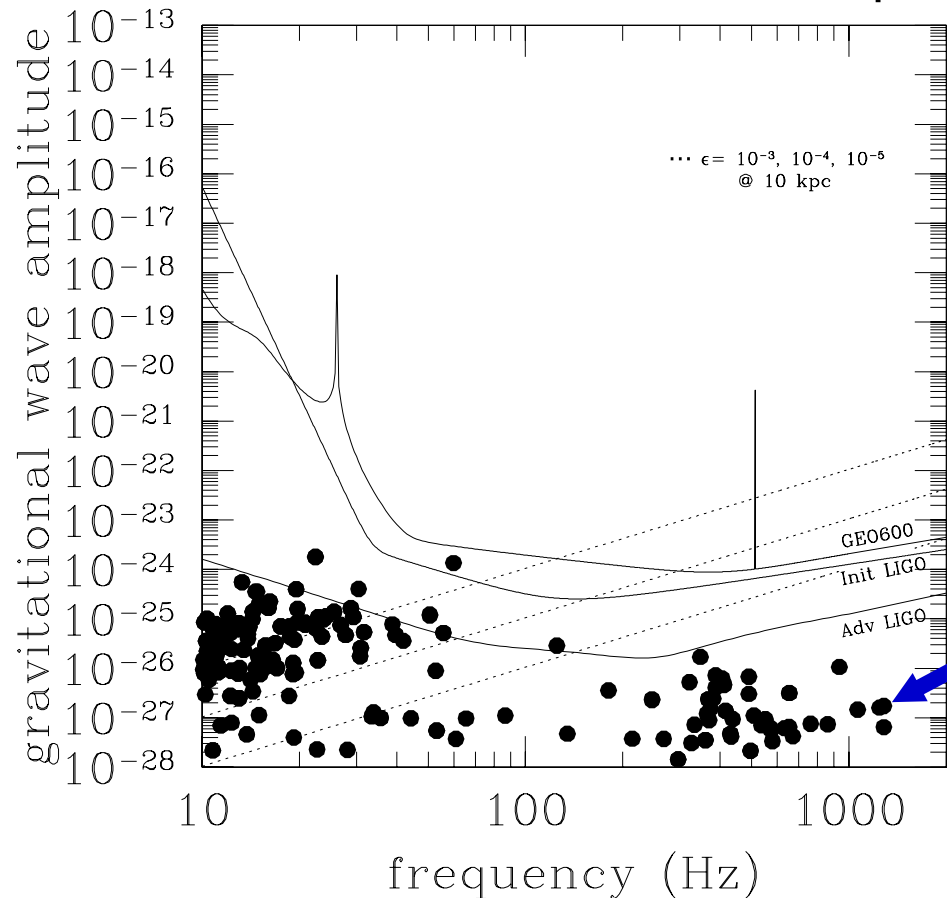
$\psi$  = polarization angle

$\iota$  = inclination angle

$\phi_0$  = phase offset



- At a 1% false alarm threshold, required amplitude for 10% false dismissal is:

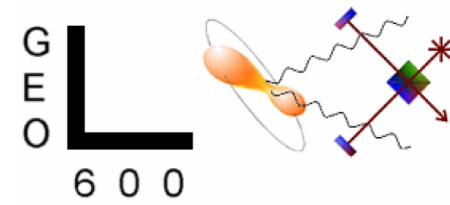


$$\langle h_0 \rangle = 11.4 \sqrt{S_h(f_{\text{gw}})/T_{\text{obs}}}$$

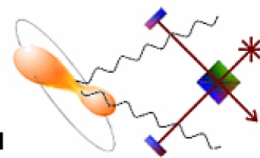
- 3 week integration
- Known pulsars shown

PSR J1939+2134  
 $f_{\text{gw}} = 1283.86\text{Hz}$

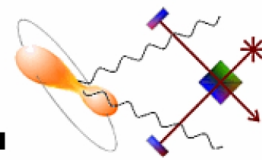
⇒ No detection expected!



