Quantum Noise Simulation

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Quantum Measurement Group

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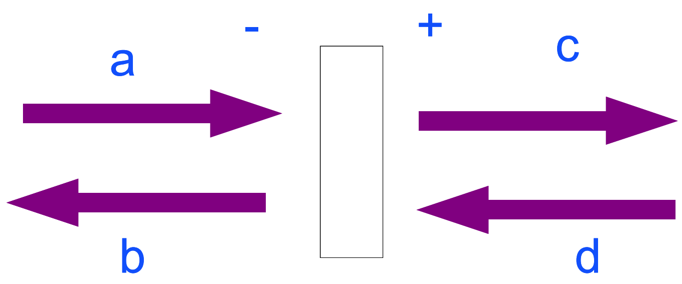
March 2004 LLO LSC Meeting

Goals

- Simulate quantum optical noise in arbitrary optical configuration.
  - Work in the frequency domain.
  - Propagate quantum fields.
  - Include radiation pressure effects.
- Calculate as much as possible
  - Propagation of noise
  - Propagation of signals
  - Propagation of laser noise
  - Include losses

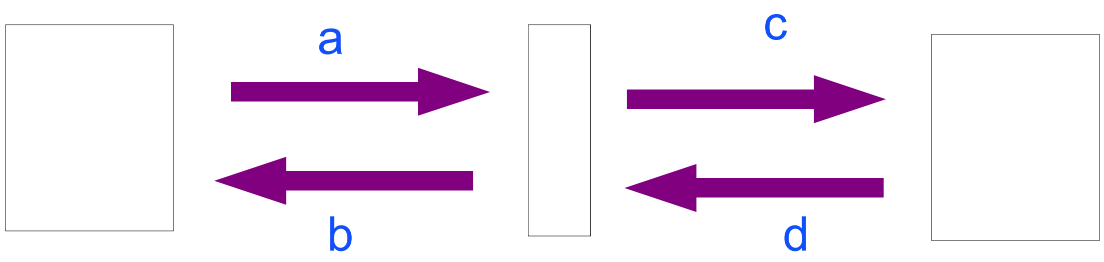
The Basics



We want to write the output fields in terms of the input fields for each object.

$$\begin{pmatrix} \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} -\rho & \tau \\ \tau & \rho \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{d} \end{pmatrix} \qquad \mathbf{a} \equiv \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Adding things together



$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ -\rho & -1 & 0 & \tau \\ \tau & 0 & -1 & \rho \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \\ \mathbf{d} \end{pmatrix} = \begin{pmatrix} -\mathbf{v}^{(1)} \\ 0 \\ 0 \\ -\mathbf{v}^{(2)} \end{pmatrix}$$

Adding losses

Losses can be added without modifying the matrix, and their propagation is calculated in the inverted matrix. (Use the same technique for GWs or transfer functions.)

$$\begin{pmatrix} \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} -\rho & \tau \\ \tau & \rho \end{pmatrix} \begin{pmatrix} \mathbf{a} + \sqrt{\frac{A}{1-A}}\mathbf{v}^{(3)} \\ \mathbf{d} + \sqrt{\frac{A}{1-A}}\mathbf{v}^{(4)} \end{pmatrix}$$

$$\rho^2 + \tau^2 + A = 1$$

$$\mathbf{M} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \\ \mathbf{d} \end{pmatrix} = \begin{pmatrix} -\mathbf{v}^{(1)} - \sqrt{\frac{A}{1-A}}\mathbf{v}^{(3)} \\ 0 \\ 0 \\ -\mathbf{v}^{(2)} - \sqrt{\frac{A}{1-A}}\mathbf{v}^{(4)} \end{pmatrix}$$

Radiation Pressure

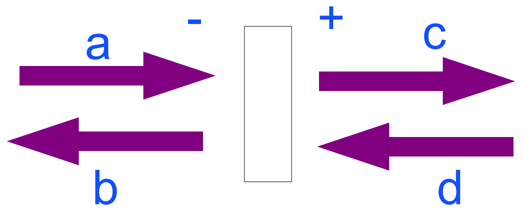
Must treat radiation pressure as generically as possible – allow for interfering fields, with arbitrary phases. In most cases, the phases can be assumed to be 0.

