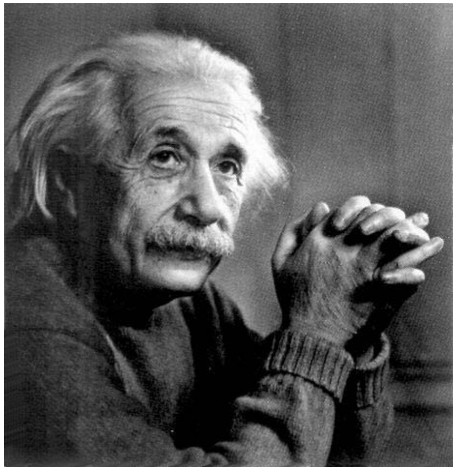
Excess power method in wavelet domain for burst searches (WaveBurst)

S.Klimenko

University of Florida

- Introduction
- Wavelets
- Time-Frequency analysis
- Coincidence
- Statistical approach
- Results for S2 playground data
- Simulation
- Summary

Newton-Einstein Theory of Gravitation



Newton's Theory
1666
"instantaneous action at a distance"
Newton's laws

Einstein's Theory
1915
"gravitational field action propagates at the speed of light"

$$\mathbf{G} + \Lambda \mathbf{g} = 8\pi (G_N/c^4)\mathbf{T}$$

G is the Einstein tensor

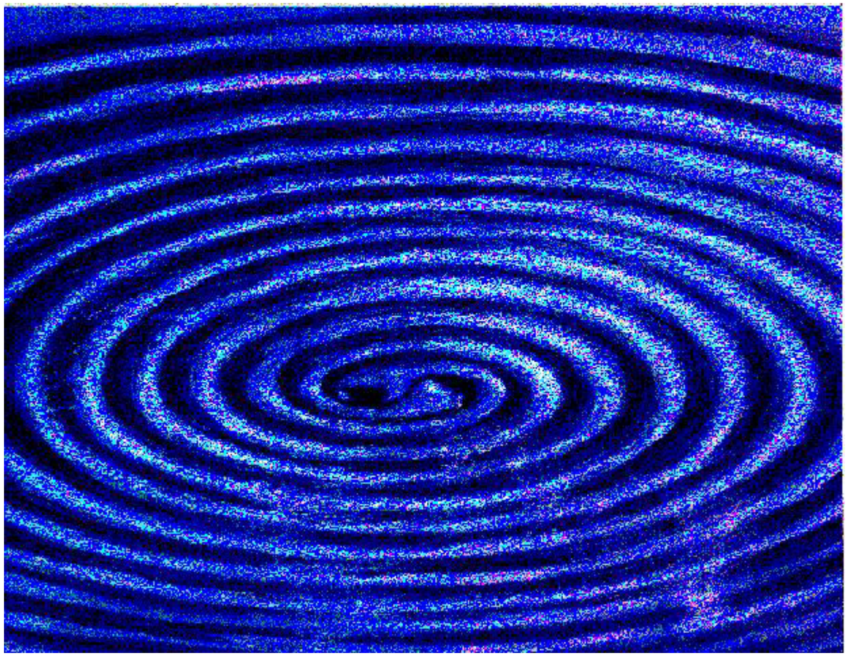
T is the stress-energy tensor

gravitational waves

- time dependent gravitational fields come from the acceleration of masses and propagate away from their sources as a space-time warpage at the speed of light

- In the weak-field limit, linearize the equation in "transverse-traceless gauge"

$$\nabla^2 h - \frac{\partial^2 h}{c^2 \partial t^2} = 16\pi \frac{G_N}{c^4} T$$



gravitational radiation
binary inspiral of compact objects

where $h_{\mu\nu}$ is a small perturbation of the space-time metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

GW strength

- Quadrupole radiation
  - monopole forbidden by conservation of E
  - dipole forbidden by mom. conservation

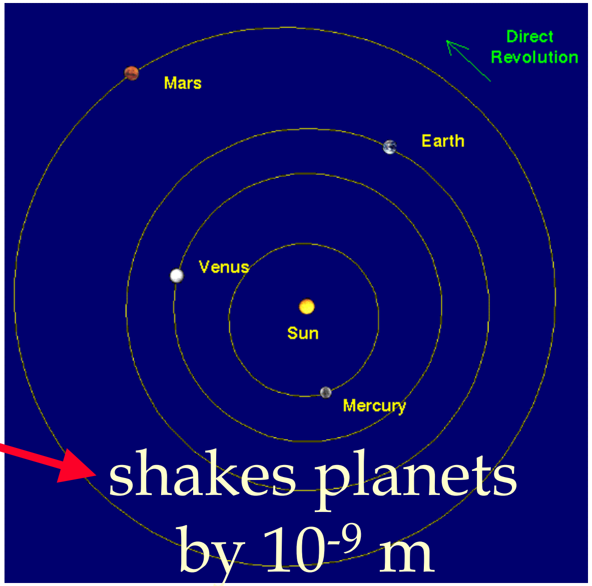
$$h \approx \frac{G_N}{c^4}\frac{\ddot{Q}}{r}$$

- For highly non-spherical source, like binary system with mass M and separation L

$$Q \approx ML^2$$

1 pc = 3 x 10^{16} m

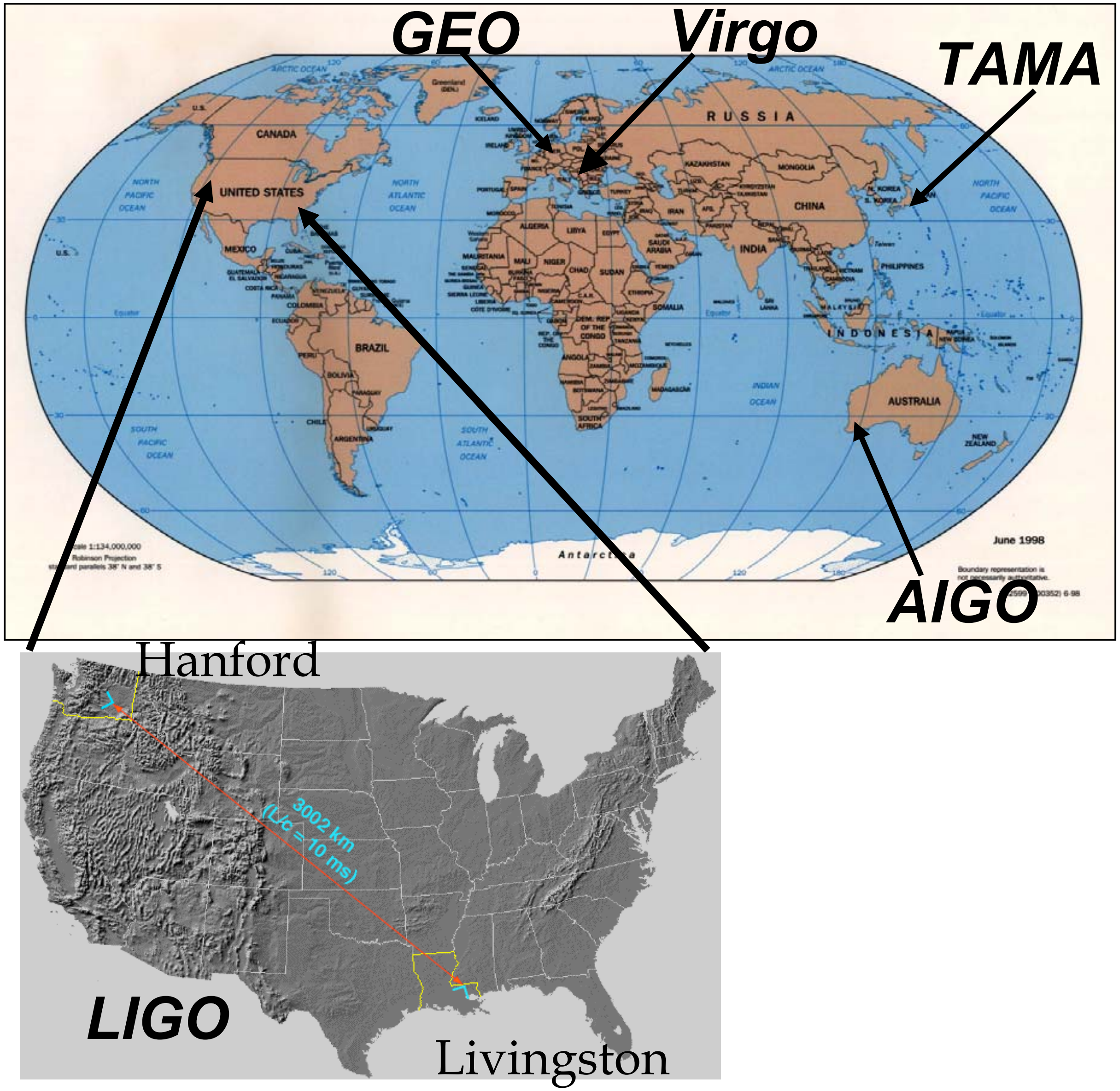
- solar mass neutron stars
  - "Solar system" (1au) $h \sim 10^{-8}$
  - Milky Way (20kpc) $h \sim 10^{-17}$
  - Virgo cluster (15Mpc) $h \sim 10^{-20}$
  - "Deep space" (200Mpc) $h \sim 10^{-21}$
  - Habble distance (3000Mpc) $h \sim 10^{-22}$



Astrophysical Sources

- Compact binary inspiral: "chirps"
  - waveforms are quite well described. Search with match filters.
- Pulsars: "periodic"
  - GW from observed neutron stars (doppler shift)
  - all sky search
- Cosmological Signals "stochastic"
  - x-correlation between several GW detectors
- Supernovae / GRBs/ BH mergers/…: "bursts"
  - triggered search – coincidence with GRB/neutrino detectors
  - un-triggered search – coincidence of GW detectors

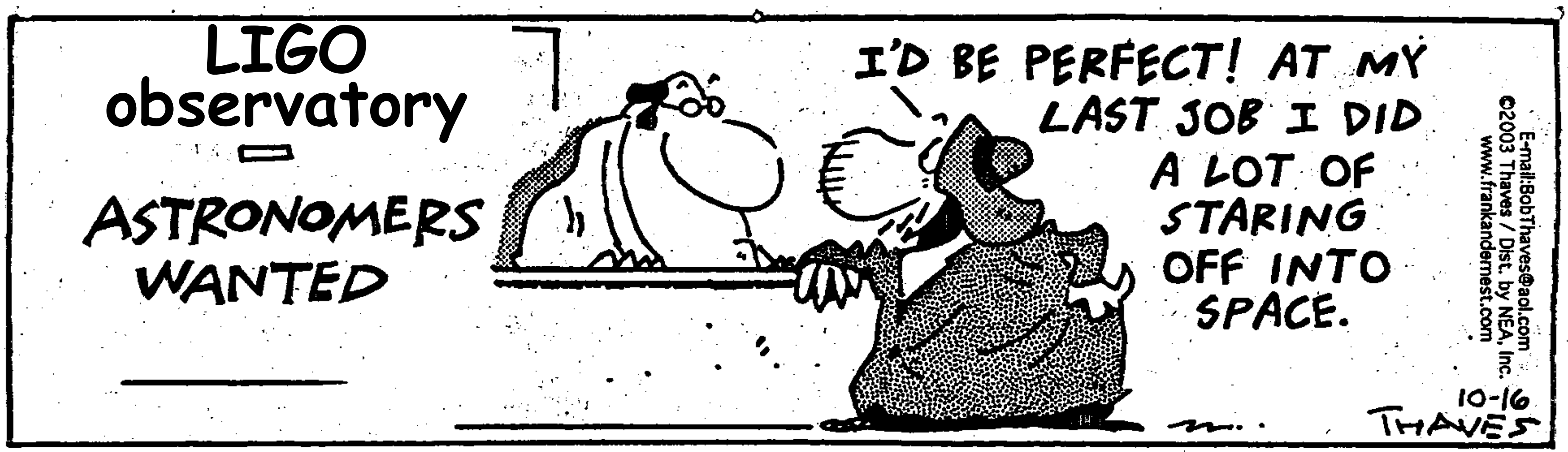
GW interferometers



Detection confidence

Direction to sources

LIGO observatory



Bursts

- **Sources:**
  - **Any short transient of gravitational radiation.**
  - **Astrophysically motivated**
    - **Unmodeled signals -- Gamma Ray Bursts, …**
    - **“Poorly modeled” -- supernova, inspiral mergers**

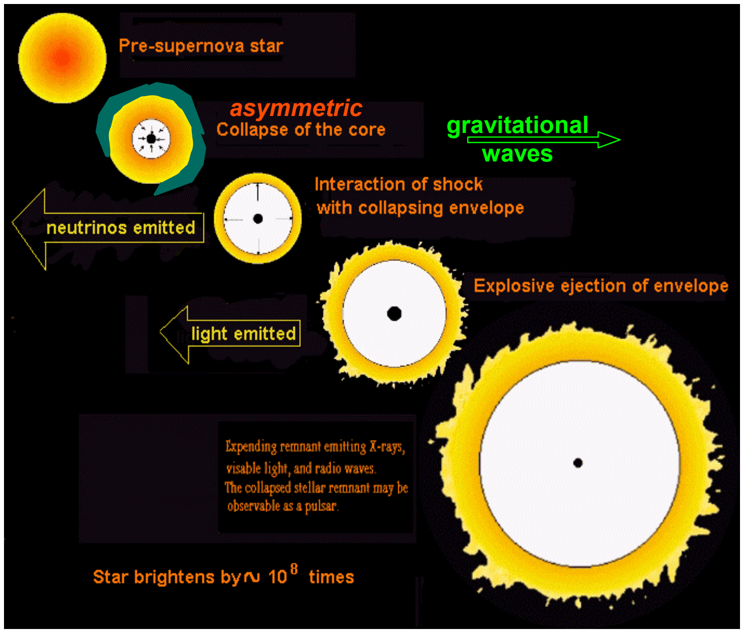
$$UL \propto \frac{N}{\varepsilon(h)T}$$

