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# Coherent Coincident Analysis of LIGO Burst Candidates

Laura Cadonati  
*Massachusetts Institute of Technology*  
*LIGO Scientific Collaboration*

8<sup>th</sup> Gravitational Wave Data Analysis Workshop  
Milwaukee, Wisconsin, December 17-20, 2003

# Post Coincidence Coherent Analysis

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- Burst candidates separately identified in the data stream of each interferometer by the Event Trigger Generators (ETG): TFclusters, Excess Power, WaveBurst, BlockNormal.
  - » Tuning maximizes detection efficiency for given classes of waveforms and a given false rate  $\sim 1-2$  Hz
- Multi-interferometer coincidence analysis:
  - » Rule of thumb: detection efficiency in coincidence  $\sim$  product of efficiency at the single interferometers. Coincidence selection criteria should not further reduce the detection efficiency. The final false rate limits how loose the cuts can be.
  - » Currently implemented: time and frequency coincidence (in general, different tolerance for different trigger generators).
  - » Amplitude/energy cut: not yet implemented.
- Cross-Correlation for coherent analysis of coincident events
  - » This is a waveform consistency test.
  - » Allows suppression of false events without reducing the detection efficiency of the pipeline.

# r-statistic Cross Correlation Test

For each triple coincidence candidate event produced by the burst pipeline (start time, duration  $\Delta T$ ) process pairs of interferometers:

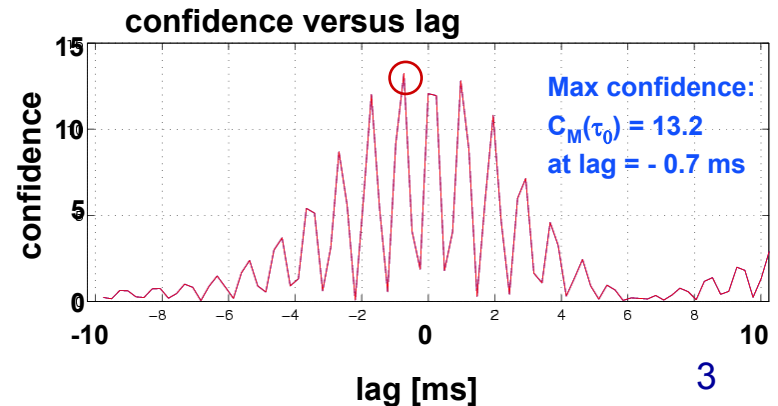
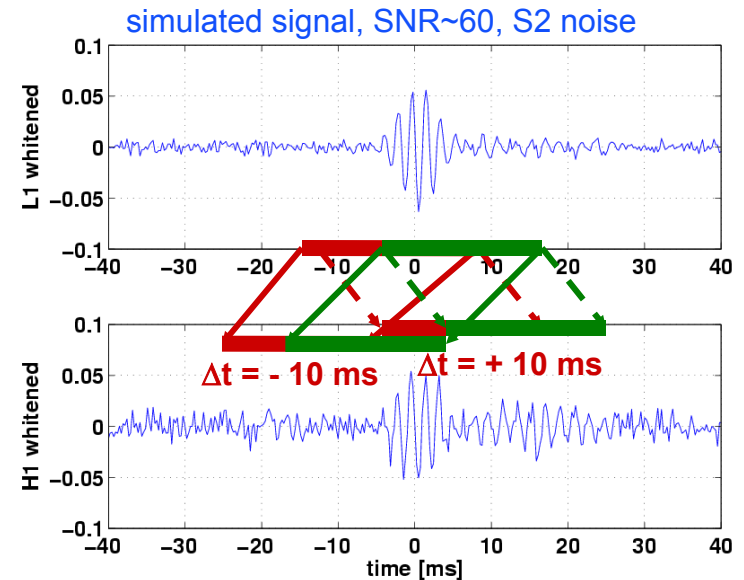
Data Conditioning:

- » 100-2048 Hz band-pass
- » Whitening with linear error predictor filters

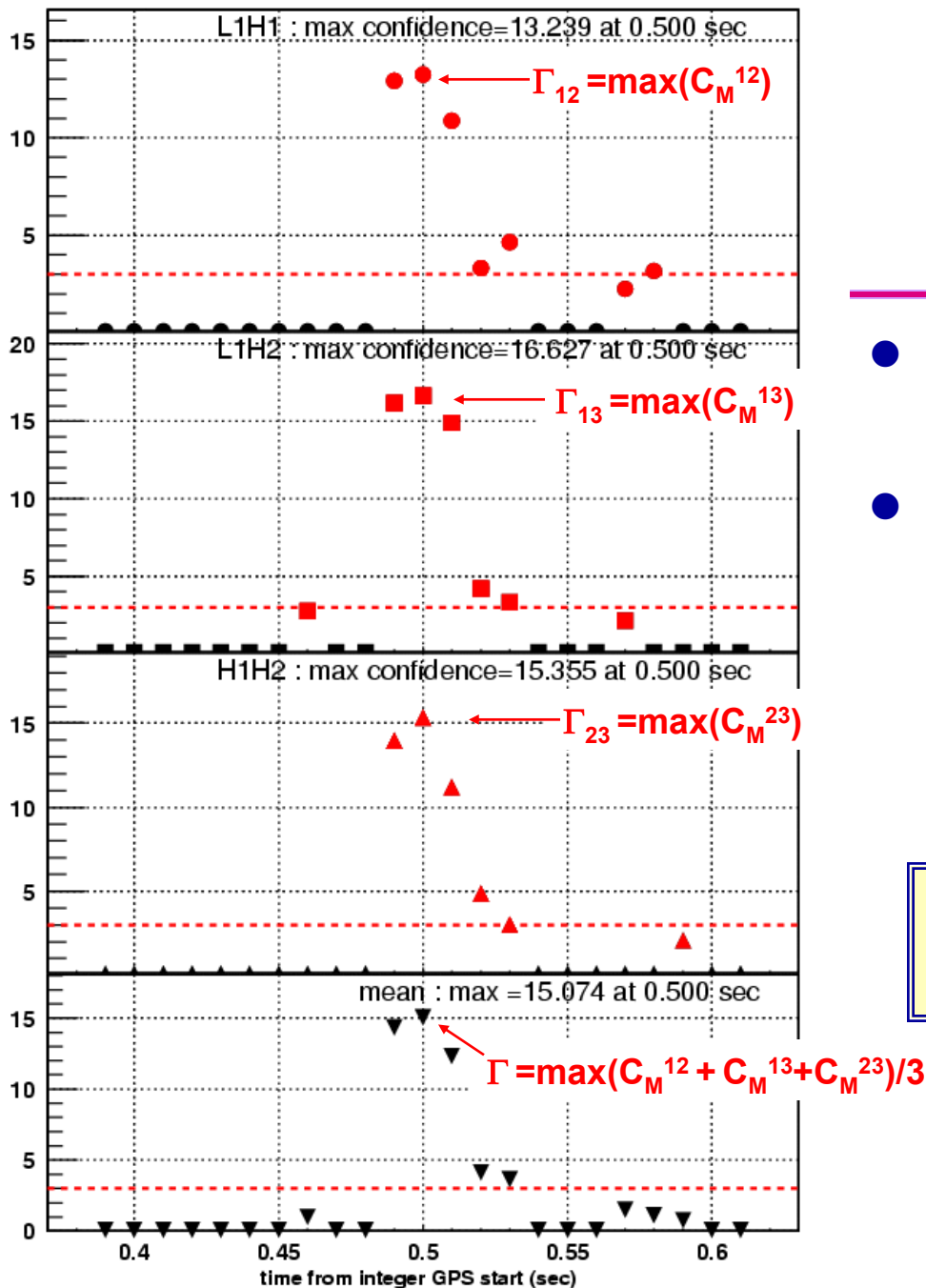
Partition the trigger in sub-intervals (50% overlap) of duration  $\tau$  = integration window (20, 50, 100 ms). For each sub-interval, time shift up to 10 ms and build an r-statistic series distribution.

