White light Cavities

Stacy Wise, Guido Mueller,
David Reitze, D.B. Tanner,

University of Florida

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Table of Content

- **Motivation**
- **Long GW-wavelength limit**
  - **Michelson Interferometer**
  - **Cavity enhanced Michelson Interferometer**
  - **Signal Recycling**
  - **White Light Cavity (WLC)**
- **Realistic GW-detector**
  - **Michelson Interferometer**
  - **Cavity enhanced MI**
  - **Signal recycling**
  - **WLC**
- **Outlook**

Gravitational Waves:

- **Generated by huge accelerated masses such as accelerated black holes in a binary system**
- **Predicted by Einstein, never detected**
- **Amplitude is a relative length change: h=dL/L**

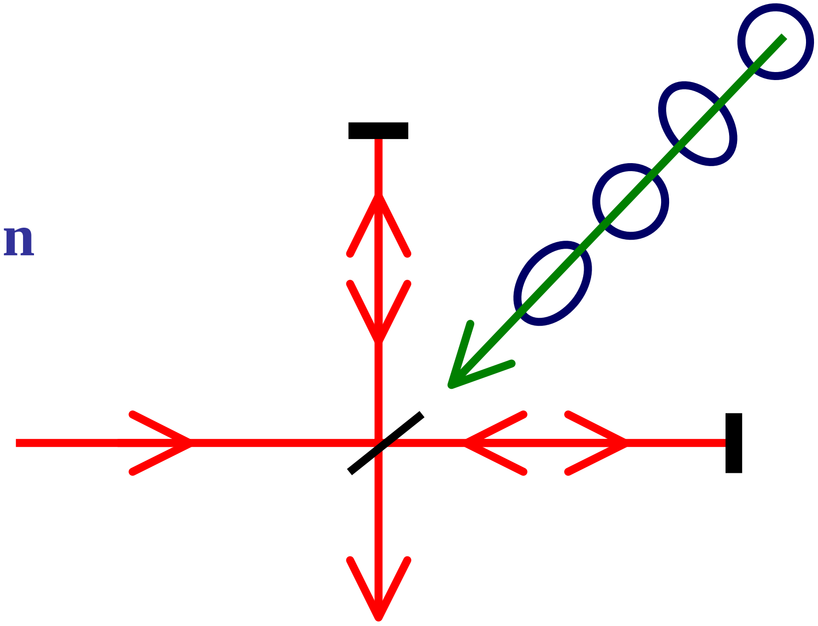
$$h \sim \frac{2G}{c^4}\frac{\ddot{Q}}{r}$$

Q: Quadrupole Tensor

Quadrupole waves:

- **Stretch one direction**
- **Squeeze orthogonal direction**

Interferometer is the ideal tool to measure gravitational waves:



But what type of sources do we expect to see ?

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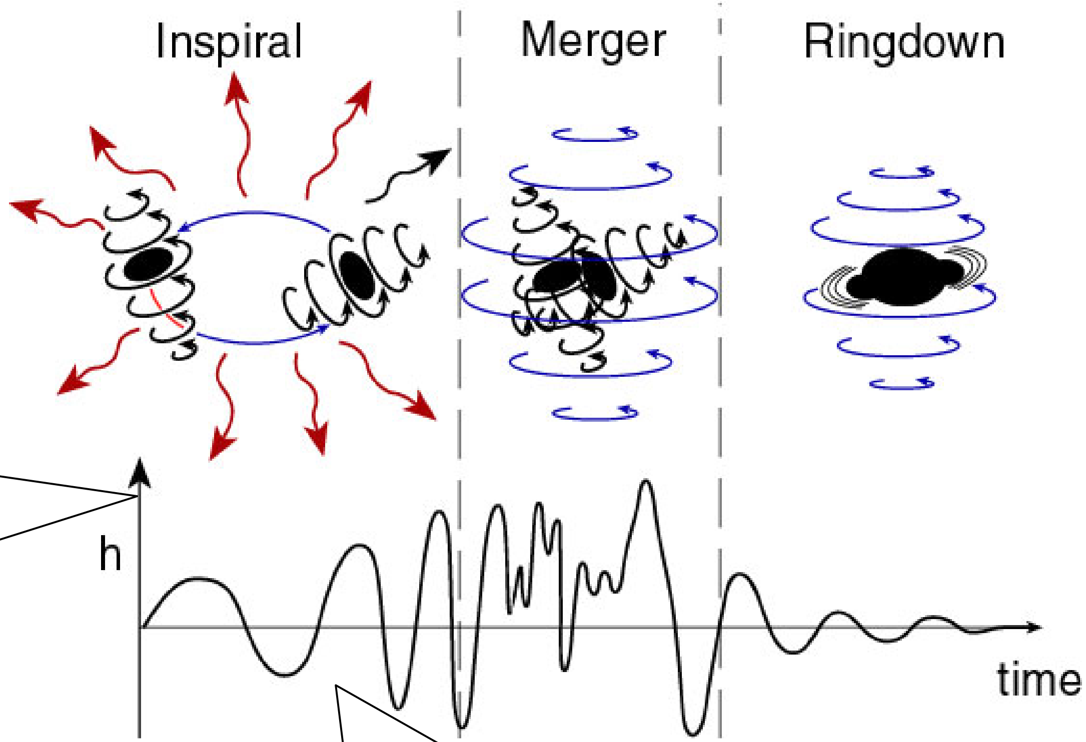
History of a big fat wedding

