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Characterization of LIGO 4-km Fabry-Perot Cavities Using High-Frequency Dynamic Responses

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Fabry-Perot dynamic responses

The response of a Fabry-Perot cavity with the length L to small variations of its length is given by

$$H_L(s) = \frac{1 - r_a r_b}{1 - r_a r_b e^{-2sT}},$$

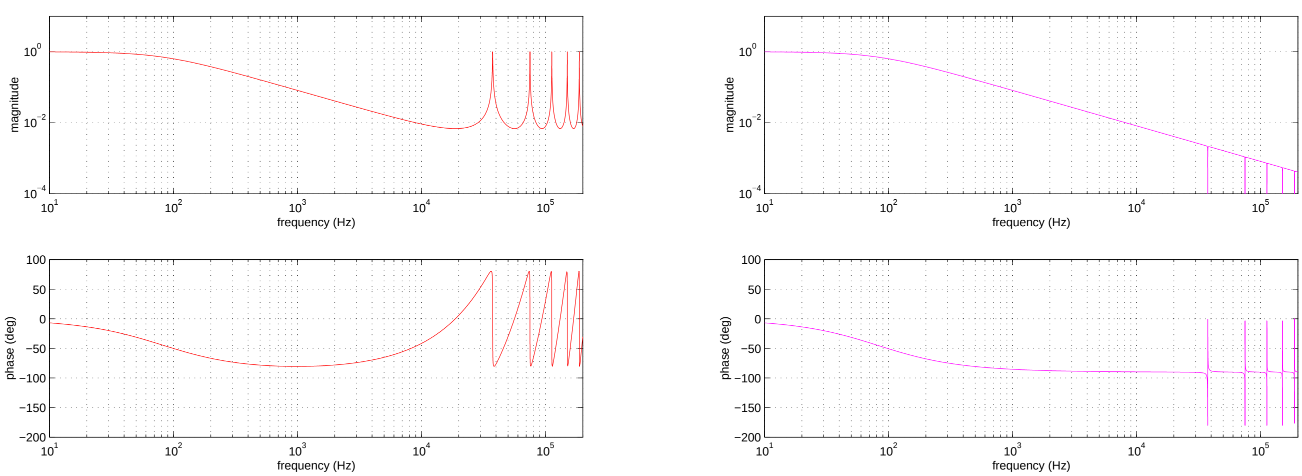
where r_a, r_b are mirror reflectivities and $T = L/c$ is the photon propagation time. The response to variations of the laser frequency is

$$H_\omega(s) = C(s)\; H_L(s),$$

where $C(s)$ is the frequency-to-length transfer function

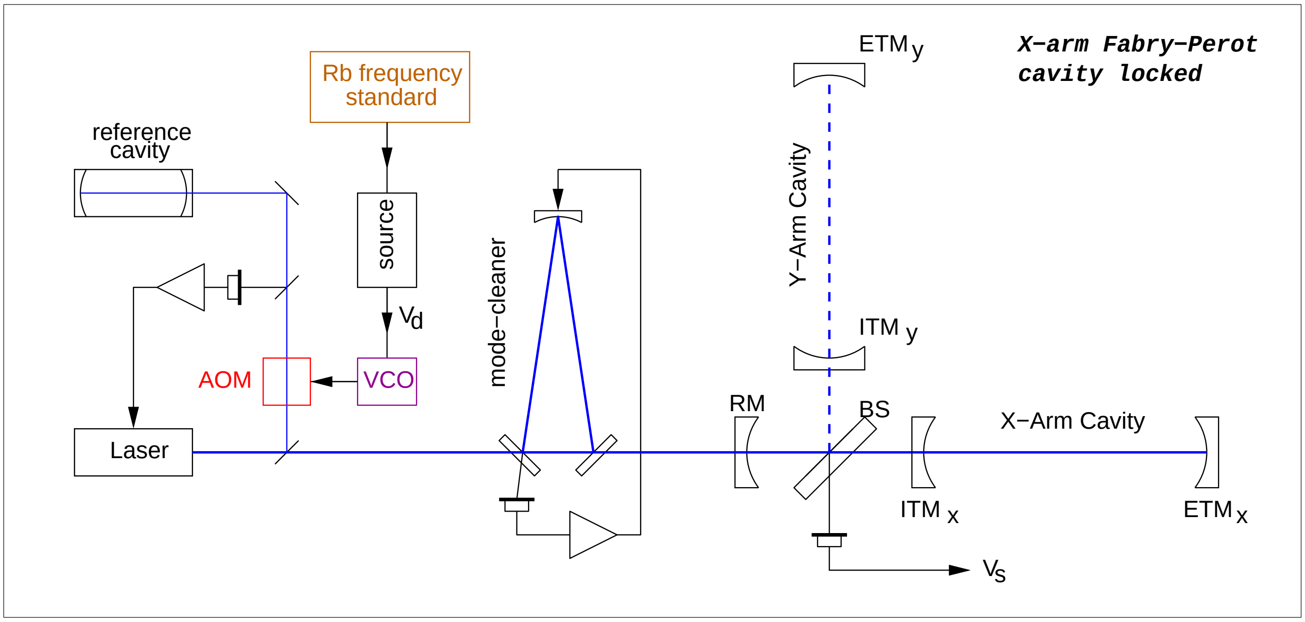
$$C(s) = \frac{1 - e^{-2sT}}{2sT}.$$

Bode plots of H_L *(left)* and H_ω *(right)* shown on the same scale.



Derivation can be found in *Phys. Lett. A 305 (2002) p.239*.

