The fast discrete Q-transform

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LIGO Scientific Collaboration Meeting

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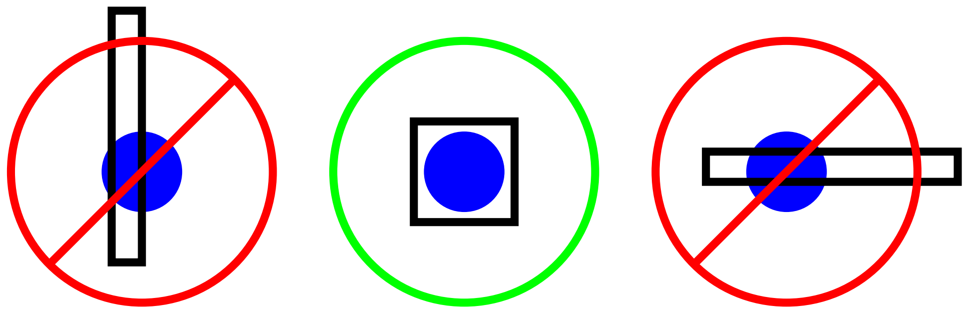
ASIS Session

Motivation

Maximize signal to noise ratio by finding the time frequency pixel which most closely matches a signal.

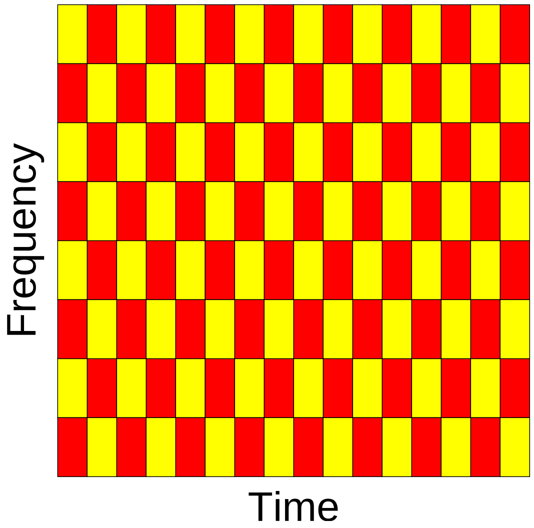


Look for gravitational-wave bursts or detector glitches with

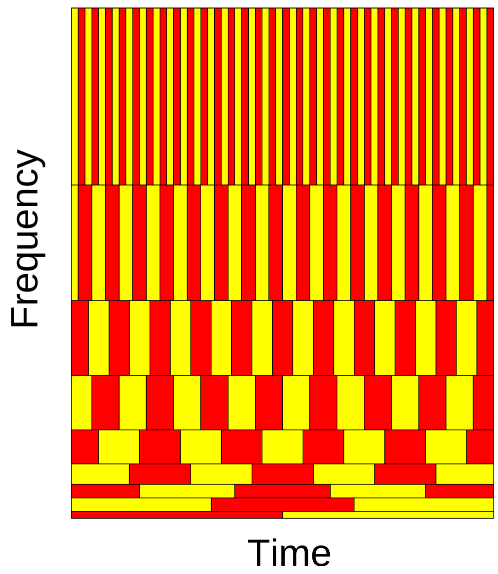
$$Q \lesssim 10.$$

Tilings

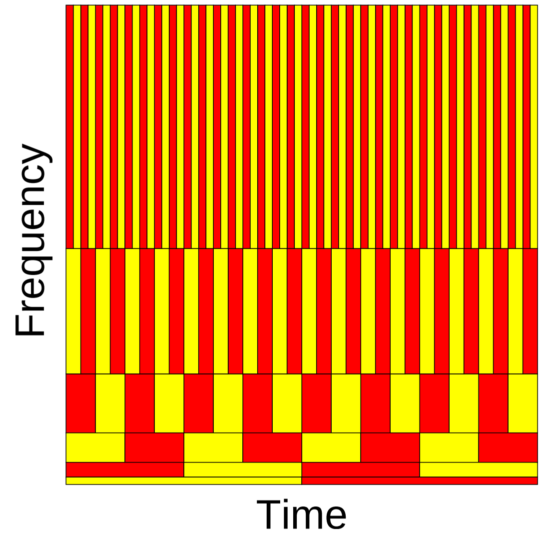
Short Time Fourier Transform