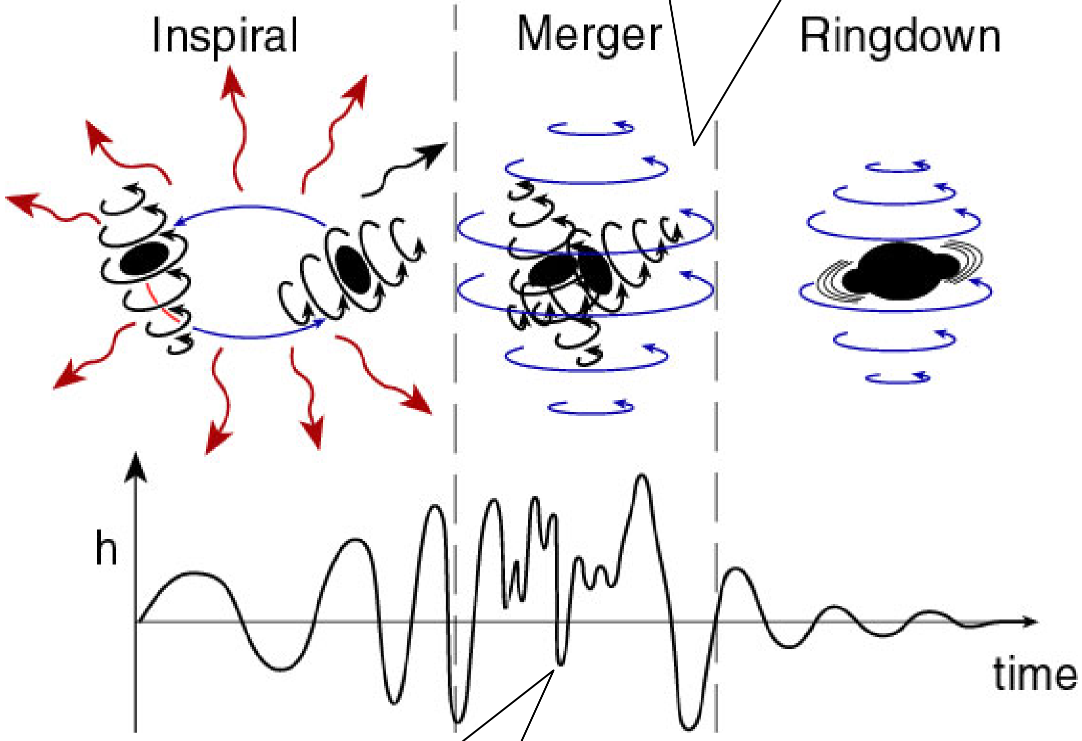


measure masses and spins of binary system

Frequency increases over time
Maximum frequency: 2kHz

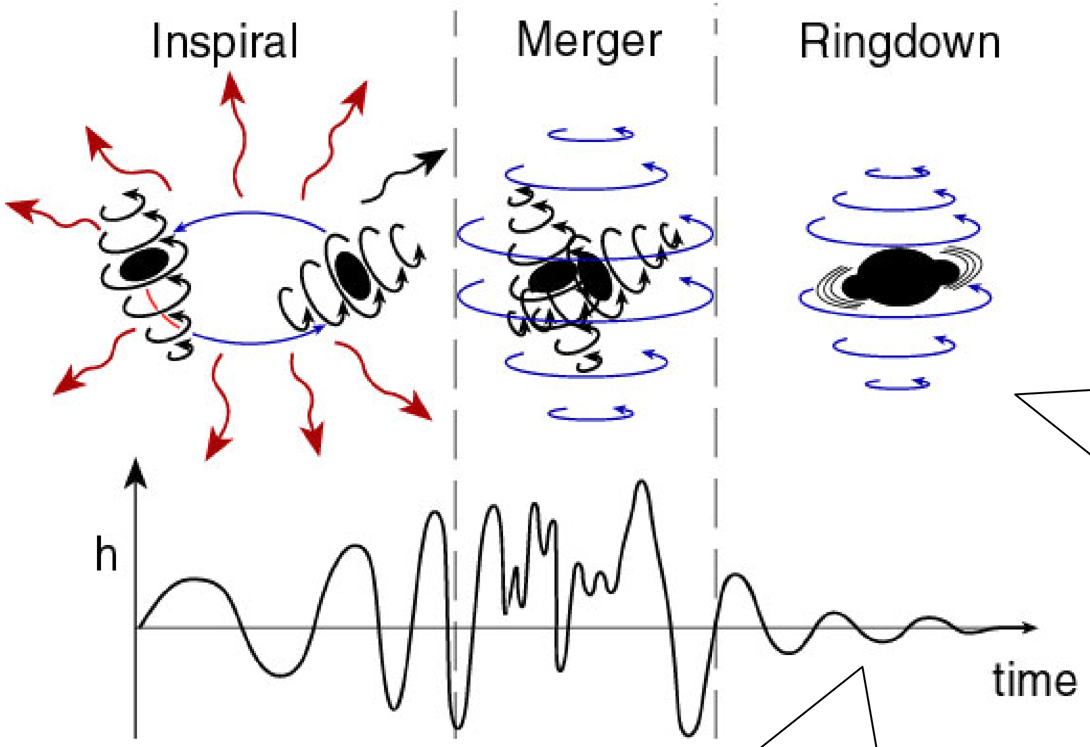
History of a big fat wedding

observe the dynamics:
spin flips and couplings…



Waveform unknown

History of a big fat wedding



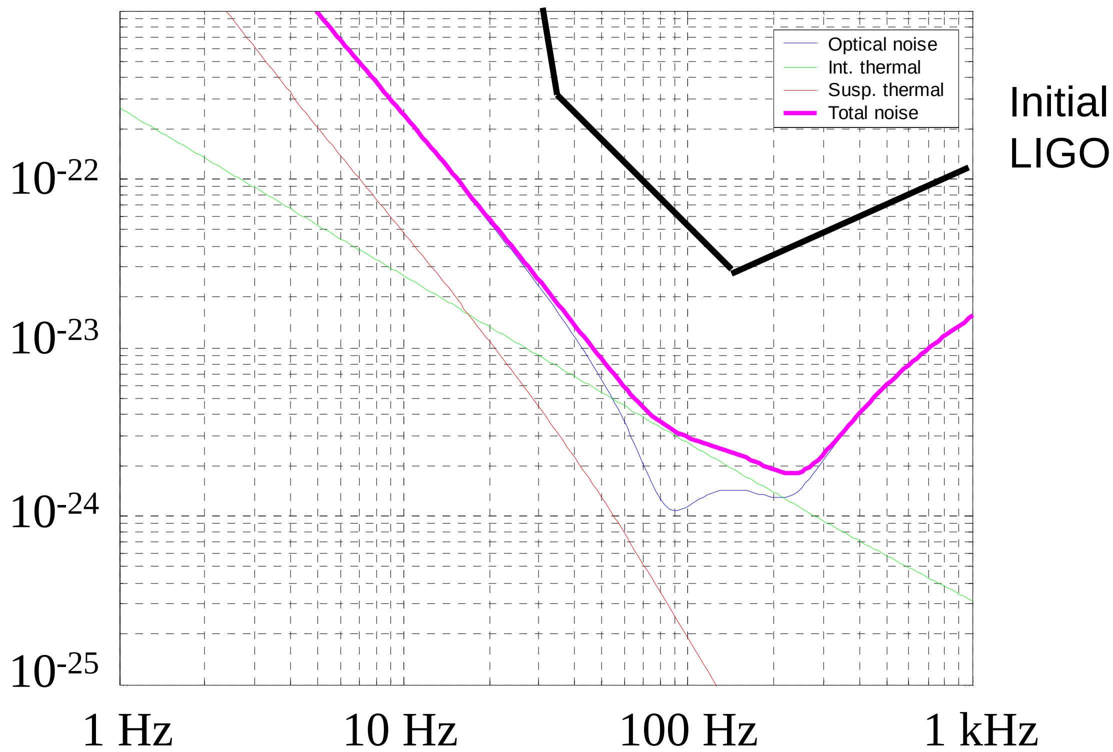
detect normal modes of ringdown to identify final NS or BH.

Ringdown frequencies in NS: 1kHz – 20kHz

Advanced Configuration

Possible Sources:

- **NS/NS mergers out to 300 Mpc**
- **BH/BH mergers**
- **Normal modes of NS (10 Mpc)**
- **Supernovae**
- **…**



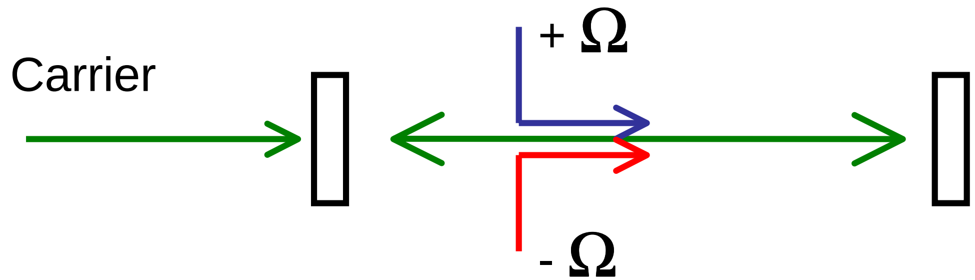
NS/NS mergers ?

- **We see the inspiral !**
- **Probably the low frequency components of the wedding night, not the entire show.**
- **And probably nothing from the ringdown.**

NEED MORE BANDWIDTH !!!

LIGO I

- Simplified picture (low frequency limit $\nu_{GW} << FSR$):
  GW modulates phase of cavity internal field !



