Parameter Estimation

Gregory Mendell
LIGO Hanford Observatory
Anah Mourant (SURF Student)
University of New Mexico

LIGO-G030398-00-W

Parameters: 1 component signal

(Triaxial ellipsoid rotating about principal axis)

Jaranowski, Krolak, and Schutz gr-qc/9804014

$$h(t) = \sum_{j=1}^{4} A_{2j} h_{2j}(t),$$

$$h_{21} = a\cos 2\Phi(t),\ h_{22} = b\cos 2\Phi(t),\ h_{23} = a\sin 2\Phi(t),\ h_{24} = b\cos 2\Phi(t),$$

$$\begin{aligned}
A_{21} &= h_0\left[\frac{1}{2}(1+\cos^2\iota)\cos 2\psi\cos 2\Phi_0 - \cos\iota\sin 2\psi\sin 2\Phi_0\right],\\
A_{22} &= h_0\left[\frac{1}{2}(1+\cos^2\iota)\sin 2\psi\cos 2\Phi_0 + \cos\iota\cos 2\psi\sin 2\Phi_0\right],\\
A_{23} &= h_0\left[-\frac{1}{2}(1+\cos^2\iota)\cos 2\psi\sin 2\Phi_0 - \cos\iota\sin 2\psi\cos 2\Phi_0\right],\\
A_{24} &= h_0\left[-\frac{1}{2}(1+\cos^2\iota)\sin 2\psi\sin 2\Phi_0 + \cos\iota\cos 2\psi\cos 2\Phi_0\right].
\end{aligned}$$

Maximum Likelihood Estimates

Jaranowski, Krolak, and Schutz gr-qc/9804014

$$\hat{A}_{21} = 2\frac{B(x||h_{21}) - C(x||h_{22})}{D},$$
$$\hat{A}_{22} = 2\frac{A(x||h_{22}) - C(x||h_{21})}{D},$$
$$\hat{A}_{23} = 2\frac{B(x||h_{23}) - C(x||h_{24})}{D},$$
$$\hat{A}_{24} = 2\frac{A(x||h_{24}) - C(x||h_{23})}{D}.$$

Anah has written LALApps code to compute these for the model of the Earth's motion in JKS. Note that JKS made approximations to get these equations. The approximations and round-off error can affect the accuracy of the results, but this does not appear to be a severe issue.

Analytic solutions for parameters in terms of A's:

These equations result in four real solutions with

$$0 \le \iota \le \pi,$$

$$-\pi/4 \le \psi \le \pi/4,$$

$$0 \le 2\Phi_0 \le 2\pi.$$

Only one solves the original equations.

$$
\begin{aligned}
\iota &= \arccos \pm \sqrt{\frac{-\wp \mp 3\aleph \pm 2\sqrt{2\aleph(\aleph \pm \wp)}}{-\wp \pm \aleph}} \\
h_0 &= \sqrt{\frac{\wp}{\frac{1}{4}\cos^4\iota + \frac{3}{2}\cos^2\iota + \frac{1}{4}}} \\
\psi &= \frac{1}{2}\arctan \frac{\frac{1}{4}h_0^2(1+\cos^2\iota)^2 - \hat{A}_{21}^2 - \hat{A}_{23}^2}{\hat{A}_{21}\hat{A}_{22} + \hat{A}_{23}\hat{A}_{24}} \\
\Phi_0 &= \frac{1}{2}\arccos \frac{\hat{A}_{21}\cos 2\psi + \hat{A}_{22}\sin 2\psi}{\frac{1}{2}h_0(1+\cos^2\iota)}
\end{aligned}
$$

$$
\begin{aligned}
\aleph &= \sqrt{\wp^2 - 4(\hat{A}_{21}\hat{A}_{24} - \hat{A}_{22}\hat{A}_{23})^2}, \\
\wp &= \hat{A}_{21}^2 + \hat{A}_{22}^2 + \hat{A}_{23}^2 + \hat{A}_{24}^2.
\end{aligned}
$$