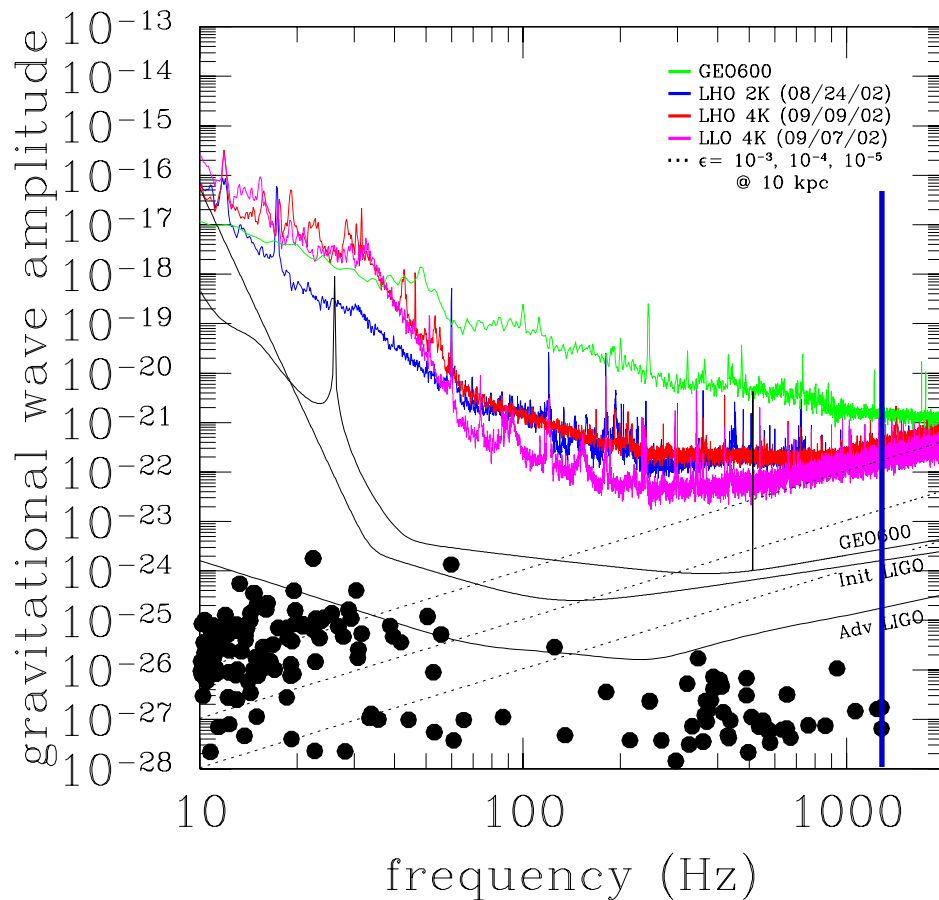
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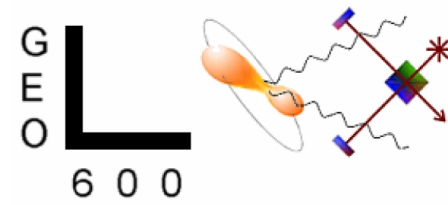
- First LIGO/GEO science run (S1): August 23 – September 9, 2002  
17 days = 408 hours
- Total of four interferometers participating:
  - ★ LIGO Livingston L1 (4 km):  
duty cycle 41.7%, total locked time: 170 hours
  - ★ LIGO Hanford H1 (4 km):  
duty cycle 57.6%, total locked time: 235 hours
  - ★ LIGO Hanford H2 (2 km):  
duty cycle 73.1%, total locked time: 298 hours
  - ★ GEO (600 m):  
duty cycle 98.5%! total locked time: 396 hours



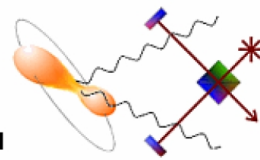
Instrumental sensitivity:



- Coincidence not important, only total uptime
  - Shorter instruments had higher uptime
- ⇒ Comparable sensitivity at frequency of interest!

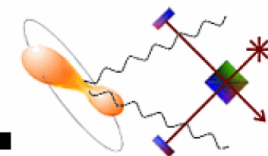


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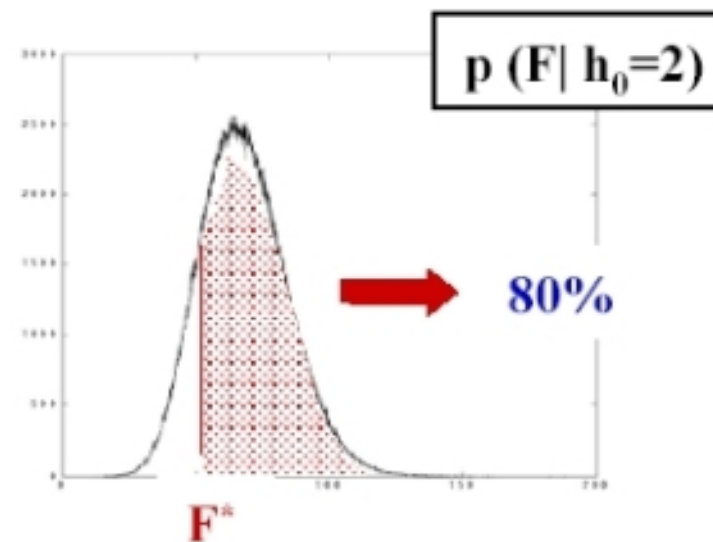
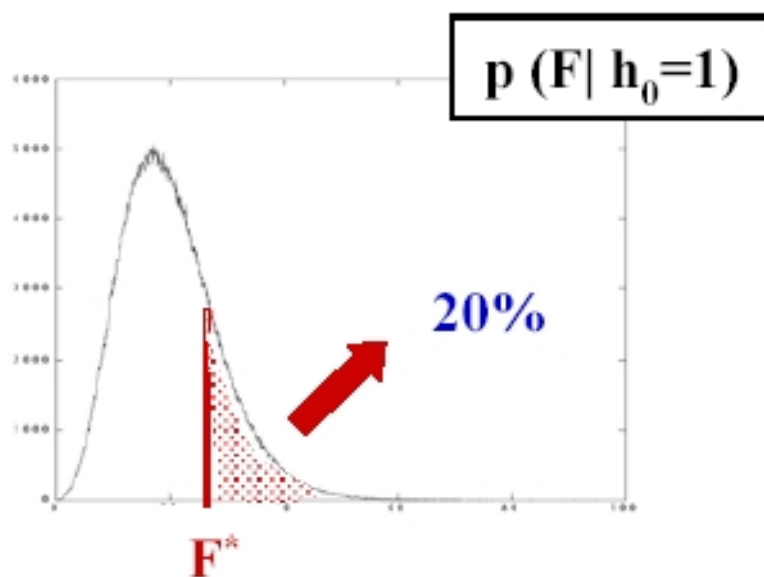