$\tau = L_{rt}/c$

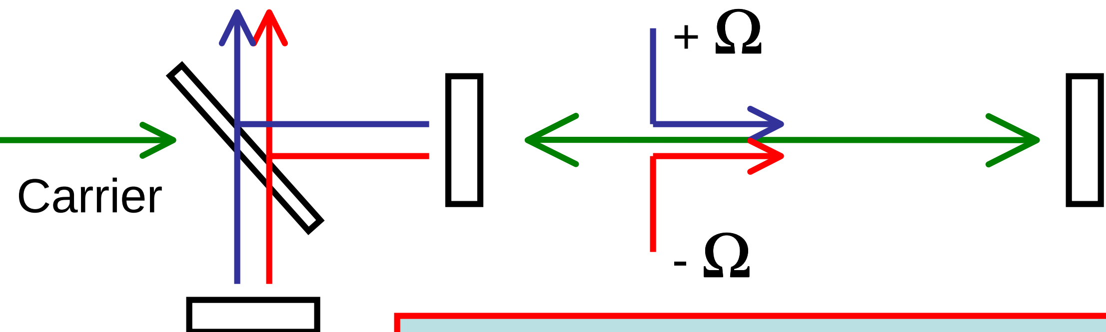
$$E_0(t) = E_C + E_+ e^{i\Omega t} - E_- e^{-i\Omega t}$$

$$E_1(t+\tau) = r_1 r_2 \left(E_C + E_+ e^{i\Omega t} - E_- e^{-i\Omega t} \right) + \left(E_C + E_+ e^{i\Omega (t+\tau)} - E_- e^{-i\Omega (t+\tau)} \right)$$

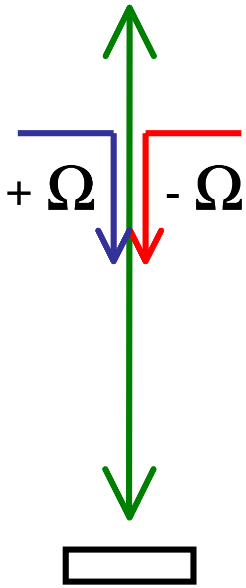
$$E_2(t+2\tau) = (r_1 r_2)^2 \left(E_C + E_+ e^{i\Omega t} - E_- e^{-i\Omega t} \right) + r_1 r_2 \left(E_C e^{-i\Phi} + E_+ e^{i\Omega (t+\tau)} - E_- e^{-i\Omega (t+\tau)} \right) + \left(E_C + E_+ e^{i\Omega (t+2\tau)} - E_- e^{-i\Omega (t+2\tau)} \right)$$

...

LIGO I



low frequency limit
$\nu_{GW} << FSR$



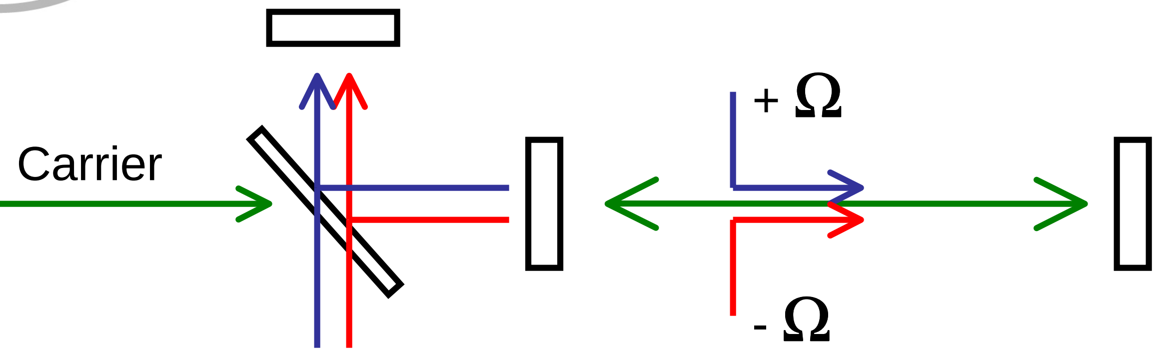
$$E_{tot} = E_C \sum (r_1 r_2)^n$$

$$+ E_+ e^{i\Omega t} \sum (r_1 r_2)^n E_+ e^{in\Omega\tau} - E_- e^{-i\Omega t} \sum (r_1 r_2)^n E_- e^{-in\Omega\tau}$$

$$= \frac{(r_1 - r_2)\, t_{MI} E_c}{(1 - r_1 r_2)} + \frac{t_1 r_{MI} E_+ e^{+i\Omega t}}{(1 - r_1 r_2 e^{i\Omega\tau})} - \frac{t_1 r_{MI} E_- e^{-i\Omega t}}{(1 - r_1 r_2 e^{-i\Omega\tau})}$$

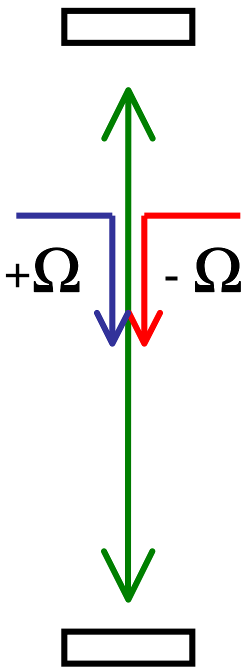
Gives LIGO I response: BW = Cavity Pol

LIGO II



low frequency limit

$\nu_{GW} \ll FSR$



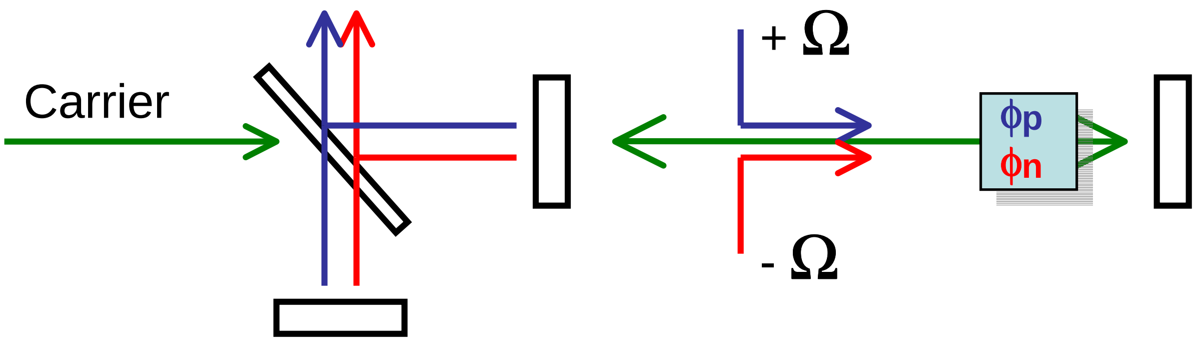
PM-sidebands:
Signal recycling changes effective reflectivity of ITM (similar to PR for carrier)

$$r_{eff}\, e^{i\Phi} = \frac{r_1 - r_S e^{i\phi_{SR}}}{1 - r_1 r_S e^{i\phi_{SR}}}$$

$$\frac{E_+ e^{+i\Omega t}}{(1 - r_{eff} r_2 e^{i\Omega\tau + \Phi})} \qquad \frac{E_- e^{-i\Omega t}}{(1 - r_{eff} r_2 e^{-i\Omega\tau + \Phi})}$$

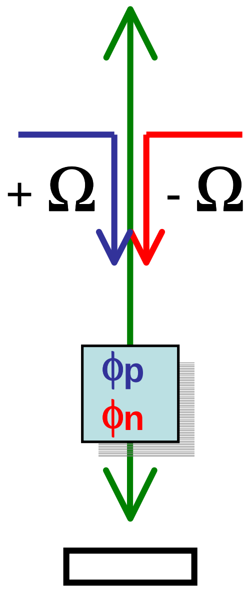
1. **Shifts resonance (peak-) frequency**
2. **r_{eff} changes build up**
3. **r_{eff} changes bandwidth**

IDEA: WLC



low frequency limit

ν_{GW} << FSR



Change reflectivity of end mirror:

