Various ways to beat the Standard Quantum Limit

Yanbei Chen
California Institute of Technology

Table of Contents

- Modifying test mass dynamics (Optical Spring)
- Modifying input-output optics (KLMTV Filters)
  - Conventional interferometers
  - Signal Recycling Interferometers
- Reshaping optical response -- Speed Meters
  - Michelson
  - Sagnac
- Feedback or Cancellation of Radiation Pressure Noise

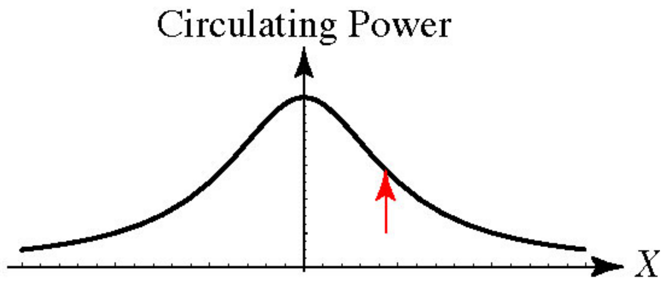
Modifying Test-Mass Dynamics

Modifying test-mass dynamics: Optical Spring

- Detuned cavity: radiation-pressure depend on mirror position
- Optomechanical resonant frequ

$$\Theta = \sqrt{\frac{8\omega_0 I_c \delta}{mLc(\gamma^2 + \delta^2)}}$$

- Free-mass SQL does not apply!



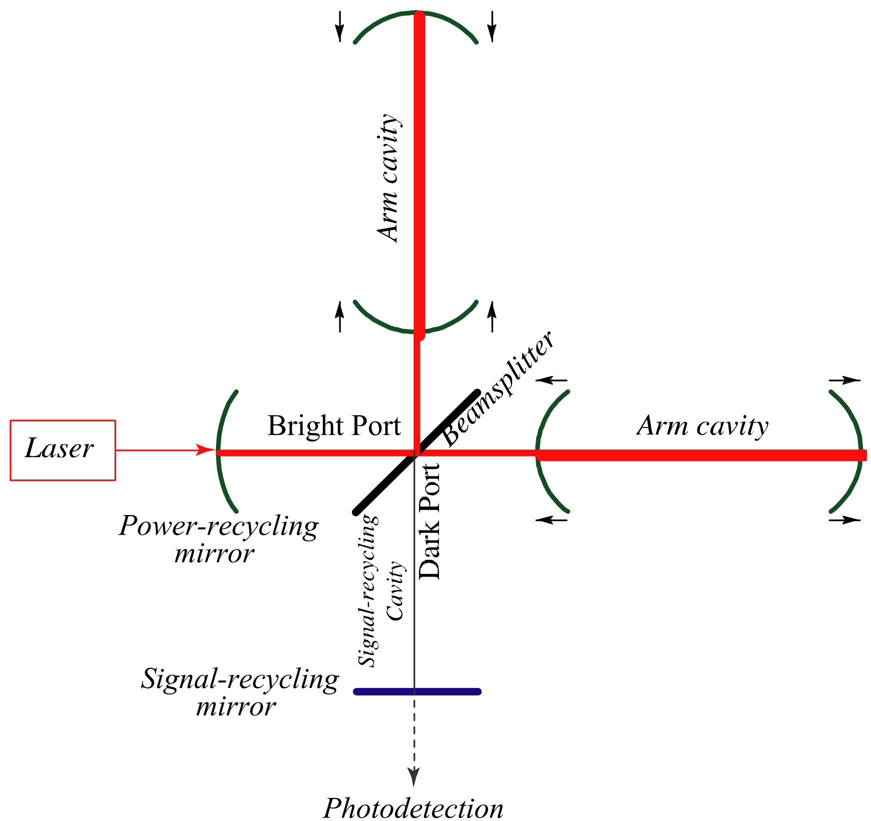
Classical Dynamics analyzed by Rakhmanov (2002), proposed for use in LIGO arm cavities.

Modifying test-mass dynamics: Optical Spring

- Detuned cavity: radiation-pressure depend on mirror position
- Optomechanical resonant frequ

$$\Theta = \sqrt{\frac{8\omega_0 I_c \delta}{mLc(\gamma^2 + \delta^2)}}$$

- Free-mass SQL does not apply!
- Signal Recycling configuration of GEO 600, AdvLIGO and LCGT: can beat SQL by moderate amount [Buonanno and Chen, 01--03; Harms 02]

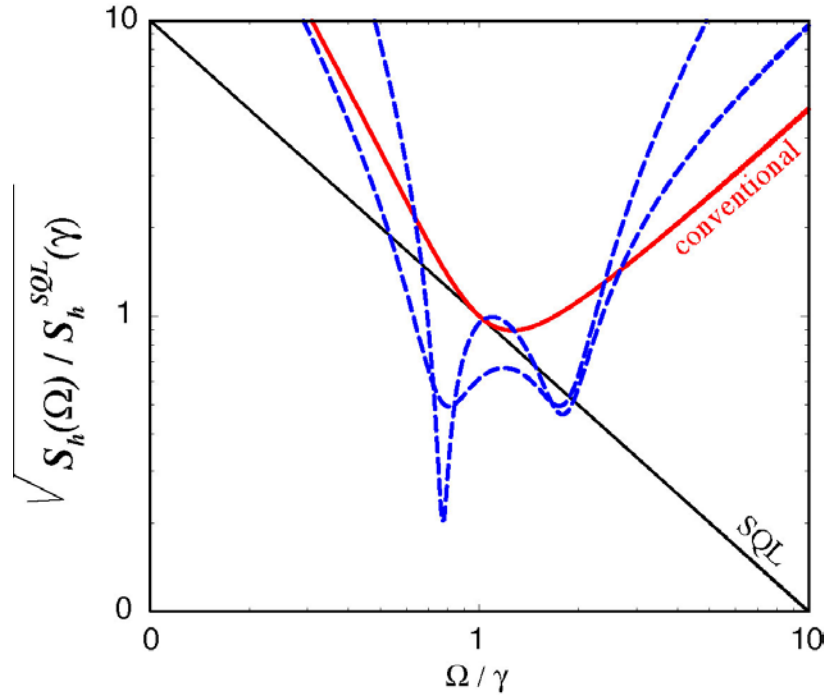


Modifying test-mass dynamics: Optical Spring

- Detuned cavity: radiation-pressure depend on mirror position
- Optomechanical resonant frequ

$$\Theta = \sqrt{\frac{8\omega_0 I_c \delta}{mLc(\gamma^2 + \delta^2)}}$$

- Free-mass SQL does not apply!
- Signal Recycling configuration of GEO 600, AdvLIGO and LCGT: can beat SQL by moderate amount [Buonanno and Chen, 01--03; Harms 02]