Swept sine of the laser frequency



A sine-wave $\delta V_d(s)$ is injected to the VCO. The AOM offsets the light frequency by $\delta\omega(s) =$ const $\delta V_d(s)$. The PSL servo forces the laser to shift its frequency in the opposite way by

$$\delta\omega'(s) = \frac{G(s)}{1 + G(s)}\,\delta\omega(s),$$

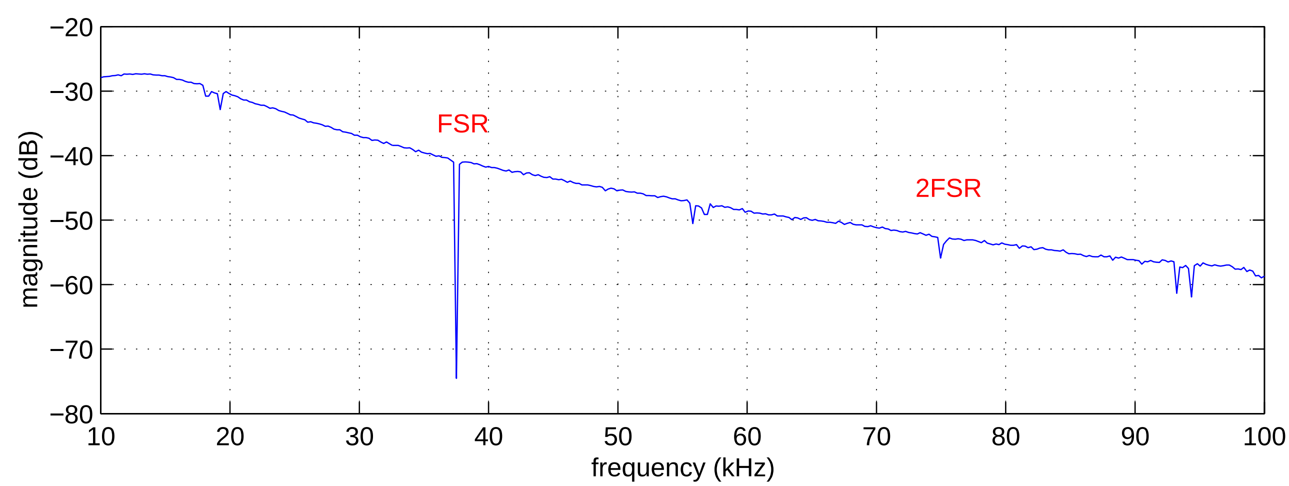
where $G(s)$ is the PSL loop gain. For large gain, $\delta\omega'(s) \approx \delta\omega(s)$.

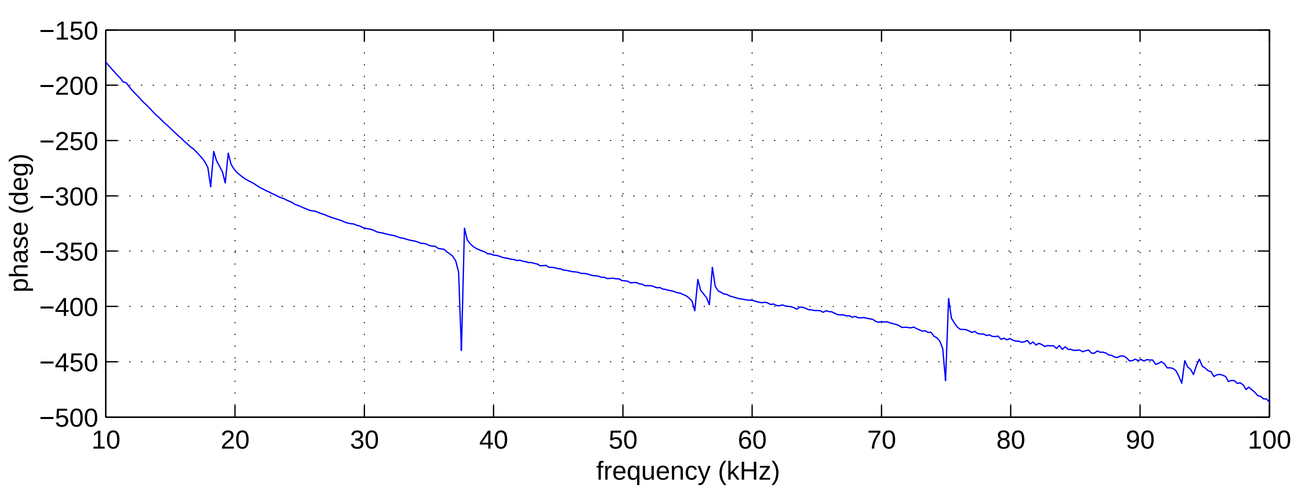
Changes in the laser frequency cause variations in the PDH-signal: $\delta V_s(s)$. The transfer function is

$$\frac{\delta V_s(s)}{\delta V_d(s)} = \frac{\delta V_s(s)}{\delta\omega(s)}\,\frac{\delta\omega(s)}{\delta V_d(s)} \approx \text{const } H_\omega(s).$$

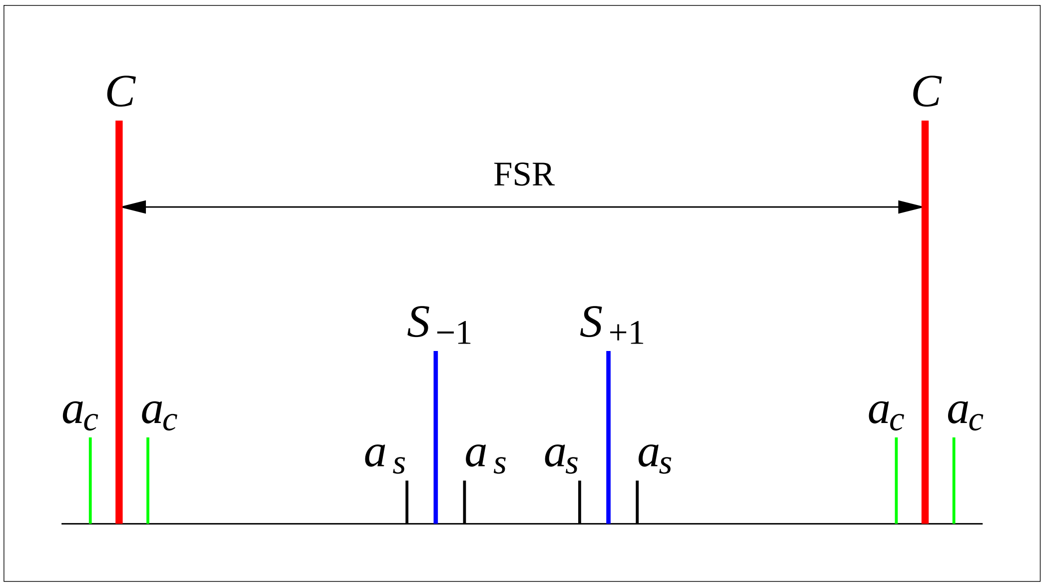
Transfer function H_ω

The measured transfer function, $\frac{\delta V_s(s)}{\delta V_d(s)}$, is shown in the figure below. It has sharp features caused by the carrier and sideband resonances. The carrier resonances occur at multiples of the FSR whereas the sideband resonances occur at roughly half-way between the FSR dips. All these resonances will be discussed in detail below.