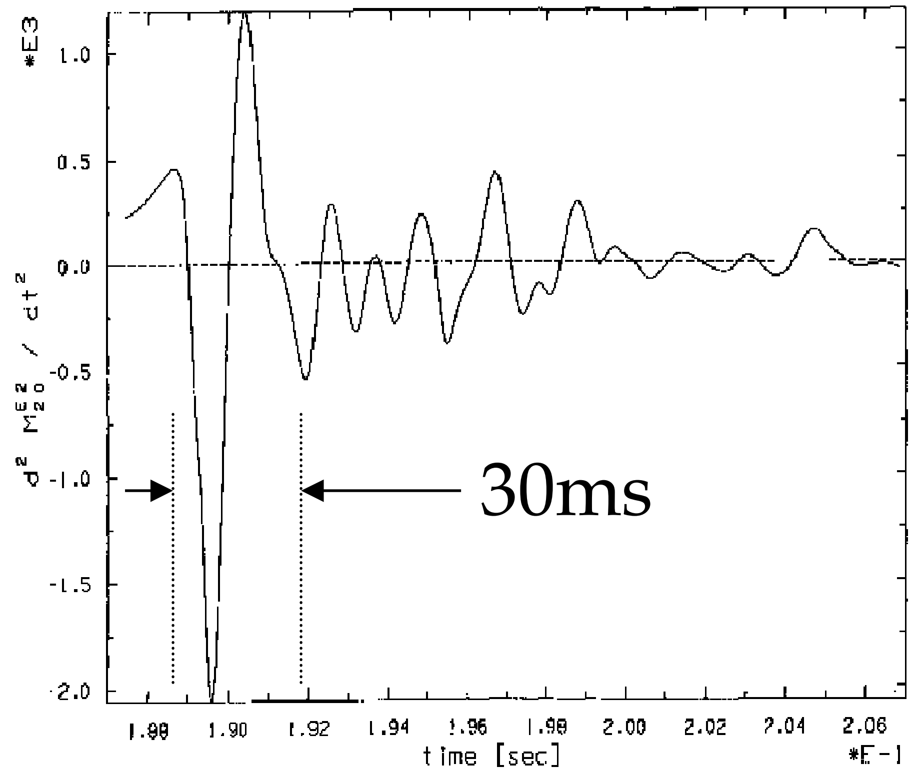
N: number observed events
ε(h): detection efficiency
T: observation time

- **Analysis goals:**
  - **Establish a bound on rates**
  - **GW burst detection**
- **Search methods**
  - **Excess power in time-frequency domain**
  - **Sudden change of the noise parameters, rise-time in time domain**
- **In all cases: coincident observations among multiple GW detectors or with external triggers (GRBs, neutrinos).**

Supernova

- **Asymmetric core collapse**

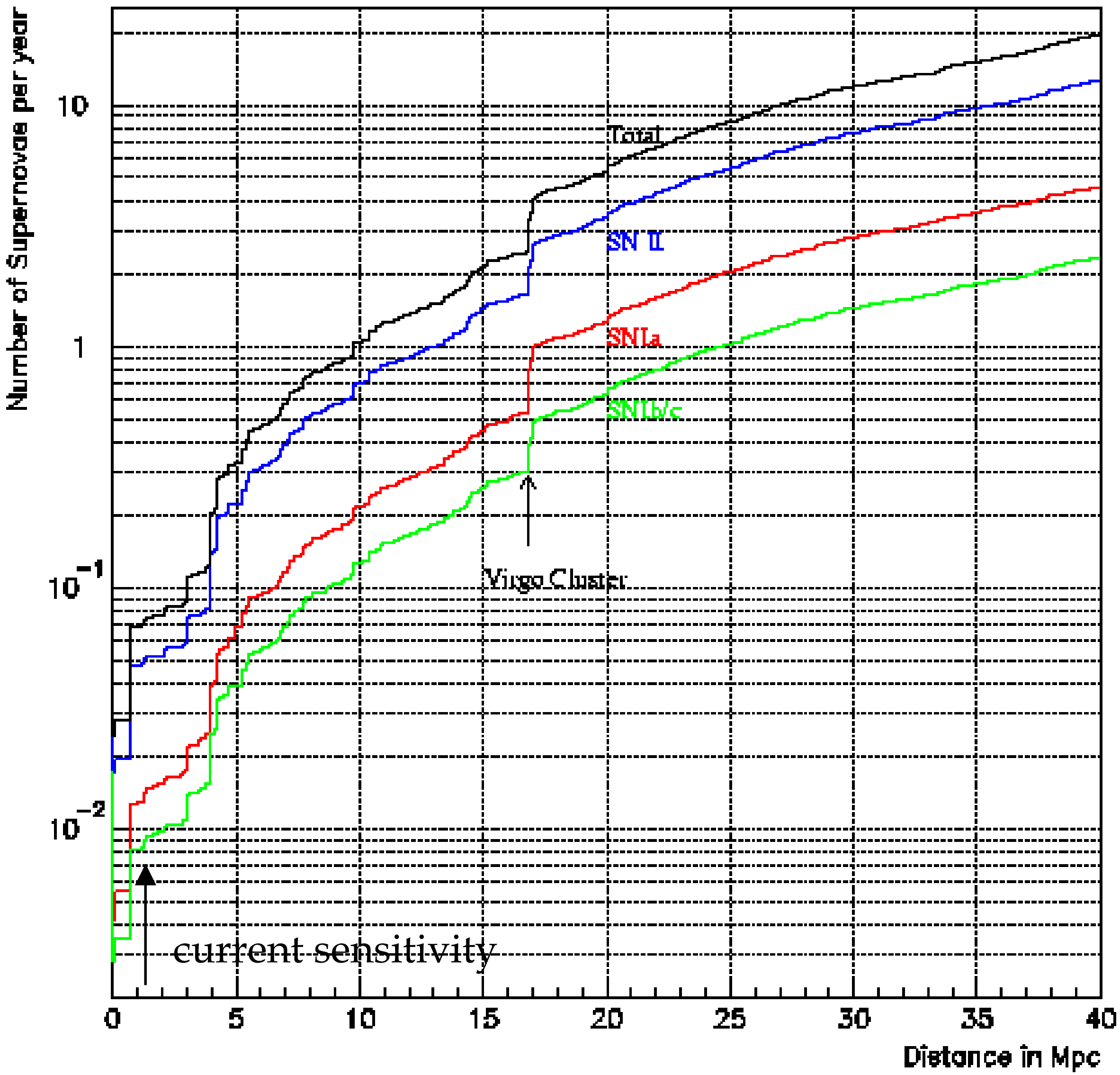




Zwerger,Muller

- Exact waveforms are not known, but any information (like signal duration) could be valuable for the analysis (classification of the waveforms)