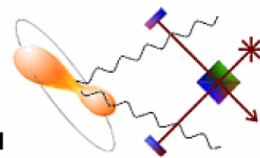
- $\mathcal{F}$ -statistic is a quadrature sum of 4 linear filters.
- In Gaussian noise, it is a *maximum likelihood* estimator of signal amplitude, implicitly maximized over  $\phi_0$ ,  $\psi$ , and  $\cos \iota$ .
  - ★  $2\mathcal{F}$  follows a  $\chi^2$  distribution with 4 degrees of freedom and non-centrality parameter  $\lambda \propto \int h(t)^2 dt$ .
- In generic noise, compute  $p(\mathcal{F}|\vec{a})$  using Monte-Carlo injections of simulated signals.
- Originally developed for pulsar *searches*: code exists to compute  $\mathcal{F}$  simultaneously over broad frequency ranges.





- *Frequentist* approach: Determine the value  $\mathcal{F}^*$  of the statistic for our source from our data.
- Determine  $p(\mathcal{F}|h_0)$  for a range of  $h_0$ .

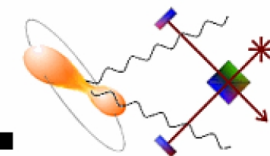




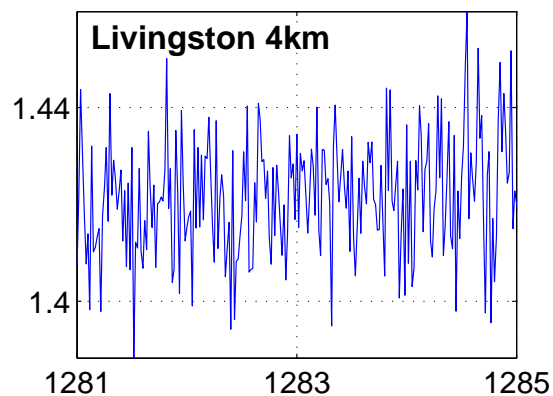
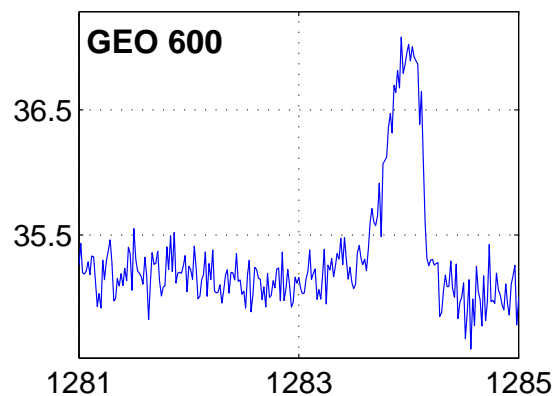
- *Frequentist* approach: Determine the value  $\mathcal{F}^*$  of the statistic for our source from our data.
- Determine  $p(\mathcal{F}|h_0)$  for a range of  $h_0$ .
- 95% frequentist upper limit  $h_{95}^*$  is the value such that, for repeated trials with a signal  $h_0 > h_{95}^*$ , we would obtain  $\mathcal{F} > \mathcal{F}^*$  more than 95% of the time:

$$0.95 = \int_{\mathcal{F}^*}^{\infty} p(\mathcal{F}|h_0 = h_{95}^*) d\mathcal{F}$$

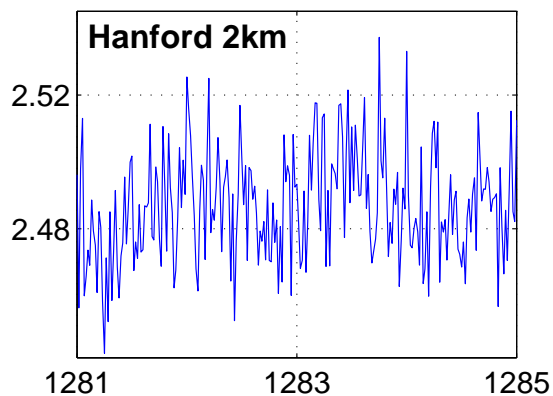
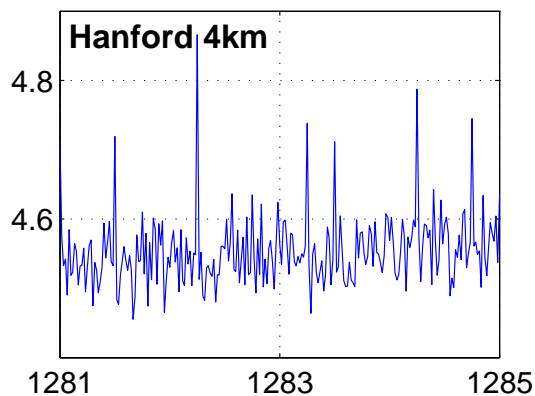
- Extra detail: When computing  $p(\mathcal{F}|h_0)$  via Monte-Carlo, inject signals with *worst possible* orientation  $\psi, \iota$ . This gives a *conservative* upper limit.