Discrete Constant Q-Transform

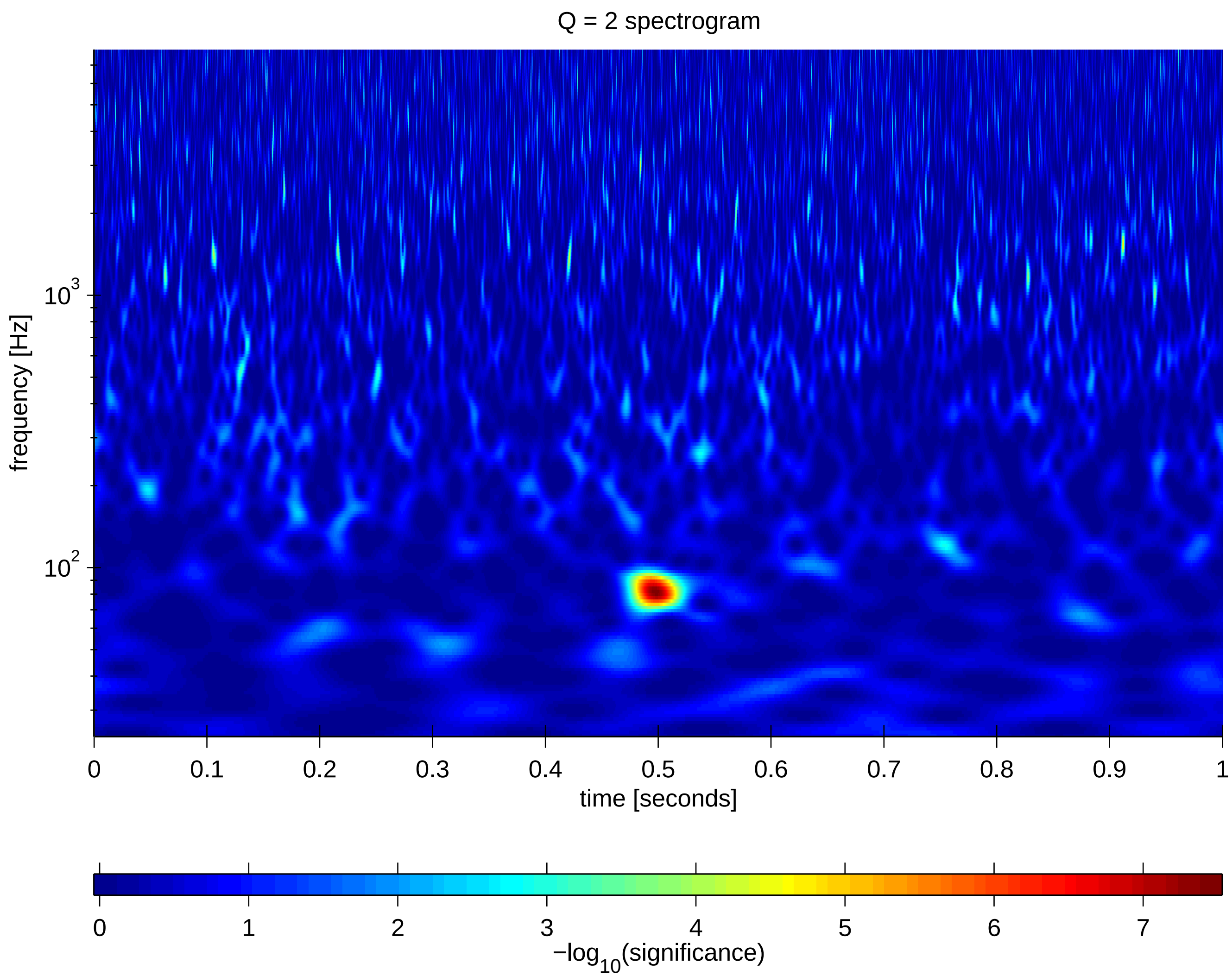


Discrete Wavelet Transform



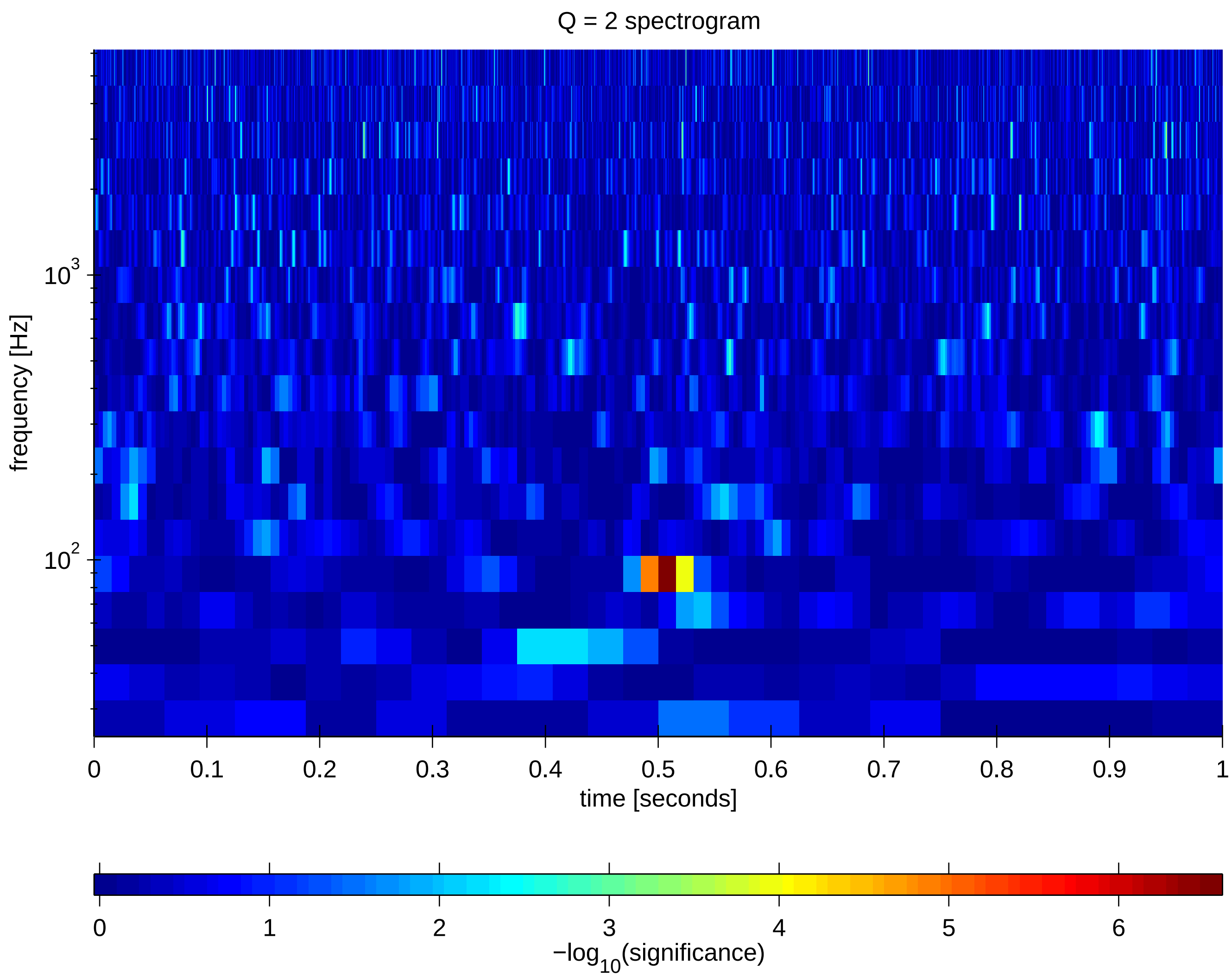
Example

Oversampling of the time-frequency plane



Example

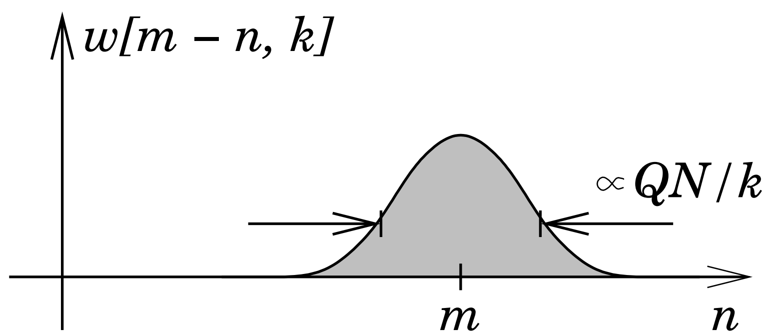
Sufficient sampling of the time-frequency plane