Supernova rate



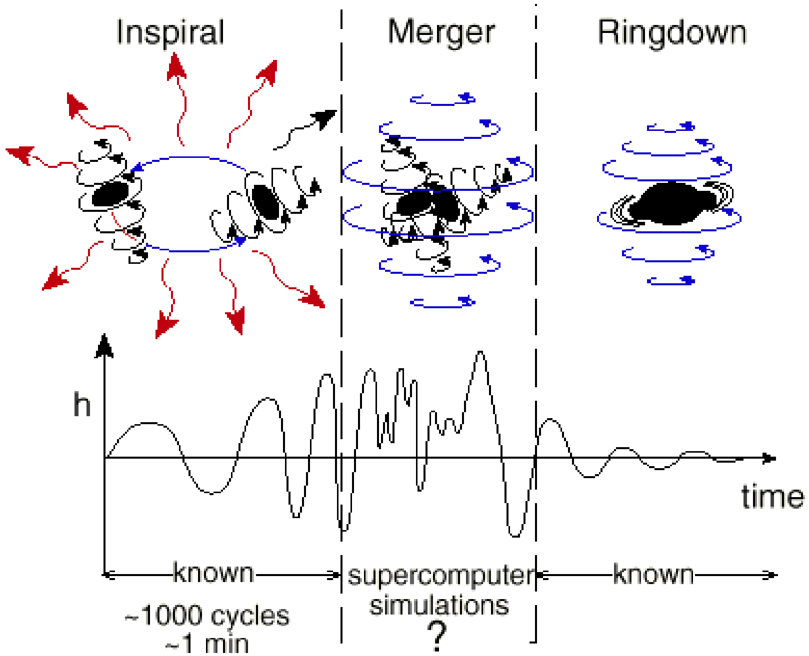
SN Rate

1/50 yr - Milky Way

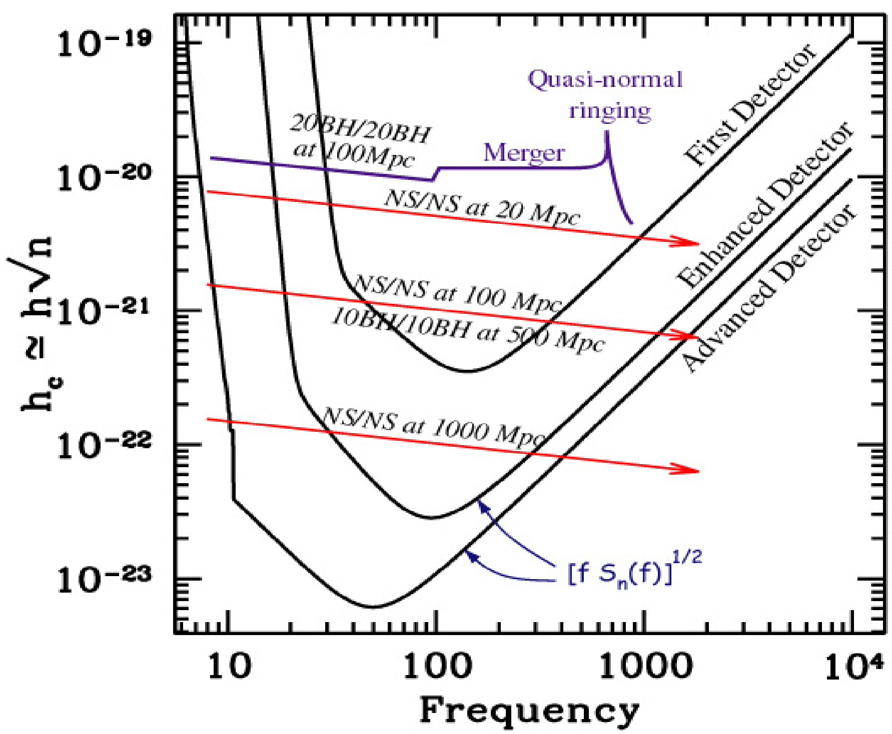
3/yr - out to Virgo cluster

Inspiral Mergers

Compact binary mergers



Sensitivity of LIGO to coalescing binaries



- **Expected merger detection rate ~40 higher then inspiral rate**

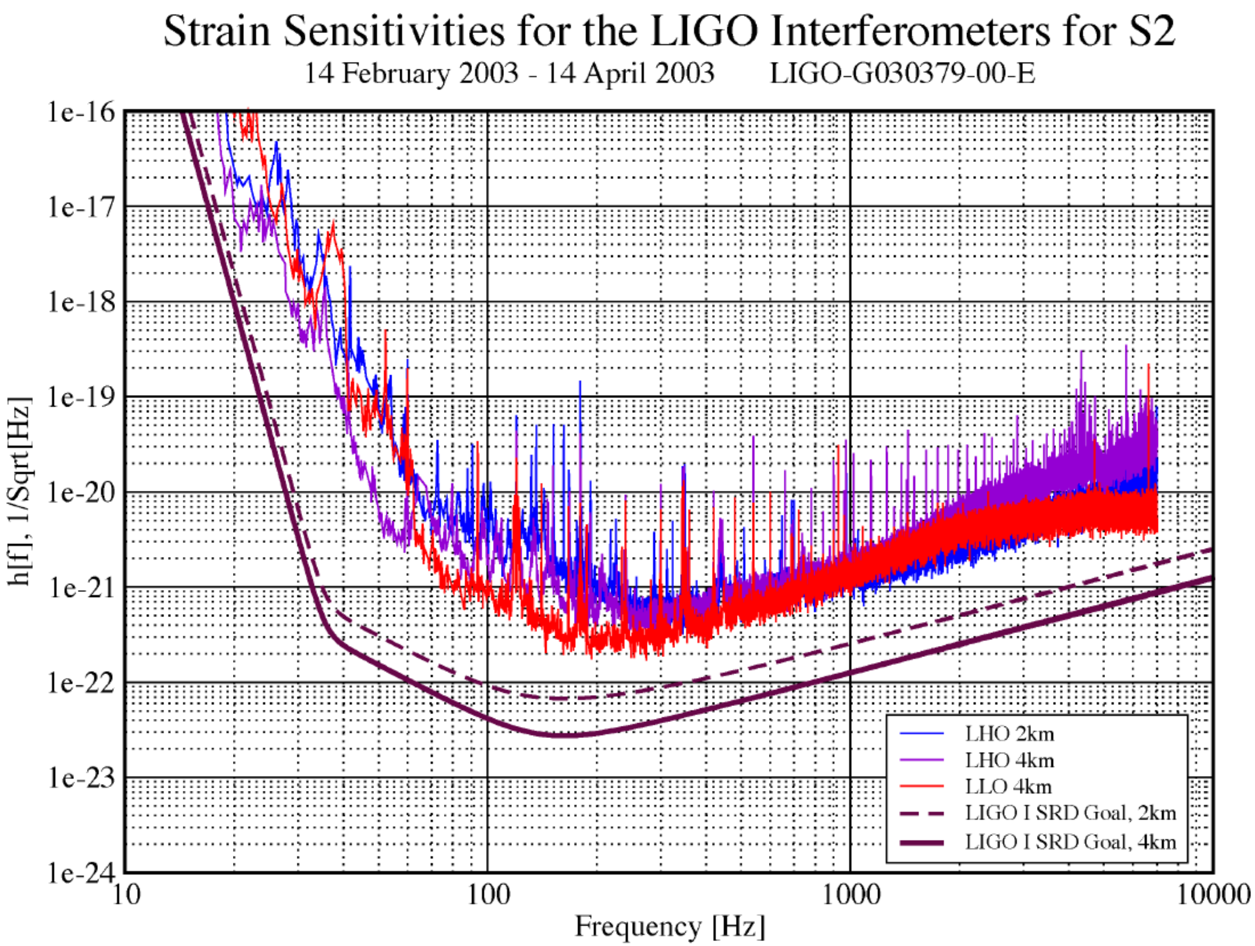
Flanagan, Hughes: gr-qc/9701039v2 1997

  - **10Mo<M<200Mo (LIGO-I) 100Mo<M<400Mo (LIGO-II)**

0.1-10 events/year → very promising analysis

S2 LIGO Sensitivity

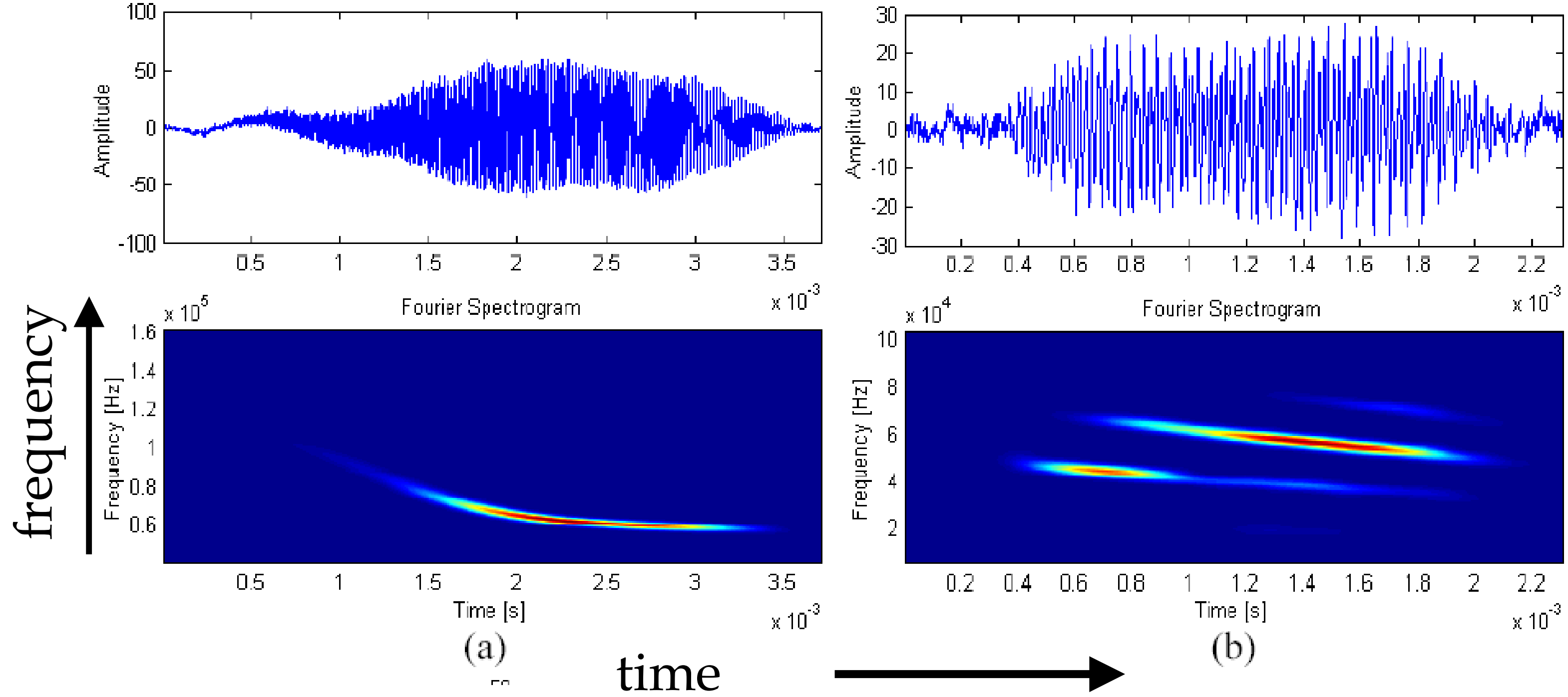
- Sensitive to bursts in
  - Milky Way
  - Magellanic Clouds
  - Andromeda
  - ……



time-frequency analysis

Ecological calls of

the Miniopterus australis **the Macroderma gigas**



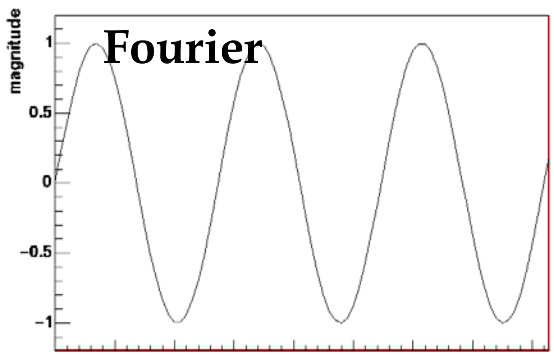
- **Classify the GW “ecological calls”**
- **Detect bursts with generic T-F properties in each class.**
- **Characterize by “strength”, duration, frequency band,...**

Wavelet basis

- basis $\{\Psi(t)\}$:
  - bank of template waveforms
  - Ψ_0 -mother wavelet
  - a=2 – stationary wavelet

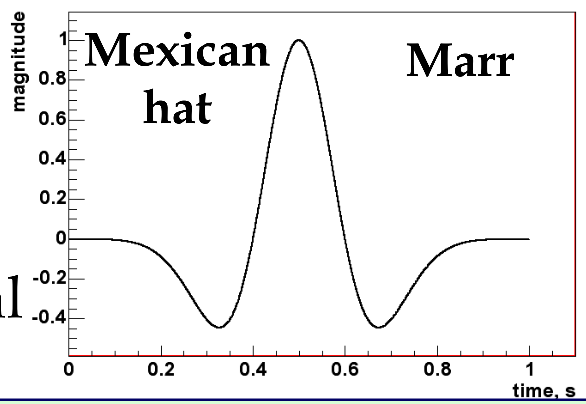
$$\Psi_{jk} = a^{j/2}\Psi_0\left(a^j t - k\right)$$

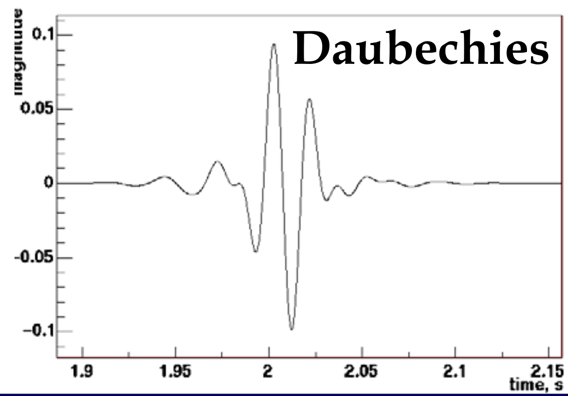
not local



local orthogonal not smooth

local, smooth, not orthogonal





local orthogonal smooth

wavelet - natural basis for bursts
fewer functions are used for signal approximation – closer to match filter