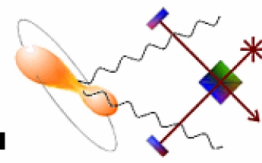


The raw data:  $\sqrt{S_h}$  ( $10^{-20}\text{Hz}^{-1/2}$ ) versus frequency in Hz.

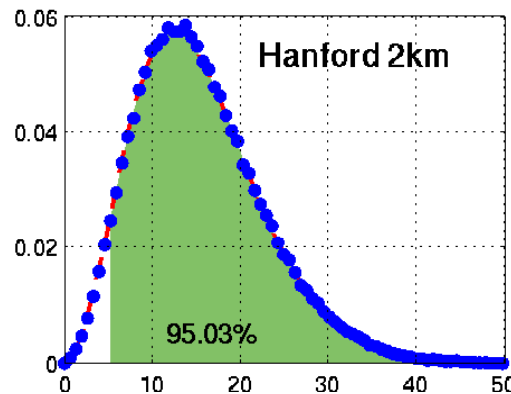
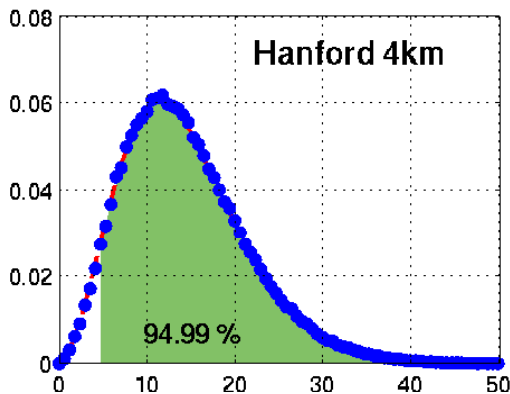
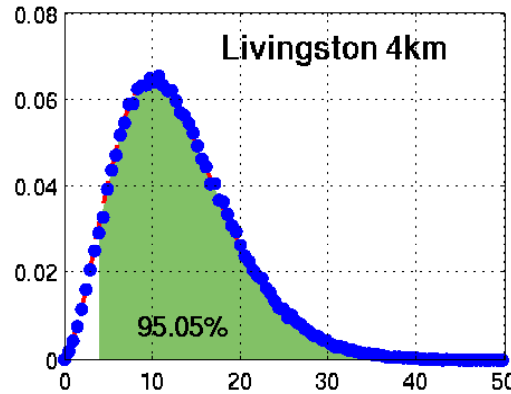
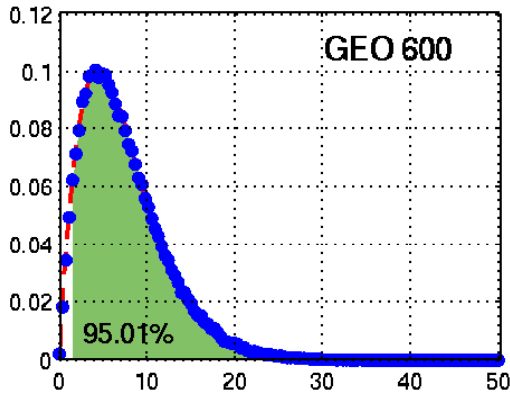


- Note spectral disturbance in GEO600



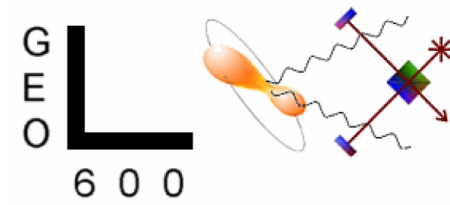


Probability distributions:

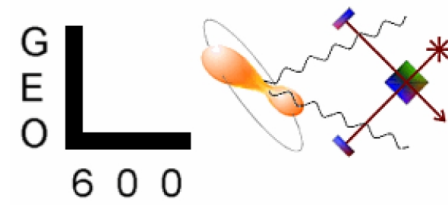


- All except GEO600 are consistent with Gaussian statistics (Kolmogorov-Smirnov test)
- 95% upper limits:

	$2\mathcal{F}^*$	$h_{95}^*$
GEO	1.5	$1.9 \times 10^{-21}$
L1	3.9	$2.8 \times 10^{-22}$
H1	4.7	$6.4 \times 10^{-22}$
H2	5.2	$4.7 \times 10^{-22}$



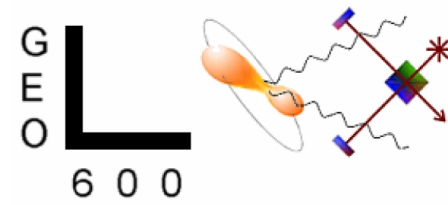
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- Signal is *heterodyned* by (known) instantaneous frequency of J1939+2134
  - ★ Reduces pulsar signal to DC
  - ★ Removes Doppler modulation from signal
- Resampled at 1/minute, and noise estimated for each minute
  - ⇒ data  $B_k \pm \sigma_k$  every minute.
- Data are then fit to a signal model:

$$y(t; \vec{a}) = \frac{1}{4} h_0 e^{2i\phi_0} [F_+(t, \psi)(1 + \cos^2 \iota) - 2F_\times(t, \psi) \cos \iota]$$

where  $\vec{a} = (h_0, \phi_0, \psi, \cos \iota)$  are unknown parameters.