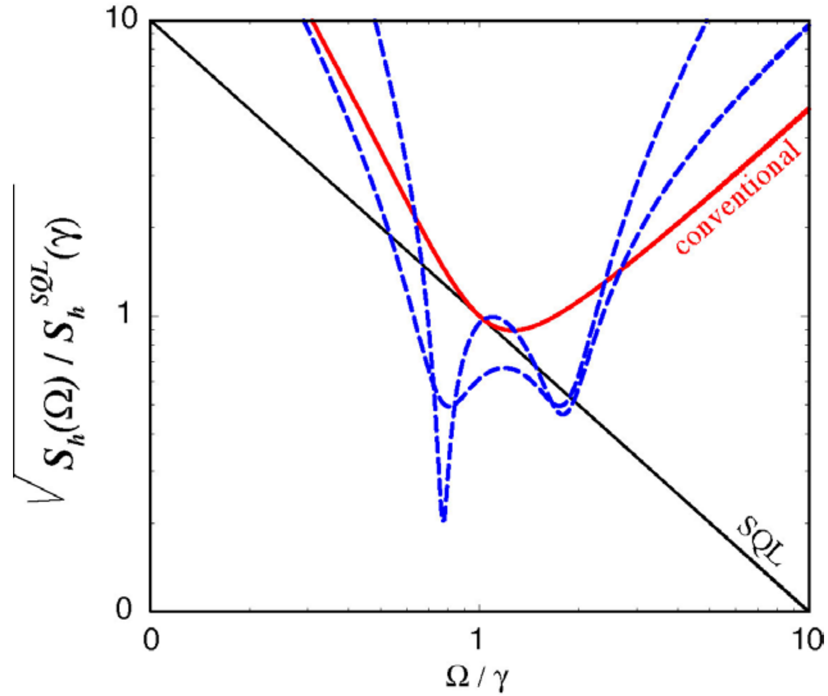


Modifying test-mass dynamics: Optical Spring

- Detuned cavity: radiation-pressure depend on mirror position
- Optomechanical resonant frequ

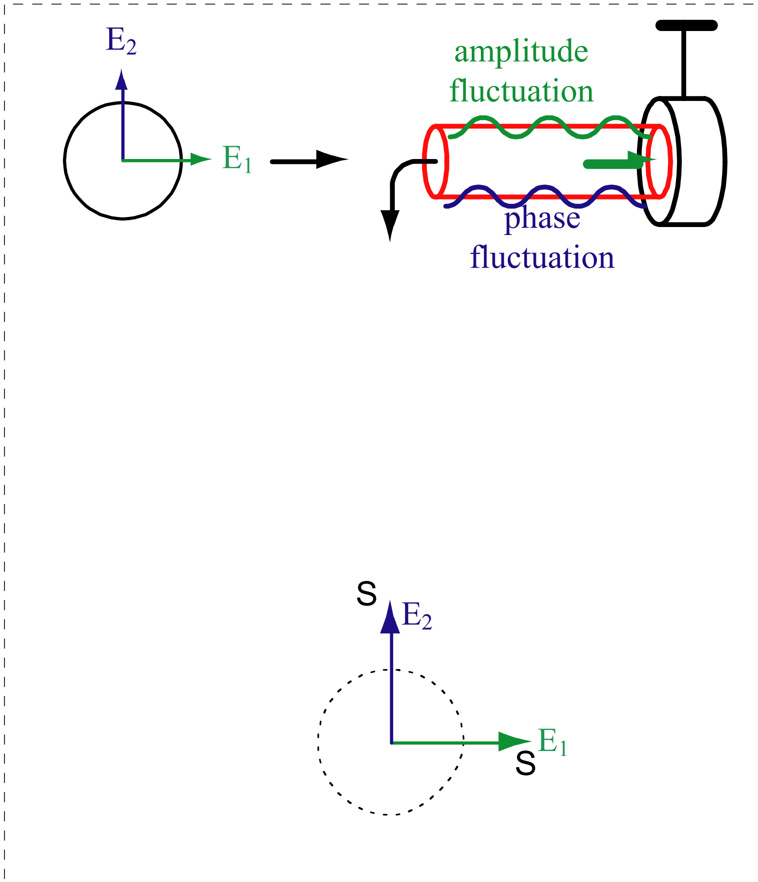
$$\Theta = \sqrt{\frac{8\omega_0 I_c \delta}{mLc(\gamma^2+\delta^2)}}$$

- Free-mass SQL does not apply!
- Signal Recycling configuration of GEO 600, AdvLIGO and LCGT: can beat SQL by moderate amount [Buonanno and Chen, 01--03; Harms 02]
- Optomechanical Instability!
- Application: ponderomotive squeezer, in a different regime of mass/power/length/finesse (talk of Thomas Corbitt)



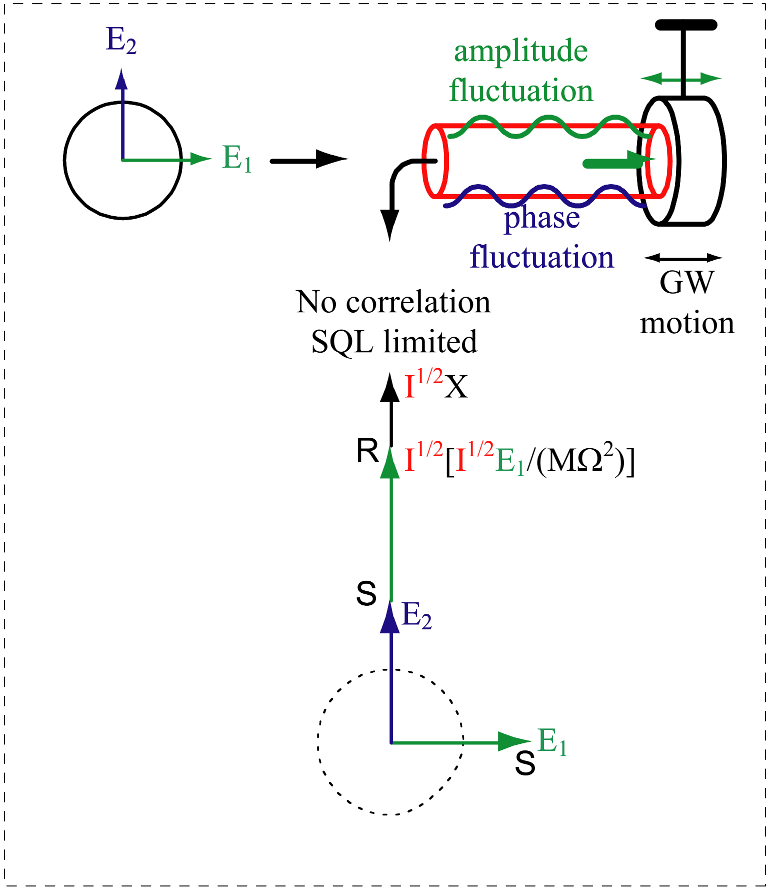
Modifying Input-Output Optics: KLMTV Filters

