Novel finite differencing techniques for numerical relativity

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An important expected source of gravitational waves for terrestrial and space based interferometers is the collision of two black holes.

The only strategy available at present to study the final orbits and plunge is to integrate the Einstein equations with supercomputers.

For many years the solution to this problem has proved elusive, mainly due to instabilities that plague the numerical codes. State of the art simulations can only follow fractions of an orbit without any significant accuracy.

Finding how to formulate and discretize the evolution equations in a way that they evolve stably is the main challenge in this field of research today.

At LSU we recently introduced in numerical relativity certain discretization techniques that have been discussed relatively recently in the numerical analysis literature.

The techniques are based on the type of concepts mathematicians use to prove well-posedness of a system of equations. Well posedness means that the solutions do not grow faster than a certain rate as a function of time and such a rate is independent of the initial data. In the case of discretized equations one also wants the rate to be independent of lattice spacing.

To prove that a system of equations is well posed, mathematicians construct an “energy” (a positive definite quantity that is a function of the variables and their derivatives) and find if its growth can be bounded. That implies that the variables of the problem (and their derivatives) are also bounded.

To establish bounds on energies is very similar to the way we all have at some point proved than an energy is conserved: one takes the time derivative, substitutes the equations of motion and integrates by parts.

If one is dealing with discrete equations, “integrating by parts” is replaced by “summation by parts”,

$$(u, Dv) - (Du, v) = u_0 v_0 - u_N v_N$$

This requires defining different stencils for “D” at the interior than at edges, corners and faces of the boundary. This is a reasonably developed subject in numerical analysis (i.e. Olsson 1995).

What we have done is to apply these techniques in numerical relativity.

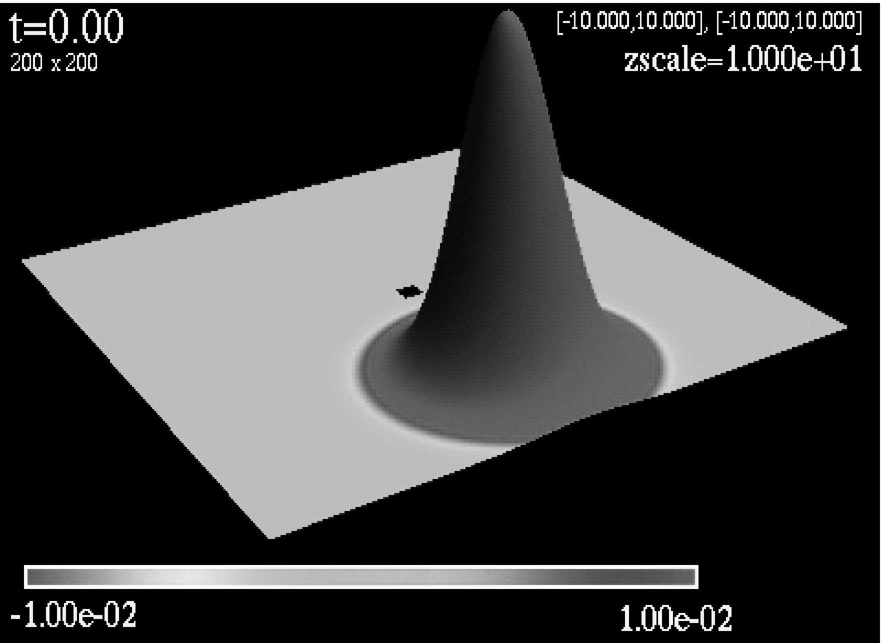
An additional issue one has to deal with in relativity is the presence of constraints. One only solves the “G_{ij}” part of the Einstein equations. The “G_{oi}” and “G_{00}” parts are the constraints. If they are satisfied initially, they should be satisfied within the domain of dependence of the initial data.

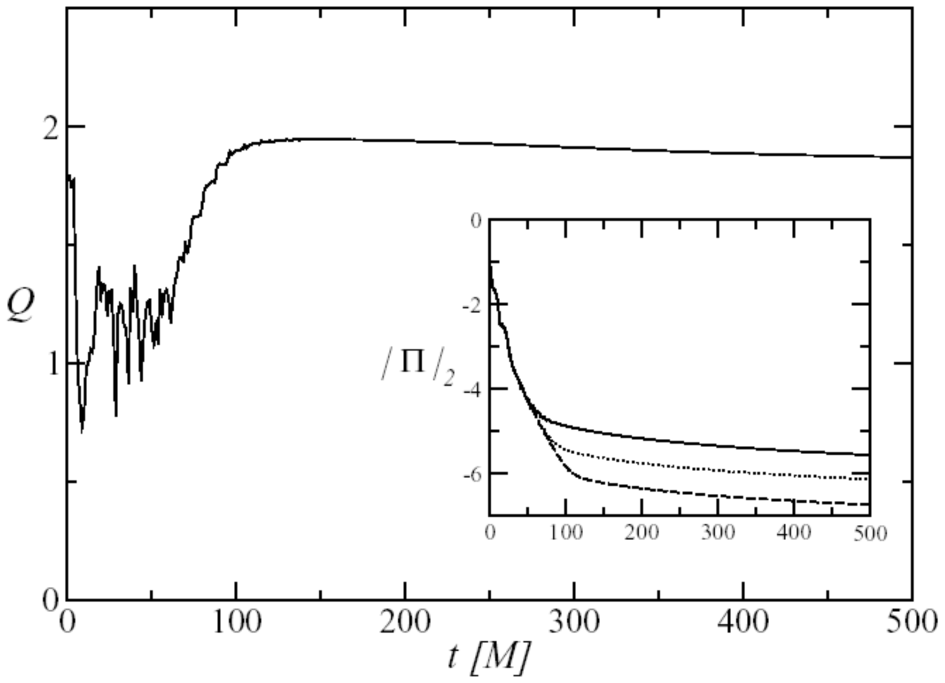
Since one wishes to have long-lived simulations, where the “time” direction of the domain is much larger than the spatial dimensions, one needs to provide boundary conditions that ensure that the constraints are satisfied in the computational domain.

This is not easy. One can only give boundary conditions to the variables of the problem. First of all, they should be such that the problem is well posed. In addition to that one should make sure they guarantee the constraints are satisfied.

Our group was the first to provide such boundary conditions in manifolds with boundaries that are not smooth (in numerical simulations the boundary is typically a box).

The first problem we studied applying all this technology was the propagation of waves on a black hole background.





One of the lessons learned in studying this problem and doing the boundaries right is that the “excision box” (the region inside the black hole the code does not evolve to avoid having to deal with the singularity) cannot be bigger than 0.7M, otherwise the problem is ill-posed.

We are currently developing a code to integrate the full Einstein equations with this technology.

References:

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Summary

- One can discretize the Einstein equations in such a way that the resulting codes are stable using techniques known in the numerical analysis literature.
- One can prescribe boundary conditions that ensure that the constraint equations are enforced.
- Hopefully this will help develop binary black hole codes in the next few years.