Location of resonances



C – resonances of the Fabry-Perot cavity. (The carrier is locked to one of the resonances. The separation between the resonances is the free spectral range (FSR).)

$S_{\pm 1}$ – the location of the first-order RF sidebands. They are 652 FSR's away from the corresponding carrier.

Modulation of the laser frequency causes audio sidebands on the light:

a_c – audio sidebands on the carrier,

a_s – audio sidebands on the RF-sidebands.

The resonances occur when either a_c or a_s coincides with multiples of the FSR.

H_ω near the FSR

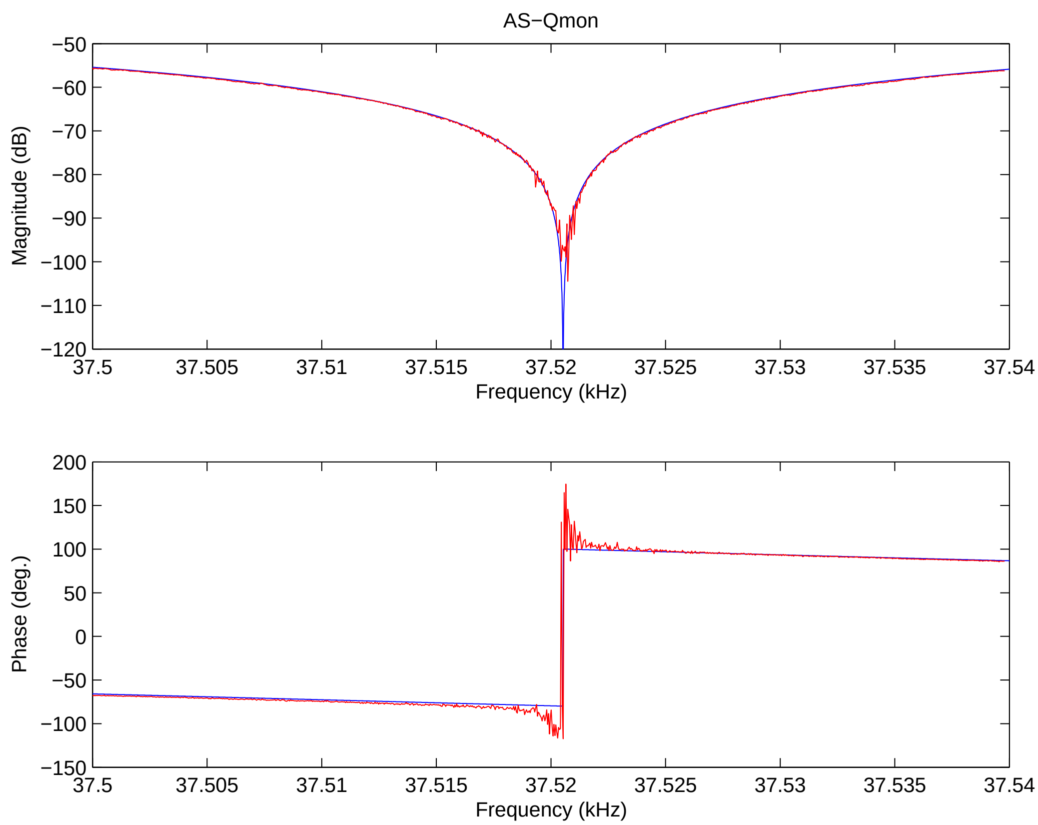
The response of the cavity to changes in the laser frequency is

$$H_\omega(s) = \left(\frac{1 - e^{-2sT}}{2sT}\right) \left(\frac{1 - r_a r_b}{1 - r_a r_b e^{-2sT}}\right).$$

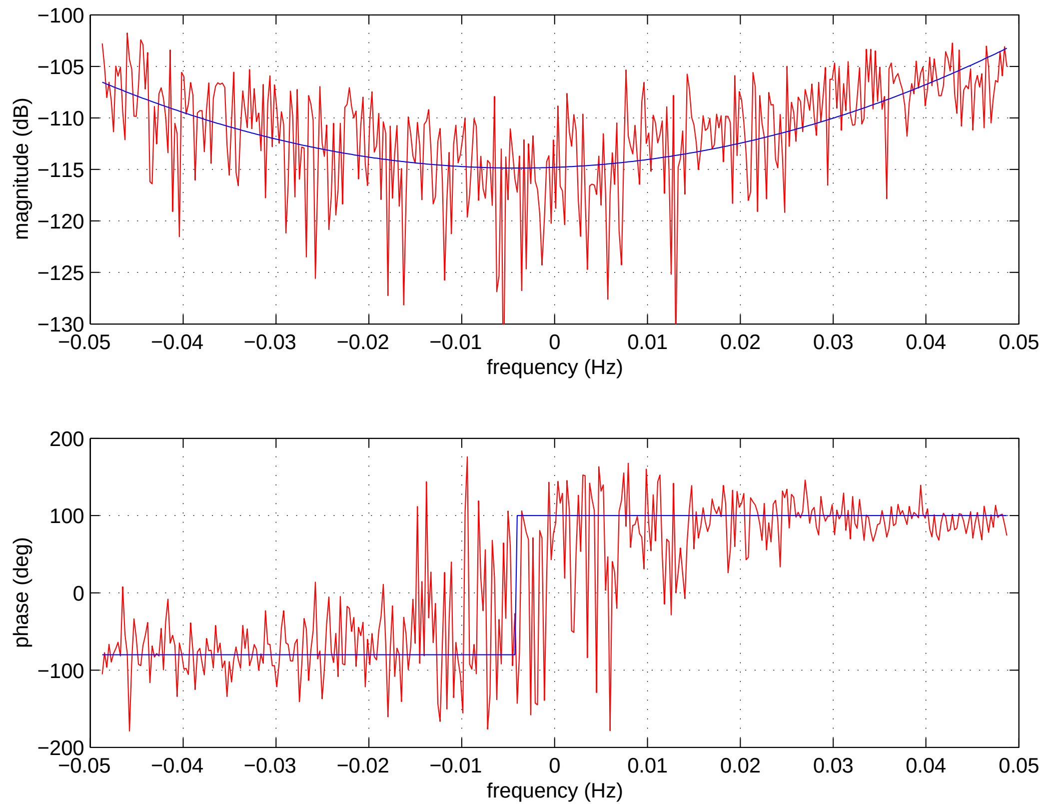
Near the FSR the transfer function can be approximated as