Utilizing Noise Correlations: Conventional Interferometers



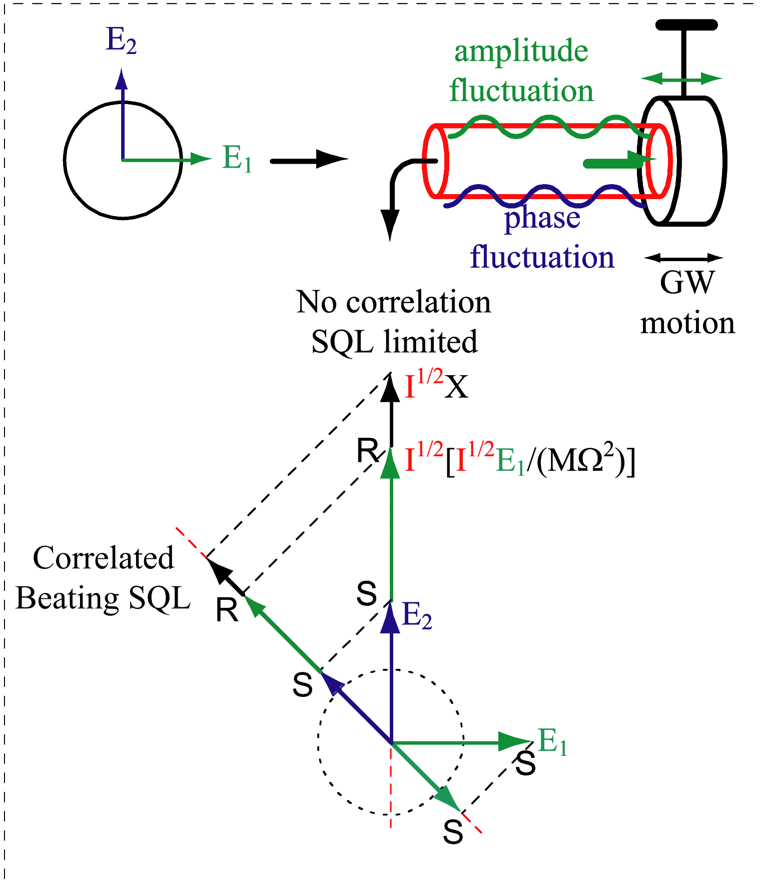
- Shot noise in both quadratures, uncorrelated

Utilizing Noise Correlations: Conventional Interferometers



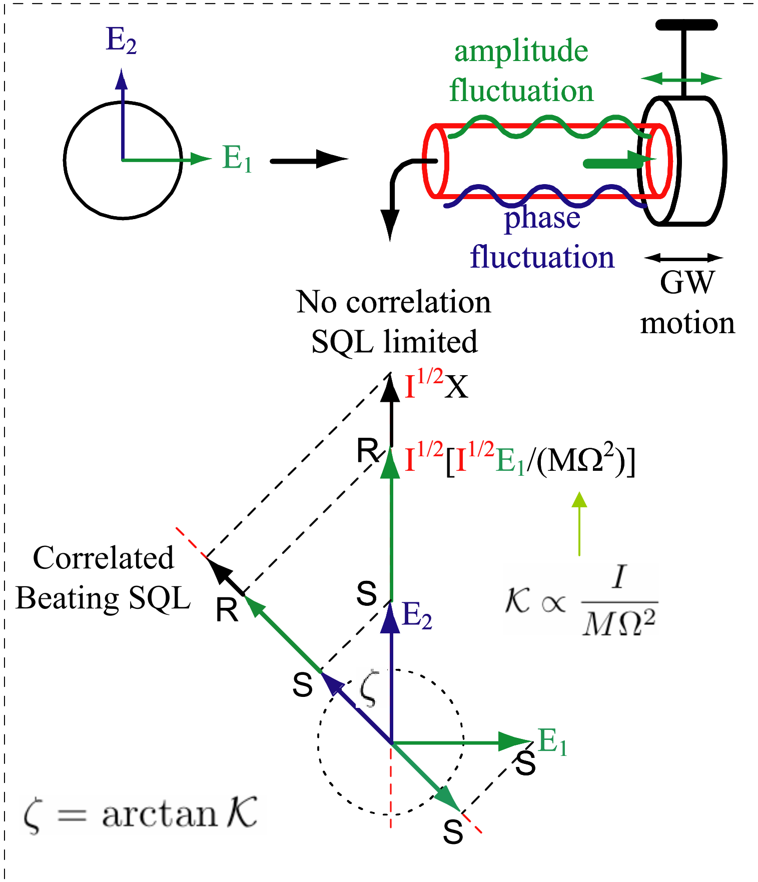
- Shot noise in both quadratures, uncorrelated
- Signal and Rad. Pres. Noise in phase quadrature
- SN & RPN uncorrelated in phase quadrature; SQL Limited

Utilizing Noise Correlations: Conventional Interferometers



- Shot noise in both quadratures, uncorrelated
- Signal and Rad. Pres. Noise in phase quadrature
- SN & RPN uncorrelated in phase quadrature; SQL Limited
- SN and RPN correlated in generic quadrature

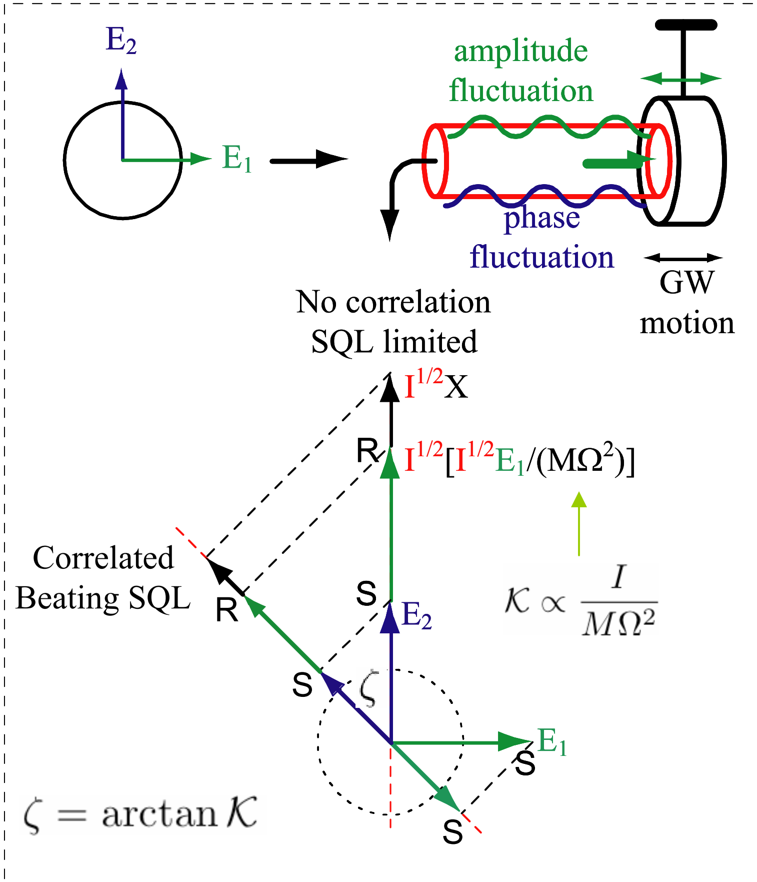
Utilizing Noise Correlations: Conventional Interferometers



- Shot noise in both quadratures, uncorrelated
- Signal and Rad. Pres. Noise in phase quadrature
- SN & RPN uncorrelated in phase quadrature; SQL Limited
- SN and RPN correlated in generic quadrature
- Complete removal of RPN is possible: optimal quadrature is frequency dependent