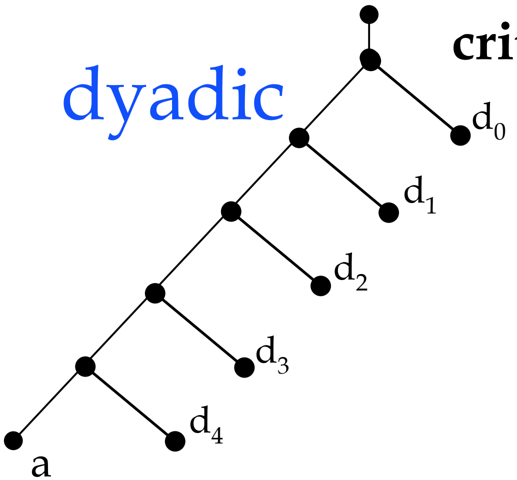
Wavelet Transform

decomposition in basis $\{\Psi(t)\}$

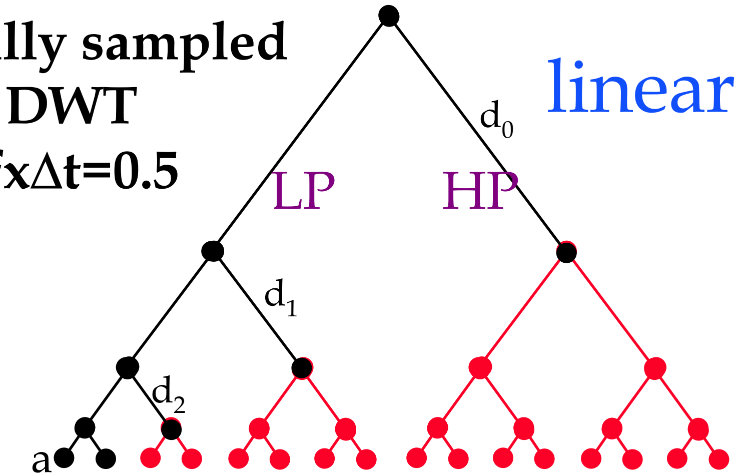
dyadic

critically sampled
DWT
$\Delta f \times \Delta t = 0.5$

linear

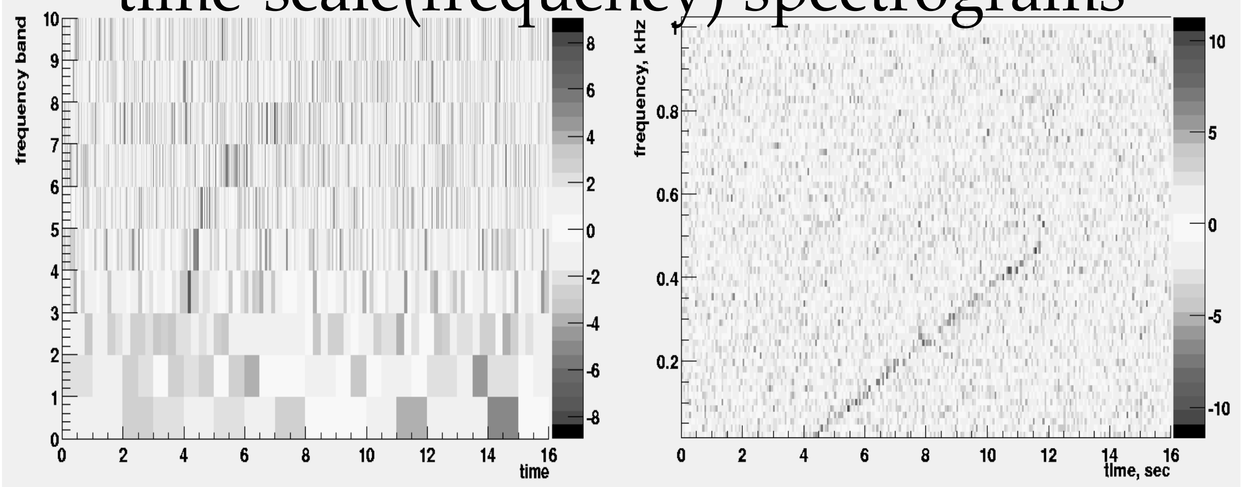


a. wavelet transform tree



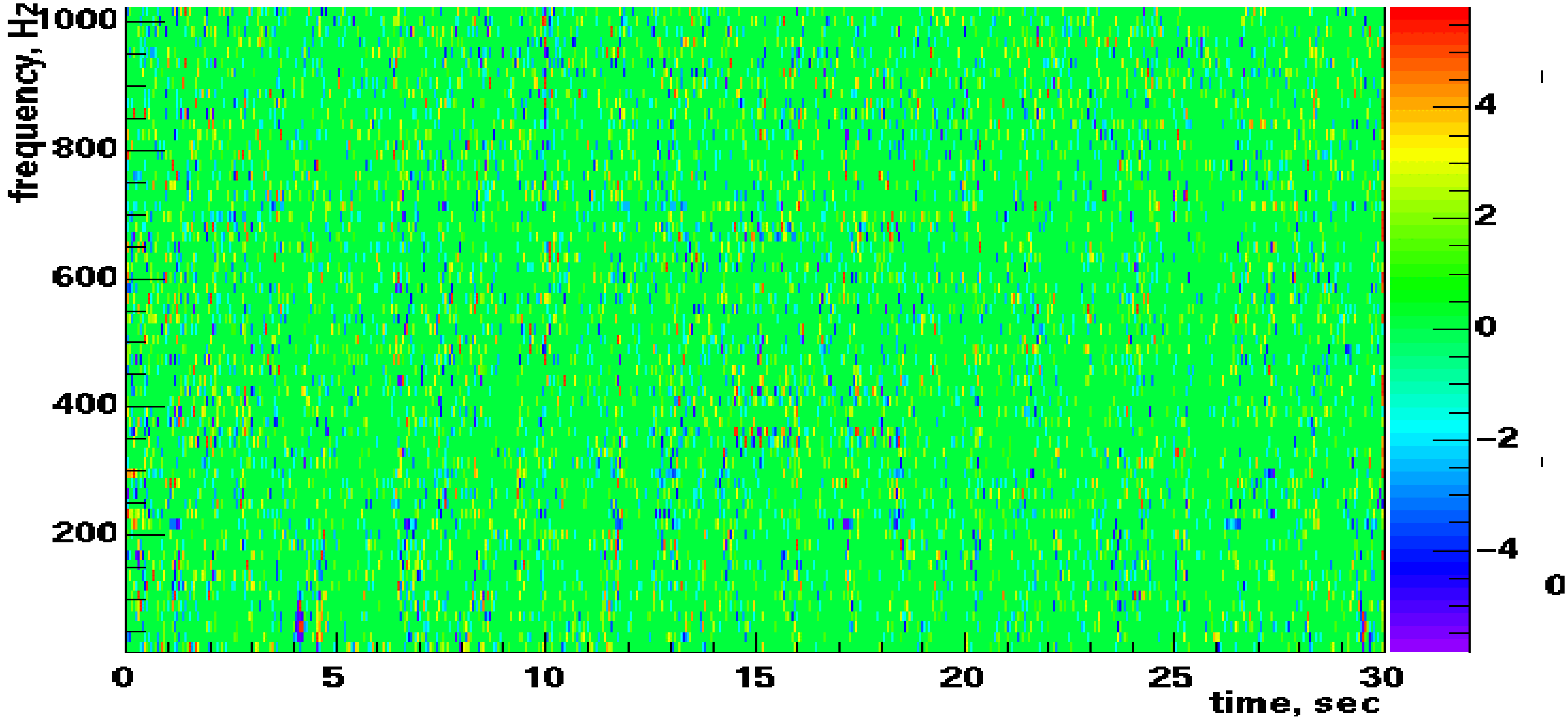
b. wavelet transform binary tree

time-scale(frequency) spectrograms



Wavelet time-scale(frequency) spectrogram

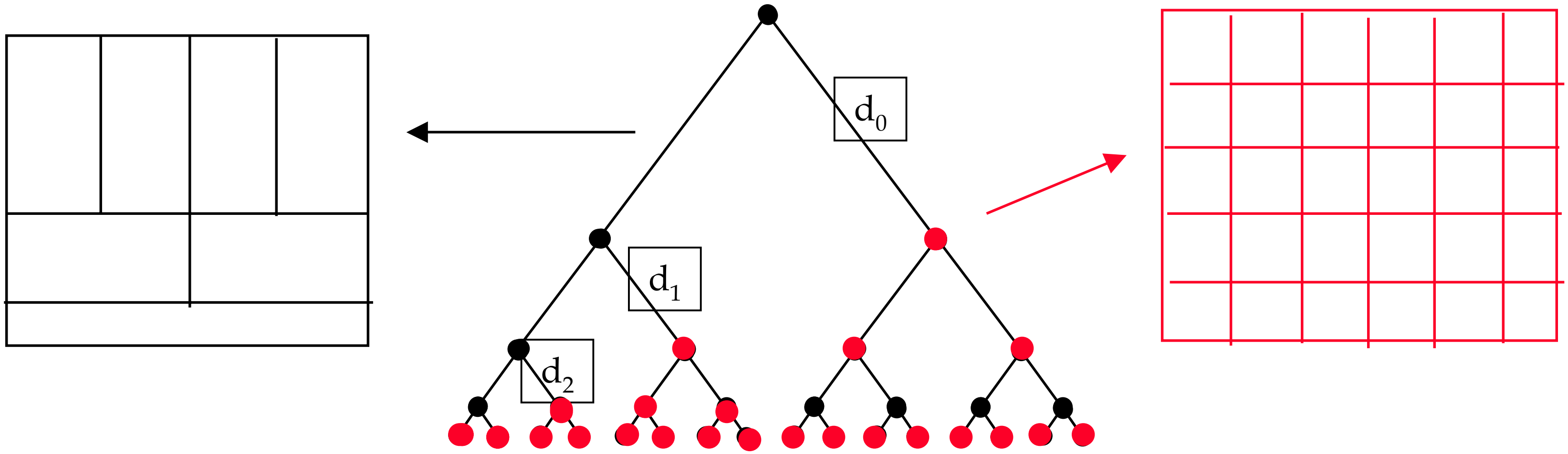
LIGO data H2:LSC-AS_Q



WaveBurst allows different tiling schemes including linear and dyadic wavelet scale resolution.

for this plot linear scale resolution is used (Δf=const)

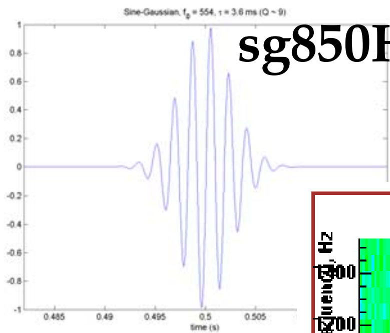
TF resolution



- depend on what nodes are selected for analysis
  - **dyadic – wavelet functions**
  - **constant**
  - **variable**

  (constant, variable:) wavelet packet – linear combination of wavelet functions
  - **multi-resolution → select significant pixels searching over all nodes and "combine" them into clusters.**

Response to sine-gaussian signals



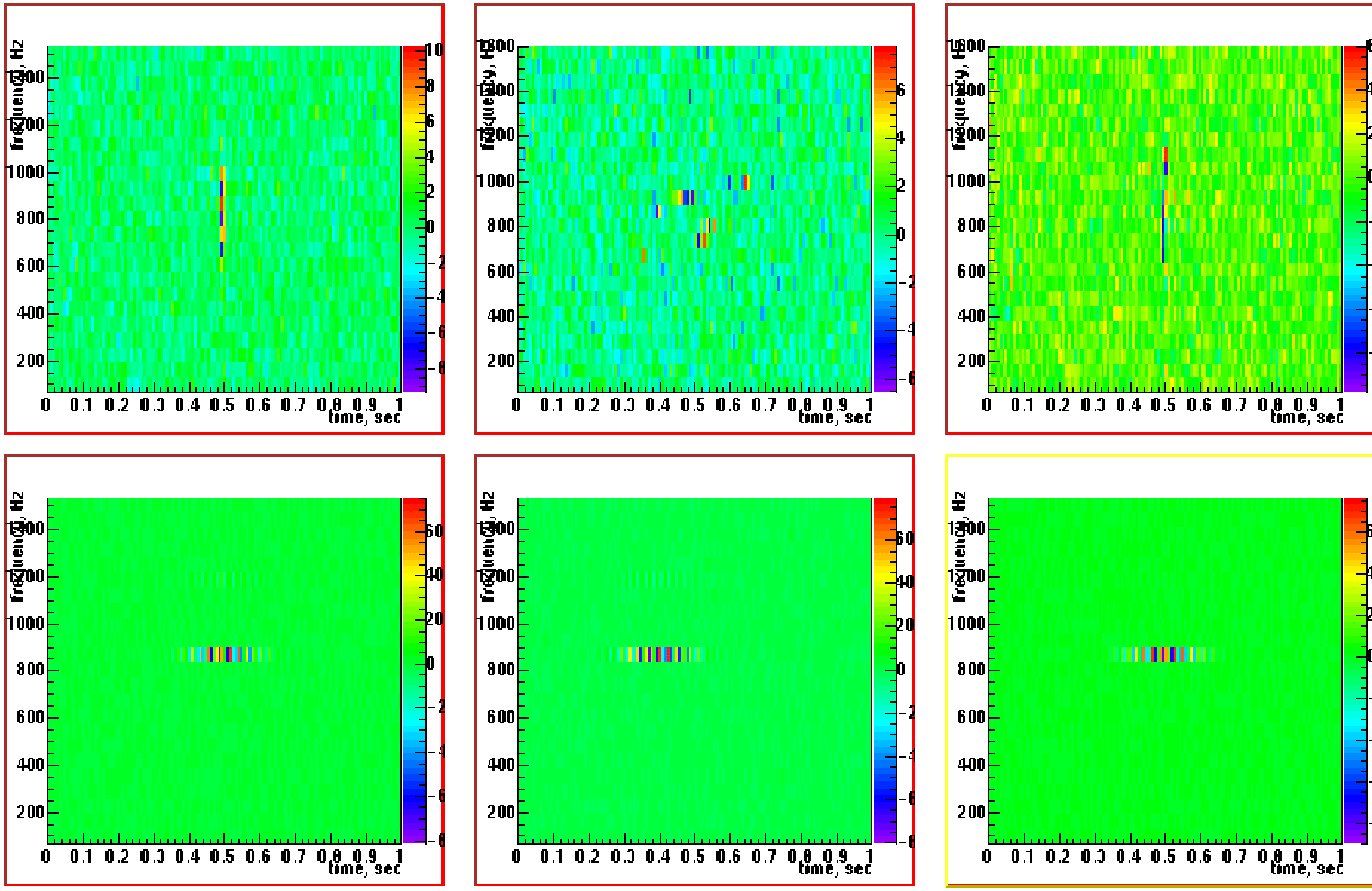
sg850Hz

wavelet resolution: 64 Hz X 1/128 sec

Symlet | Daubechies | Biorthogonal

τ=1 ms

τ=100 ms

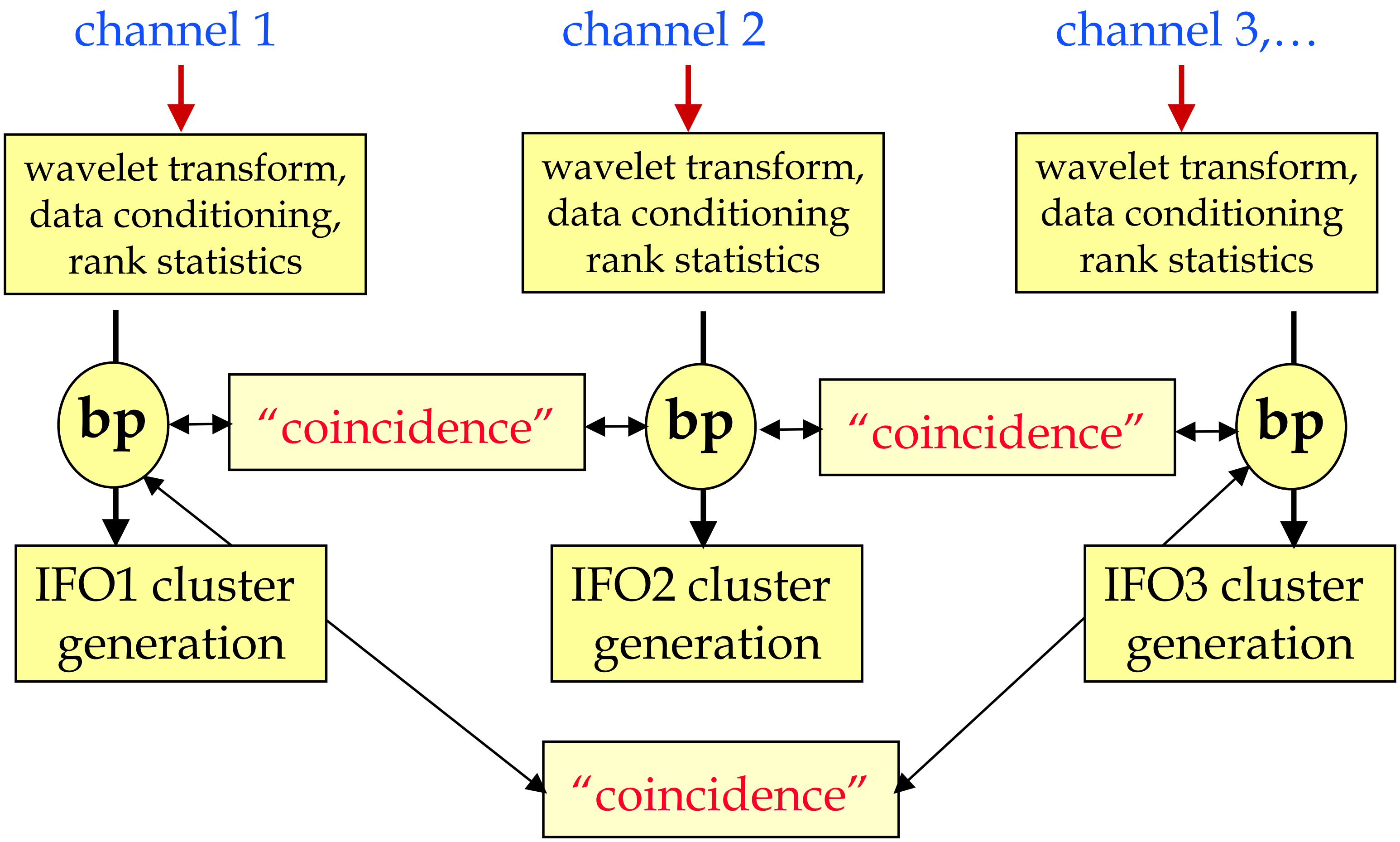


WaveBurst analysis method

detection of excess power in wavelet domain

- use wavelets
  - flexible tiling of the TF-plane by using wavelet packets
  - variety of basis waveforms for bursts approximation
  - low spectral leakage
  - wavelets in DMT, LAL, LDAS: Haar, Daubechies, Symlet, Biorthogonal, Meyers.
- use rank statistics
  - calculated for each wavelet scale
  - robust
- use local T-F coincidence rules
  - coincidence at pixel level applied before triggers are produced
  - works for 2 and more interferometers

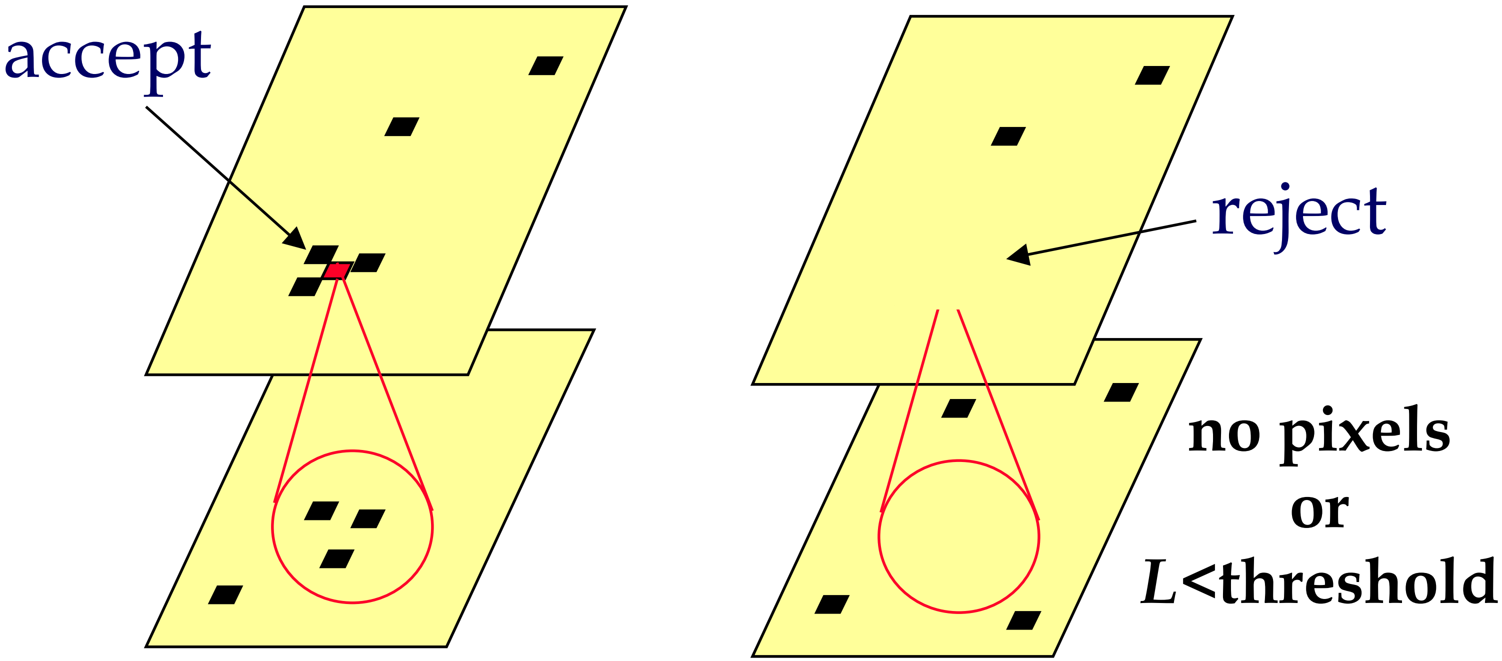
Analysis pipeline



bp →selection of loudest (black) pixels
(black pixel probability *P*~10% - 1.64 GN rms)