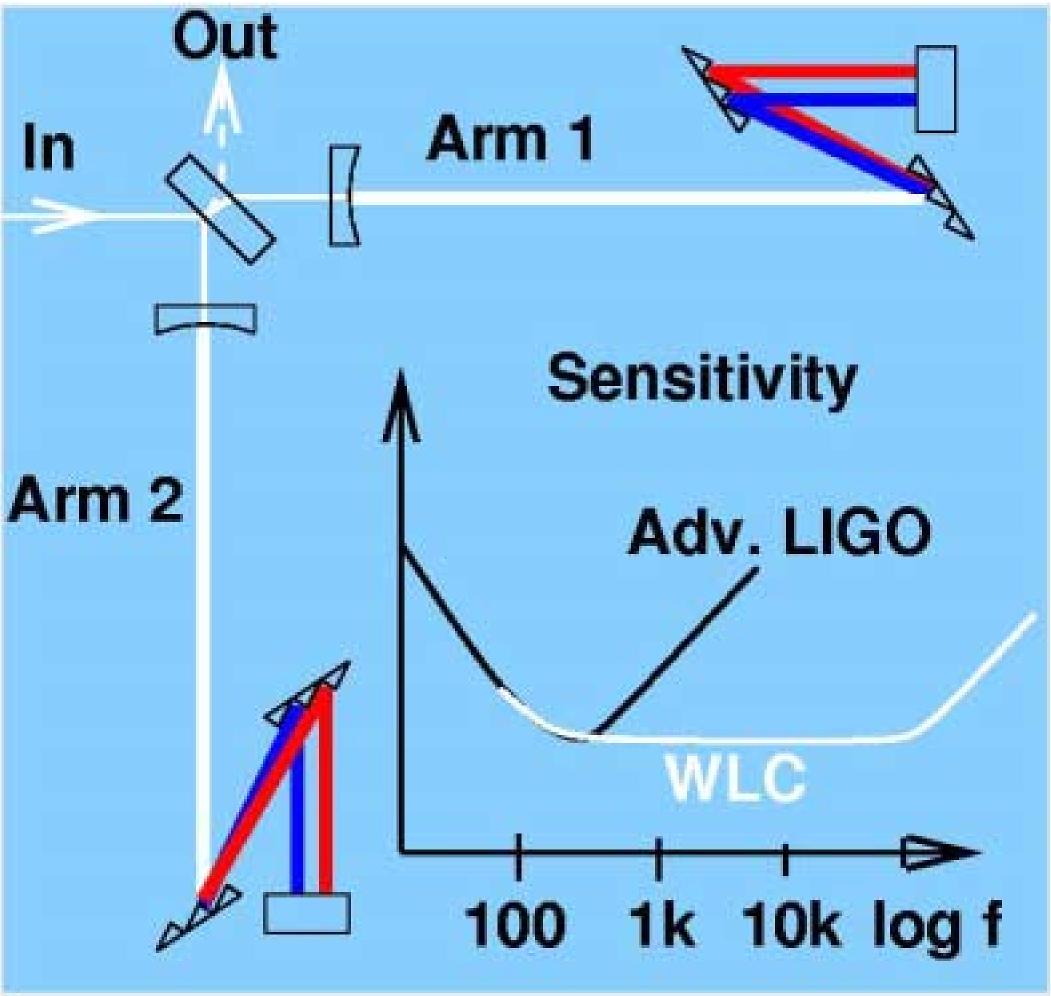
$$r_2 \longrightarrow r_2 e^{-i\Omega\tau}$$

$$\frac{E_+ e^{i\Omega t}}{(1 - r_1 r_2)} \qquad \frac{E_- e^{-i\Omega t}}{(1 - r_1 r_2)}$$

1. Resonant at all frequency
2. Build up does not change BW

White light cavity

Basic Idea:



$$\frac{L(\lambda)}{\lambda} = \frac{\delta L(\lambda)}{\delta \lambda}$$

Or

Make Cavity longer for longer wavelength

$$\Longrightarrow$$

***Unlimited* Bandwidth**

LIGO

$\phi(\Omega) = \Omega\tau$ How ?

$$L_0(\nu) = -\nu_0 \frac{\delta L}{\delta \nu} \quad \text{or} \quad \frac{L(\lambda)}{\lambda} = \frac{\delta L(\lambda)}{\delta \lambda}$$

Several methods:

1. **Atomic resonances (Wicht-paper)**
   Index of refraction in resonantly pumped two level system (see *lasing without inversion*)
2. **Angular Dispersion**
   - **a) Prisms (not dispersive enough ?)**
   - **b) Gratings**
   - **c) misaligned triangular cavities (tricky)**

Original Idea published in: A. Wicht, K. Danzmann, M. Fleischhauer, M.O. Scully, G. Mueller, R.-H. Rinkleff Opt. Comm. 134 (1997), pg 431-439

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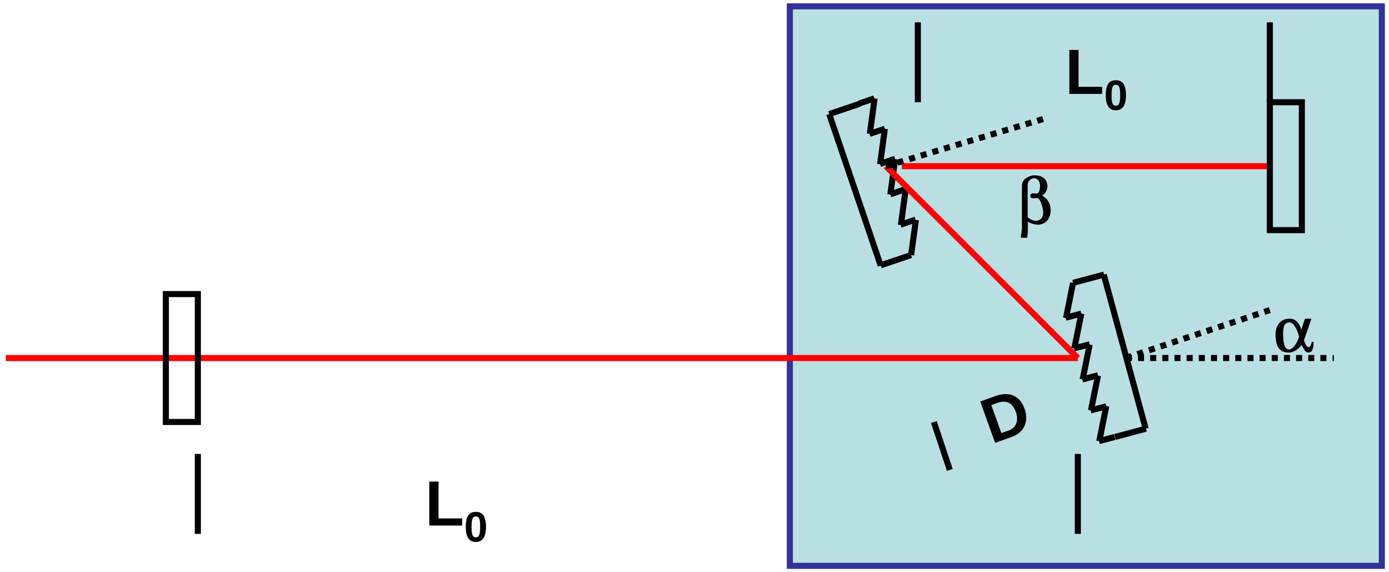
White light cavity



New end Mirror !

Cavity length:

$$L(\lambda) = L_0 + \frac{D\,[1+\sin\alpha\,\sin\beta(\lambda)]}{\cos\beta(\lambda)}$$

WLC requires:

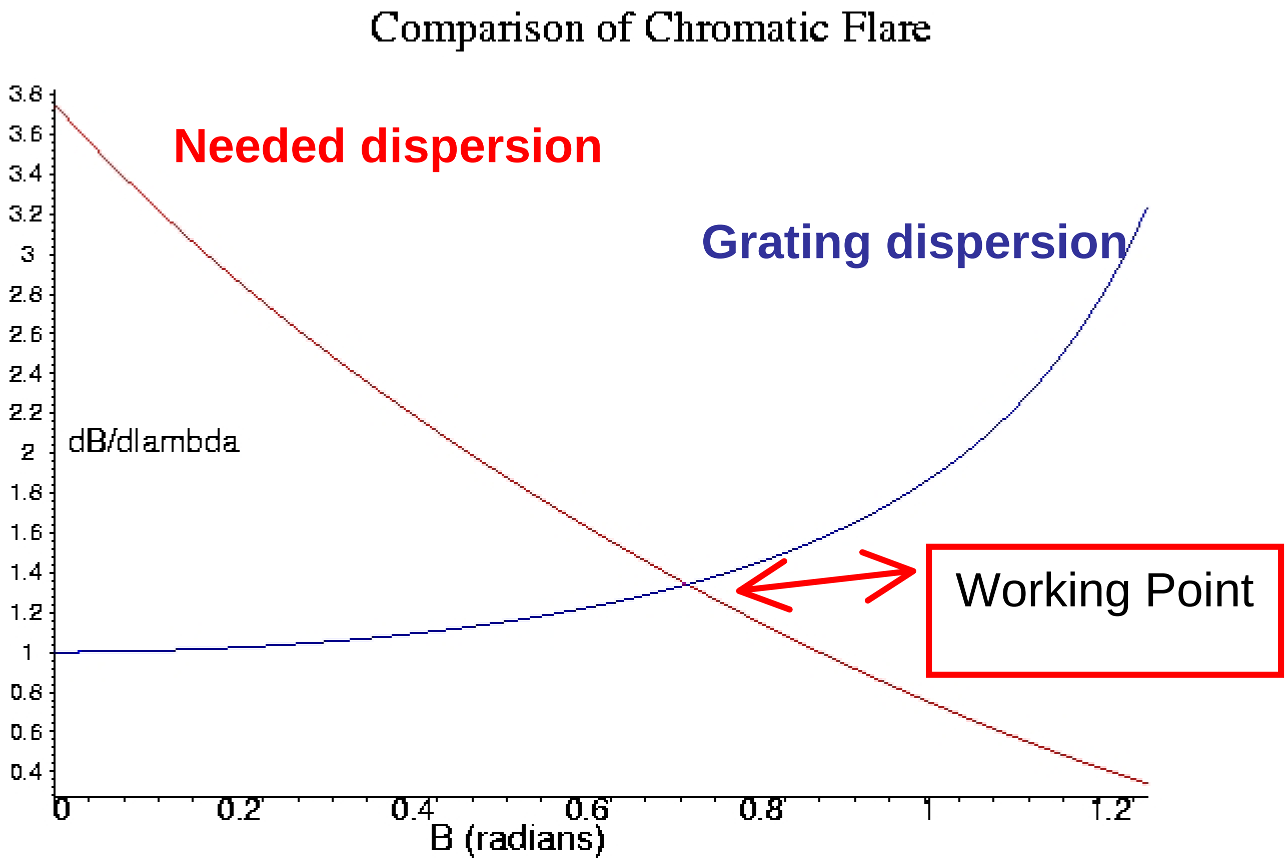
$$\frac{L(\lambda)}{\lambda} \stackrel{!}{=} \frac{\delta L(\lambda)}{\delta\lambda} = \frac{\delta L}{\delta\beta}\,\frac{\delta\beta}{\delta\lambda}$$

with:

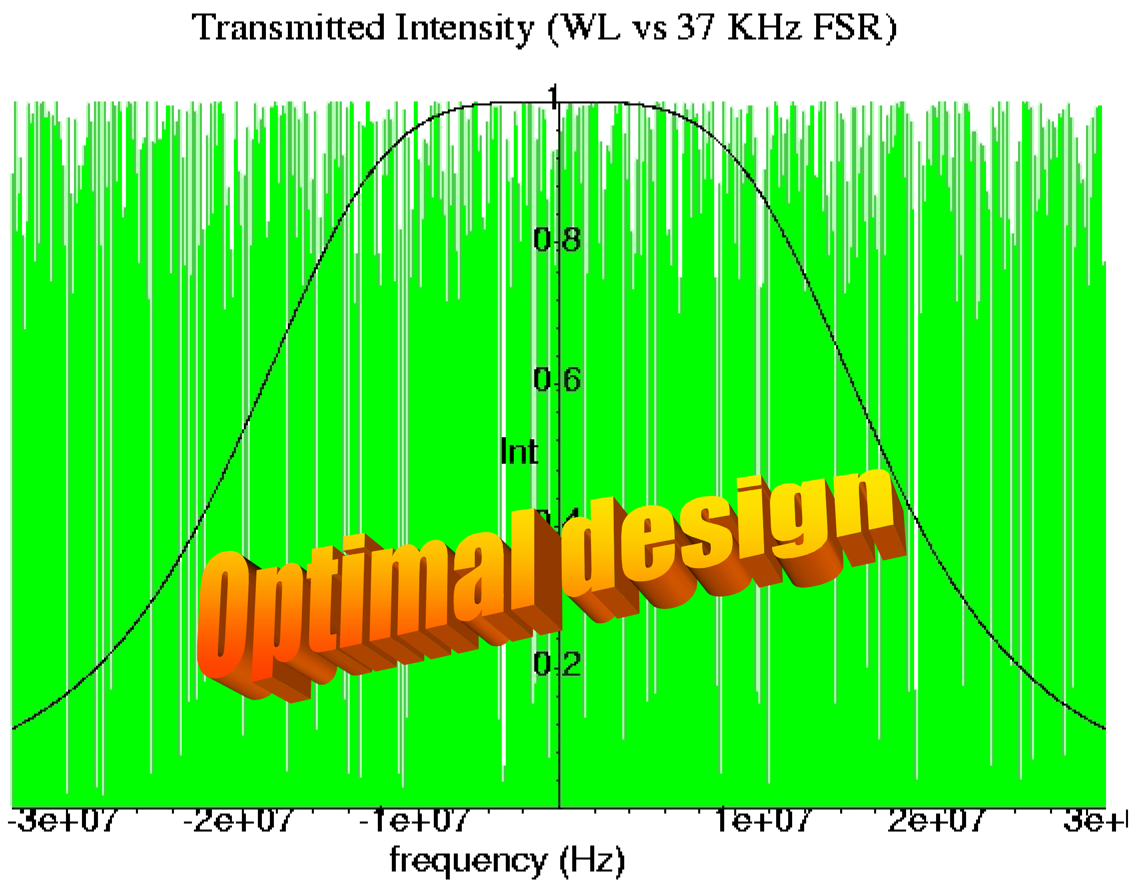
$$\frac{\delta L}{\delta\beta} = \frac{D}{\cos^2\beta}\,\frac{\lambda}{d} \qquad \frac{\delta\beta}{\delta\lambda} = \frac{m}{d\cos\beta}$$

Does this work ?

Is the dispersion in a grating large enough ?



Final Bandwidth



Bandwidth increased from 60 Hz to 36 MHz

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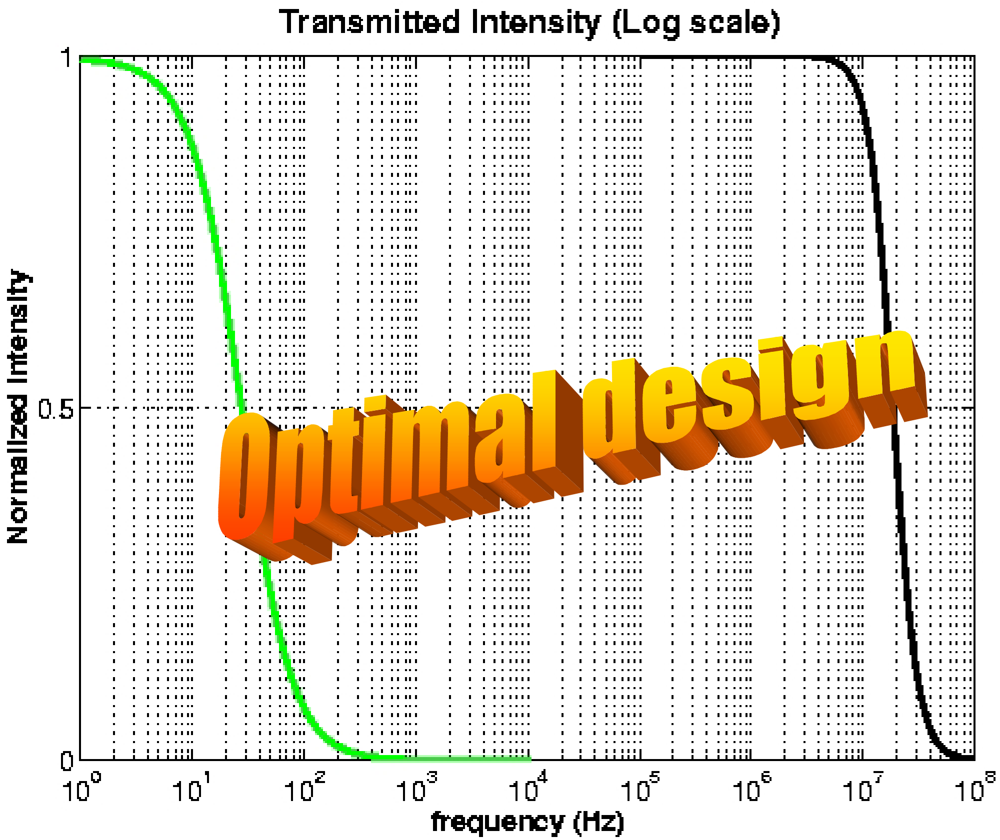
Final Bandwidth



This is the optical line width, not the GW-response !