  $\Rightarrow$ Variational-Readout Interferometer

Utilizing Noise Correlations: Conventional Interferometers

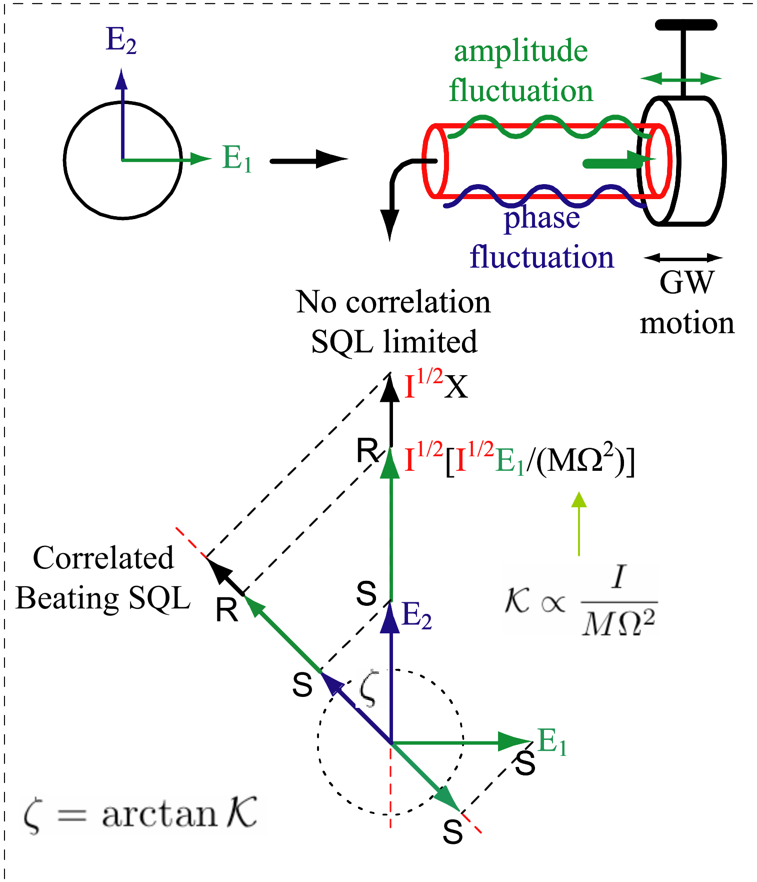


- Shot noise in both quadratures, uncorrelated
- Signal and Rad. Pres. Noise in phase quadrature
- SN & RPN uncorrelated in phase quadrature; SQL Limited
- SN and RPN correlated in generic quadrature
- Complete removal of RPN is possible: optimal quadrature is frequency dependent

  ⇒ Variational-Readout Interferometer
- QND also possible detecting just phase quadrature, if input state is squeezed, with frequency dependent squeeze angle

  ⇒ Squeezed-Input Interferometer

Utilizing Noise Correlations: Conventional Interferometers



- Shot noise in both quadratures, uncorrelated
- Signal and Rad. Pres. Noise in phase quadrature
- SN & RPN uncorrelated in phase quadrature; SQL Limited
- SN and RPN correlated in generic quadrature
- Complete removal of RPN is possible: optimal quadrature is frequency dependent

  ⇒ Variational-Readout Interferometer
- QND also possible detecting just phase quadrature, if input state is squeezed, with frequency dependent squeeze angle

  ⇒ Squeezed-Input Interferometer
- Desired: Frequency-Dependent quadrature-rotation technique!

KLMTV filters for frequency-dependent homodyne detection/squeezing

- [Kimble, Levin, Matsko, Thorne and Vyatchanin(KLMTV), PRD **65**, 022002 (2001)]
  - Use detuned FP cavity to achieve frequency dependent quadrature rotation
  - Applied to conventional interferometers
- Maximum capability of a sequence of FP cavities [Appendix of Purdue and Chen, PRD **66**, 122004 (2002)]

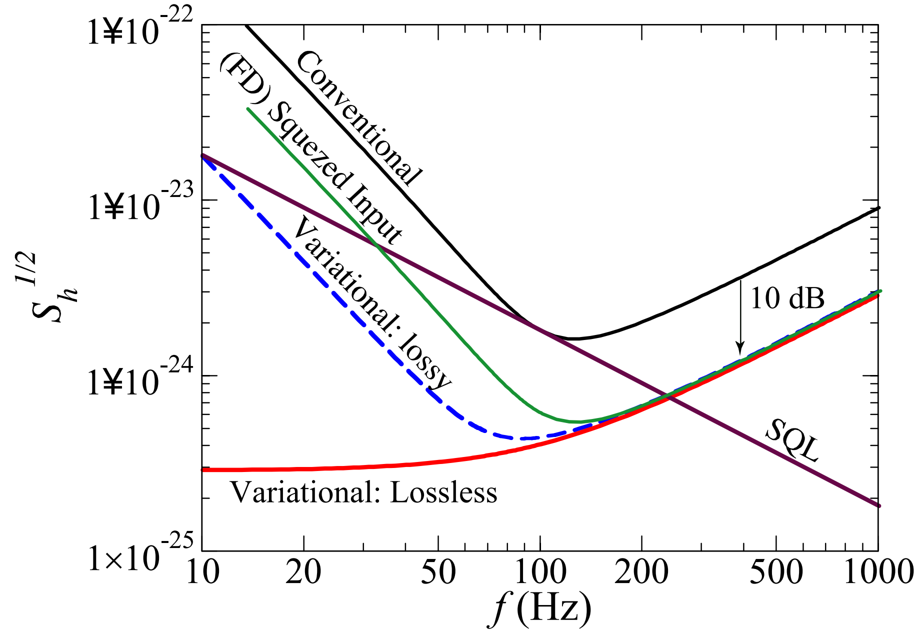
$$\tan\zeta = \frac{\sum_{k=1}^{n} A_k \Omega^{2k}}{\sum_{k=1}^{n} B_k \Omega^{2k}} \quad \Leftarrow \quad \sum_{k=0}^{n} (A_k - iB_k)\Omega_J^{2k} = 0\,, \; J = 1, 2, \ldots, n$$

- Further applications:
  - Signal Recycling Interferometers [Harms et al., PRD 68, 042001 (2003); Buonanno and Chen, gr-qc/0310026, to appear on PRD]
  - Speed Meters (optional) [Purdue and Chen, PRD **66**, 122004 (2002)]
- Filters expected to be narrowbanded (~100Hz), either long or high finesse!

Squeezed-Input and Variational-Readout interferometers

- Squeezed-Input:
  - Global improvement by squeeze factor
  - Less susceptible to losses
- Squeezed-Variational:
  - More improvement, esp. at low frequencies
  - More susceptible to losses

- Warning: Short-filter approximation ($\Omega L/c \ll 1$) breakdown for Squeezed-Variational interferometers [Buonanno and Chen, gr-qc/0310026, to appear on PRD]

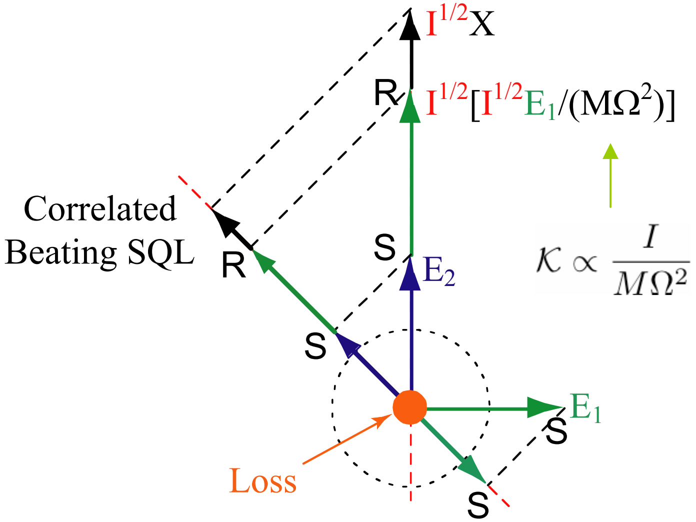


800kW, 10dB squeezing and ~1% total loss [filters: 4km long, 20ppm round-trip loss]