Calculate momentum flow from each field:

$$\dot{P}_j(\Omega) = \sqrt{\frac{\hbar\omega}{c^2}} \mathbf{D}_j^T \mathbf{j}(\Omega)$$

$$\mathbf{D}_j \equiv \sqrt{2I_j} \begin{pmatrix} \cos\theta_j \\ \sin\theta_j \end{pmatrix}$$

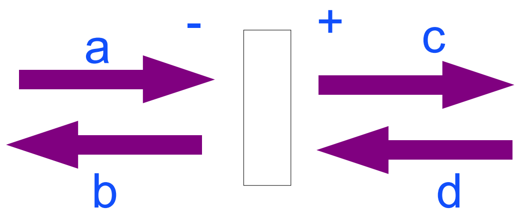
Quadrature representation of carrier field



Account for direction of propagation

Use equation of motion to find the mirror displacement: $-M\Omega^2 X = \sum_j \eta_j \dot{P}_j$

Radiation pressure (Mirror)



$$\mathbf{v}^* \equiv \begin{pmatrix} v_2 \\ -v_1 \end{pmatrix}, \quad \text{for } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

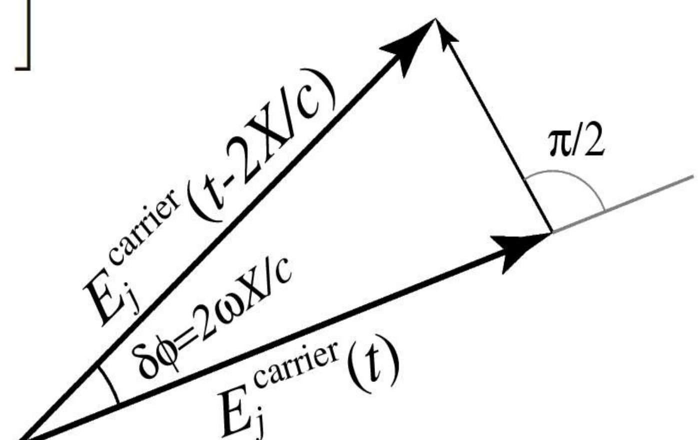
$$\mathbf{D}_j \equiv \sqrt{2I_j} \begin{pmatrix} \cos\theta_j \\ \sin\theta_j \end{pmatrix}$$

$$\left[\mathbf{I} + \Pi \begin{pmatrix} \mathbf{D}_a^* \\ \mathbf{D}_d^* \end{pmatrix} \begin{pmatrix} \mathbf{D}_b^T & -\mathbf{D}_c^T \end{pmatrix}\right] \begin{bmatrix} \mathbf{b} \\ \mathbf{c} \end{bmatrix}$$

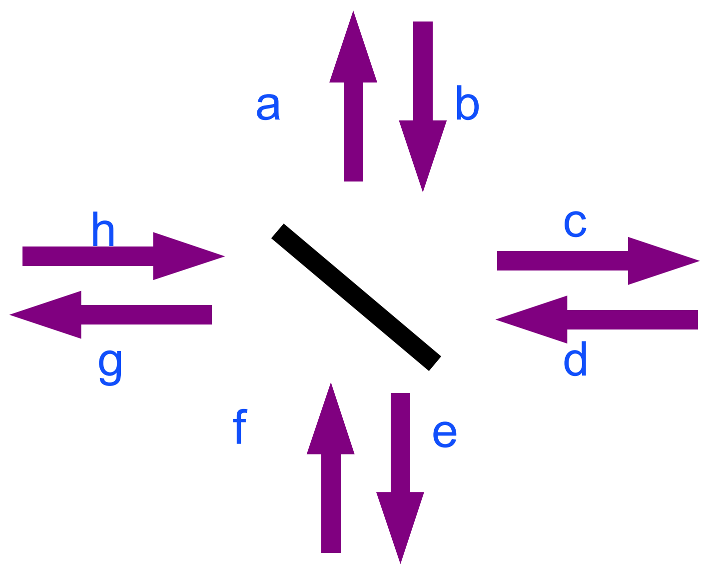
$$= \left[\mathbf{M}_{\text{mirror}} - \Pi \begin{pmatrix} \mathbf{D}_a^* \\ \mathbf{D}_d^* \end{pmatrix} \begin{pmatrix} \mathbf{D}_a^T & -\mathbf{D}_d^T \end{pmatrix}\right] \begin{bmatrix} \mathbf{a} \\ \mathbf{d} \end{bmatrix}$$

where

$$\Pi \equiv \frac{2\rho\omega}{M\Omega^2 c^2}$$



Radiation Pressure(beamsplitter)