Not the Same !

White light cavity

Basic Idea:

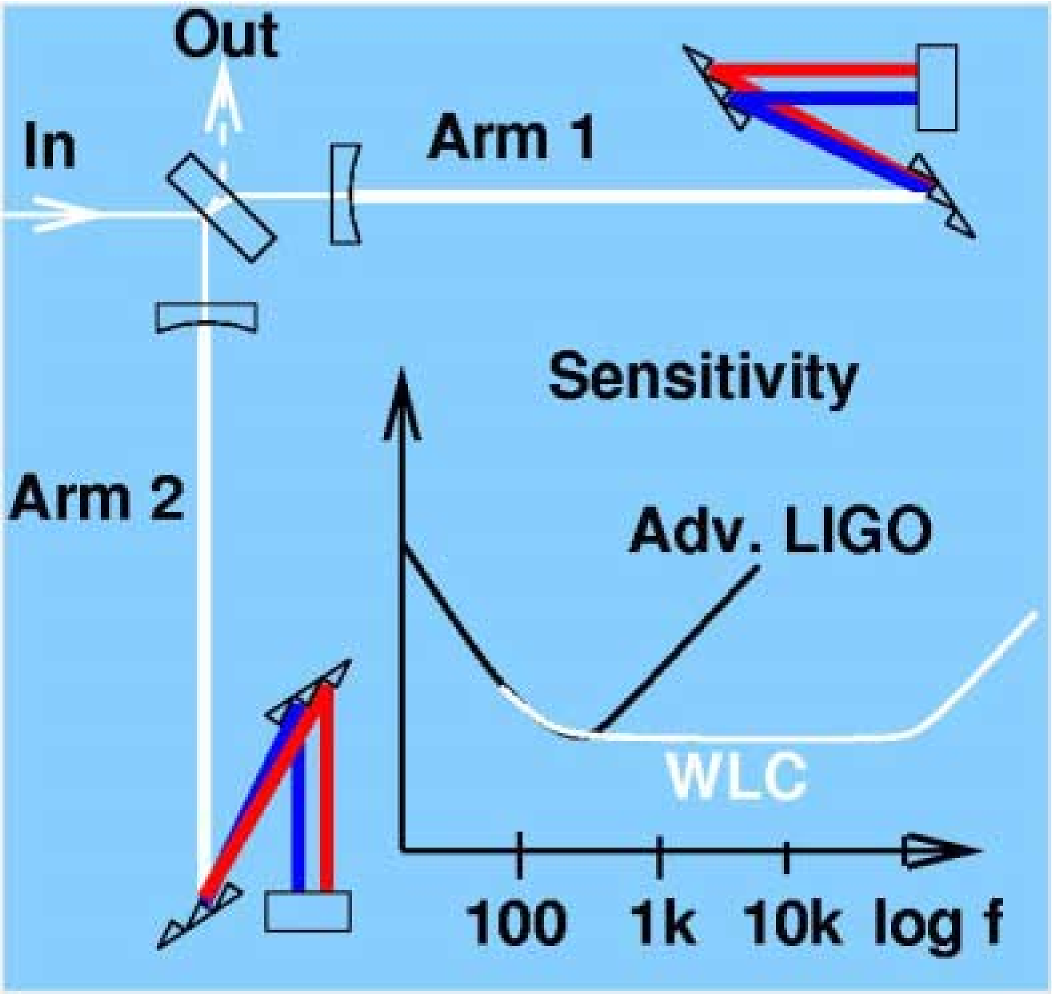
$$\frac{L(\lambda)}{\lambda} = \frac{\delta L(\lambda)}{\delta \lambda}$$

Or

Make Cavity longer for longer wavelength

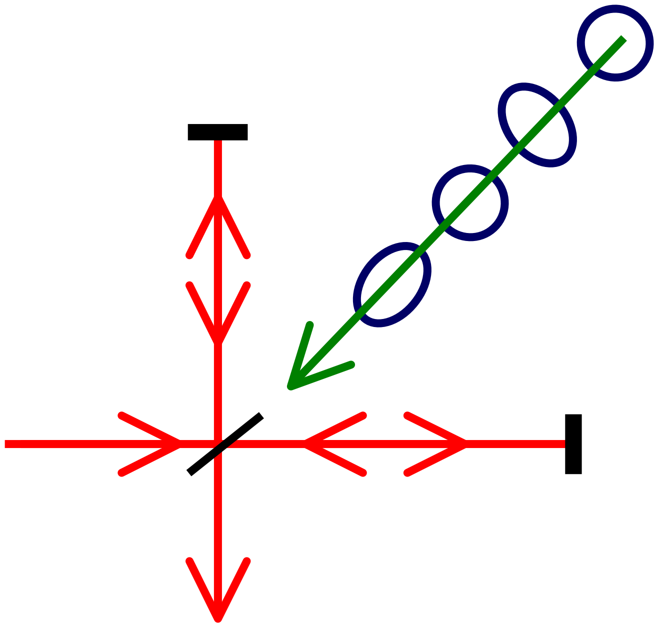
$\Longrightarrow$

***Unlimited* Bandwidth**



But assumption
$\nu_{GW} << c/L$
not longer valid !

Michelson Interferometer



Response based on:

$$ds^2 = 0 = c^2dt^2 + [1+h(t)]^2dx^2 + [1-h(t)]^2dy^2$$

- **Propagation of GW: z-direction**
- **Polarization of GW: + (optimum)**

Light travel time in X-arm seen by beam splitter:

$$c\int dt = \int (1+0.5h_0\sin(\Omega t+kx))dx \longrightarrow T_x = \frac{L}{c}\left[1+\frac{h_0}{2}\,\frac{\sin(\Omega t-kL)-\sin(\Omega t)}{\Omega}\right]$$

Y-arm: $$T_y = \frac{L}{c}\left[1-\frac{h_0}{2}\,\frac{\sin(\Omega t-kL)-\sin(\Omega t)}{\Omega}\right]$$

Phase difference at BS: $\Delta\phi = \omega(T_x - T_y)$

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Cavities

Cavity field at ITM_x:

$$E_{cav} = i t_1 r_2 E_{in} e^{i\omega t} \sum (r_1 r_2)^n e^{-i\omega\tau_n}$$

L: round trip length
$\omega L/c = N2\pi$

$$\tau_n = (n+1)\frac{L}{c} + \frac{h_0}{2\Omega}\left[\sin(\Omega t-(n+1)kL)-\sin(\Omega t)\right]$$

Carrier:

$$E_{cav} = i t_1 r_2 E_{in} e^{i\omega t}\left[\frac{\mathbf{1}}{\mathbf{1-r_1 r_2}}\right.$$

Sideband:

$$-\frac{i\omega h_0}{4FSR}\,\frac{e^{-i\Omega/2FSR}\,\mathrm{sinc}(\Omega/2FSR)}{(1-r_1r_2)(1-r_1r_2e^{i\Omega/FSR})}\,e^{-i\Omega t}$$

Sideband:

$$\left.-\frac{i\omega h_0}{4FSR}\,\frac{e^{-i\Omega/2FSR}\,\mathrm{sinc}(\Omega/2FSR)}{(1-r_1r_2)(1-r_1r_2e^{-i\Omega/FSR})}\,e^{i\Omega t}\right]$$