Losses in variational-readout/squeezed input interferometers

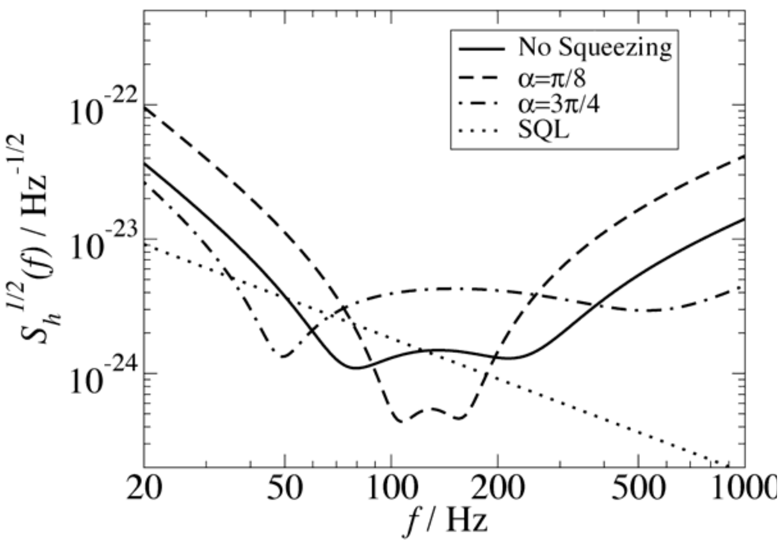
- Squeezed-Input Interferometer (detecting phase quadrature, with FD input squeezing)

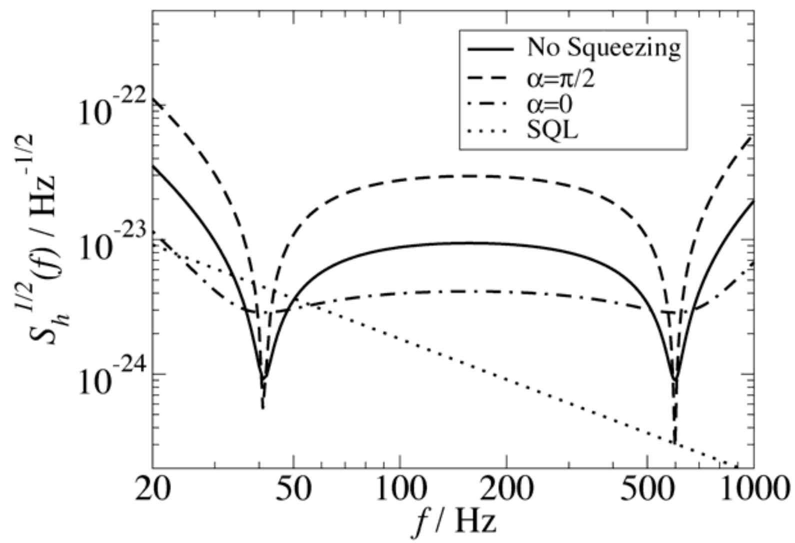
  filter loss must be much lower than power squeeze factor. 1% more than enough for 10dB squeezing (= 0.1 in power). [Maybe push filters to be few hundred meters?]
- Variational-output interferometer has more stringent requirement: low signal content in optimal detection quadrature. [The lower the frequency, lower the signal content!!]



Frequency Independent Squeezing and Readout for Detuned Signal Recycling Interferometers

Gain sensitivity in some frequencies, lose in other frequencies

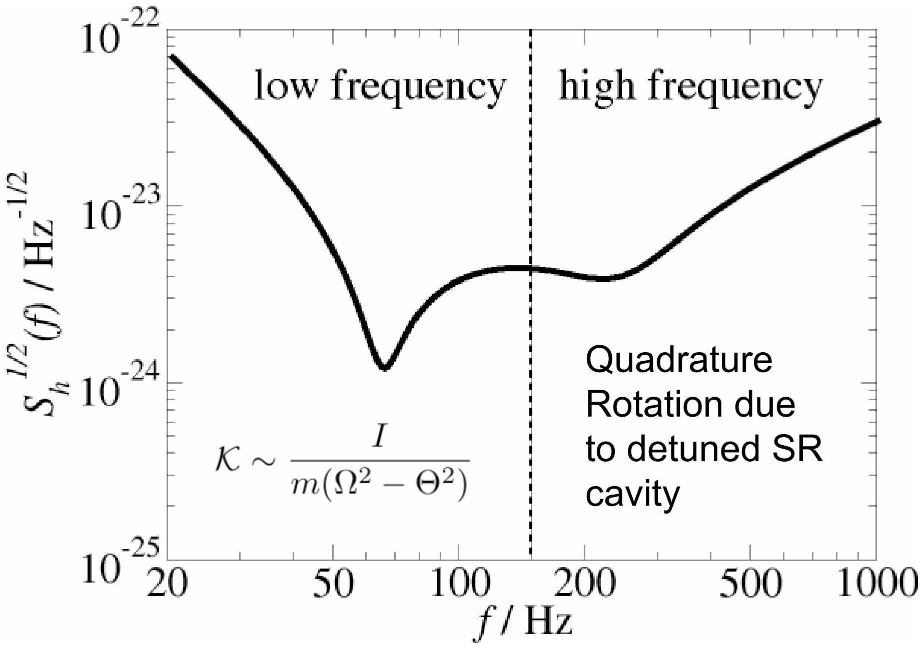


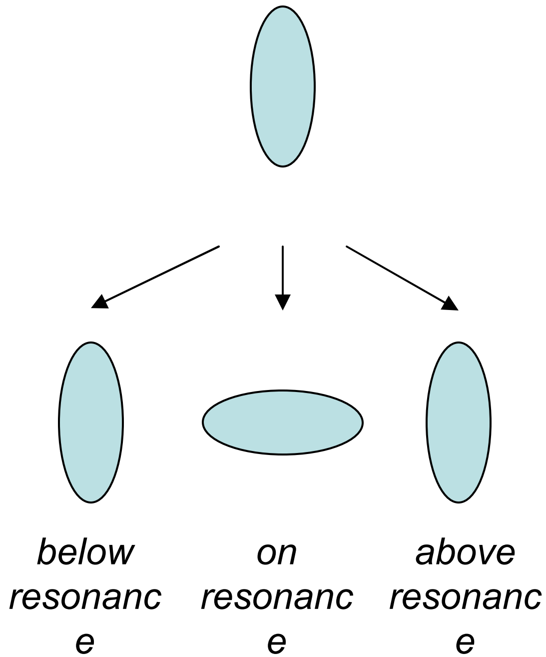


In practice, wideband configuration without filters can make improvements!