$$\left[\mathbf{I}+\frac{\Pi}{2}\begin{pmatrix}\mathbf{D}_d^*\\ \mathbf{D}_b^*\\ \mathbf{D}_h^*\\ \mathbf{D}_f^*\end{pmatrix}\begin{pmatrix}\mathbf{D}_a^T & \mathbf{D}_c^T & -\mathbf{D}_e^T & -\mathbf{D}_g^T\end{pmatrix}\right]\begin{pmatrix}\mathbf{a}\\ \mathbf{c}\\ \mathbf{e}\\ \mathbf{g}\end{pmatrix}=\left[\mathbf{M}_{\mathrm{BS}}-\frac{\Pi}{2}\begin{pmatrix}\mathbf{D}_d^*\\ \mathbf{D}_b^*\\ \mathbf{D}_h^*\\ \mathbf{D}_f^*\end{pmatrix}\begin{pmatrix}\mathbf{D}_d^T & \mathbf{D}_b^T & -\mathbf{D}_h^T & -\mathbf{D}_f^T\end{pmatrix}\right]\begin{pmatrix}\mathbf{d}\\ \mathbf{b}\\ \mathbf{h}\\ \mathbf{f}\end{pmatrix}$$

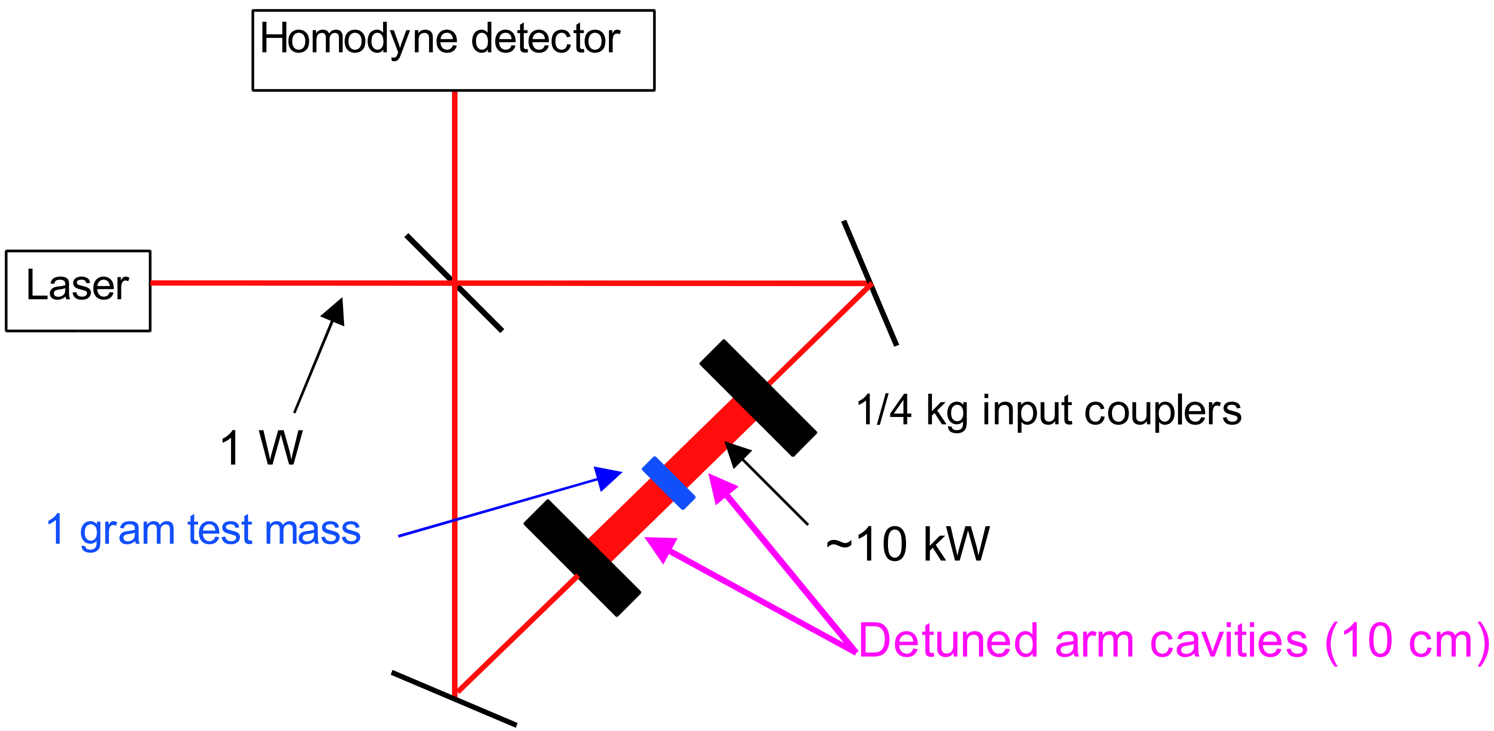
Signals

Adding signals is easily accomplished – add another source term (as is done with losses) in the phase quadrature.