$$r_k = \frac{\sum_i (x_i - \bar{x})(y_{i+k} - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_{i+k} - \bar{y})^2}}$$

If the distribution of the r-statistic is inconsistent with the no-correlation hypothesis: find the time shift yielding maximum correlation confidence  $C_M(j)$  ( $j$ =index for the sub-interval)



# $C_M(j)$ plots



- Each point: max confidence  $C_M(j)$  for an interval  $\tau$  wide (here:  $\tau = 20\text{ms}$ )

- Threshold on  $\Gamma$ :

2 interferometers:

$$\Gamma = \max_j(C_M(j)) > \beta_2$$

3 interferometers:

$$\Gamma = \max_j(C_M^{12} + C_M^{13} + C_M^{23}) / 3 > \beta_3$$

In general, we can have  $\beta_2 \neq \beta_3$

$\beta_3=3$ : 99.9% correlation probability in each sub-interval

Testing 3 integration windows:  
 20ms ( $\Gamma_{20}$ ) 50ms ( $\Gamma_{50}$ ) 100ms ( $\Gamma_{100}$ )  
 in OR:  $\Gamma = \max(\Gamma_{20}, \Gamma_{50}, \Gamma_{100})$

# Triple Coincidence Performance Analysis in S2

Exploring the test performance for triple coincidence detection, independently from trigger generators and from previous portions of the analysis pipeline:

- Add simulated events to real noise at random times in the 3 LIGO interferometers, covering 10% of the S2 dataset (*in LIGO jargon: triple coincidence playground*)
- apply r-statistic test to 200 ms around the simulation peak time

*Definition of quantities used to characterize a burst signal:*

$$h_{\text{rss}} = \sqrt{\int_0^{\infty} |h(t)|^2 dt} = \sqrt{\int_{-\infty}^{\infty} |\tilde{h}(f)|^2 df}$$

*Total energy in the burst (units: strain/rHz)  
[directly comparable to sensitivity curves]*

$$\text{SNR} = \sqrt{2 \int_0^{\infty} \frac{|\tilde{h}(f)|^2}{S_h(f)} df} \approx \frac{h_{\text{rss}}}{\sqrt{S_h(f_c)}}$$

*SNR definition for excess-power techniques in the burst search =  $\text{SNR}_{\text{matched filtering}} / \sqrt{2}$*

