## Sources in the Medium Frequency Band

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## Motivation and Outline

- Motivation
- Look for possible continuous sources in the band
- Quasi-radial oscillations of rotating white dwarfs
- Outline
- Identify frequency range
- Determine gravitational radiation
- Estimate stochastic background level


## Identifying the Frequency Range



## White Dwarf Properties and Resonant Frequencies

| $\rho_{\mathrm{c}}\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ | $\mathrm{M}_{0}\left(\mathrm{M}_{\odot}\right)$ | $\mathrm{M}\left(\mathrm{M}_{\odot}\right)$ | $\Omega_{\max }$ | $Q^{0}{ }_{\max }\left(\mathrm{kg} \mathrm{m}^{2}\right)$ | $\omega$ |
| :---: | :--- | :--- | :--- | :--- | :---: |
| $1.716 \times 10^{6}$ | 0.498 | 0.572 | 0.196 | $3.872 \times 10^{42}$ | 0.757 |
| $1.544 \times 10^{7}$ | 0.867 | 0.976 | 0.476 | $2.315 \times 10^{42}$ | 0.766 |
| $5.377 \times 10^{7}$ | 1.049 | 1.164 | 0.768 | $1.338 \times 10^{42}$ | 1.077 |
| $1.287 \times 10^{8}$ | 1.145 | 1.254 | 1.063 | $8.095 \times 10^{41}$ | 1.399 |
| $7.036 \times 10^{8}$ | 1.245 | 1.34 | 2.042 | $2.687 \times 10^{41}$ | 2.001 |
| $2.091 \times 10^{9}$ | 1.257 | 1.339 | 3.105 | $1.217 \times 10^{41}$ | 1.299 |

## Determining Strain Amplitude

- Assume oblate shape due to rotation
- Oscillation is self-similar and is described by:

$$
x_{\alpha}=x_{\alpha}^{0}(1+\eta \sin (\omega t))
$$

- Quadrupole moment given by

$$
Q_{i j}=\int \rho\left(x_{i} x_{j}-\frac{1}{3} x^{2} \delta_{i j}\right) d V=Q_{i j}^{0}(1+\eta \sin (\omega t))
$$

Choose $z$-axis along rotation axis: $Q_{z z}^{0}=-2 Q_{x x}^{0}=-2 Q^{0}{ }_{y y}=-2 Q^{0}$

## Polarizations

In TT gauge with $z$-axis along the wave vector:

$$
\begin{aligned}
& h_{+}=h_{x x}-h_{y y}=\frac{4 G Q^{0} \eta \omega^{2}}{c^{4} r} \sin ^{2} \theta \sin (\omega t) \\
& h_{\times}=2 h_{x y}=0
\end{aligned}
$$

where $\theta$ is the angle between the wave vector and the white dwarf axis of rotation

## Estimating the Background Level

- Numerical calculations give $Q^{0}{ }_{\text {max }}$ for select white dwarfs rotating at $\Omega_{\max }$
- Results are from an expansion in a dimensionless parameter proportional to $\Omega^{2}$
- Use $Q^{0}=\left(\Omega / \Omega_{\text {max }}\right)^{2} Q^{0}{ }_{\text {max }}$
- Integrate over $\theta \Rightarrow \sin ^{2} \theta \rightarrow 1 / 2$

$$
h_{+}=5.35 \times 10^{-65}\left(\frac{1 \mathrm{pc}}{r}\right)\left(\frac{\Omega}{\Omega_{\max }}\right)^{2} Q_{\max }^{0} \omega^{2} \eta
$$

## White Dwarf Properties and Resonant Frequencies

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## Expected Strain Amplitudes

- Maximum values (assume $\Omega=\Omega_{\max }$ )
- Assume $r=1 \mathrm{pc}, \eta=10^{-4}$



## Observed Rotation Rates

Kawaler (astro-ph/0301539)

- Spectroscopic methods:
- M ~ $0.6 \mathrm{M}_{\odot}$
- $\Omega \sim 0.0036 \mathrm{~Hz}$
- $\mathrm{h} \sim 4.1 \times 10^{-26}$
- Asteroseismic methods:
- $\mathrm{M} \sim 0.6 \mathrm{M}_{\odot}$
- $\Omega \sim 5.6 \times 10^{-5} \mathrm{~Hz}$
- $\mathrm{h} \sim 9.7 \times 10^{-30}$


## Conclusions

- Vibrating white dwarfs are potential sources of gravitational radiation in the mid-frequency range.
- Assuming a mean distance of $r \mathrm{pc}$ to nearest vibrating white dwarf, an upper limit of $\sim 10^{-23} / r$ can be placed on the expected level.
- If the high rotational rates of Kawaler are used, a more reasonable upper limit would be $\sim 10^{-26} / r$.
- If the low rotational rates are used, the upper limit is an almost negligible $\sim 10^{-30} / r$.
- Source of energy needs to be determined.
- Standard inflation gives $h \sim 10^{-27}-10^{-29}$ in this frequency range.