(Talk of Nergis)

Two Regimes for SR interferometers with Separated Resonances

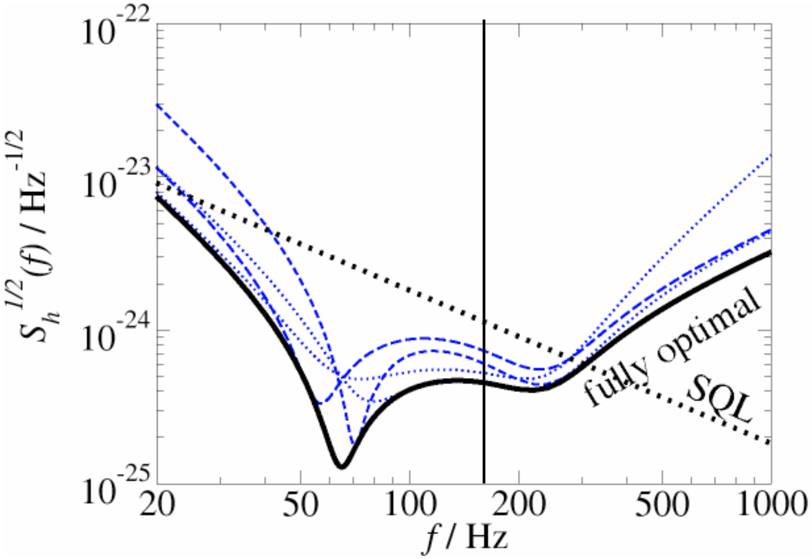




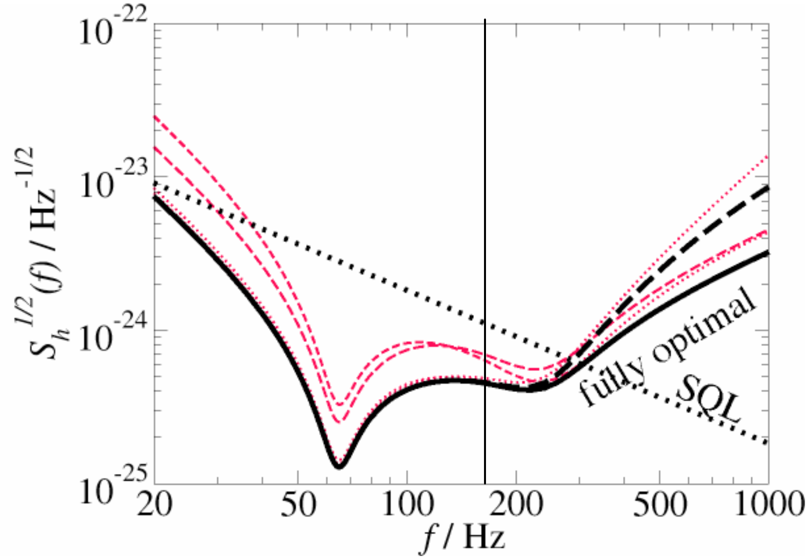
- Low frequencies: (opto)mechanical resonance dominates. Ponderomotive effect (squeezing) important [this can be compared to conventional interferometers!]
- High frequencies: optical resonance dominates. Frequency-dependent quadrature rotation due to detuned cavity [this obviously requires filters!]

Applying KLMTV Filters to Signal Recycling Interferometers

Combining **optomechanical dynamics** and **input-output modification**



Frequency dependent input squeezing
& frequency independent readout
[squeezed-input]
[Harms et al. PRD **68**, 042001 (2003)]



Frequency independent input squeezing
& frequency dependent readout
[squeezed-variational]
[Buonanno and Chen, gr-qc/0310026, PRD accepted]

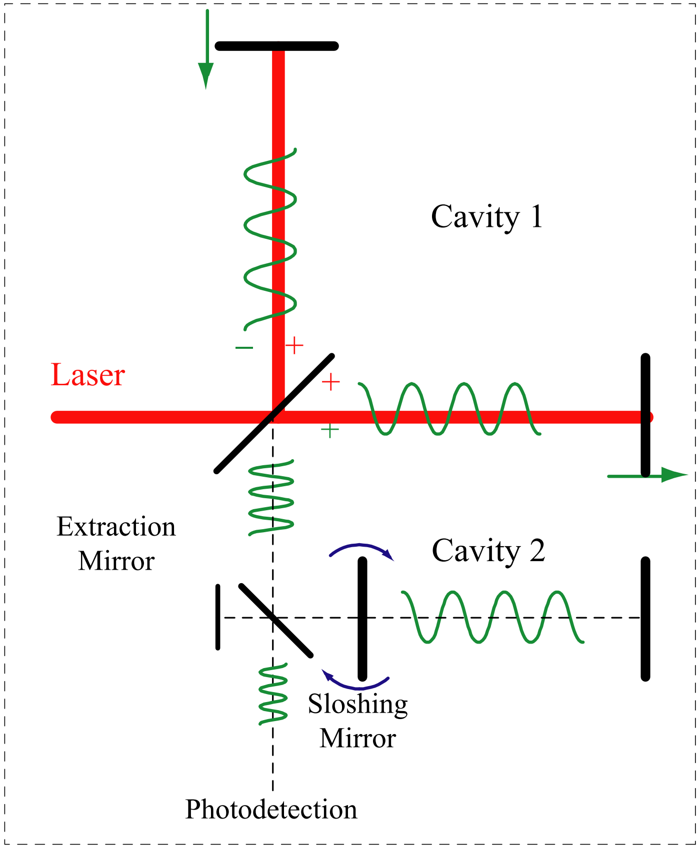
Speed Meters:
Reshaping Optical Response

The Speed Meter: Michelson Topology

- Michelson Speed Meter, one more km-scale cavity [Purdue and Chen, PRD **66**, 122004 (2002), based on conceptual design of Braginsky and Khalili]
- Sideband sloshing between two cavities at period τ_s
  - Each time flips phase by π
  - Sideband field in cavity1 generated by:

$$\underbrace{X(t)}_{\text{directly}} - \underbrace{X(t - t_{\text{slosh}})}_{\text{back from cavity 2}} + \ldots - \ldots$$

  - No sensitivity to displacement if $f<f_s$
  - Broadband sensitivity to speed obtained when $\tau_{ext} \sim \tau_s$



The Speed Meter: Michelson Topology

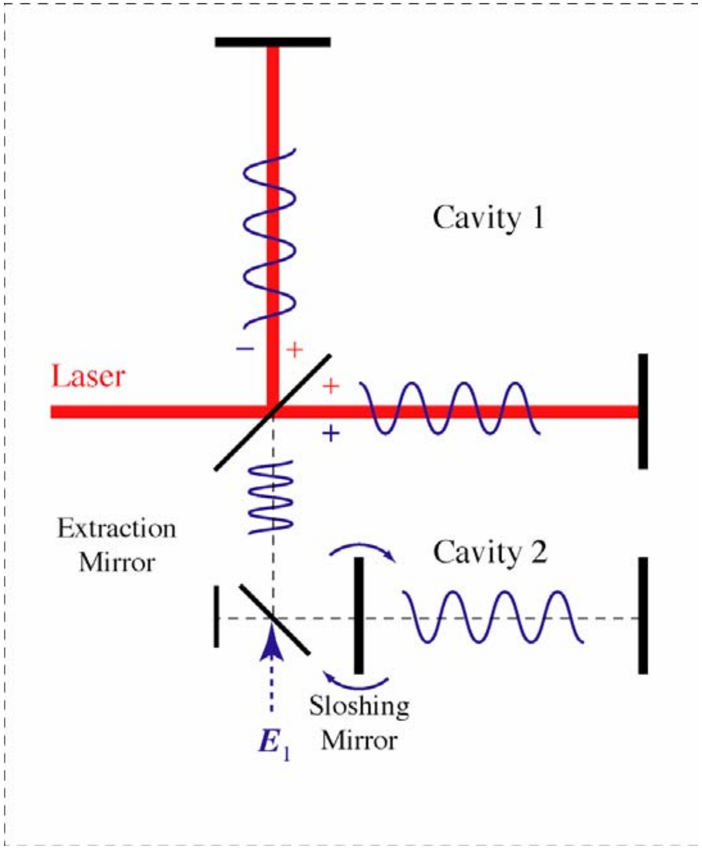
- Radition pressure noise caused by E_1 of the input vacuum field
- Like signal sideband, E_1 also sloshes between the two cavities, so
- Force acting on mirror in cavity 1

$$\sqrt{I_c}\Big[\underbrace{E_1(t)}_{\text{direct}} - \underbrace{E_1(t-t_{\text{slosh}})}_{\text{back from cavity 2}} + \ldots - \ldots\Big]$$