For narrow-band bursts with central frequency  $f_c$

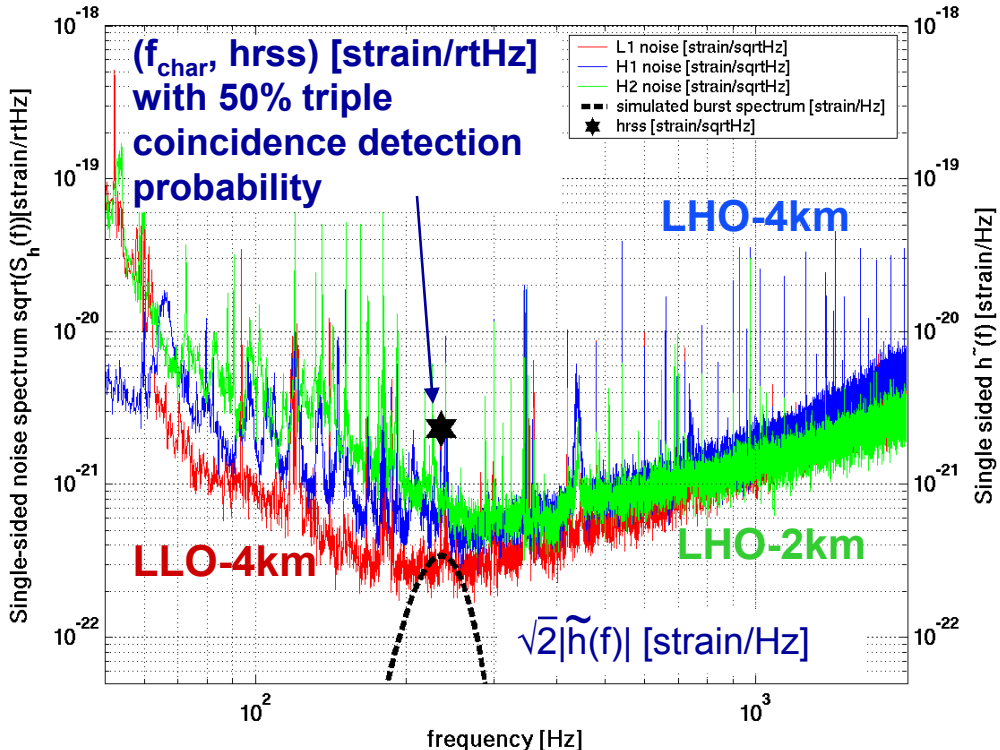
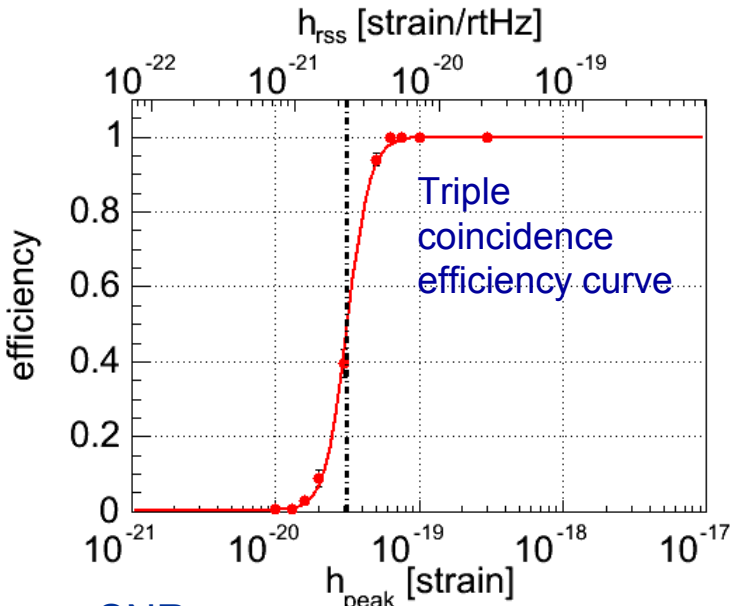
$S_h(f)$ =single-sided reference noise in the S2 Science Run  
 $\Rightarrow$  reference S2 SNR for a given amplitude/waveform

Sine-Gaussian waveform  $f_0=254\text{Hz}$   $Q=9$   
 linear polarization, source at zenith

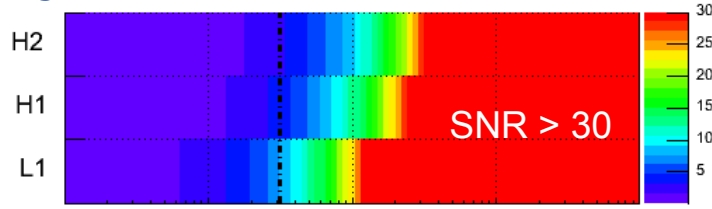
$$h(t) = h_{\text{peak}} \sin(2\pi f_0(t-t_0)) e^{-(t-t_0)^2/\tau^2} \quad Q = \sqrt{2\pi}\tau f_0$$

$$h_{\text{rss}} = \sqrt{\int_0^\infty |h(t)|^2 dt} = \sqrt{\int_{-\infty}^\infty |\tilde{h}(f)|^2 df} = \sqrt{\frac{Q}{4\sqrt{\pi}f_0}} h_{\text{peak}}$$

$$\text{SNR} = \sqrt{2 \int_0^\infty \frac{|\tilde{h}(f)|^2}{S_h(f)} df} \approx \frac{h_{\text{rss}}}{\sqrt{S_h(f_c)}}$$