- *Bayesian* approach: Compute joint probability distribution over all of  $\vec{a}$ , using *uniform* priors on  $h_0$ ,  $\phi_0$ ,  $\psi$ ,  $\cos \iota$ :

$$\begin{array}{ccccc}
 p(\vec{a}|\{B_k\}) & \propto & p(\vec{a}) & \cdot & p(\{B_k\}|\vec{a}) \\
 \uparrow & & \uparrow & & \uparrow \\
 \text{posterior} & & \text{prior} & & \text{likelihood}
 \end{array}$$

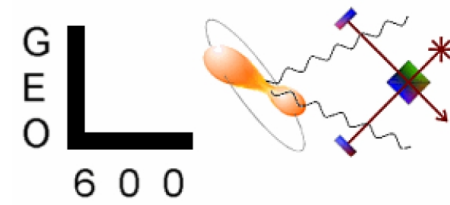
In Gaussian noise, likelihood  $\propto e^{-\chi^2/2}$ , where  $\chi^2(\vec{a}) = \sum_k \left| \frac{B_k - y(t_k; \vec{a})}{\sigma_k} \right|^2$

- To get probability distribution on  $h_0$ , *marginalize* over other parameters:

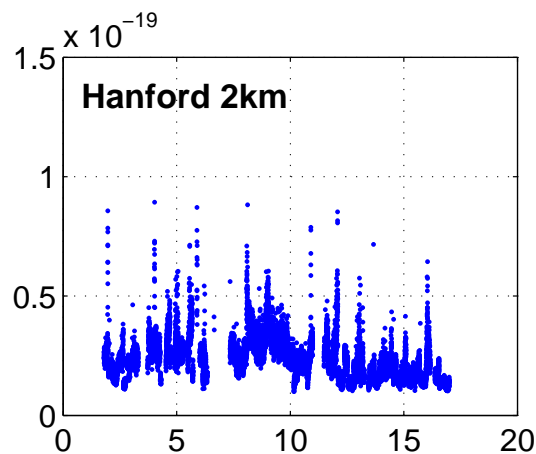
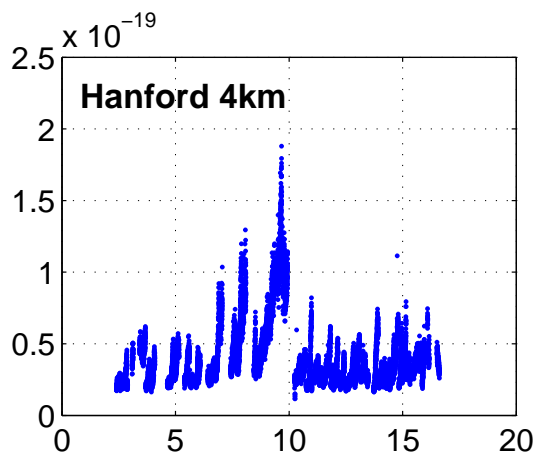
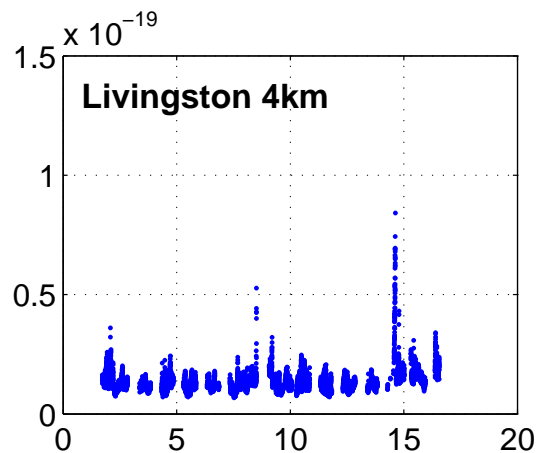
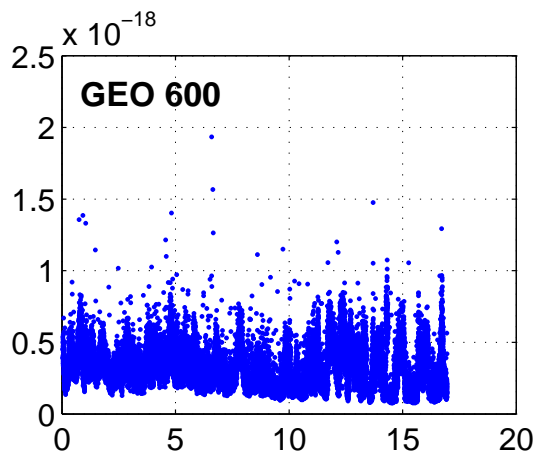
$$p(h_0|\{B_k\}) \propto \int d\phi_0 \int d\psi \int d\cos \iota e^{-\chi^2/2}$$

- 95% confidence upper limit  $h_{95}$  defined by:

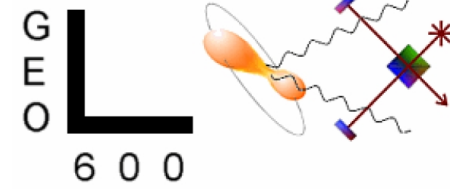
$$0.95 = \int_0^{h_{95}} dh_0 p(h_0|\{B_k\})$$