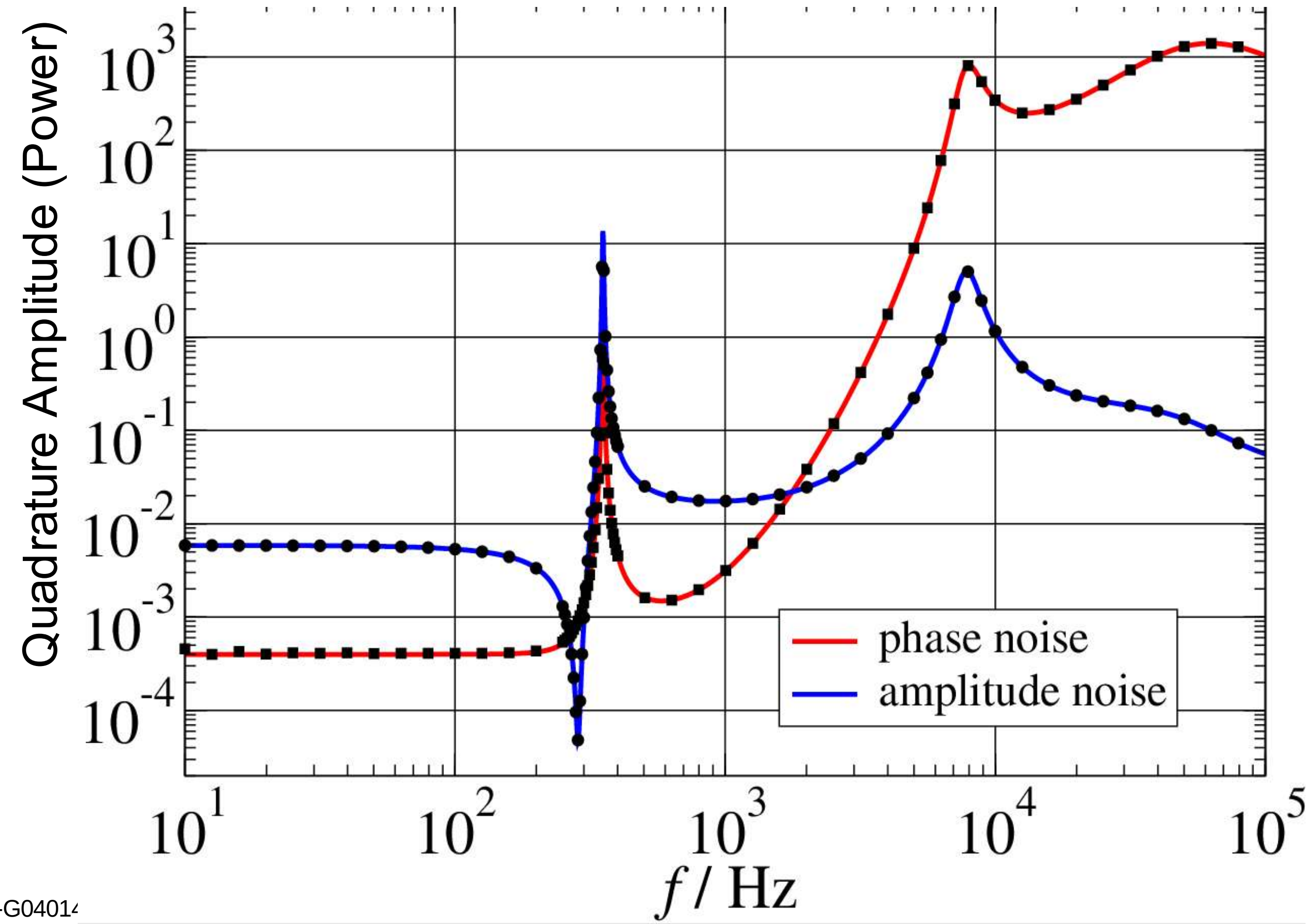
Add other terms to this vector

$$
\mathrm{M}\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\\ \mathbf{c}\\ \mathbf{d}\end{pmatrix} = \begin{pmatrix} -\mathbf{v}^{(1)} - \sqrt{\frac{A}{1-A}}\mathbf{v}^{(3)} \\ 0 \\ 0 \\ -\mathbf{v}^{(2)} - \sqrt{\frac{A}{1-A}}\mathbf{v}^{(4)} \end{pmatrix}
$$