$$H_\omega(s) \approx \frac{s - z_1}{s - p_1}.$$

The zero and pole are $z_1 = 2\pi i$ FSR and $p_1 = z_1 - \frac{1}{\tau}$, where $\tau = 1.7$ ms is cavity storage time.



H_ω near the FSR (0.1-Hz span)



horizontal axis: $x = f - f_c$, where $f_c = 37520.164237$ Hz.

LSQ fit yields the FSR and the length:

$$
\begin{aligned}
f_0 &= 37520.1602 \quad \text{Hz}, \\
L &= 3995.084996 \quad \text{m}.
\end{aligned}
$$

The errors are found using the matrix-inversion method and also R. Coldwell's technique,

$$
\begin{aligned}
\delta f_0 &= 0.0011 \quad \text{Hz}, \\
\delta L &= 0.00012 \quad \text{m}.
\end{aligned}
$$

$H_\omega + \Delta H_\omega$ near $\frac{1}{2}$FSR

Correction to the PDH signal from the 1st order RF side-bands excited in the cavity is

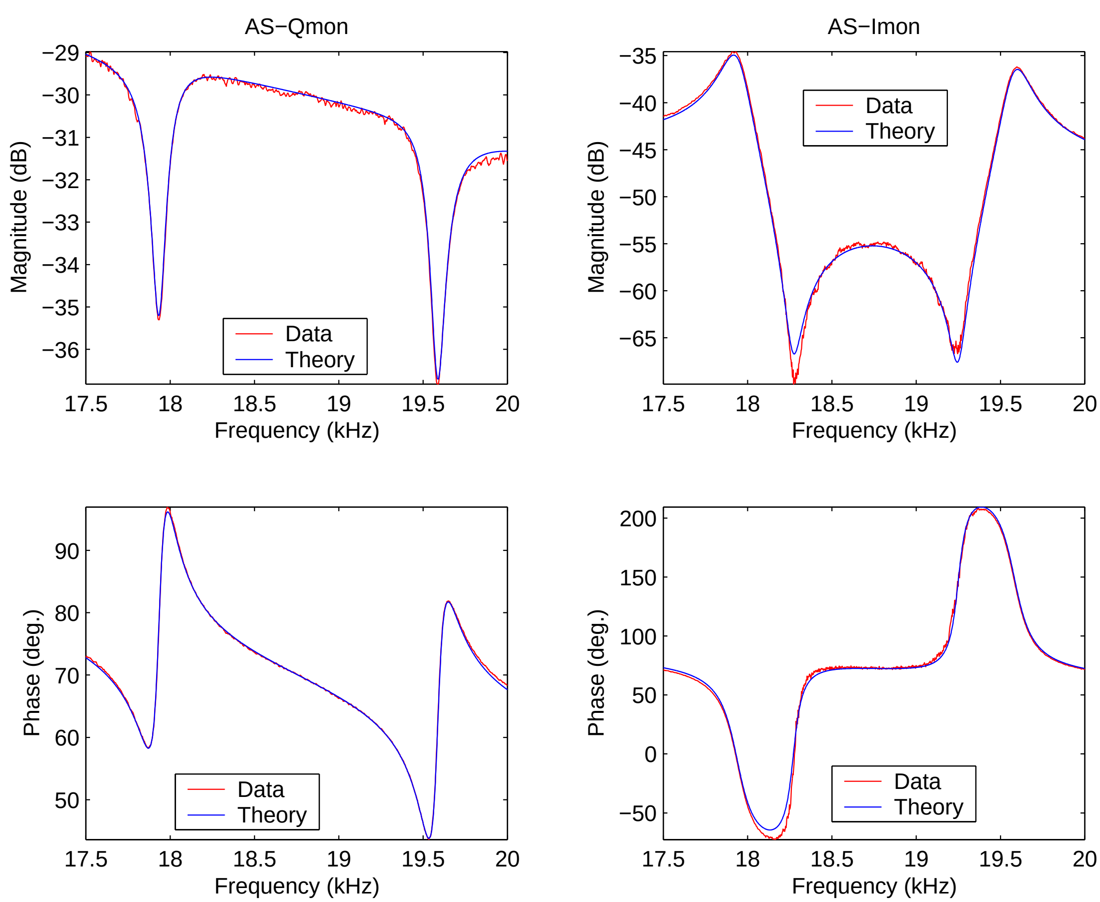
$$\Delta H_\omega(s) = -e^{i\gamma}\frac{q\rho}{1-q}H_1(s) + e^{-i\gamma}\frac{q^*\rho}{1-q^*}H_{-1}(s).$$

Here $H_{\pm 1}$ are the sideband transfer functions:

$$H_{\pm 1}(s) = \frac{1}{1 - q_{\pm 1}\, e^{-2sT}}, \quad \text{where} \quad q_{\pm 1} = r_a r_b\, e^{\pm 2i\psi},$$

and ψ is the sideband propagation phase in the arm cavity.