Toy Model Study

$$h(t) = A\cos(2pft) + B\sin(2pft)$$

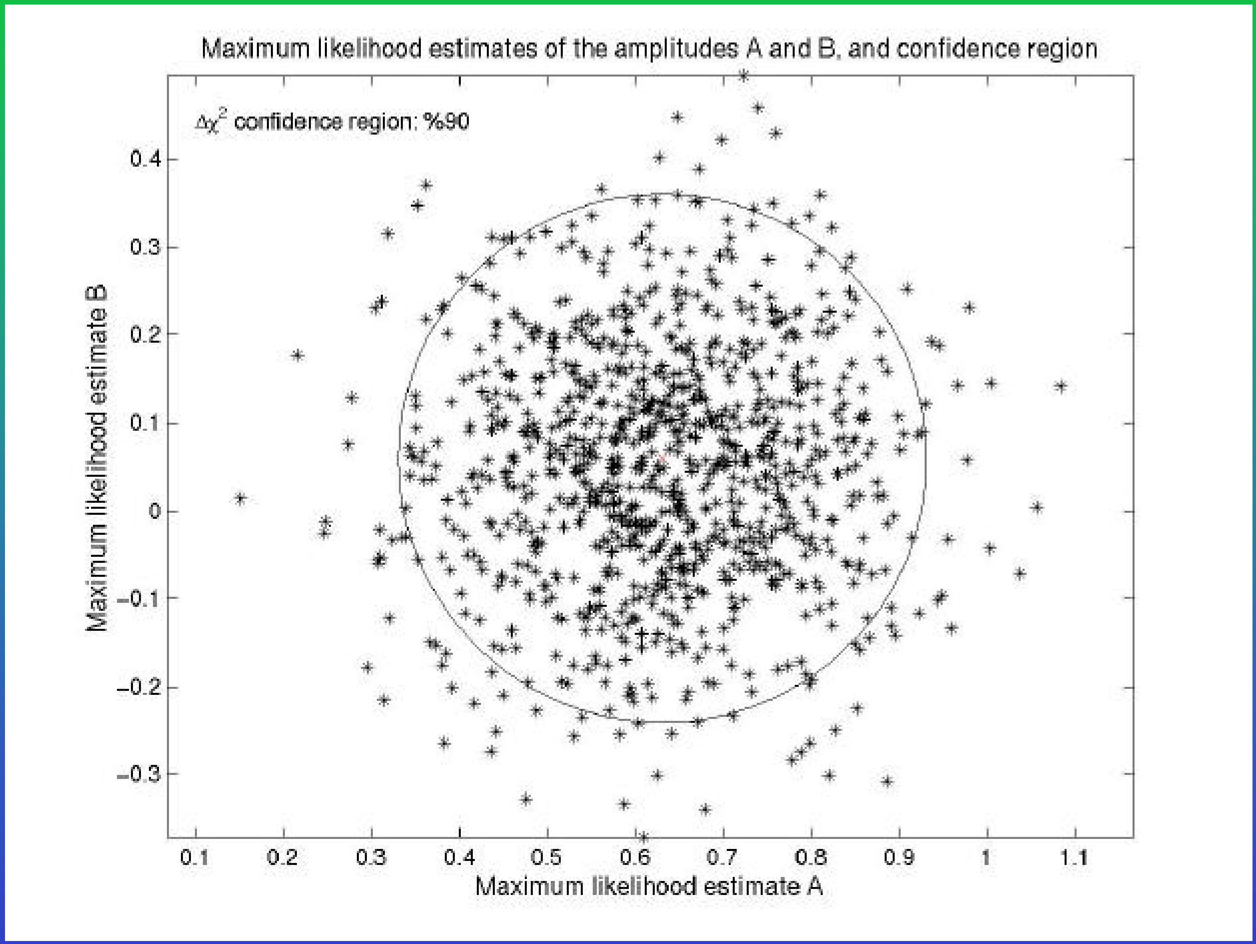
Anah has written Matlab code that sets Frequentist and Bayesian confidence intervals on A and B for this toy model.

Time permitting she will apply the same types of analysis to parameters estimated via JKS.