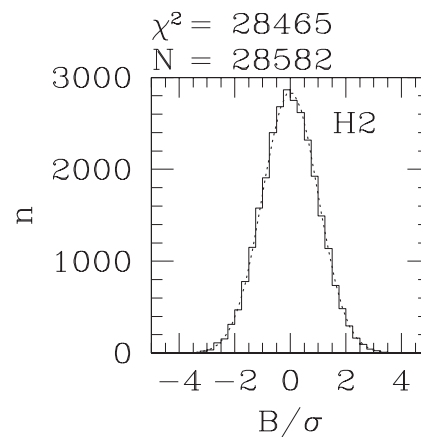
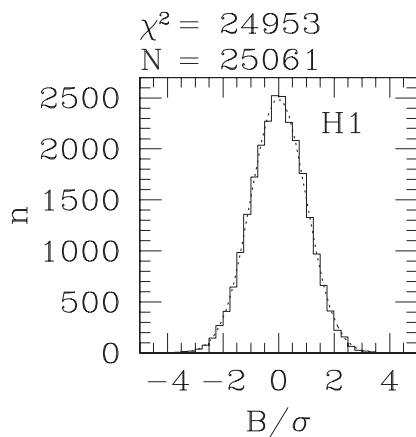
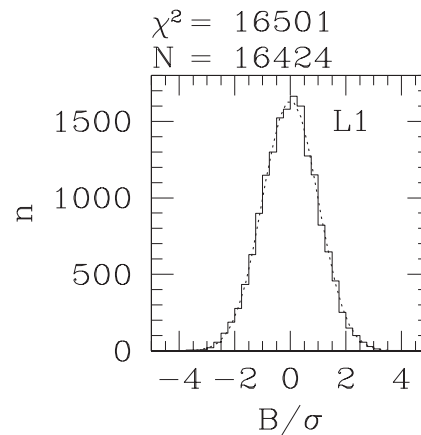
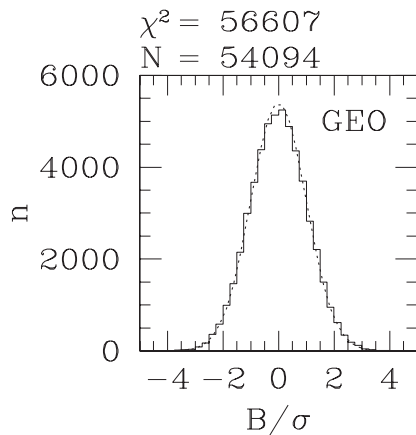
The raw data:  $\sqrt{S_h}$  ( $\text{Hz}^{-1/2}$ ) versus time in days



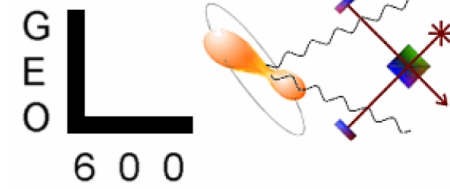




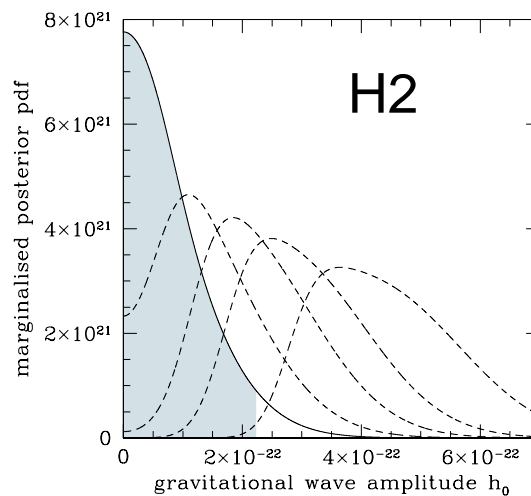
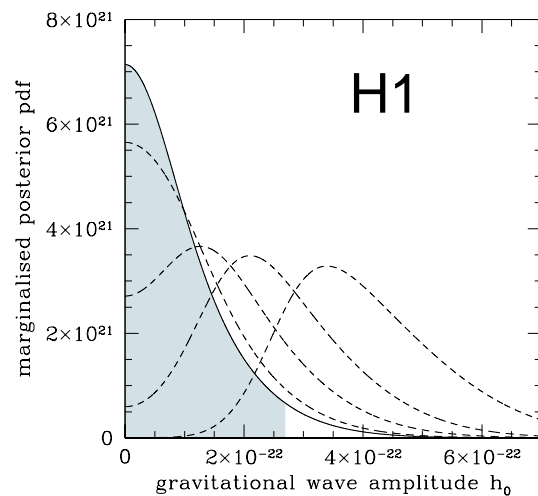
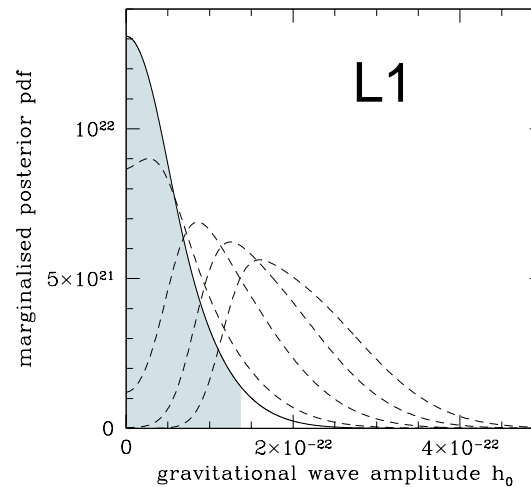
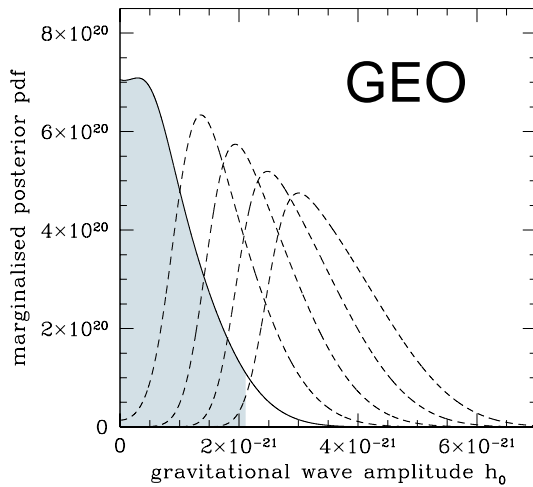
Gaussianity of resampled data  $B_k$ :