Bode plots of $H_\omega + \Delta H_\omega$ in the vicinity of $\frac{1}{2}$FSR



$H_\omega + \Delta H_\omega$ near $\frac{3}{2}$FSR

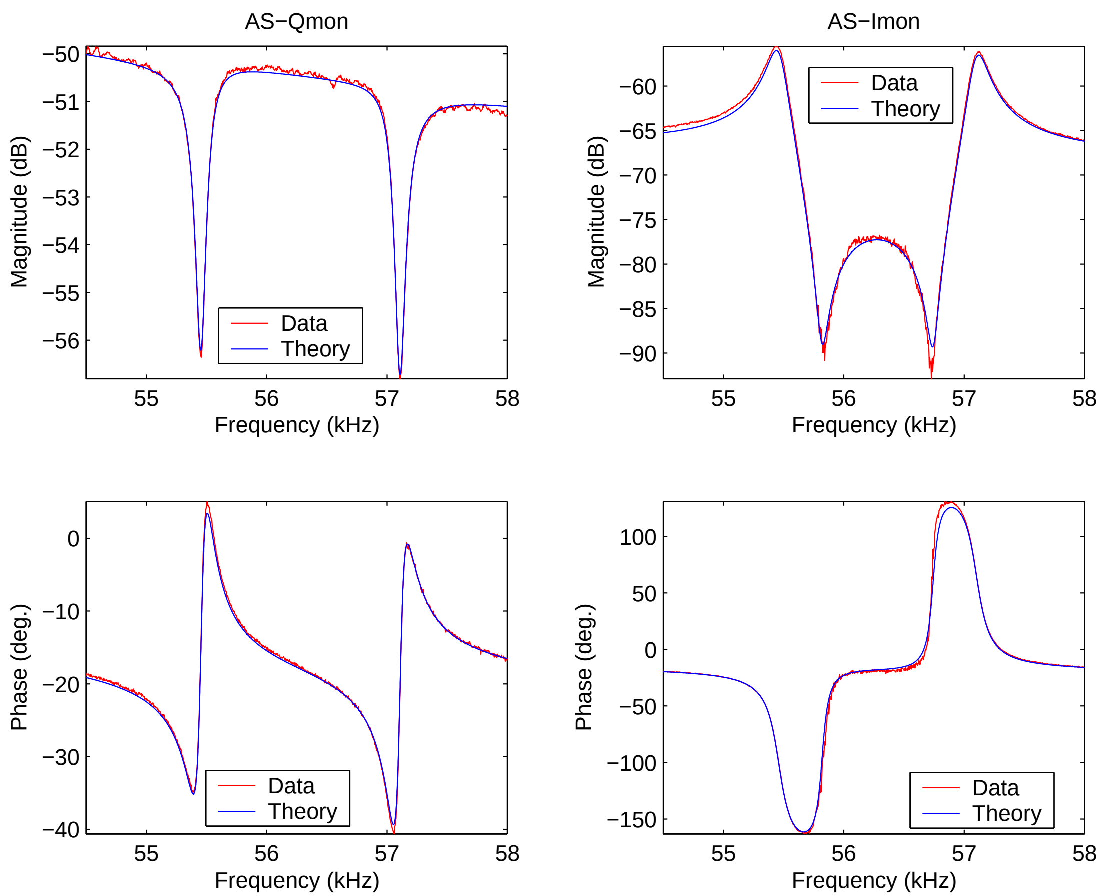
The modulation frequency is $f_{\text{mod}} = 24,481,326$ Hz. The corresponding propagation phase is

$$\psi = 2\pi f_{\text{mod}} T.$$

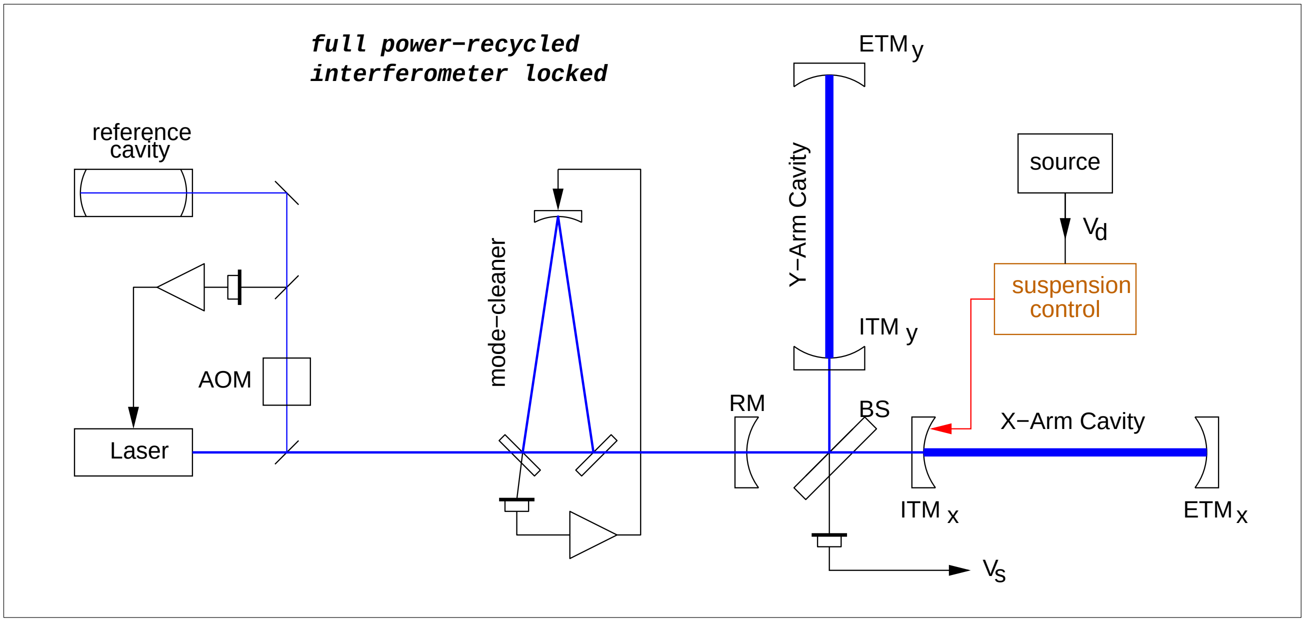
Frequency calibration was done with the Rubidium Frequency Standard SRS FS725.

Correction to the frequency array on the spectrum analyzer was 0.404 Hz.

Bode plots of $H_\omega + \Delta H_\omega$ in the vicinity of $\frac{3}{2}$FSR



Swept sine of the cavity length



A sine-wave $\delta V_d(s)$ is injected to the suspension control module to produce the force on the mass:

$$\delta F(s) = \text{const}\ \delta V_d(s).$$

The force generates displacement:

$$\delta L(s) = \frac{1}{m} H_p(s)\delta F(s),$$

where $H_p(s)$ is the pendulum transfer function. For high frequencies, $H_p(s) \approx 1/s^2$.