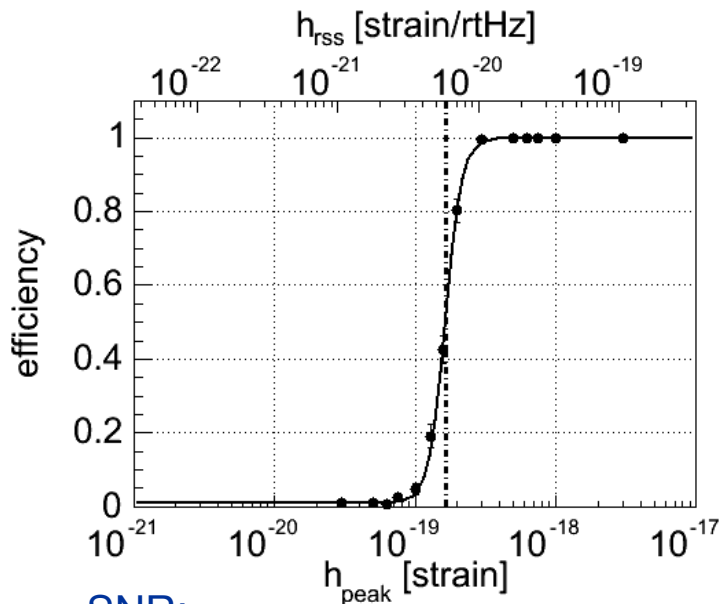
SNR:



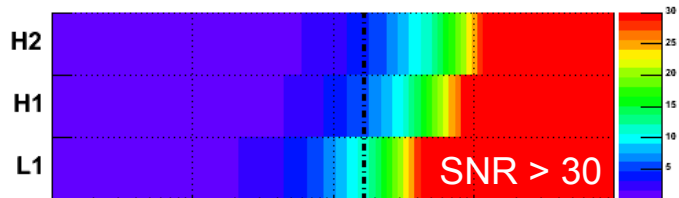
50% triple coincidence detection probability:  
 $h_{\text{peak}} = 3.2 \times 10^{-20}$  [strain]     $h_{\text{rss}} = 2.3 \times 10^{-21}$  [strain/rtHz]  
 SNR: LLO-4km=8    LHO-4km=4    LHO-2km=3

Gaussian waveform  $\tau=1\text{ms}$   
 linear polarization, source at zenith

$$h(t) = h_{\text{peak}} e^{-(t-t_0)^2/\tau^2}$$



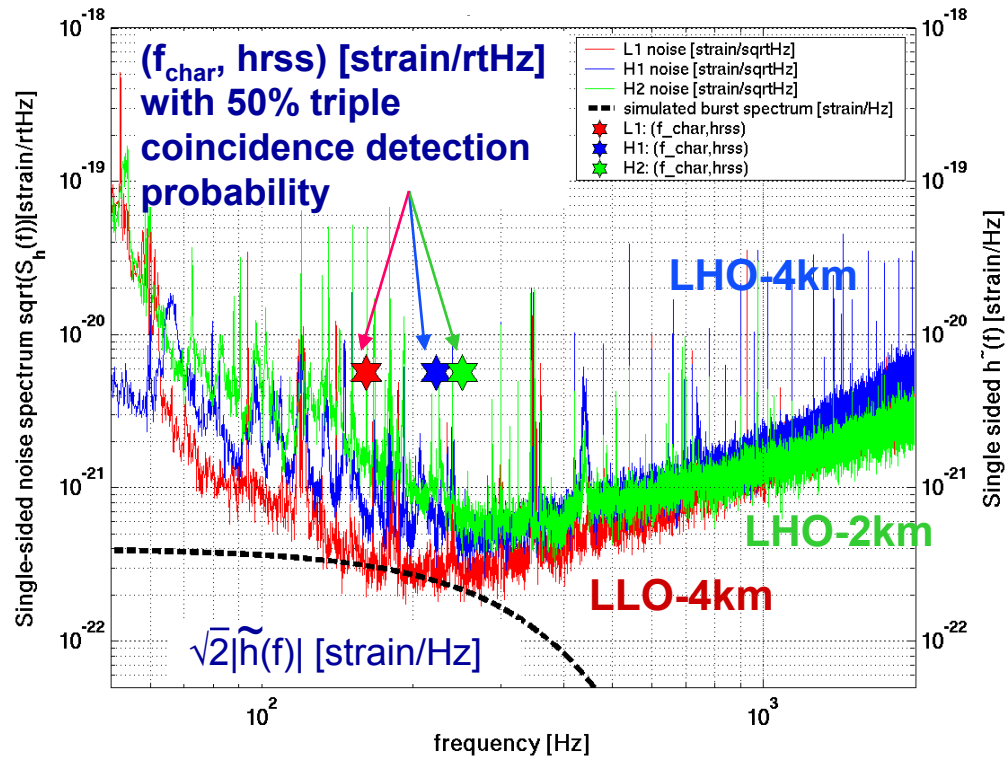
SNR:



50% triple coincidence detection probability:

$h_{\text{peak}} = 1.6\text{e-}19$  [strain]     $h_{\text{rss}} = 5.7\text{e-}21$  [strain/rtHz]

SNR: LLO-4km=11.5    LHO-4km=6    LHO-2km=5



$$\text{SNR} = \sqrt{2 \int_0^{\infty} \frac{|\tilde{h}(f)|^2}{S_h(f)} df}$$

# R.O.C.

Receiver-Operator  
Characteristics

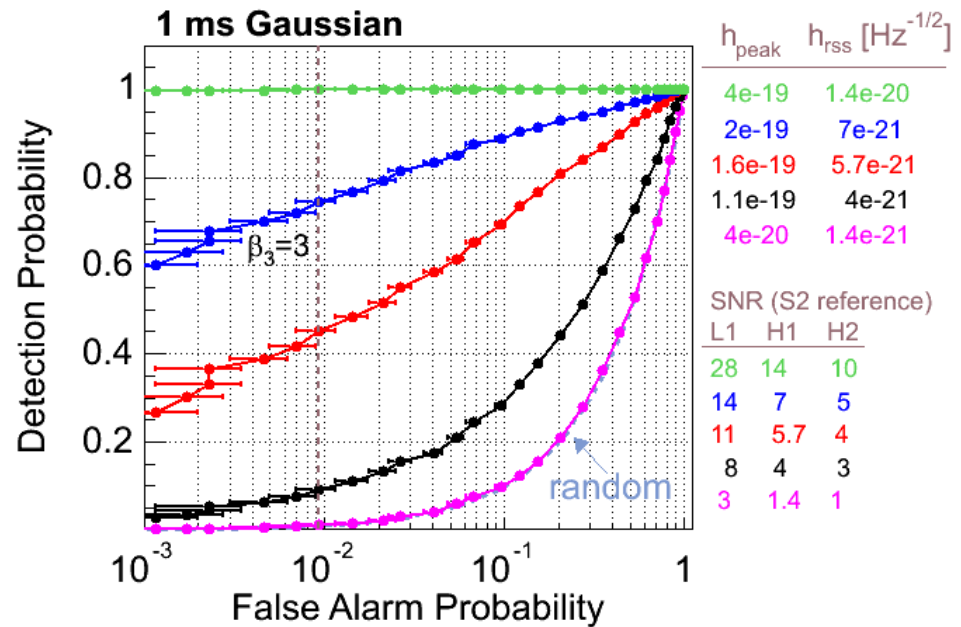
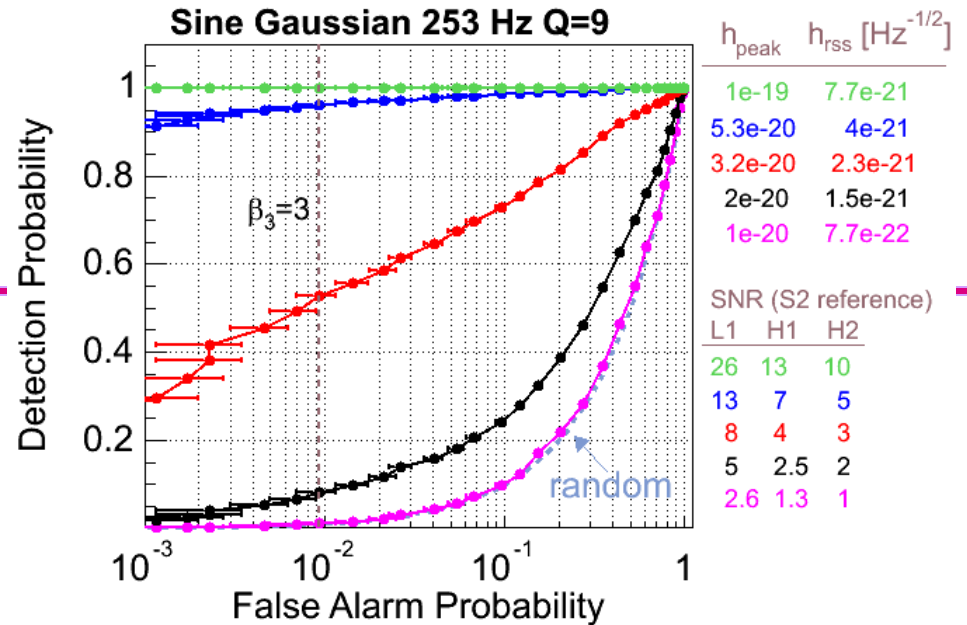
Detection Probability versus  
False Alarm Probability.  
Parameter: triple coincidence  
confidence threshold  $\beta_3$