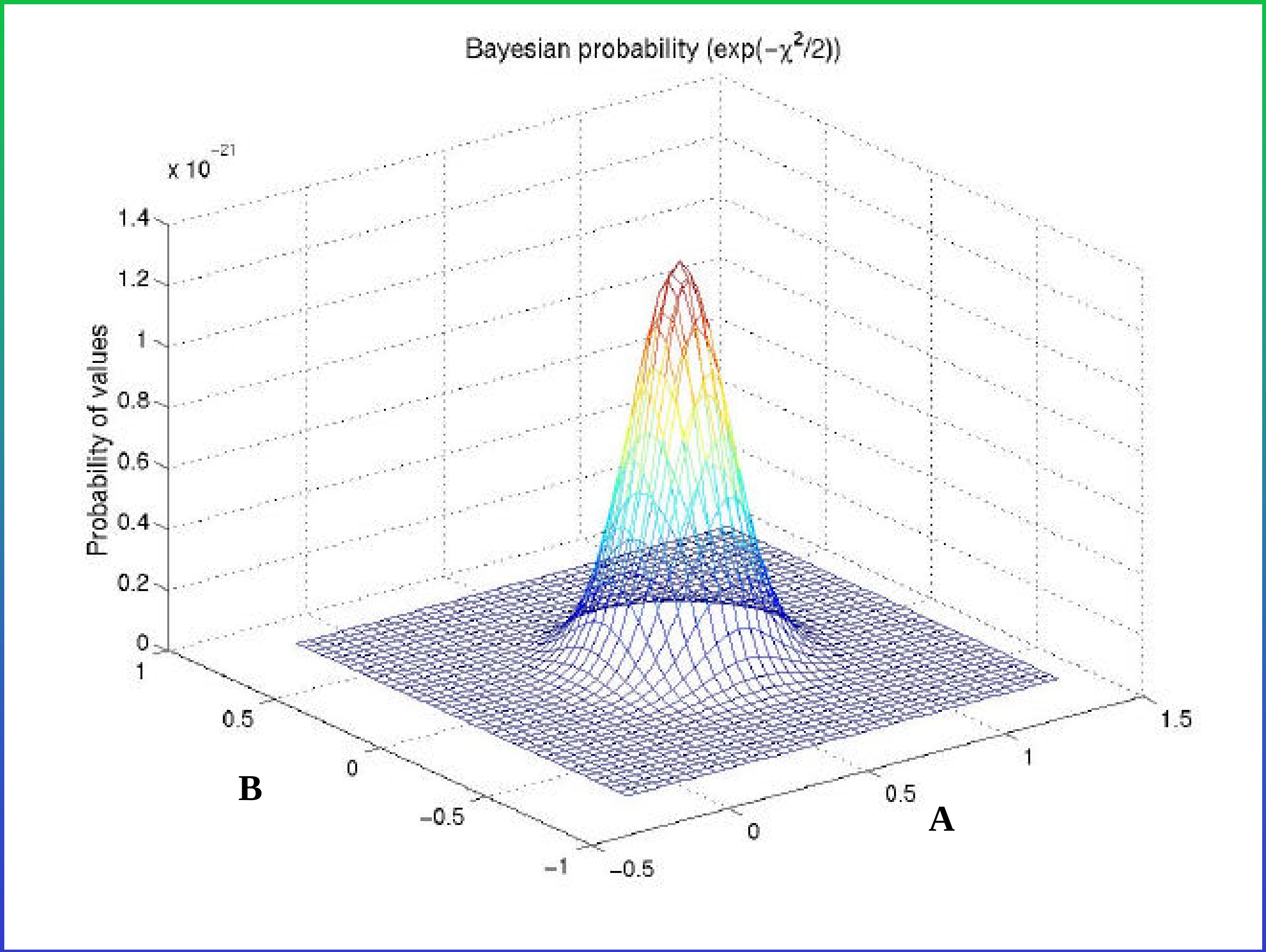
Example Frequentist Result

The figure shows results of a Monte Carlo of synthetic data sets, each with signal for estimated parameters.

Radius of 90% confidence region = 0.3



Chi-Squared Plot



Example Bayesian Result

Radius of 90% confidence region = 0.5 for uniform prior on A and B.