Variations in the PDH-signal: $\delta V_s(s)$. The transfer function:

$$\frac{\delta V_s(s)}{\delta V_d(s)} = \frac{\delta V_s(s)}{\delta L(s)} \frac{\delta L(s)}{\delta V_d(s)} \approx \frac{\text{const}}{m\omega_{\text{FSR}}^2} H_L(s).$$

Transfer function H_L

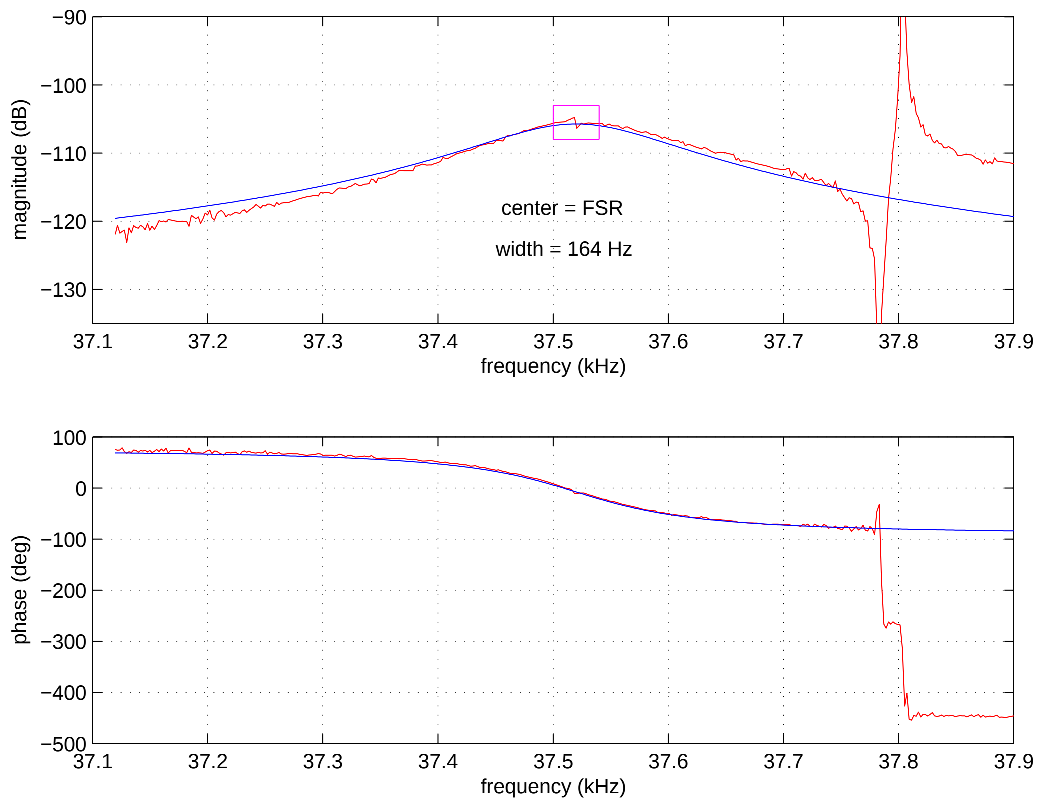
The response of a Fabry-Perot cavity to changes of its length is given by

$$H_L(s) = \frac{1 - r_a r_b}{1 - r_a r_b e^{-2sT}}.$$

Near the FSR the transfer function can be approximated as

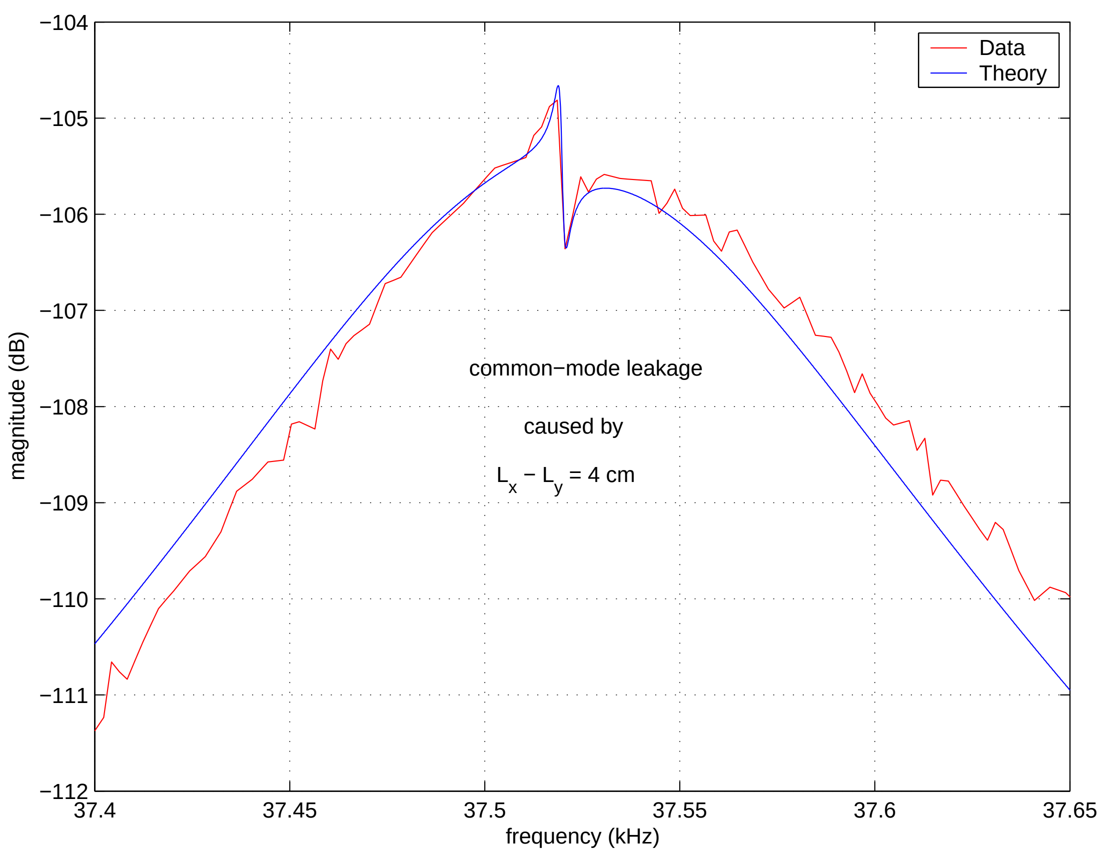
$$H_\omega(s) \approx \frac{1}{s - p_1},$$

where $p_1 = 2\pi i$ FSR $- \frac{1}{\tau}$ is a complex pole and and $\tau =$ 1.7 ms is cavity storage time.