Simulated 1730 events at fixed  $h_{\text{peak}}$ ,  $h_{\text{rss}}$   
(10 events uniformly distributed in each S2 "playground" segment)

Tested cross correlation over 200 ms  
around the peak time

Operating condition:  $\beta_3=3$

chosen from first principles (99.9% correlation probability  
in each event sub-interval for a pair of interferometers),  
corresponds to a ~1% false alarm probability for triple  
coincidence events with duration 200 ms.





# Suppression of Accidental Coincidences from the Pipeline

*In general: depends on the Event Trigger Generator and the nature of its triggers.*

*In particular: typical distribution of event duration (larger events have more integration windows).*

*Shown here: TFCLUSTERS 130 - 400 Hz (presented in Sylvestre's talk)*

<i>Triple Coincidence Playground. T=88800 s (24.7 hours)</i>	<i>Singles</i>
<b>LLO-4km (L1)</b>	2.5 Hz
<b>LHO-4km (H1)</b>	2 Hz
<b>LHO-2km (H2)</b>	2 Hz

Coincident numbers reported here are averages of 6 background measurements:

LLO-LHO =  $\pm 8, \pm 6, \pm 4$  sec (H1-H2 together)

**PRELIMINARY!!**

“Loose” coincidence cuts

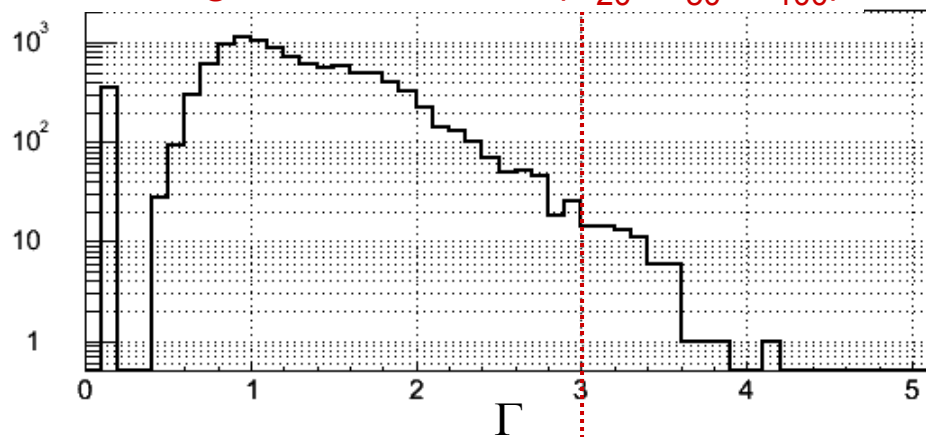
<i>coincidence</i>	<i>triple coincident clusters (<math>\Delta t = 30</math> ms)</i>	<i>after frequency cut (200Hz tolerance)</i>	<i>after r-statistic test (<math>\beta_3 = 3</math>)</i>	<i>Rejection efficiency</i>
<b>L1-H1-H2</b>	20 mHz	15 mHz	0.1 mHz	(99.35 $\pm$ 0.08)%