LIGO-G040038-00-D

Coincidence

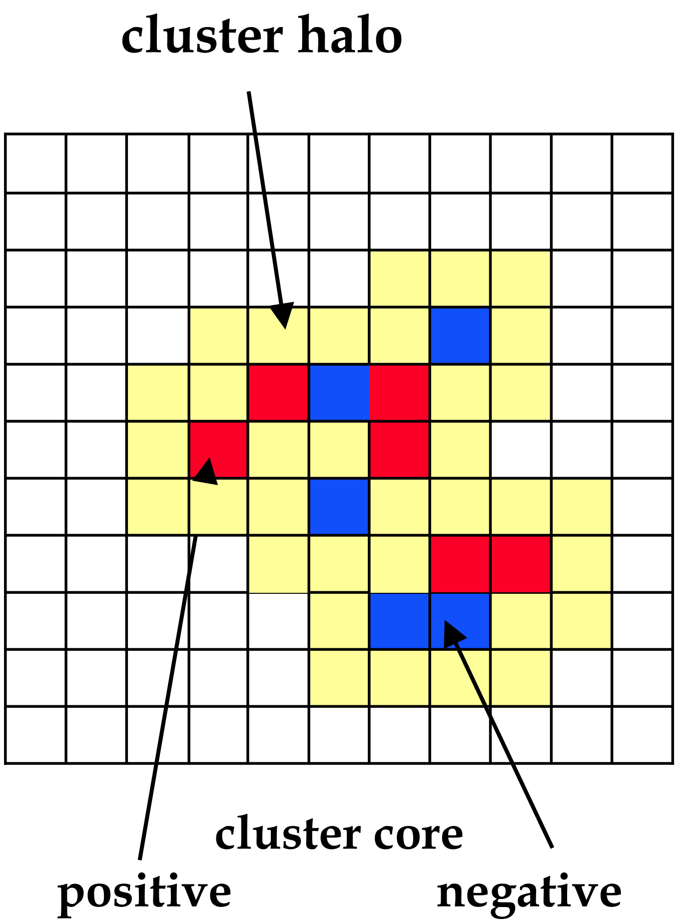


- **Given local occupancy P(t,f) in each channel, after coincidence the black pixel occupancy is** $$P_C(t,f) \propto P^2(t,f)$$

  for example if P=10%, average occupancy after coincidence is 1%
- **can use various coincidence policies → allows customization of the pipeline for specific burst searches.**

Cluster Analysis (independent for each IFO)

cluster → T-F plot area with high occupancy



Cluster Parameters

size – number of pixels in the core
volume – total number of pixels
density – size/volume
amplitude – maximum amplitude
power - wavelet amplitude/noise rms
energy - power x size
asymmetry – (#positive - #negative)/size
confidence – cluster confidence
neighbors – total number of neighbors
frequency - core minimal frequency [Hz]
band - frequency band of the core [Hz]
time - GPS time of the core beginning
duration - core duration in time [sec]

Statistical Approach

- statistics of pixels & clusters (triggers)
- parametric
  - Gaussian noise
  - pixels are statistically independent
- non-parametric
  - pixels are statistically independent
  - based on rank statistics:

data: $\{x_i\}$: $|x_{k1}| < |x_{k2}| < \ldots < |x_{kn}|$

rank: $\{R_i\}$: n n-1 1

$$y_i = \eta(R_i) \cdot u(x_i)$$

η – *some* function

u – sign function

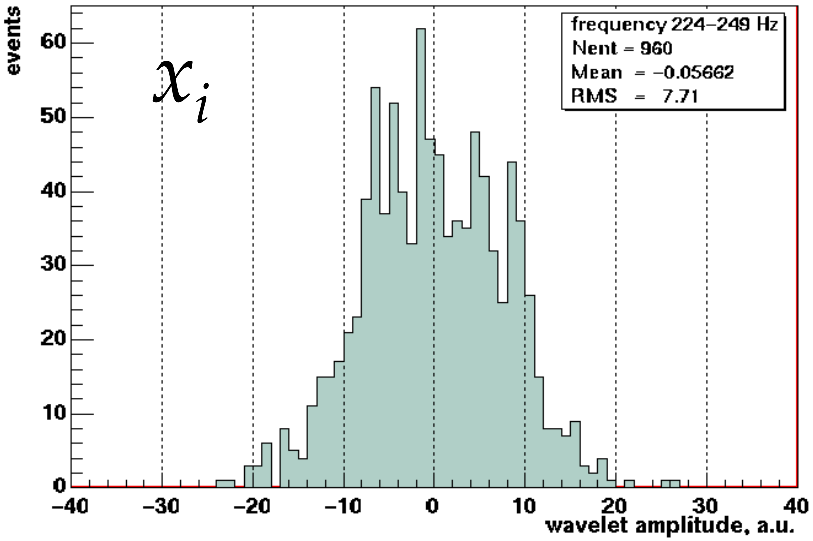
example: Van der Waerden transform, R→G(0,1)

non-parametric pixel statistics

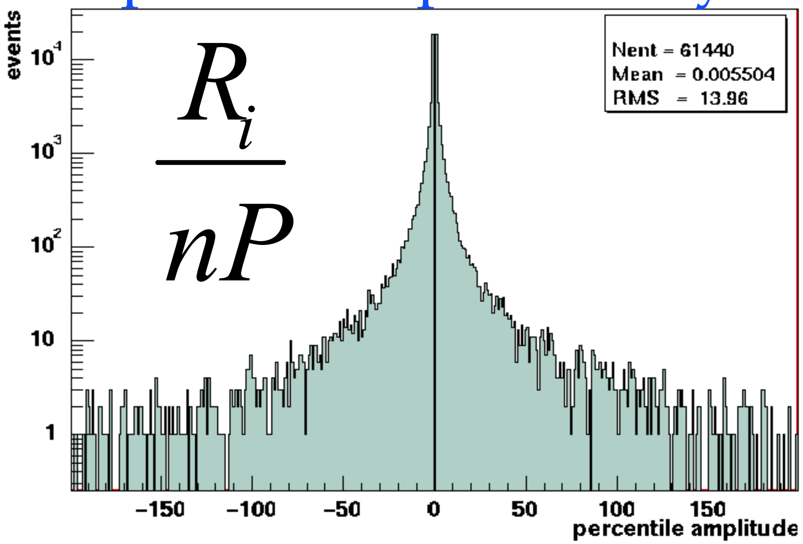
- calculate pixel likelihood from its rank:

$$y_i = -\ln\left(\frac{R_i}{nP}\right)\cdot \mathrm{u}(x_i)$$

- Derived from rank statistics → *non-parametric*
- likelihood *pdf* - exponential



percentile probability

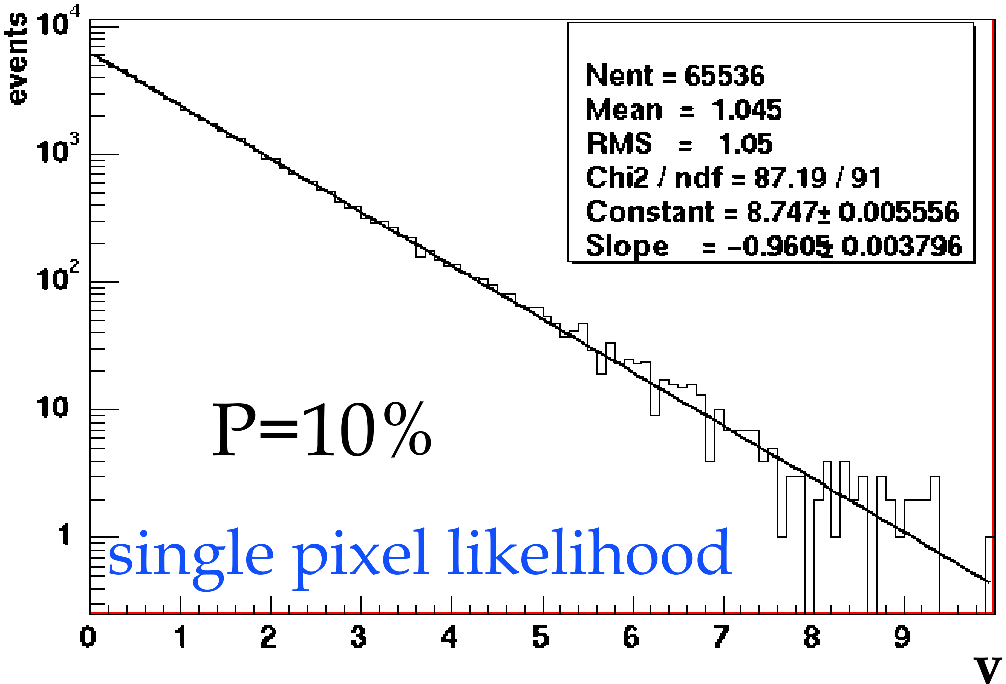


statistics of filter noise (non-parametric)

- **non-parametric cluster likelihood**

$$Y_k = -\sum_{i=0}^{k} \ln\left(\frac{R_i}{nP}\right)$$

- **sum of k (statistically independent) pixels has *gamma* distribution**



$$pdf(Y_k) = \frac{Y_k^{\,k-1} e^{-Y_k}}{\Gamma(k)}$$

statistics of filter noise (parametric)

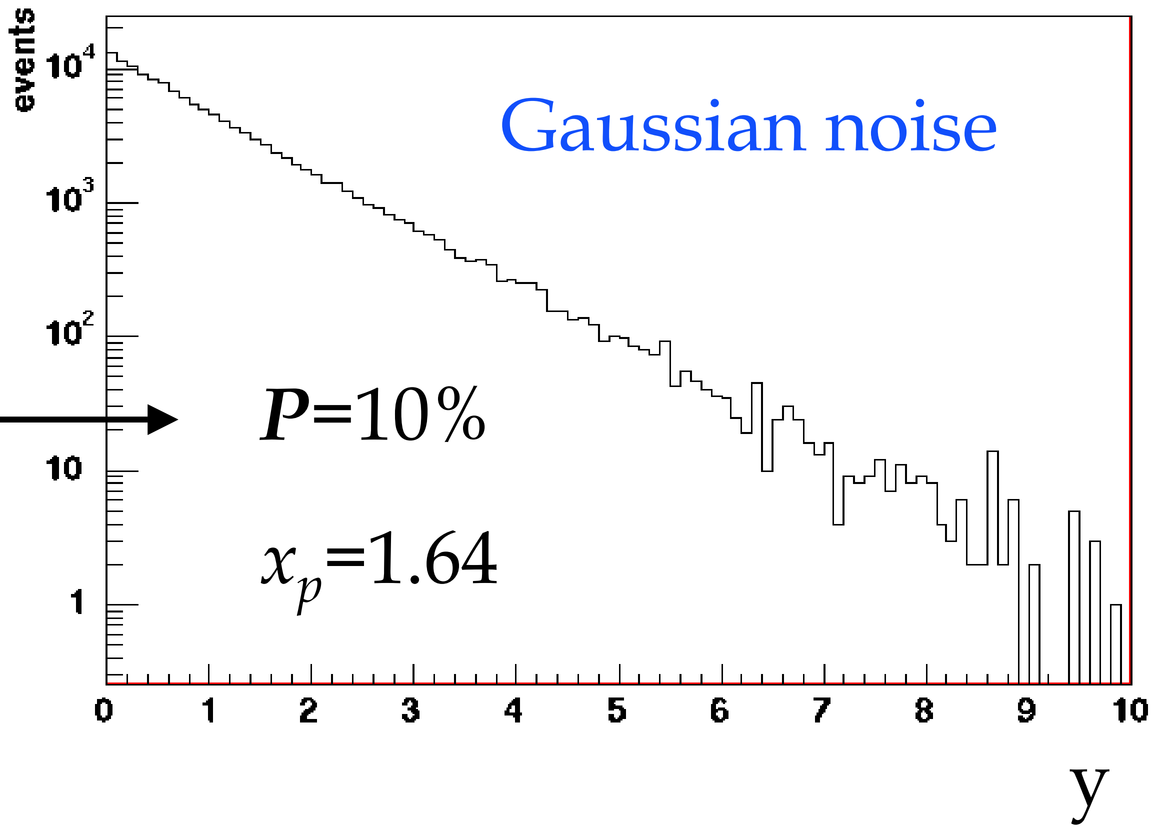
- x: assume that detector noise is gaussian
- y: after black pixel selection ($|x|>x_p$)→ gaussian tails
- Y_k: sum of k independent pixels distributed as Γ_k