Cavities

Cavity field at ITM$_x$:

$$E_{cav} = i t_1 r_2 E_{in} e^{i\omega t} \sum (r_1 r_2)^n e^{-i\omega \tau_n}$$

$$\tau_n = (n+1)\frac{L}{c} + \frac{h_0}{2\Omega}\left[\sin(\Omega t-(n+1)kL)-\sin(\Omega t)\right]$$

Interpretation:

$$\frac{i\omega h_0}{4FSR}\ \boxed{\mathrm{sinc}(\Omega/2FSR)}\ e^{-i\Omega/2FSR}\ \boxed{\frac{1}{(1-r_1 r_2)}}\ \boxed{\frac{1}{(1-r_1 r_2 e^{i\Omega/FSR})}}\ e^{-i\Omega t}$$

- $\mathrm{sinc}(\Omega/2FSR)$ ← **GW**
- $\frac{1}{(1-r_1 r_2)}$ ← **Carrier build up**
- $\frac{1}{(1-r_1 r_2 e^{i\Omega/FSR})}$ ← **Signal build up**

Signal recycling changes $r_1 \longrightarrow r_{eff} e^{i\Phi}$ (not r_1)

White Light Cavity ?

Cavity field at ITM$_x$:

$$E_{cav} = it_1 r_2 E_{in} e^{i\omega t} \Sigma(r_1 r_2)^n e^{-i\omega\tau_n}$$

$$\tau_n = (n+1)L/c + \mathbf{?}$$

Work in progress …

Hope for:

$$r_2 \longrightarrow r_2 e^{-i\Omega/FSR}$$

Carrier:

$$E_{cav} = it_1 r_2 E_{in} e^{i\omega t} \Bigg[\frac{1}{1-r_1 r_2}$$

Sideband:

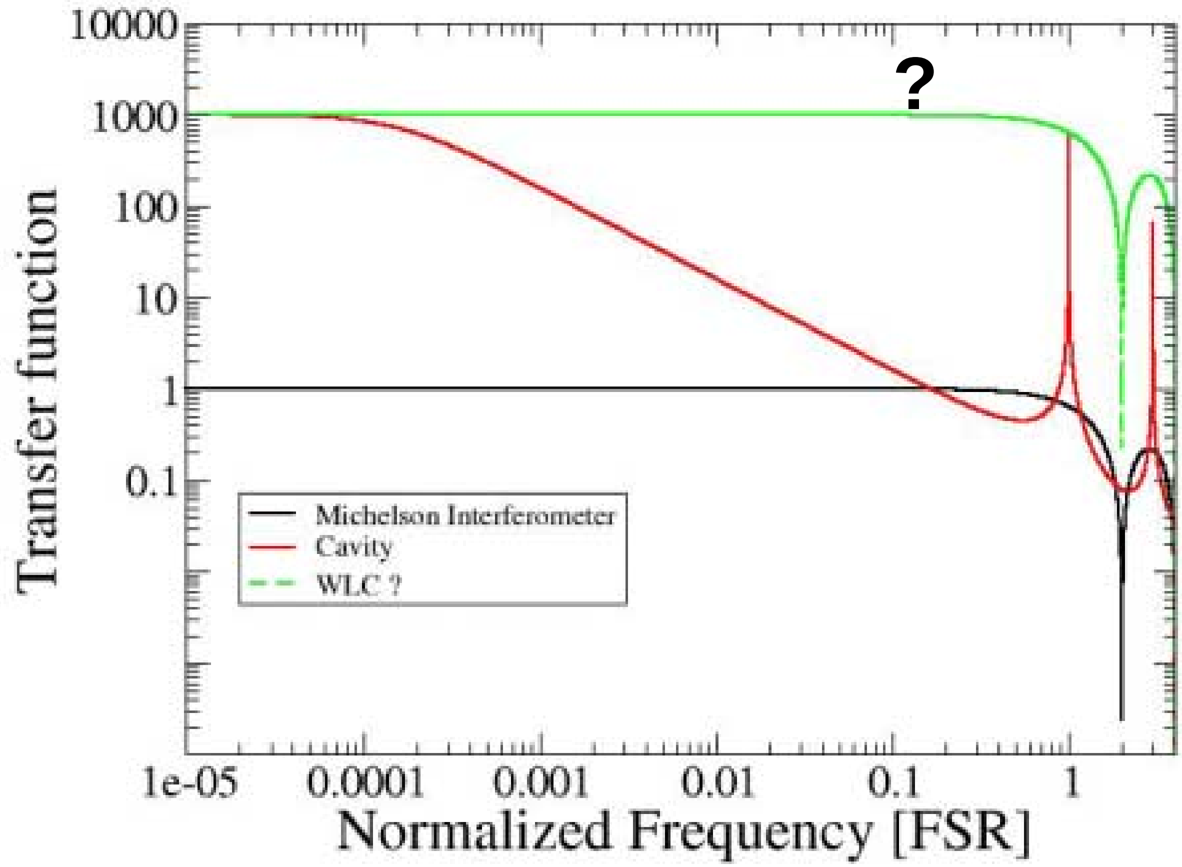
$$- \frac{i\omega h_0}{4FSR} \frac{e^{-i\Omega/2FSR}\,\mathrm{sinc}(\Omega/2FSR)}{(1-r_1 r_2)(1-r_1 r_2)} e^{-i\Omega t}$$

Sideband:

$$- \frac{i\omega h_0}{4FSR} \frac{e^{-i\Omega/2FSR}\,\mathrm{sinc}(\Omega/2FSR)}{(1-r_1 r_2)(1-r_1 r_2)} e^{i\Omega t} \Bigg]$$

White Light Cavity

Transfer function for optimum angle of incidence and optimum polarization



All sky average will average out the sharp peaks.

Comments:

- Low frequency limit should be OK.
- If Not, we should check SR, too.
- Higher frequencies: ?

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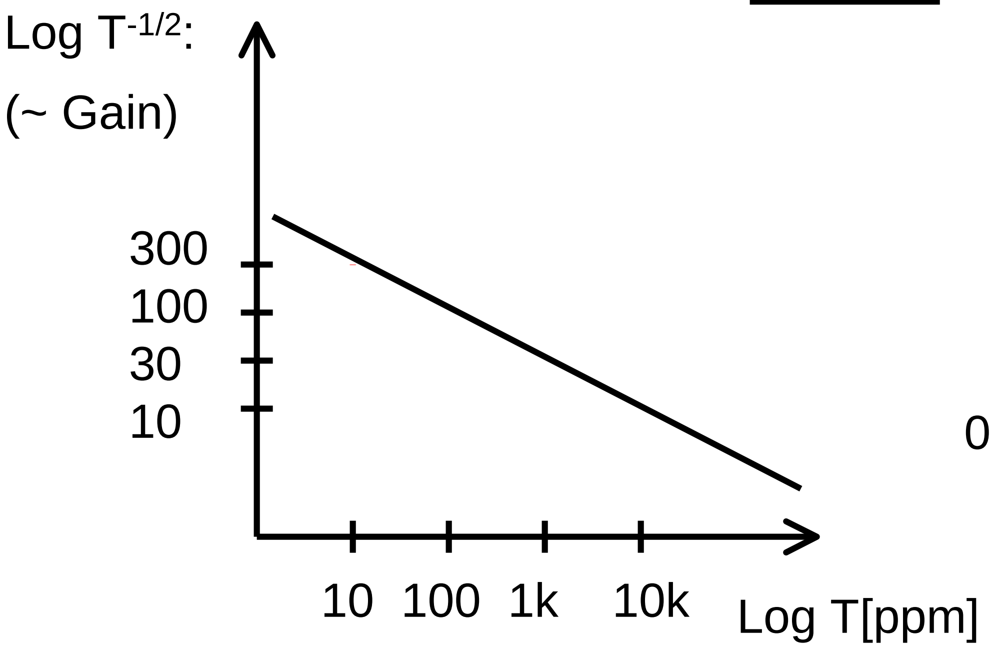
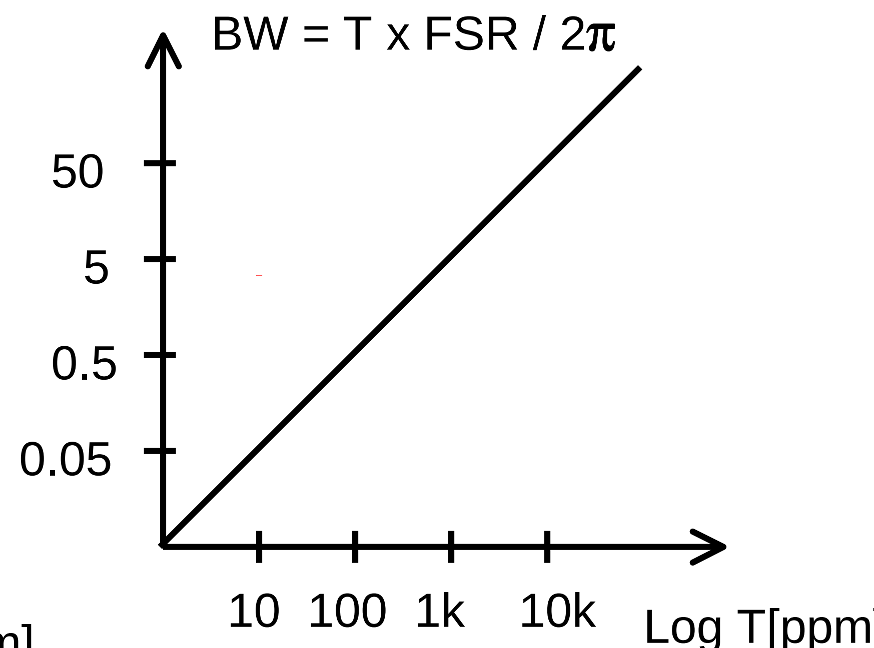
Compare with Adv. LIGO