“Tight” coincidence cuts

<i>coincidence</i>	<i>triple coincident clusters (<math>\Delta t = 15</math> ms)</i>	<i>after frequency cut (75Hz tolerance)</i>	<i>after r-statistic test (<math>\beta_3 = 3</math>)</i>	<i>Rejection efficiency</i>
<b>L1-H1-H2</b>	6 mHz	1 mHz	0.01 mHz (1/day)	(98.8 $\pm$ 0.4)%

# False Probability versus Threshold

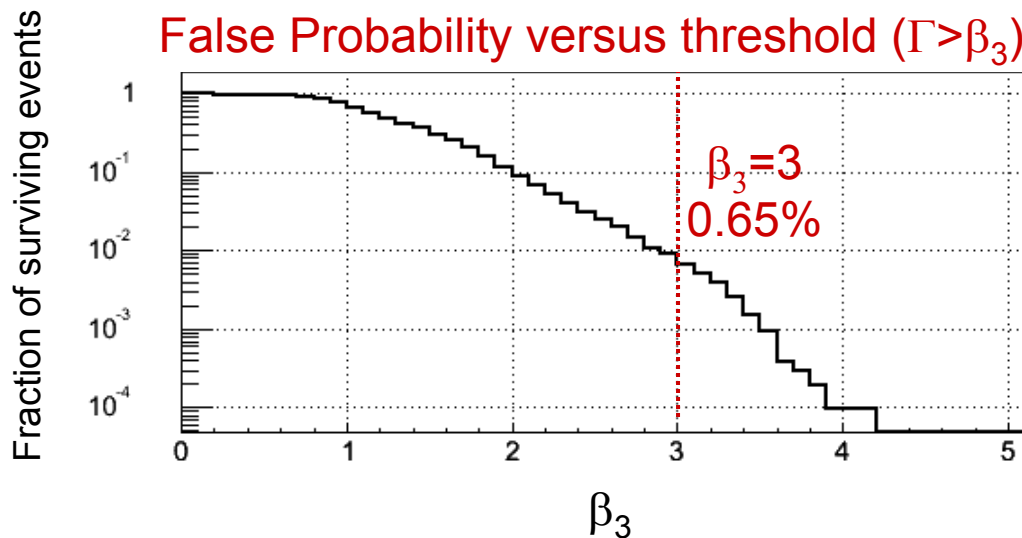
Histogram of  $\Gamma = \max(\Gamma_{20}, \Gamma_{50}, \Gamma_{100})$



Entries	10485
Mean	1.28
RMS	0.5277
Underflow	0
Overflow	0

*In general: depends on the trigger generators and the previous portion of the analysis pipeline (typical event duration, how stringent are the selection and coincidence cuts)*

False Probability versus threshold ( $\Gamma > \beta_3$ )



*Shown here:  
TFCLUSTERS 130-400 Hz with  
"loose" coincidence cuts*

# Conclusions

- The LIGO burst S1 analysis exclusively relied on event trigger generators and time/frequency coincidences.
- The search in the second science run (S2) includes a new module of coherent analysis, added at the end of the burst pipeline:
  - r-statistic test for cross correlation in time domain
    - » Assigns a confidence to coincidence events at the end of the burst pipeline
    - » Verifies the waveforms are consistent
    - » Suppresses false rate in the burst analysis, allowing lower thresholds
- Tests of the method, using simulated signals on top of real noise, yield 50% triple coincidence detection efficiency for narrow-band and broad-band bursts at SNR=3-5 in the least sensitive detector (LHO-2km) with a false probability ~1%.
- Currently measuring global efficiency and false rate for the S2 pipeline (event analysis + coherent analysis).