$$y = \frac{x^2 - x_p^2}{2\alpha},$$

$$\alpha = \left(1 + x_p^{-2}\right)^{-1}$$

$$pdf(y) \approx e^{-y},$$

$$Y_k = \sum_0^k y_i$$

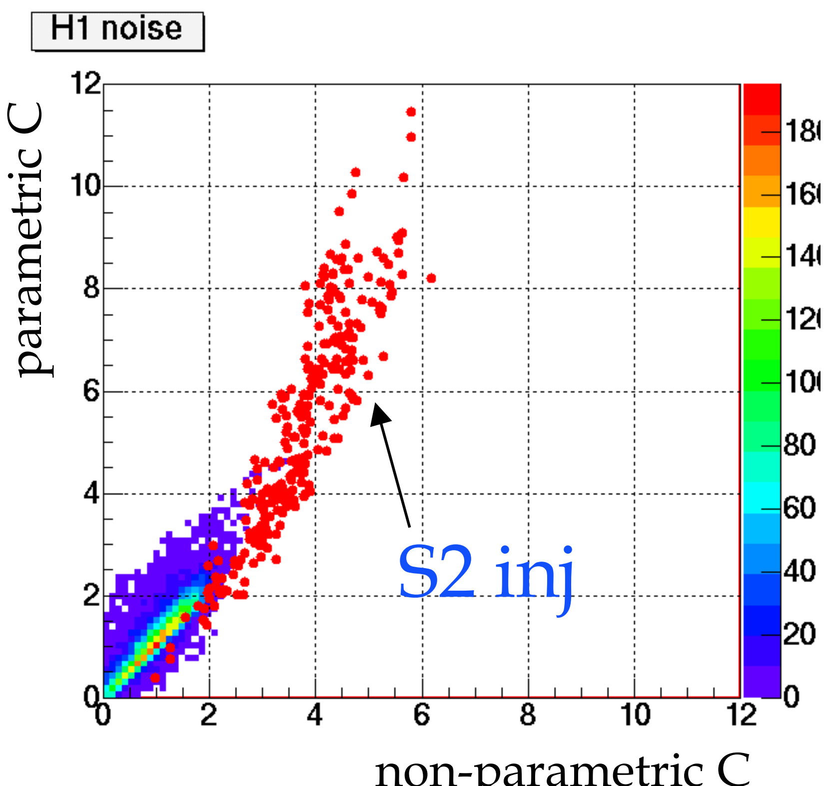


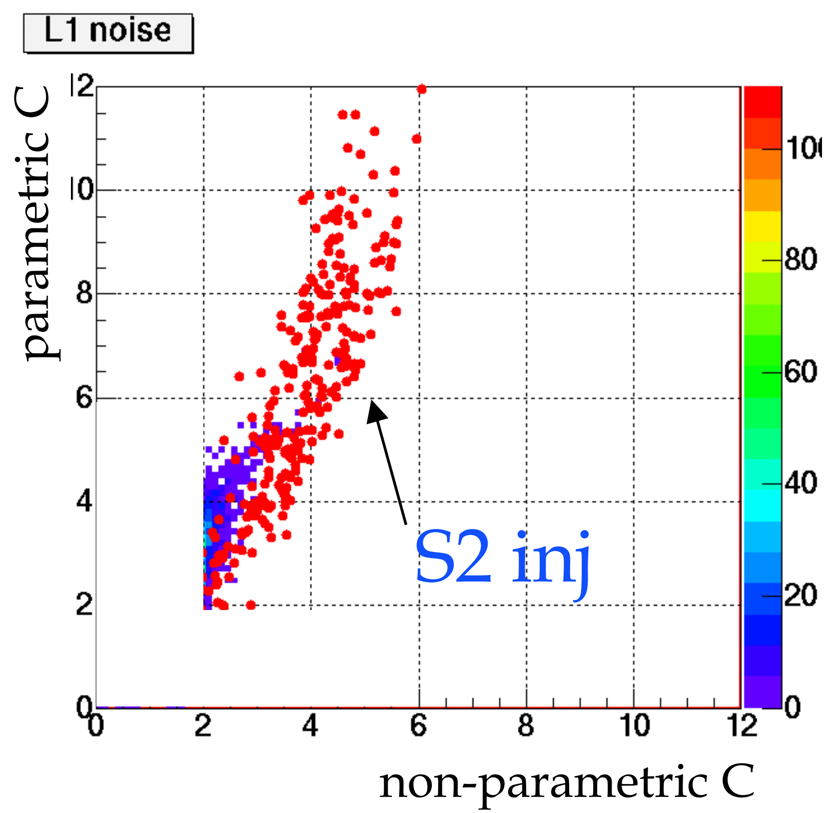
cluster confidence

- ***cluster confidence:*** ***C*** **= -*ln*(survival probability)**

$$C(Y_k) = -\ln\left(\frac{1}{\Gamma(k)} \int_{Y_k}^{\infty} x^{k-1} e^{-x} dx \right)$$

- ***pdf*(*C*) is exponential regardless of *k*.**





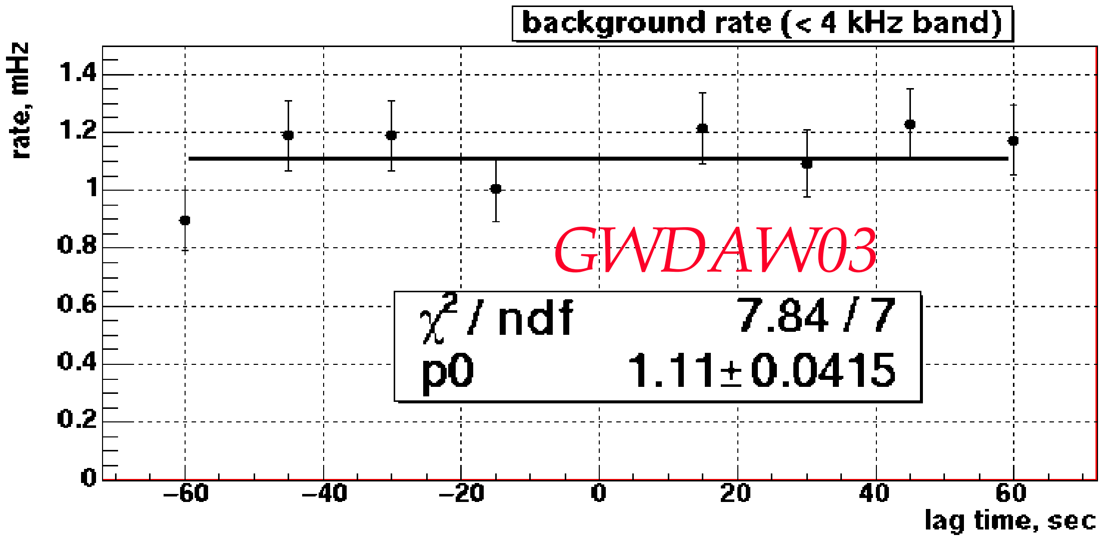
Coincidence Rates

double coincidence samples (S2 playground)

ifo pair	L1-H1	H1-H2	H2-L1
triggers	29346	22469	36956
lock,sec	94652	98517	93699
rate, Hz	0.31	0.23	0.39

off-time samples are produced during the production stage independent on GW samples

raw triple coincidence rates

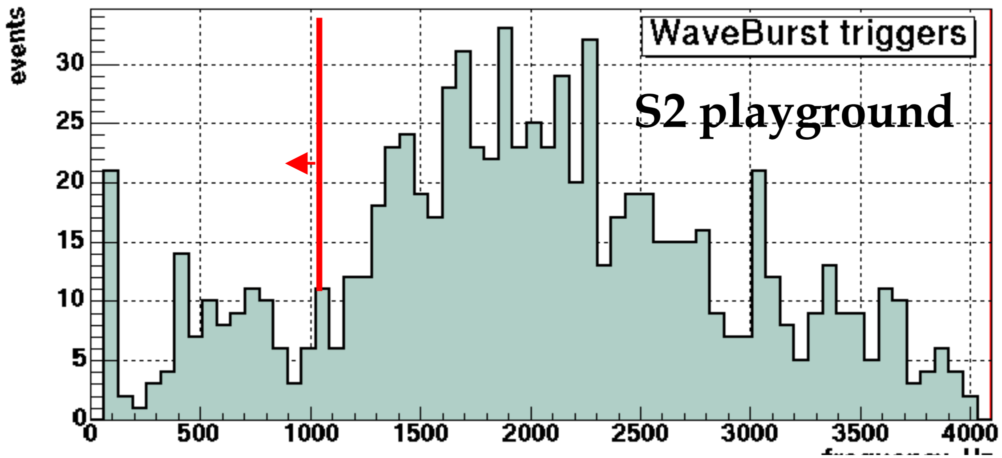


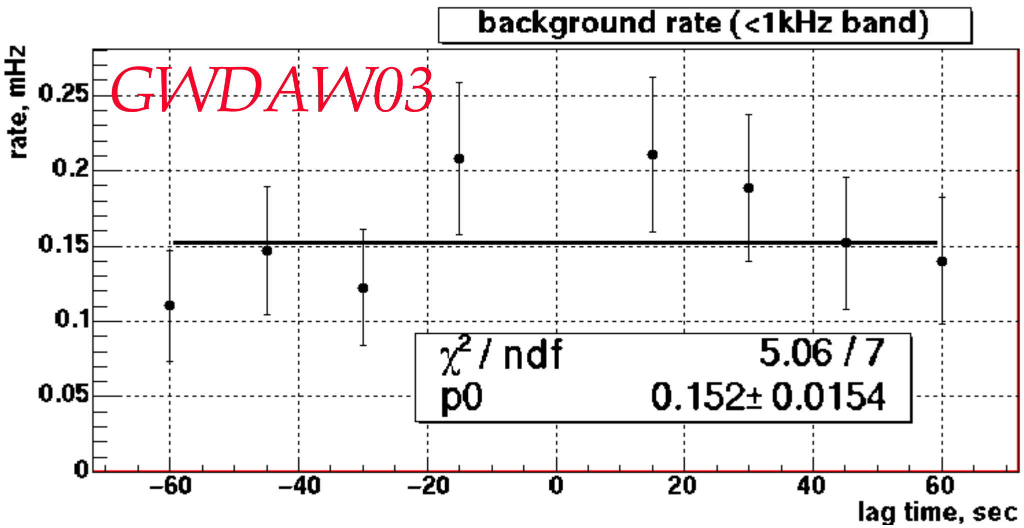
triple coincidence:
time window: 20 ms
frequency gap: 0 Hz
→ 1.10 ± 0.04 mHz

- **expect reduce background down to <20 μHz using post-processing selection cuts: triple event confidence, veto, …**

"BH-BH merger" band

off-time triple coincidence sample





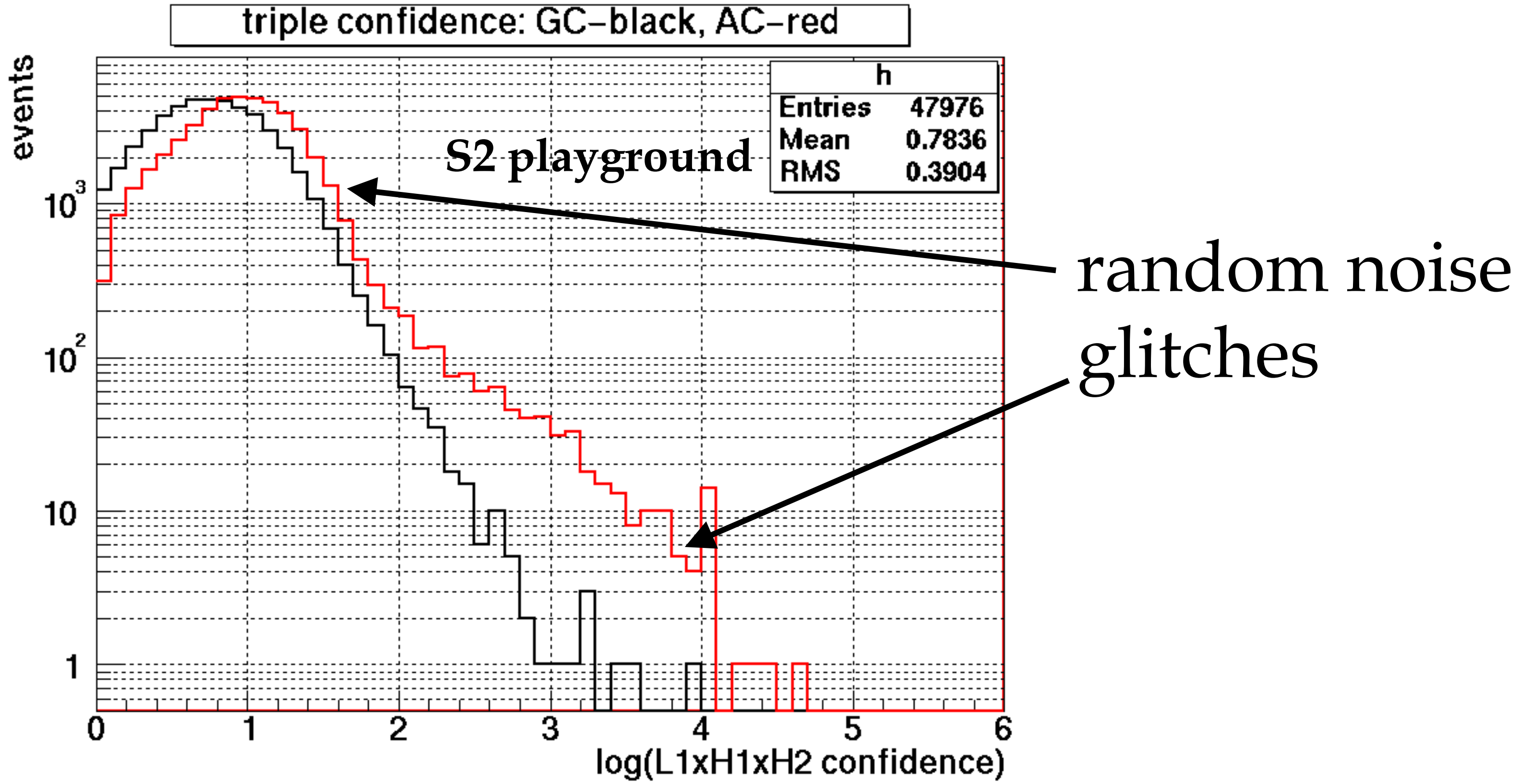
expect BH-BH mergers (masses >10 Mo) in frequency band < 1 kHz (BH-BH band)

background of **0.15 ± 0.02 mHz**

expect **<1 μHz** after post-processing cuts

confidence of triple coincidence event

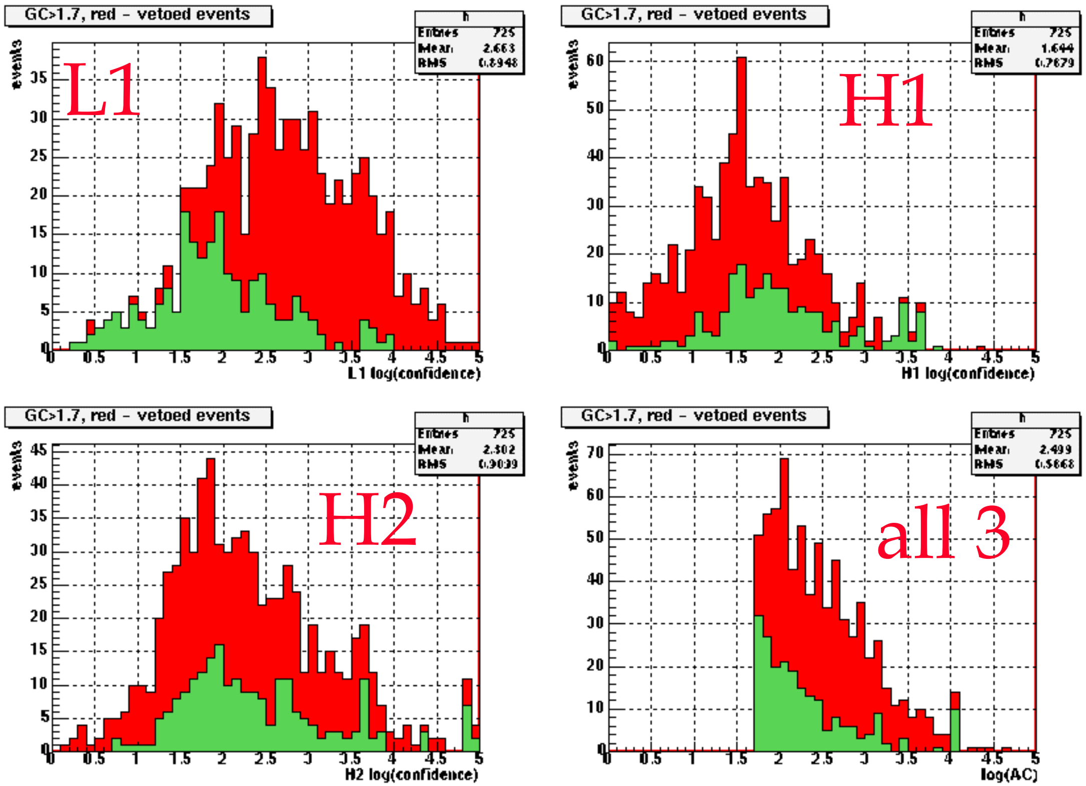
- **"arithmetic"** $AC = (C_{L1} + C_{H1} + C_{H2})/3$
- **"geometric"** $GC = (C_{L1} \cdot C_{H1} \cdot C_{H2})^{1/3}$