- Broadband sensitivity to speed obtained when $\tau_{ext} \sim \tau_s$ --- at the same time,

$$F = m\dot{V}_{\text{BA}} \propto \sqrt{I_c}\dot{E}_1(t)$$

$$\Rightarrow mV_{\text{BA}} \propto \sqrt{I_c}E_1(t)$$



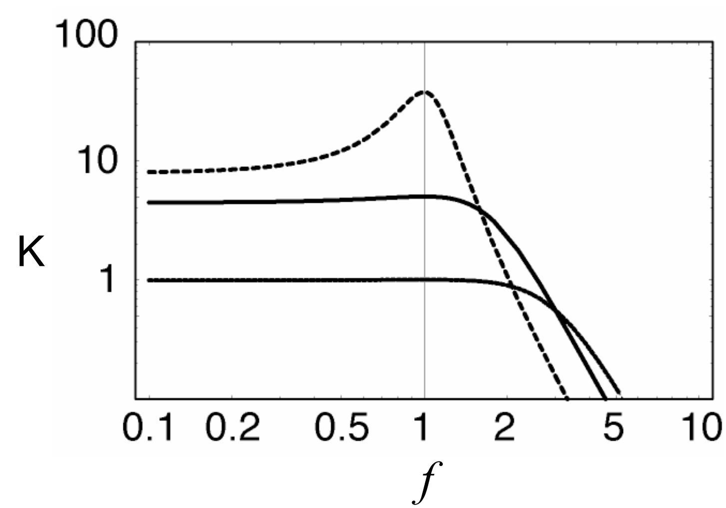
Speed Meter: Structure of Response

- For frequencies below f_s

$$\text{phase quadrature} \quad [\textit{max signal}] \quad E_2^{\text{out}} = E_2^{\text{in}} + \sqrt{I}\Big[\overbrace{\frac{\sqrt{I}}{M}E_1^{\text{in}}}^{V_{\text{BA}}} + V_{\text{GW}}\Big]_{\text{total } V}$$

$$\text{amplitude quadrature} \quad [\textit{pure noise}] \quad E_1^{\text{out}} = E_1^{\text{in}}$$

- Constant K below f_s:
  - Constant optimal quadrature for Rad. Pres. Noise removal
  - Constant optimal squeeze quadrature
- **No need for KLMTV filters for QND**
- Reflection: Ω^{-2} (Newton`s law) become Ω^{0}
  - Measuring d X / dt instead of X
  - Rad Pres Noise ∝ d E / dt

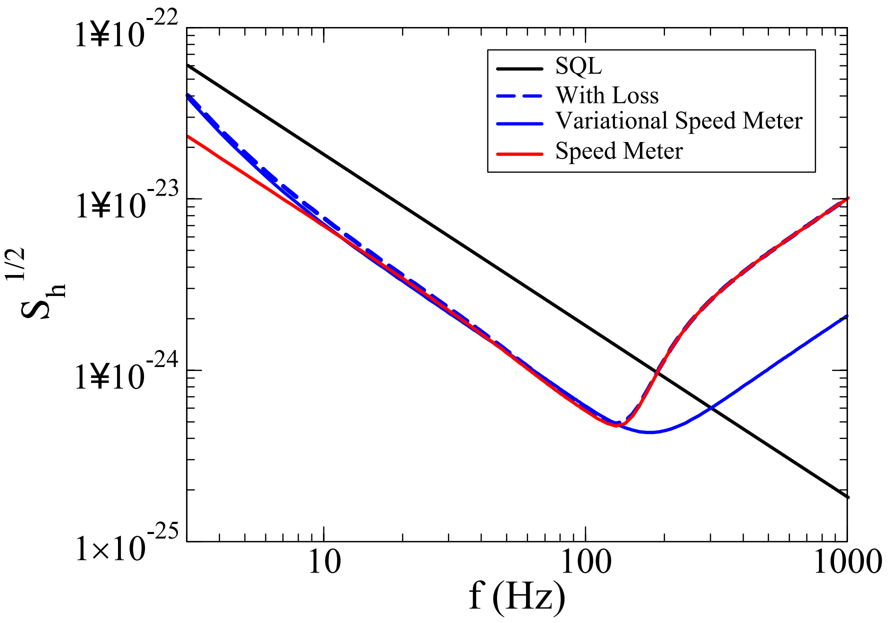


The Speed Meter: Performance

- Beating SQL by uniform factor below *sloshing frequency*, even if
  - Frequency independent quadrature measured
  - Frequency independent squeezing used
- Less susceptible to optical losses
- ... all come from the modified frequency response!
- Loss Limit:

$$\sqrt{S_h} \geq (e^{-2R}\epsilon)^{1/4} h_{\mathrm{SQL}}$$

[obeyed also by conventional interferometers, but not with equal sign!]

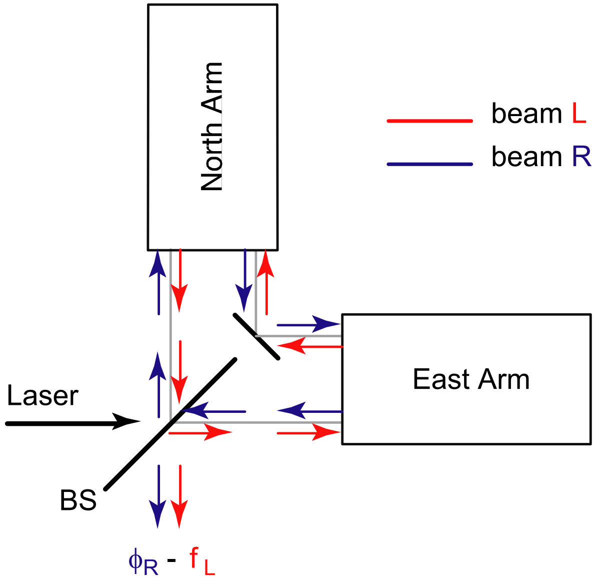


800kW, 10dB squeezing and ~1% total loss [sloshing cavity and filters 4km long, round-trip loss 20ppm]

Formal Theory of Speed Meters

- Motivation: Momentum is QND observable ⇒ No Standard Quantum Limit
  [Braginsky and Khalili, Phys. Lett. A**147**, 251 (1990)]
  - Measuring momentum kicks position (canonoical conjugate)
  - But position noise does not enter future evolution of momentum
- [... Photons are kicking the mirrors ⇒ There is still radiation-pressure noise!]
- Formally reconciles: [Khalili, gr-qc/0211088]
  - Distinguishing canonical momentum (p) and kinetic momentum (mv).