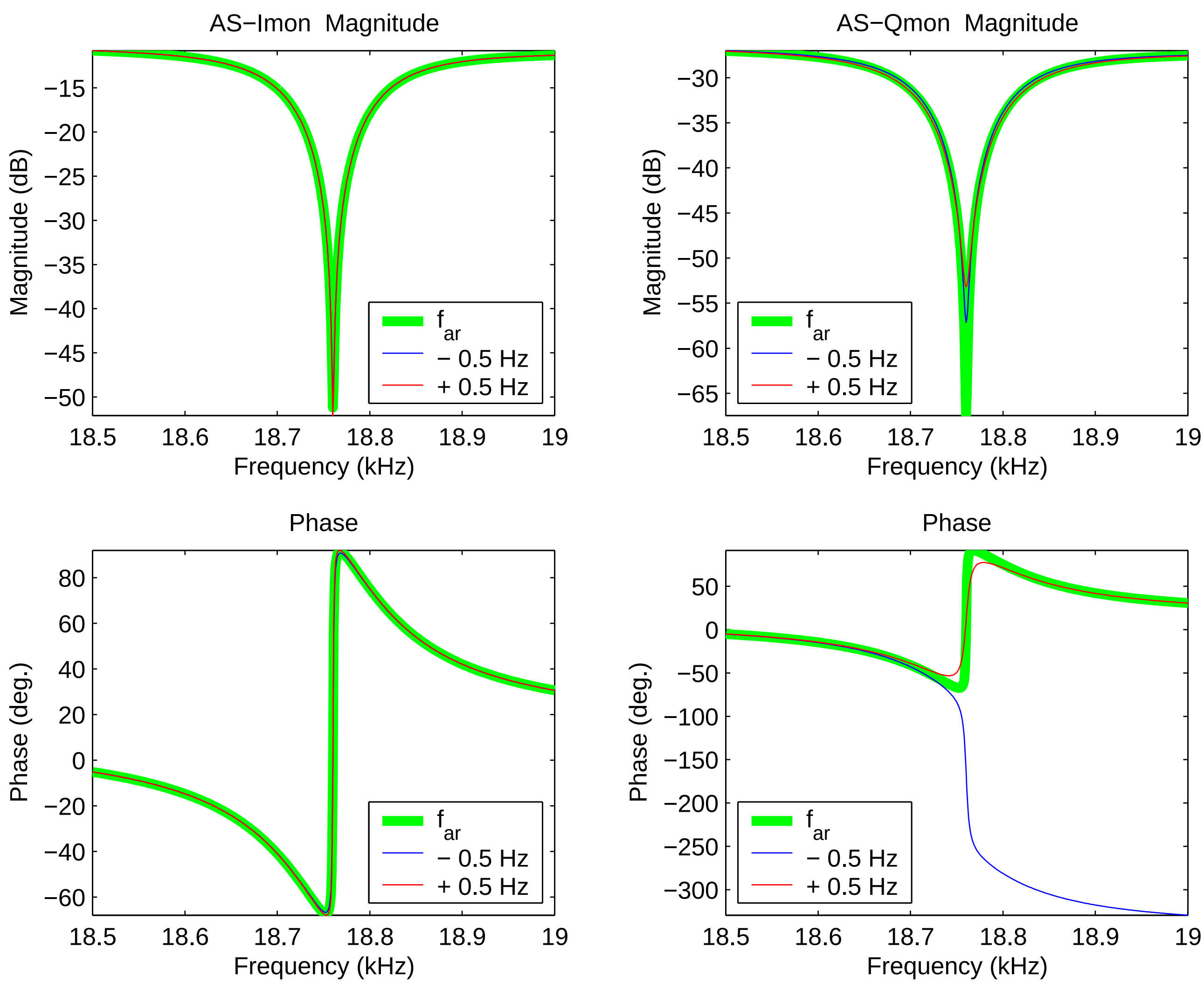
Arm-length mismatch

The magnitude of the transfer function has a small wiggle near the top. This feature is reproduced by numerical models (*Finesse* and *e2e*) and indicates a difference in the arm lengths of 3–4 cm (Y arm shorter). The simulation results (below) are obtained with the arm-length mismatch = 4 cm.



Anti-resonance of RF-sidebands

If the modulation frequency is tuned across the anti resonance point ($\frac{1}{2}$FSR), transfer function H_ω changes in an abrupt way. Its phase can have either steady roll-offs or fast switch-backs. This abrupt change in the transfer function can be used to determine the arm lengths.