- **Clean up the pipeline output by setting threshold on triple GC**

VETO

- anti-coincidence with environmental & control channels
  - 95% of LIGO data
- generated with GlitchMon and WaveMon (DMT monitors)



green – WaveBurst triggers with GC>1.7 after WaveMon VETO (~55 L1 channels) is applied

dead time frac: ~5%
veto efficiency: 76%

LIGO veto system is working !

address veto safety issue before use in the analysis

WaveBurst false alarm summary

- expect reduce background down to
  - <10 μHz for frequency band of 64-4096 Hz
  - < 1 μHz for frequency band of 64-1024 Hz

  by using post-processing selection cuts:
  - triple event confidence
  - veto
- false alarm of 1 event per year is feasible with the use of the x-correlation cut.
- expect <1 background events for all S2 (no veto)

→ WaveBurst is low false alarm burst detection pipeline

- What is the pipeline sensitivity?

Simulation

- hardware injections
- software injection into all three interferometers:
  - waveform name
  - GPS time of injection
  - {θ, φ,Ψ} - source location and polarization angle
  - T {L1,H1,H2} - LLO-LHO delays
  - F+{L1,H1,H2} - + polarization beam pattern vector
  - Fx {L1,H1,H2} - x polarization beam pattern vector
- use exactly the same pipeline for processing of GW and simulation triggers.
- sine-Gaussian injections
  - 16 waveforms: 8-Q9 and 8-Q3
  - F+ {1,1,1} , Fx {0,0,0}

$$Q = \sqrt{2}\pi\tau f_0$$

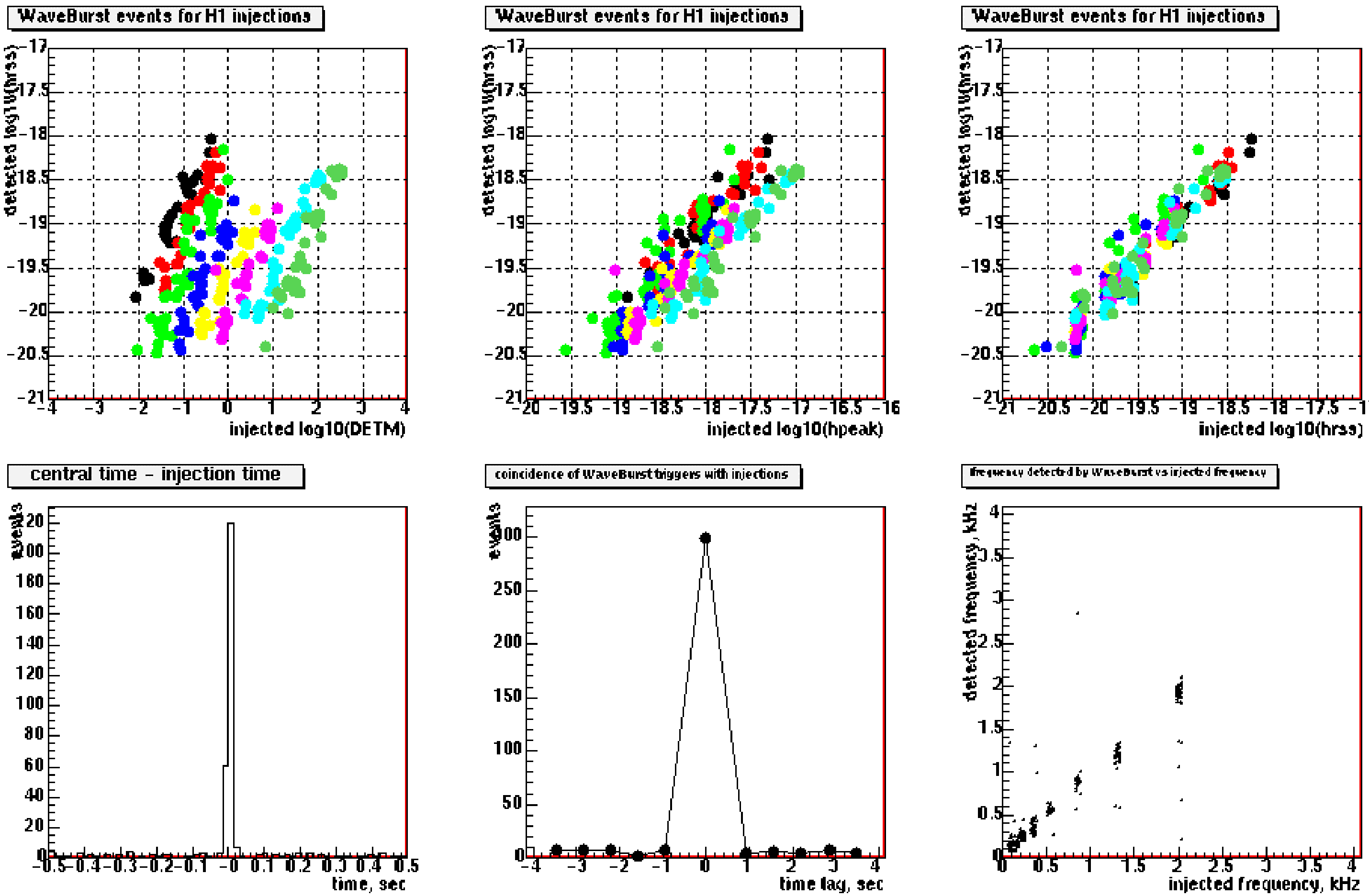
τ –duration

f_0-central frequency

- BH-BH mergers (10-100 Mo)
  - 10 pairs of Lazarus waveforms {h+,hx}
  - all sky uniform distribution with calculation {F+,Fx} for LLO,LHO

hardware injections

SG injections [100Hz, 153Hz , 235Hz, 361Hz, 554Hz, 850Hz, 1304Hz 2000Hz]



good agreement between injected and reconstructed hrss
good time and frequency resolution

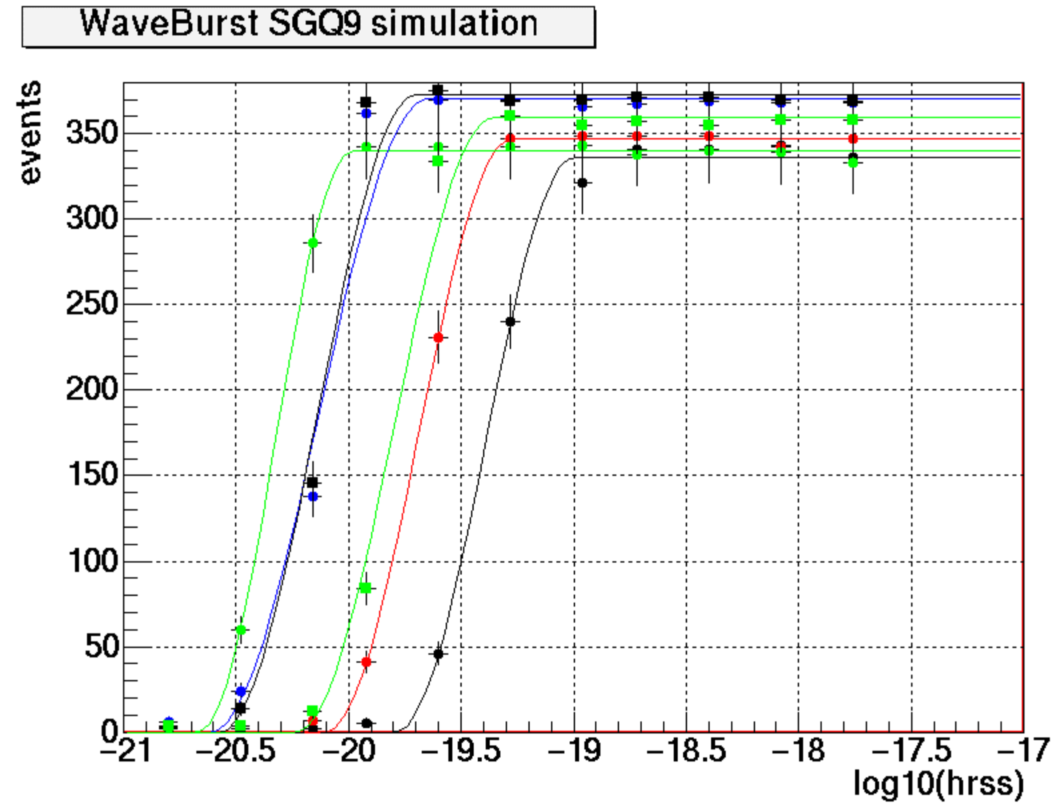
H1H2 pair

detection efficiency vs hrss

hrss(50%)

$$4-5\cdot 10^{-21}\ \frac{strain}{\sqrt{Hz}}$$

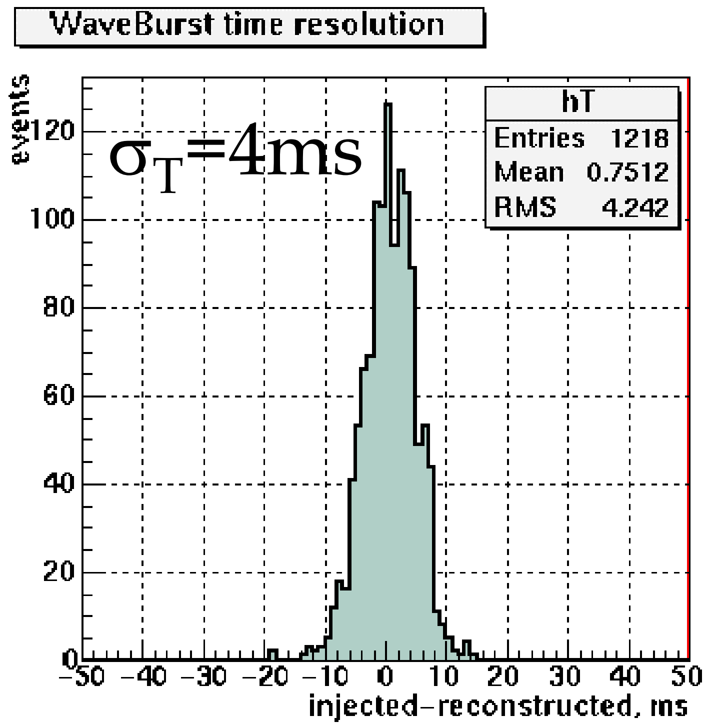
@235 Hz
robust
with respect
to waveform Q

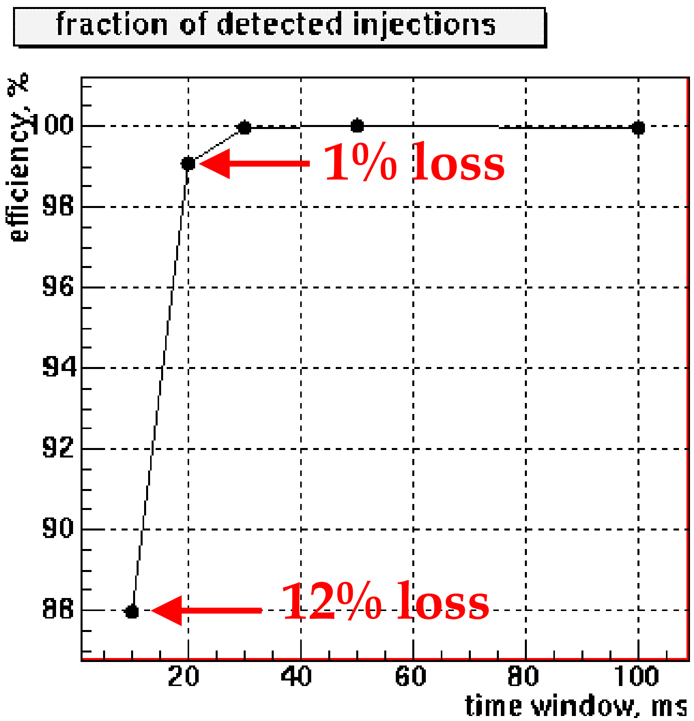


f_o, Hz	100	153	235	361	554	850	1034	2000	
$h_{50\%}$, Q9	40.	20.	4.8	7.5	7.2	-	16.	-	$\times 10^{-21}$
$h_{50\%}$, Q3	36.	14.	6.0	6.6	8.6	10.	17.	30.	$\times 10^{-21}$
	●	●	●	●	■		■		

timing resolution

S2 playground simulation sample



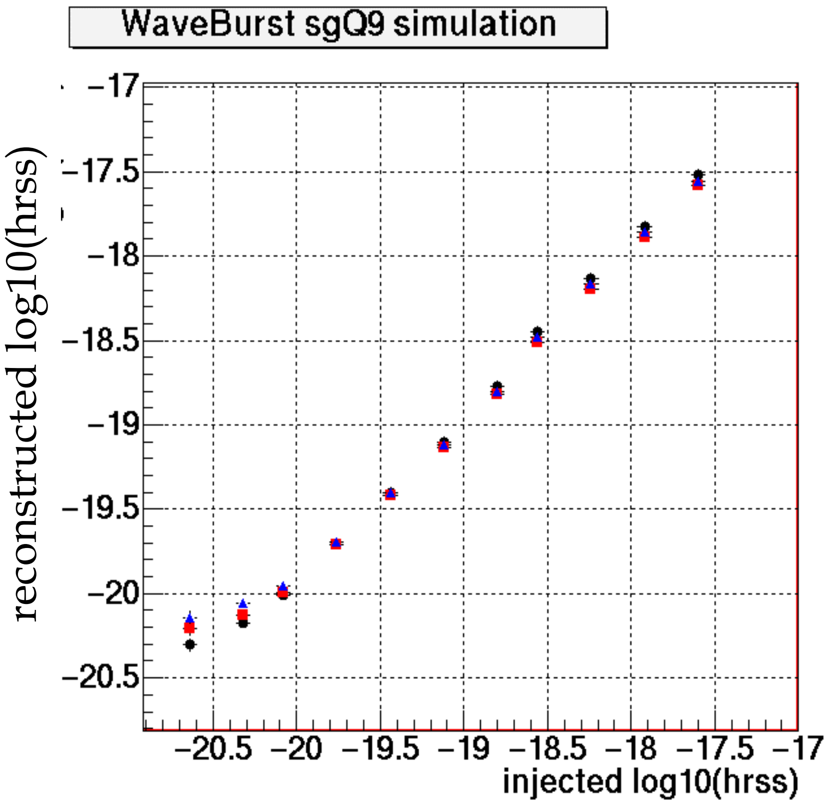


- time window >= 20 ms → negligible loss of simulated events (< 1%)