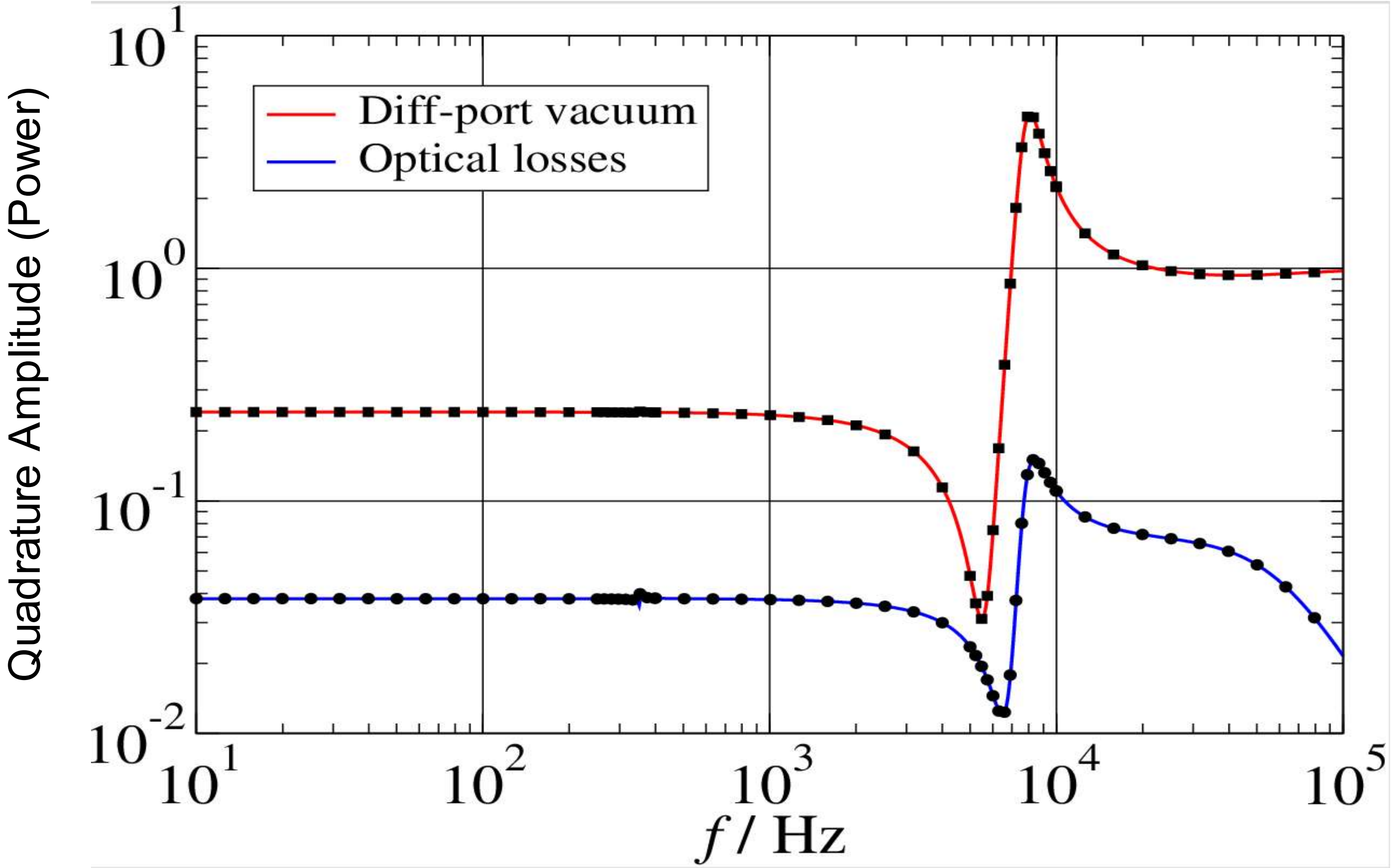
Testing the results



Analytical versus Code



Analytical versus Code



Limitations

- Only works with single frequency of bright light, so schemes that need sidebands or multiple frequencies (crystal squeezing) are not simulated. This could be added in the future.
- Servo loops are not supported. This may also be added in the future.
- Supported objects:
  - Mirrors
  - Beamsplitters
  - Cavities
  - Squeezers (input only)
  - Beam blocks
  - Detectors
  - Lasers