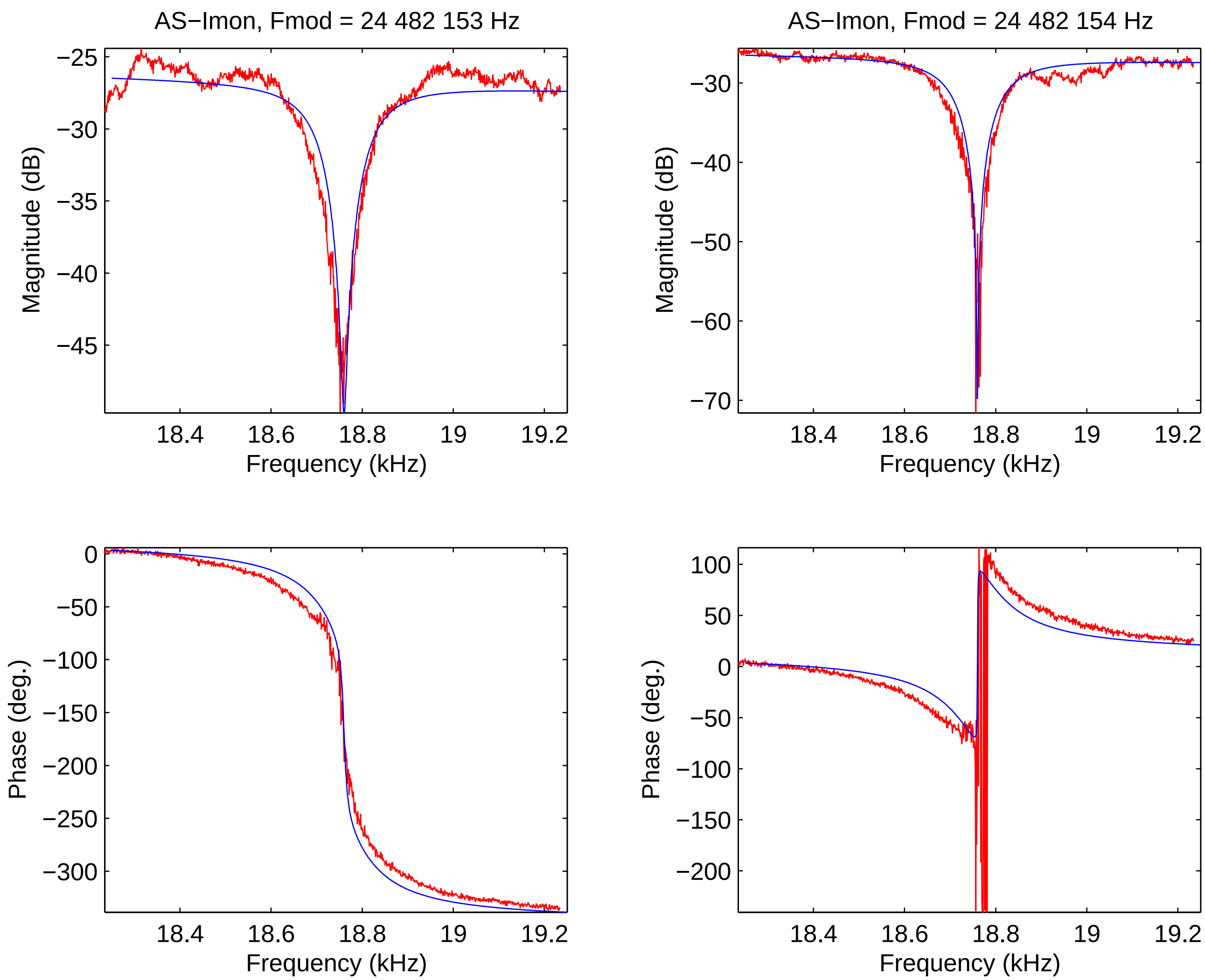


Precision length measurement

The precision is limited by the minimum step size (1 Hz) of the frequency synthesizer. The cavity lengths thus measured are

$$
\begin{aligned}
L_X &= 3995084.18 \pm 0.08 \text{ mm}, \\
L_Y &= 3995044.37 \pm 0.08 \text{ mm}.
\end{aligned}
$$

The 39.81 mm arm-length difference is consistent with the measurement of H_L shown above.



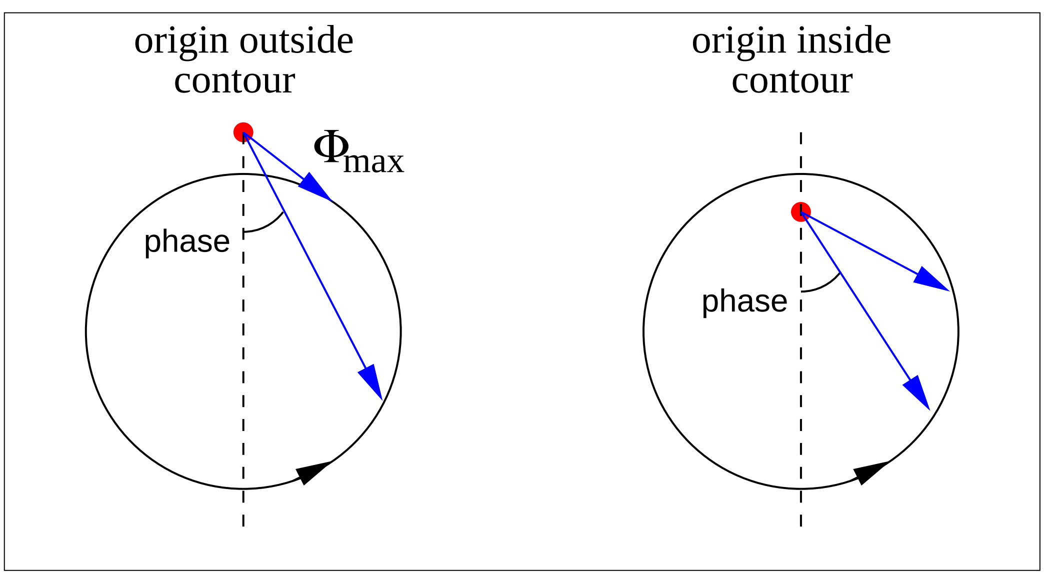
Discontinuity in the phase

The abrupt change in the phase of the transfer function can be understood using phasor representation. As the frequency changes, the tip of the phasor travels along a circular contour. (For simplicity, it is shown by a circle.)

- if the origin is outside the contour the phase changes from 0 to some maximum value (Φ_m) and back to 0. The maximum value is defined by proximity of the origin to the contour. The phasor is moving retrograde between the two extreme angles.

- If the origin is inside the contour, the phase monotonically changes from 0 to 360^0.

Nyquist diagram



Characterization of LIGO FP cavities

The present work is a part of a larger effort aimed at complete characterization of the LIGO arm cavities. The measurements described above are closely related to other investigations, some of which are

- fix 4-km Schnupp asymmetry
- match the arm cavity lengths
- measure and set the demodulation phases
- calibrate and tune the modulation frequency

Other experiments that can be done using the techniques and concepts described above are

- monitor long-term (tidal) changes of the arm lengths
- measure the arm-cavity g-factors
- extract the mirror radii of curvature
- study mechanical resonances of the mirrors
- search for GW at frequencies near the first FSR