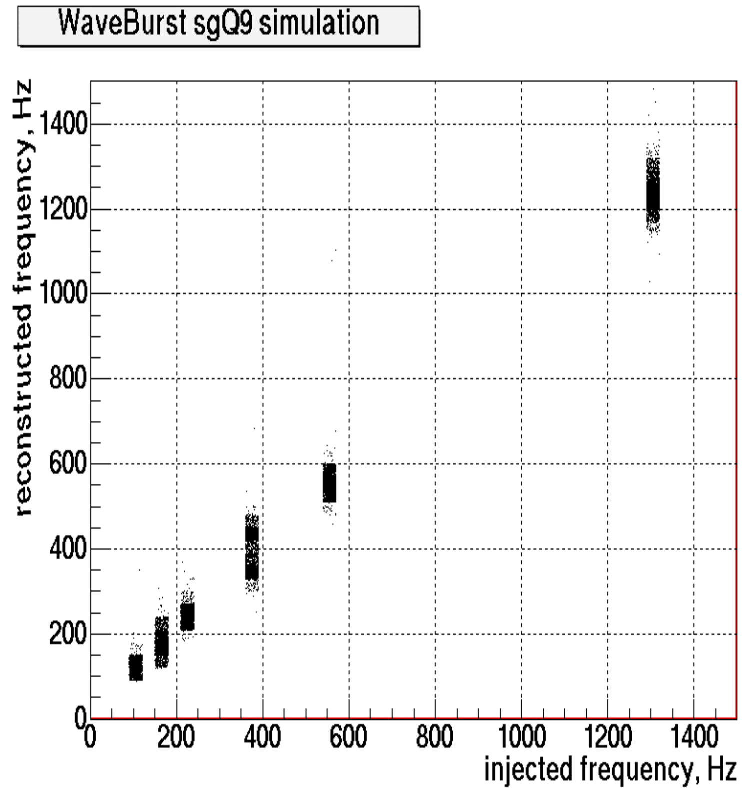
Signal reconstruction

- Use orthogonal wavelet (energy conserved) and calibration.

mean amplitude



frequency



BH-BH merger injections

- **BH-BH mergers** **(Flanagan, Hughes: gr-qc/9701039v2 1997)**

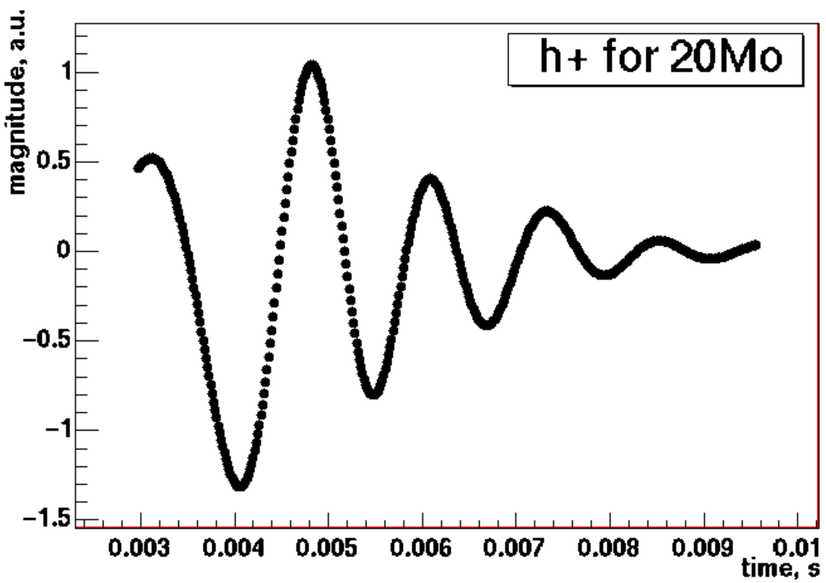
duration : $$\tau \approx 50M = 5ms \cdot \left(\frac{M}{20M_o}\right)$$

start frequency : $$f_{start} \approx \left(\frac{0.02}{M}\right) = 205Hz \cdot \left(\frac{20M_o}{M}\right)$$

bandwidth: $$\Delta f \sim f_{qnr} \approx \left(\frac{0.13}{M}\right) = 1300Hz \cdot \left(\frac{20M_o}{M}\right)$$

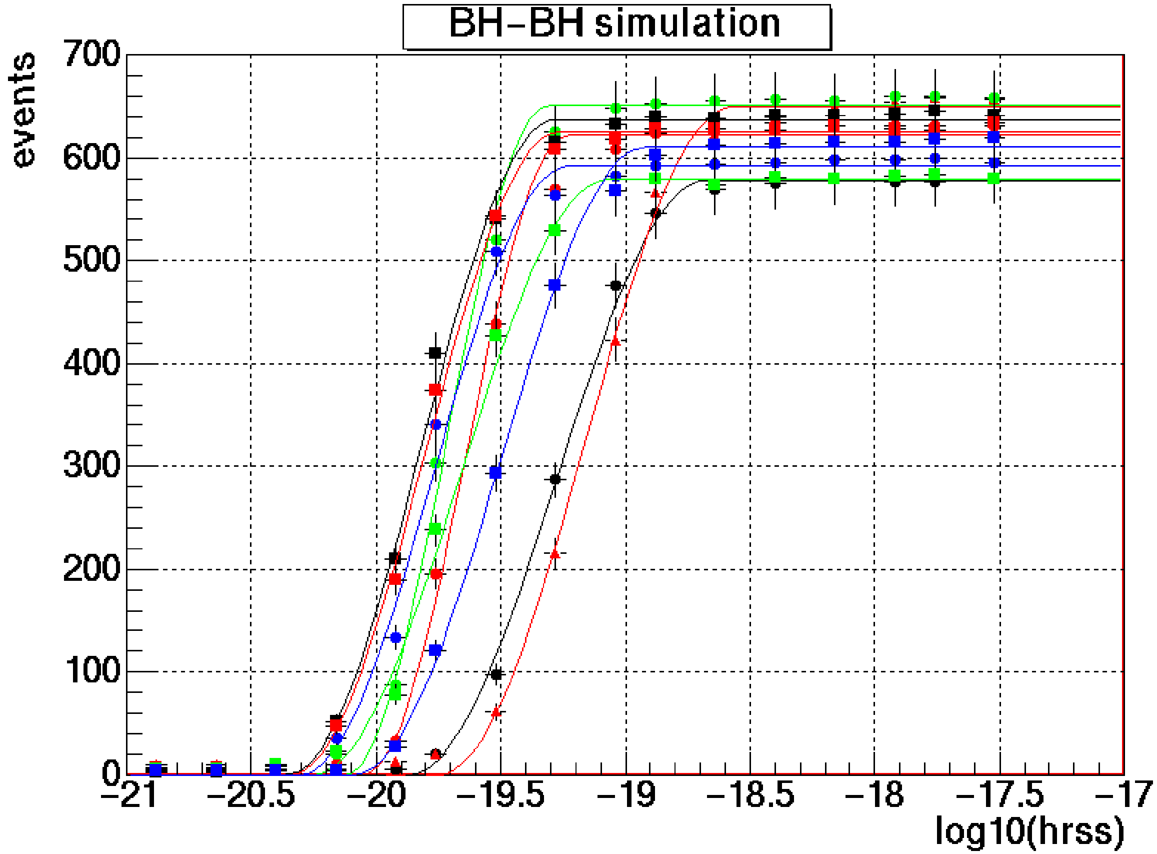
- **Lazarus waveforms**
  (J.Baker et al, astro-ph/0202469v1)
  (J.Baker et al, astro-ph/0305287v1)

all sky simulation using two polarizations and L & H beam pattern functions



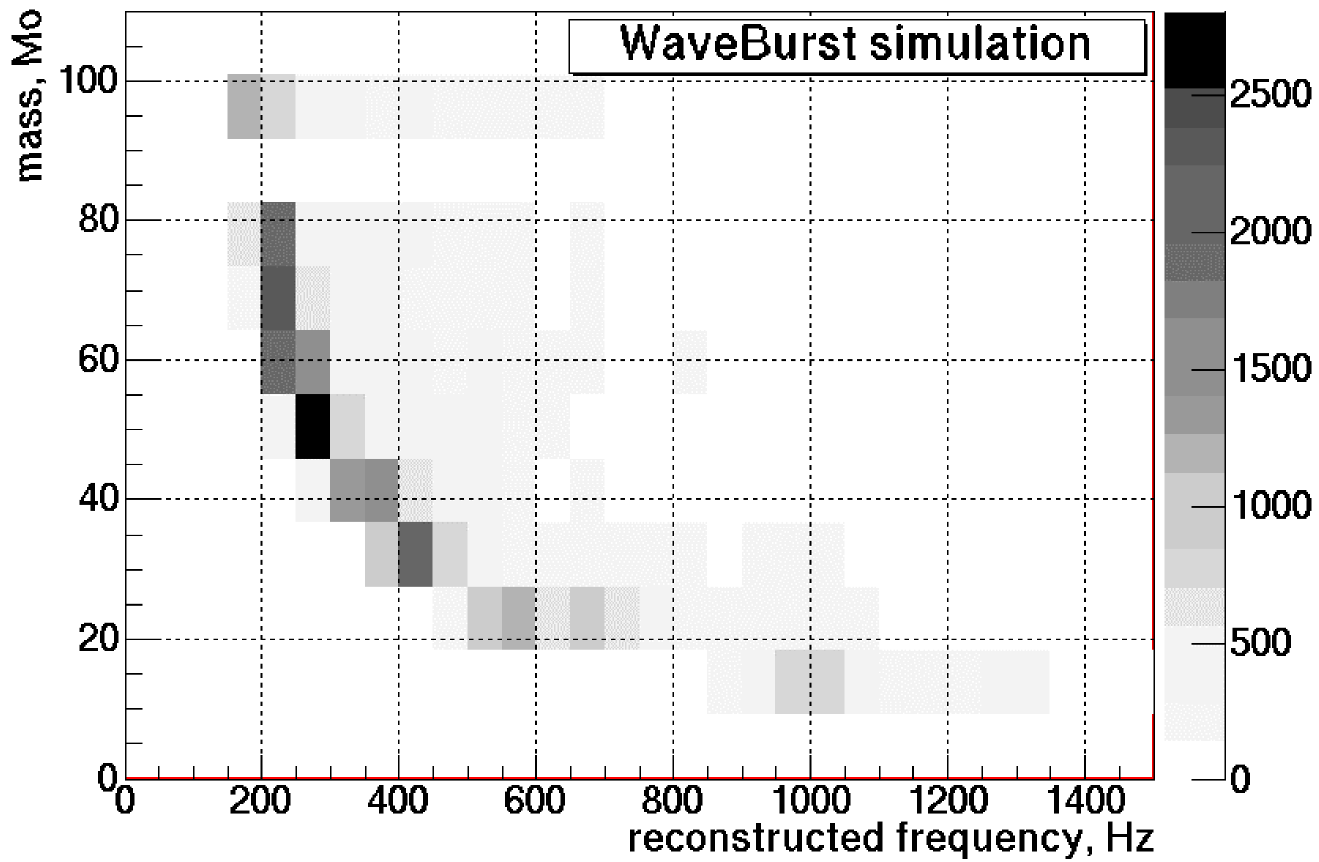
Lazarus waveforms: efficiency

all sky search:
hrss(50%)
$\sim 2\cdot 10^{-20}\ \frac{strain}{\sqrt{Hz}}$



mass, Mo	10	20	30	40	50	60	70	80	100
hrss(50%) x 10^{-20}	4.5	2.4	2.0	1.8	1.5	1.7	2.2	3.4	7.1

Lazarus waveforms: frequency vs mass



- **expected BH-BH frequency band – 100-1000 Hz**

WaveBurst pipeline status

- WaveBurst ETG: stable, fully operational, tuned
- S2 production: complete (Feb 8), ready to release triggers
- Post-production
  - time, frequency coincidence: fully operational, tuned
  - trigger selection: fully operational, tuned
  - off-time analysis: ready to go
- VETO analysis
  - feasible, good veto efficiency (87%)
  - need to finish production of WaveMon H1 and H2 triggers
  - requires cleaning-up veto sample and some tuning to reduce DTF
  - address more accurate veto safety with software injections
- Simulation
  - All sky SG,BH-BH mergers, Gaussians: complete

ready to produce S2 result before the LSC meeting

Summary

- WaveBurst -low false alarm burst detection by using
  - Wavelet transform with low spectral leakage
  - TF coincidence at pixel level
  - Non-parametric statistics
  - Combined triple event confidence
  - Efficient VETO analysis
- at the same time maintaining high detection efficiency