Discrete Q-Transform

Project $x[n]$ onto time-shifted windowed sinusoids, whose widths are inversely proportional to their center frequencies.

$$X_Q[m,k] = \sum_{n=0}^{N-1} x[n] e^{-i2\pi nk/N} w[m-n,k]$$



Discrete Q-Transform

Efficient computation is possible via the FFT

$$X_Q[m,k] = \sum_{l=0}^{N-1} \tilde{X}[l+k]\tilde{W}[l,k]e^{-i2\pi ml/N}$$

$$\tilde{X}[l] = \sum_{n=0}^{N-1} x[n]e^{-i2\pi nl/N}$$

$$\tilde{W}[l] = \sum_{n=0}^{N-1} w[n,k]e^{-i2\pi nl/N}$$

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Energy

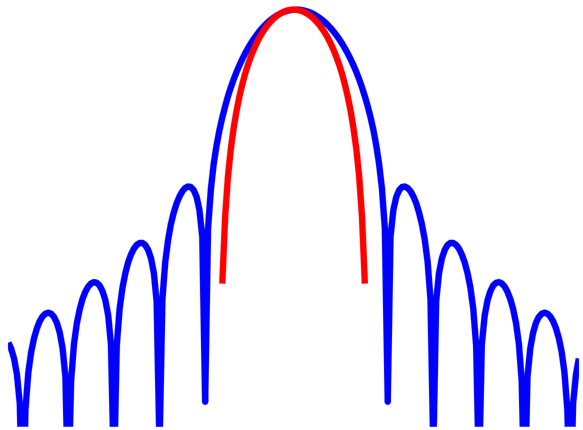
The window normalization is chosen to obey a generalized Parseval's theorem.

$$\frac{1}{f_s}\sum_{m=0}^{N-1}\sum_{k=0}^{N-1}\left|X_Q[m,k]\right|^2 = \frac{1}{N}\sum_{n=0}^{N-1}|x[n]|^2 = \sigma_x^2$$

The square root of the reported pixel energy yields the sum of the background noise amplitude spectral density and the signal root sum square in units of Hz$^{-1/2}$.

Computation Efficiency

- Hanning window in frequency
- Automatic 50 percent overlap
- Oversampling for efficiency
- Computational order?



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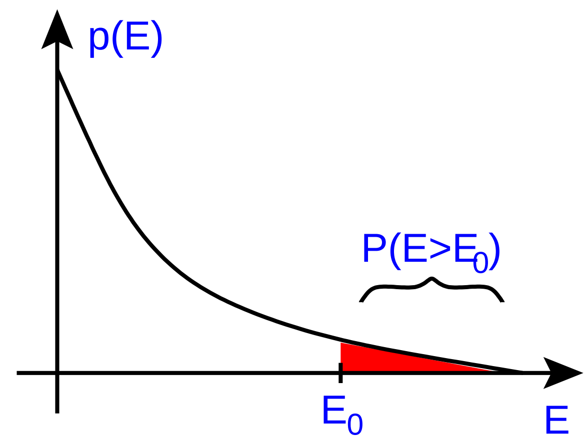
Guassian Statistics

Assuming white Gaussian noise, the distribution of pixel energies at a particular frequency follows an exponential distribution.



