# Sources in the Medium Frequency Band

M. Benacquista Montana State University-Billings

D.Sedrakian, M. Hairapetyan, K. Shahabasyan, A. Sadoyan Yerevan State University

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Gravitational Wave Advanced Detectors Workshop

## Motivation and Outline

- Motivation
  - Look for possible continuous sources in the band
  - Quasi-radial oscillations of rotating white dwarfs
- Outline
  - Identify frequency range
  - Determine gravitational radiation
  - Estimate stochastic background level

#### Identifying the Frequency Range



#### White Dwarf Properties and Resonant Frequencies

$\rho_{\rm c} \left( {\rm g/cm^3} \right)$	$M_0 (M_{\odot})$	$M (M_{\odot})$	$\Omega_{ m max}$	$Q^0_{\rm max}$ (kg m <sup>2</sup> )	ω
$1.716 \times 10^{6}$	0.498	0.572	0.196	$3.872 \times 10^{42}$	0.757
$1.544 \times 10^{7}$	0.867	0.976	0.476	$2.315 \times 10^{42}$	0.766
$5.377 \times 10^{7}$	1.049	1.164	0.768	$1.338 \times 10^{42}$	1.077
$1.287 \times 10^{8}$	1.145	1.254	1.063	$8.095 \times 10^{41}$	1.399
$7.036 \times 10^{8}$	1.245	1.34	2.042	$2.687 \times 10^{41}$	2.001
$2.091 \times 10^{9}$	1.257	1.339	3.105	$1.217 \times 10^{41}$	1.299

## Determining Strain Amplitude

QuickTime<sup>™</sup> and a Animation decompressor are needed to see this picture.

- Assume oblate shape due to rotation
- Oscillation is self-similar and is described by:

$$x_{\alpha} = x_{\alpha}^{0} \left( 1 + \eta \sin(\omega t) \right)$$

• Quadrupole moment given by  $Q_{ij} = \int \rho(x_i x_j - \frac{1}{3} x^2 \delta_{ij}) dV = Q_{ij}^0 (1 + \eta \sin(\omega t))$ 

Choose z-axis along rotation axis:  $Q_{zz}^0 = -2Q_{xx}^0 = -2Q_{yy}^0 = -2Q_{yy}^0$ 

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#### Polarizations

In TT gauge with *z*-axis along the wave vector:

$$h_{+} = h_{xx} - h_{yy} = \frac{4GQ^{0}\eta\omega^{2}}{c^{4}r}\sin^{2}\theta\sin(\omega t)$$
$$h_{\times} = 2h_{xy} = 0$$

where  $\theta$  is the angle between the wave vector and the white dwarf axis of rotation

## Estimating the Background Level

- Numerical calculations give  $Q^0_{max}$  for select white dwarfs rotating at  $\Omega_{max}$
- Results are from an expansion in a dimensionless parameter proportional to  $\Omega^2$
- Use  $Q^0 = (\Omega / \Omega_{\text{max}})^2 Q^0_{\text{max}}$
- Integrate over  $\theta \Rightarrow \sin^2 \theta \to \frac{1}{2}$  $h_+ = 5.35 \times 10^{-65} \left(\frac{1 \text{ pc}}{r}\right) \left(\frac{\Omega}{\Omega_{\text{max}}}\right)^2 Q_{\text{max}}^0 \omega^2 \eta$

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#### White Dwarf Properties and Resonant Frequencies

$ ho_{c}$ (g/cm <sup>3</sup> )	$M_0 (M_{\odot})$	$M (M_{\odot})$	$\Omega_{\max}$	$Q^0_{\rm max}$ (kg m <sup>2</sup> )	ω
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## Expected Strain Amplitudes

• Maximum values (assume  $\Omega = \Omega_{max}$ )

• Assume 
$$r = 1$$
 pc,  $\eta = 10^{-4}$ 



## **Observed Rotation Rates**

Kawaler (astro-ph/0301539)

- Spectroscopic methods:
  - $M \sim 0.6 M_{\odot}$
  - $\Omega \sim 0.0036 \text{ Hz}$
  - $h \sim 4.1 \times 10^{-26}$
- Asteroseismic methods:
  - $M \sim 0.6 \ M_{\odot}$
  - $\Omega \sim 5.6 \times 10^{-5} \text{ Hz}$
  - $h \sim 9.7 \times 10^{-30}$

## Conclusions

- Vibrating white dwarfs are potential sources of gravitational radiation in the mid-frequency range.
- Assuming a mean distance of *r* pc to nearest vibrating white dwarf, an upper limit of  $\sim 10^{-23}/r$  can be placed on the expected level.
- If the high rotational rates of Kawaler are used, a more reasonable upper limit would be  $\sim 10^{-26}/r$ .
- If the low rotational rates are used, the upper limit is an almost negligible  $\sim 10^{-30}/r$ .
- Source of energy needs to be determined.
- Standard inflation gives  $h \sim 10^{-27} 10^{-29}$  in this frequency range.

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