Assume: 1MW in each arm cavity (infinite mirror masses):

$$S_h \sim \frac{5\times10^{-23}}{T^{1/2}\,\mathrm{Hz}^{1/2}}$$ for a non recycled MI (T transmission of ITM)

at DC





SR: shifts to other frequencies, Gain, BW: replace T by T_{eff}

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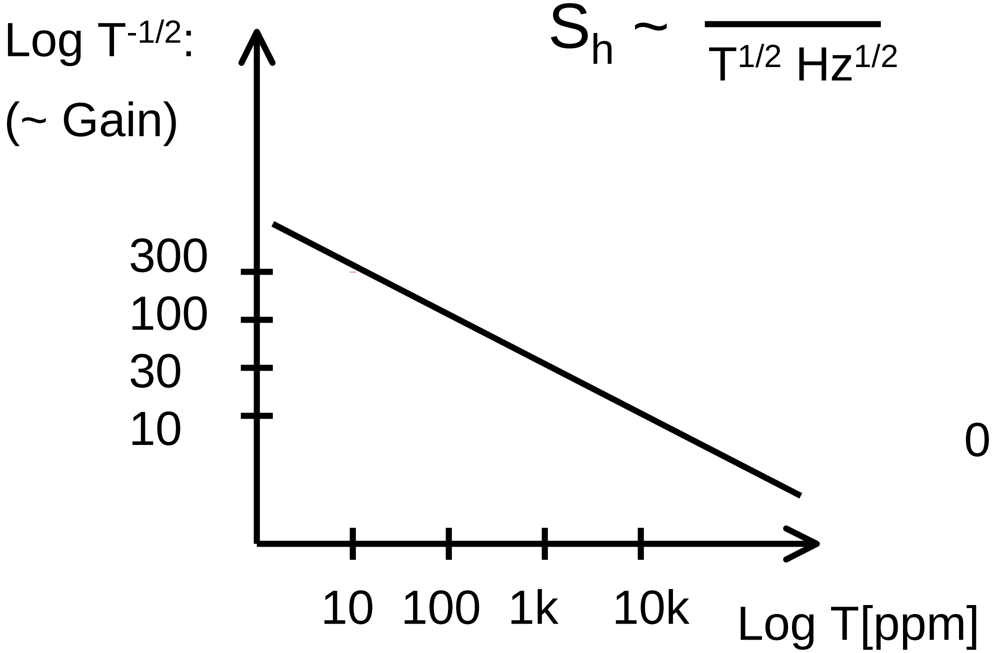
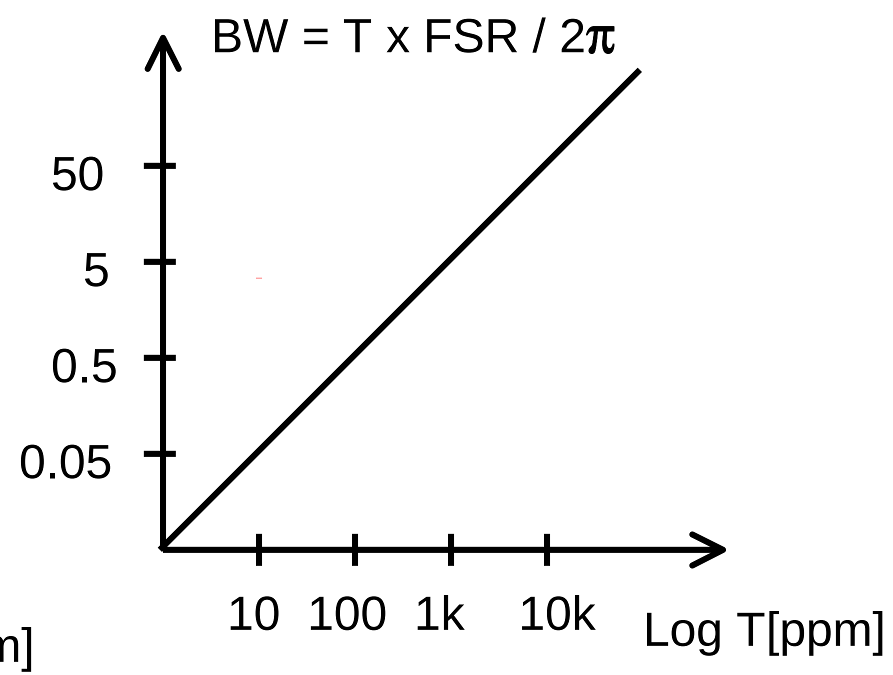
Scaling

$$S_h \sim \frac{5\times10^{-23}}{T^{1/2}\ \mathrm{Hz}^{1/2}}$$





Choices: want $S_h \sim 5\times10^{-25}$ → BW ~ 0.5 Hz

want $S_h \sim 5\times10^{-24}$ → BW ~ 50 Hz

Example for Grating:	L = 10k ppm	$S_h \sim 5\times10^{-24}$,	BW ~ MHz	(P_{in} = 10kW)
(T = Losses = L)	L = 1k ppm	$S_h \sim 1.6\times10^{-24}$,	BW ~ MHz	(P_{in} = 1kW)
	L = 100 ppm	$S_h \sim 5\times10^{-25}$,	BW ~ MHz	(Pin = 100W)

Remarks

- WLC reduces shot noise limit above cavity pol.
- Quadrature components of the quantum noise are uncoupled (no optical spring…).
- Radiation pressure noise will push on mirrors and noise will depend on mass of mirrors.
- Losses in gratings need to be below ~200ppm for grating, otherwise build up to low.
- Should be set up in an all reflective design.
- Nontransmissive materials for test masses possible: Silicon

Outlook

- **Gratings with 97% losses from Uni Jena arrived**
- **They produced already gratings with >99% efficiency**
- **Stacy started to model gratings (preliminary: 99.6%)**
- **Designed tabletop with expected optical linewidth of 10GHz in 23cm cavity.**
- **GW-bandwith ? (need to study experimental setup)**

RPN+thermal noise:

assumes equal masses
and same materials
in both cases.

All reflective optics
enables us to use new
materials (Silicon):
Larger masses,
better thermal properties
will reduce both noise
sources.