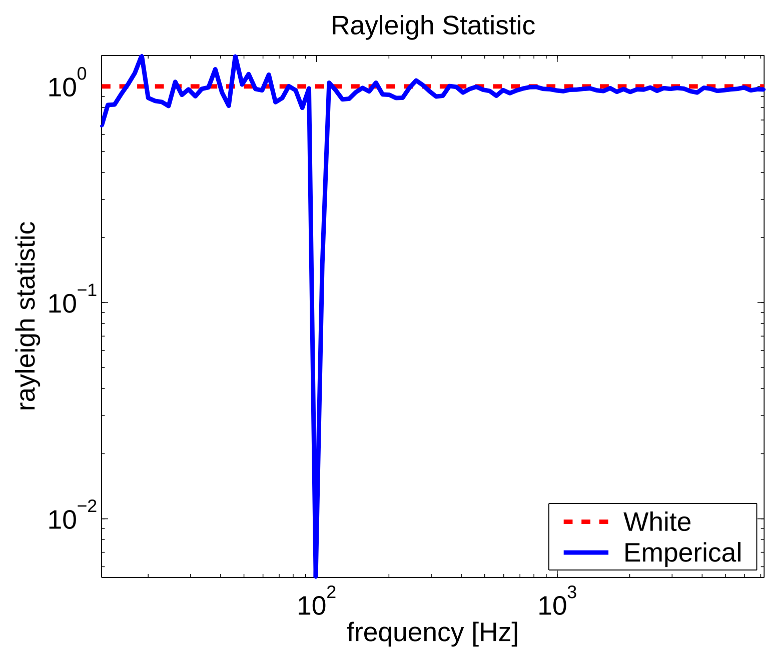
$$p(E)\,dE = \exp\left(-\frac{E}{E_n}\right)\frac{dE}{E_n} \qquad P(E > E_0) = \exp\left(-\frac{E_0}{E_n}\right)$$

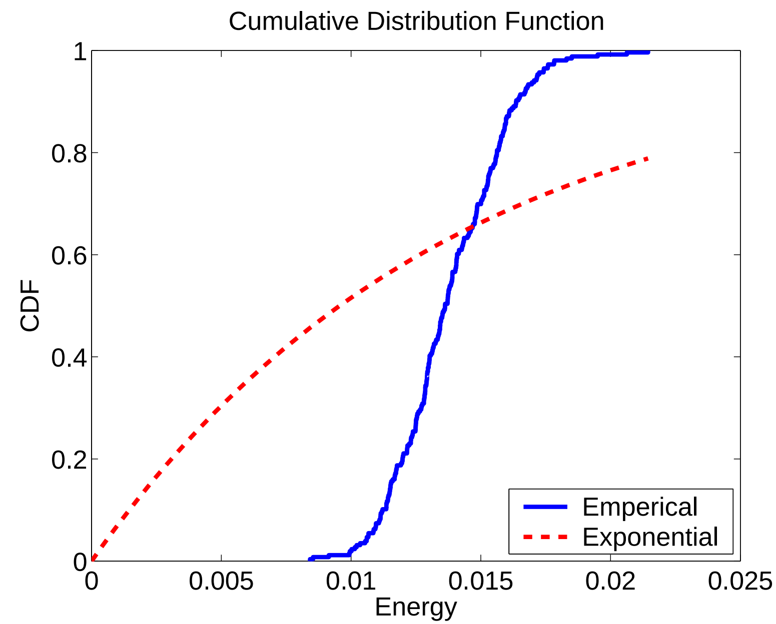
$$\text{significance} = -\log_{10} P(E > E_0)$$

$$\text{SNR} = \left(\frac{E - E_n}{E_n}\right)^{1/2} \qquad \text{RSS} = (E - E_n)^{1/2}$$