- GEO is not in fact consistent with Gaussian distribution.
  - ★ Spectral disturbance near this frequency
  - ★ Might raise our upper limit by about  $\times 1.5$
  
- LIGO detectors are consistent with Gaussian distribution.



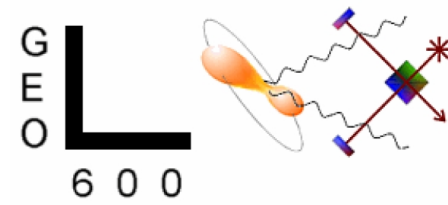
## Posterior probability distributions:



- 95% upper limits:

GEO	$2.1 \times 10^{-21}$
L1	$1.4 \times 10^{-22}$
H1	$2.7 \times 10^{-22}$
H2	$2.2 \times 10^{-22}$

- Can inject simulated signal to see how PDF changes.

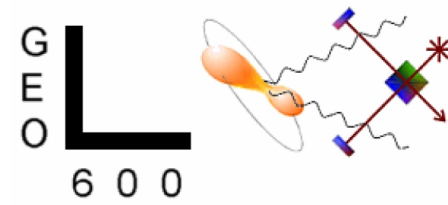


- Can also compute joint probability distribution:

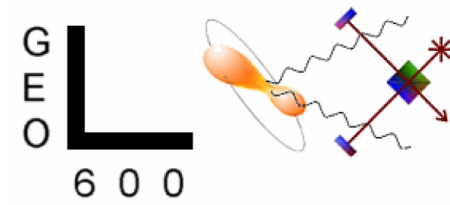
$$p(\vec{a}|\text{all data}) = p(\vec{a}|\text{GEO}) \cdot p(\vec{a}|\text{L1}) \cdot p(\vec{a}|\text{H1}) \cdot p(\vec{a}|\text{H2})$$

- Marginalizing gives:

$$h_{95} = 1.0 \times 10^{-22}$$

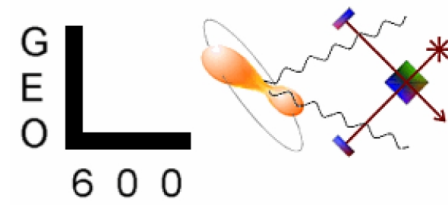


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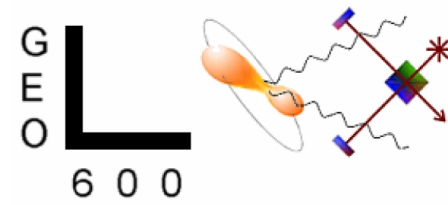


	Frequentist UL $h_{95}^*$	Bayesian UL $h_{95}$
GEO	$1.9 \times 10^{-21}$	$2.1 \times 10^{-21}$
H1	$6.4 \times 10^{-22}$	$2.7 \times 10^{-22}$
H2	$4.7 \times 10^{-22}$	$2.2 \times 10^{-22}$
L1	$2.8 \times 10^{-22}$	$1.4 \times 10^{-22}$
Joint	—	$1.0 \times 10^{-22}$

- PSR J1939+2134 is at 3.6 kpc  
 $\Rightarrow$  ellipticity  $\epsilon \leq 7.5 \times 10^{-5}$

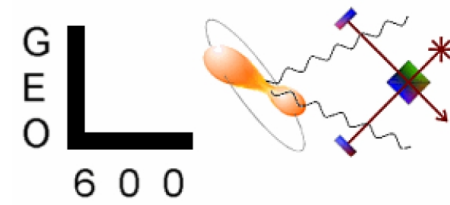


- Bayesian and frequentist analyses answer *two different questions*:
  - ★ Bayesian: Given our model and priors, for what value  $h_{95}$  are we 95% sure that the true  $h_0$  lies below this level?
    - ⇒ Threshold on  $p(h_0|\text{data, priors})$
  - ★ Frequentist: Given the measured value of  $\mathcal{F}^*$ , for what value  $h_{95}^*$  would a signal with  $h_0 > h_{95}^*$  yield  $\mathcal{F} > \mathcal{F}^*$  95% of the time?
    - ⇒ Threshold on  $p(\text{data}|h_0, \text{orientation})$
- It is therefore not surprising that the values  $h_{95}$  and  $h_{95}^*$  do not in general agree.
- Discrepancy largely due to *worst-case* (conservative) orientation chosen for frequentist approach.