LPEF Whitening

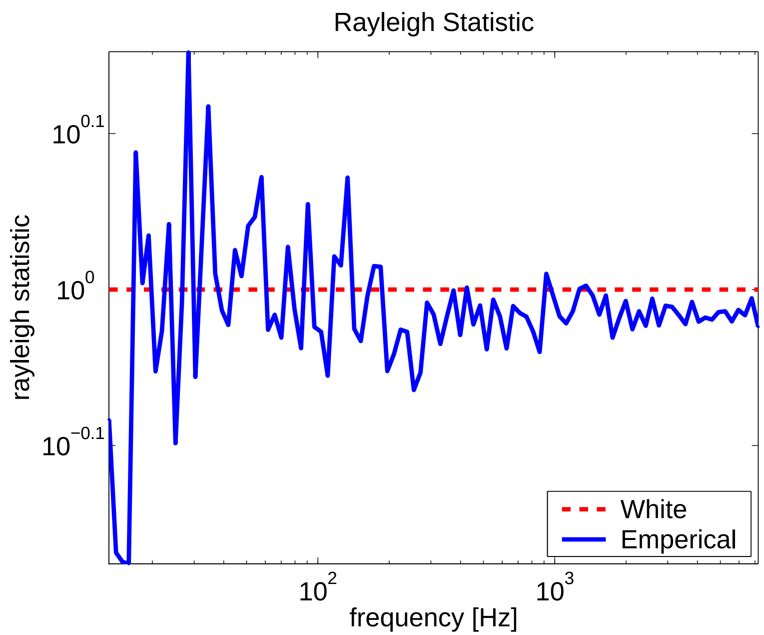
Coherent signal is Rician distributed

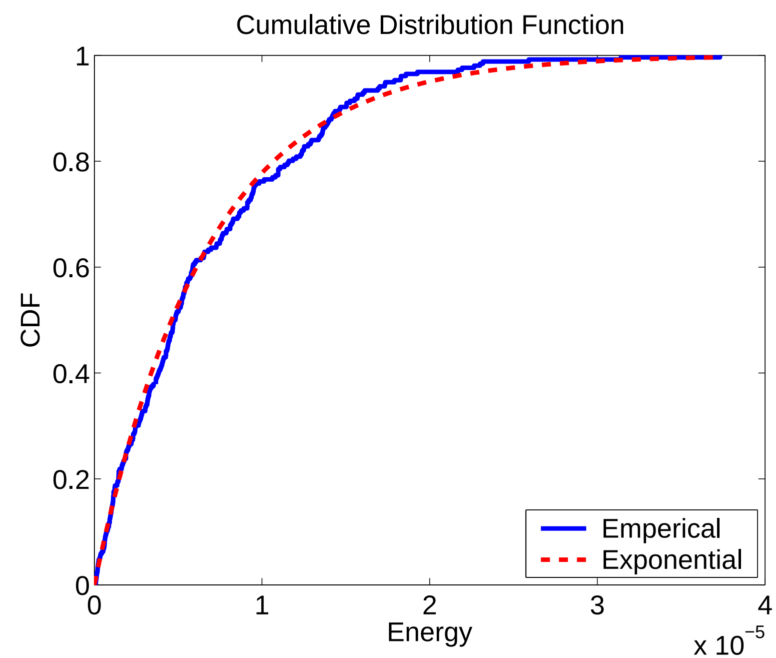




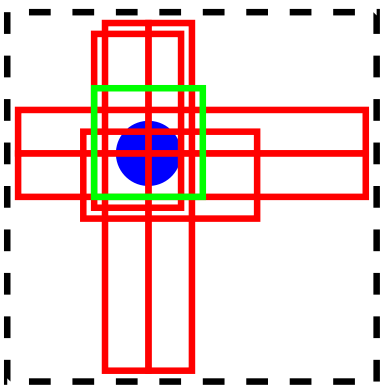
LPEF Whitening

Linear Prediction restores Exponential distribution





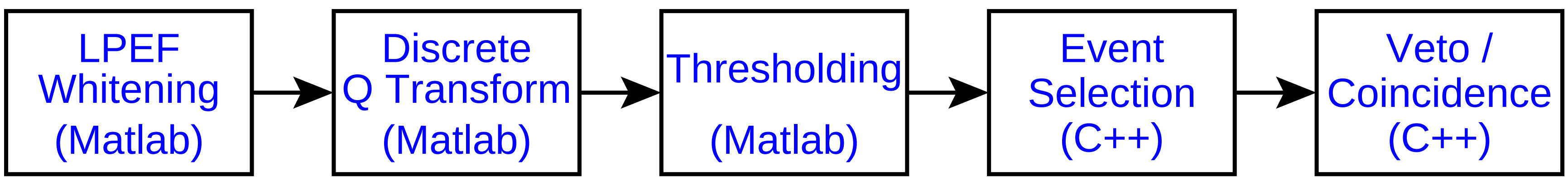
Event Selection



- choose white noise false rate
- yields significance threshold
- identify significance pixels
- group overlapping pixels
- select most significant pixel
- report pixel parameters

Pipeline Implementation

A prototype pipeline has been implemented in Matlab and C++



http://ligo.mit.edu/ shourov/q/

Future Plans

- Detection efficiency vs. false rate
  - hardware injections†
  - software injections (requires calibration)
- Veto efficiency and safety studies†
- Post processing (requires calibration)
  - amplitude estimation
  - amplitude and waveform consistency
- Migrate to other platforms?
  - DMT
  - LDAS (BurstDSO)

†Friday Burst group meeting?