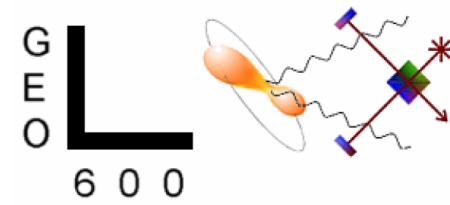
Other experimental results:

- Best UL on continuous signals is from a bar detector:  $2.9 \times 10^{-24}$  around 921.3 Hz from Galactic centre
  - ★ but no known pulsar at that frequency/location.
- Best previous UL on PSR J1939+2134 is  $1 \times 10^{-20}$  (using a divided bar).
- Indirect observational UL is  $2 \times 10^{-27}$  based on spindown rate.

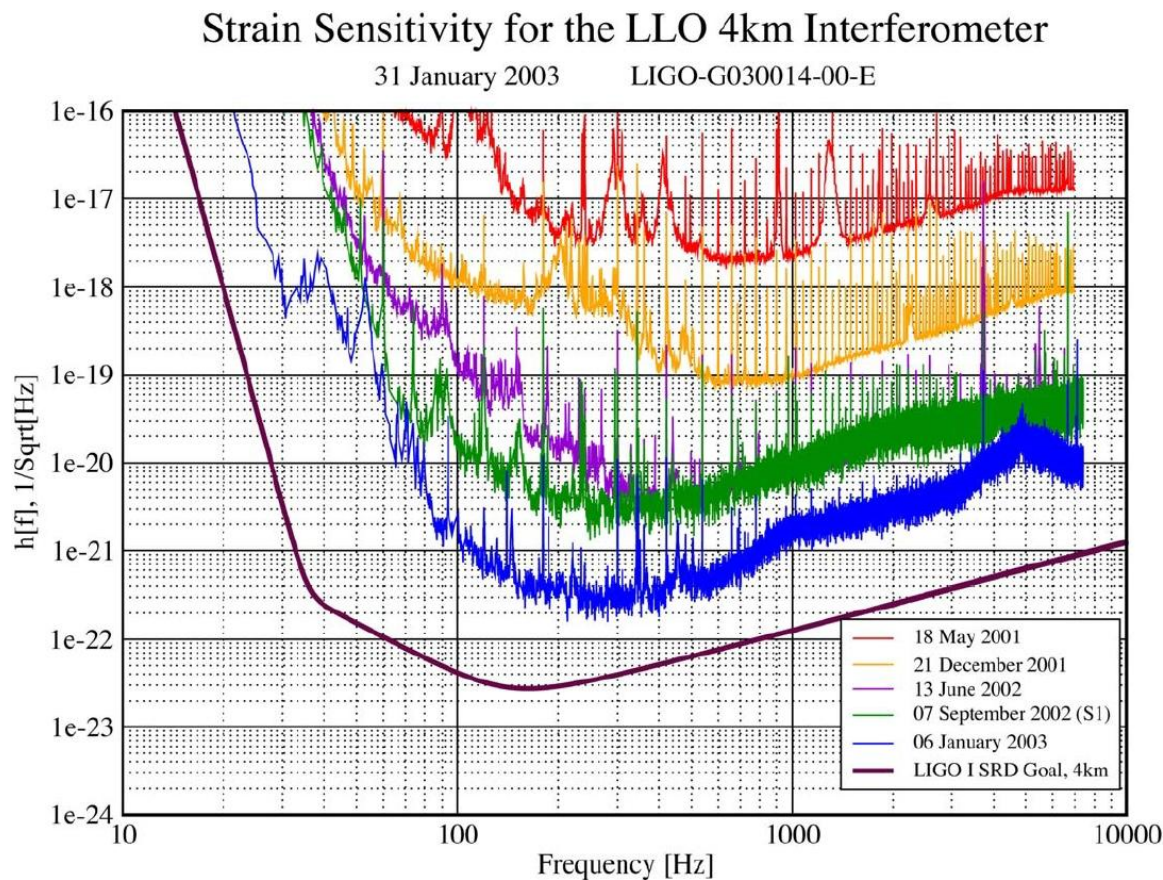


- I. Gravitational waves from pulsars
- II. LIGO and GEO during S1
- III. Frequency-domain analysis method
- IV. Time-domain analysis method
- V. Comparison of results
- VI. Future searches

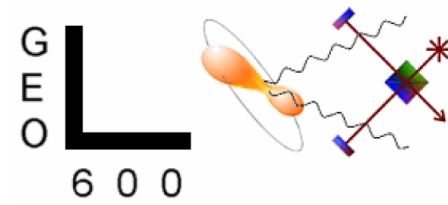




Second science run (S2) has just completed.



- Order of magnitude improvement in sensitivity!
- We want to start in on new data as soon as possible.



- Targeted searches on all known pulsars.
- Directed searches on known systems with unknown phase evolution (e.g. xray binaries).
- Broad-band wide-area searches.  
⇒ Set upper limits on *unknown* sources.
- As instruments continue to improve, we may make actual *detections* of gravitational emissions!