Speed Meter: Sagnac Interferometer



For each individual beam:

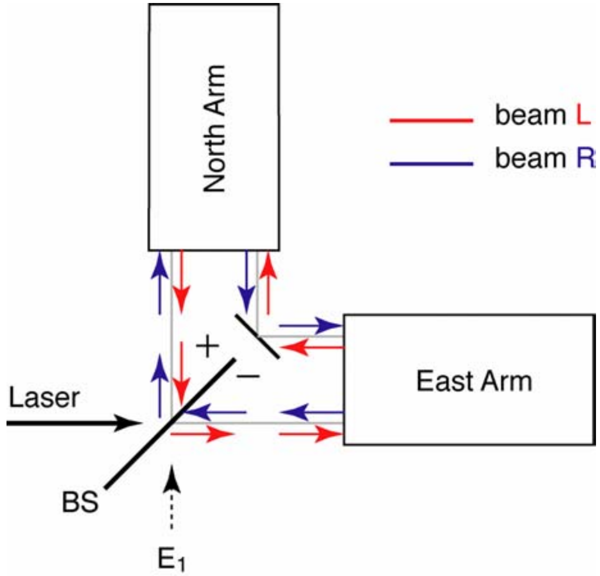
$$\begin{aligned}\phi_R &= X_N(t-\tau) + X_E(t) \\ \phi_L &= X_E(t-\tau) + X_N(t)\end{aligned}$$

Upon recombination:

$$\begin{aligned}& \phi_R - \phi_L \\ = & \left[X_E(t) - X_E(t-\tau)\right] - \left[X_N(t) - X_N(t-\tau)\right] \\ \sim & \dot{X}_{\rm diff}\end{aligned}$$

- Sagnac is automatically speed meter --- without extra cavities!

Speed Meter: Sagnac Interferometer



Now Radiation Pressure Noise:

$$\left[m\dot{V}_E(t)\right]_{\rm BA} = \sqrt{I_c}\Big[\underbrace{-E_1(t)}_{\text{L beam}}\underbrace{+E_1(t-\tau)}_{\text{R beam}}\Big]$$

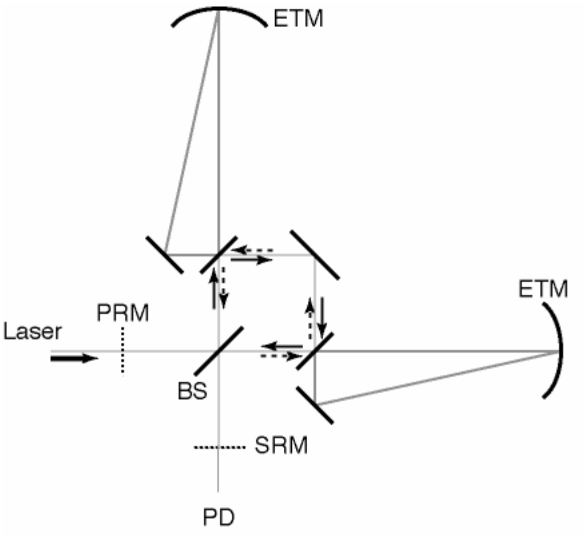
$$\left[m\dot{V}_N(t)\right]_{\rm BA} = \sqrt{I_c}\Big[\underbrace{+E_1(t)}_{\text{R beam}}\underbrace{-E_1(t-\tau)}_{\text{L beam}}\Big]$$

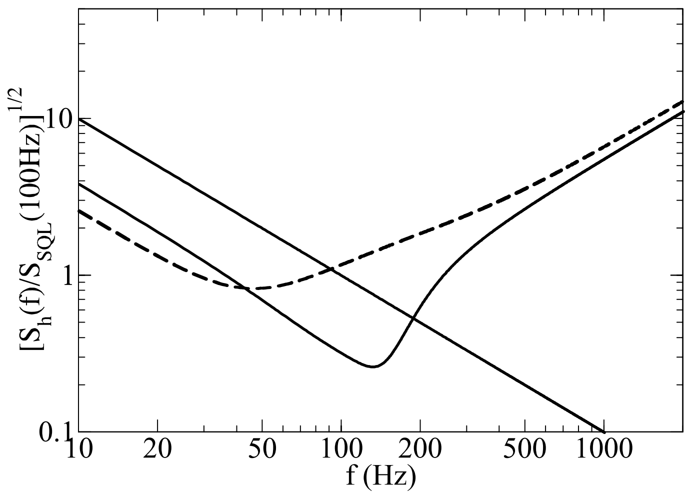
$$\Rightarrow \left[m\dot{V}_{\rm diff}\right]_{\rm BA} = 2\sqrt{I_c}\Big[-E_1(t)+E_1(t-\tau)\Big] \approx -2\tau\sqrt{I_c}\dot{E}_1(t)$$

Similar to Michelson Speed Meter!

- Sagnac is automatically speed meter --- without extra cavities!

Speed Meter: Sagnac Interferometer

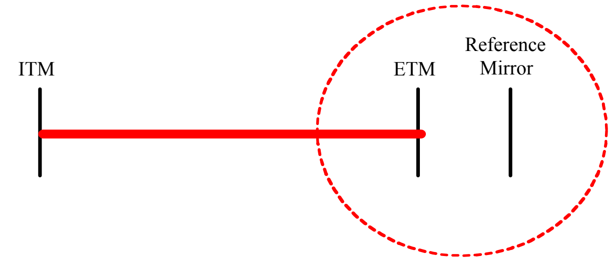


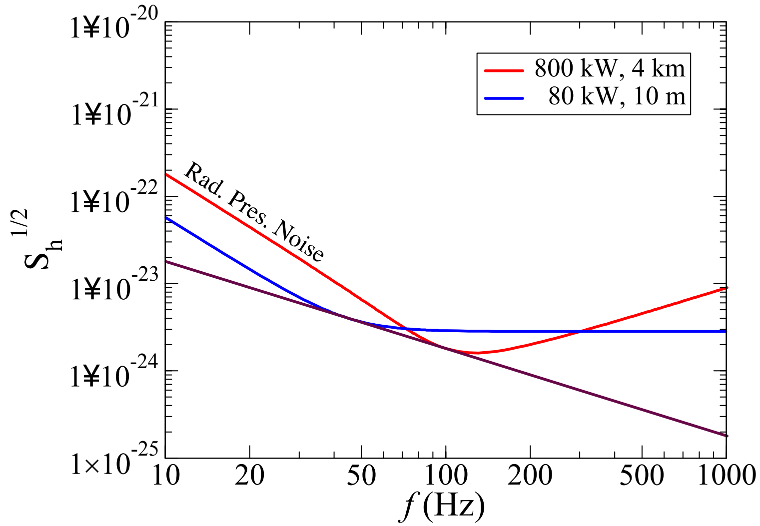


- Sagnac is automatically speed meter --- without extra cavities!
- Confirmation of QND performance for simple configurations: mathematically equivalent to Michelson Speed Meter [Chen, PRD 67, 122004 (2003)]
- Further study with another configuration, including losses [Danilishin, gr-qc/0312016]

Feedback/Cancellation Approach

- In the proper reference frame of a *free mass* near the End Test Mass: the ETM motion is **only** Radiation Pressure Noise, so
- If one can measure that motion using ...
  - { Auxiliary cavity with lower power, then large fraction of the RPN can be removed ⇒ Follow the SQL [Kawamura, private communication, 2001; Courty, Heidmann and Pinard, PRL **90**, 083601 (2003).]
  - { Auxiliary QND meter ⇒ Removal of RPN to below the SQL [Courty, Heidmann and Pinard, Europhys. Lett. **63**, 226-232 (2003) .]





Summary

- Modifying test mass dynamics (Optical Spring)
  - Circumventing free-mass SQL by spring resonance
- Modifying input-output optics (KLMTV Filters)
  - Conventional interferometers
    - Noise Correlations; Frequency-dependent qudrature rotation
    - Filter Parameter Calculation
    - Importance of losses
  - Signal Recycling Interferometers
    - Two regimes: mechanics/optics
- Reshaping optical coupling constant (K) -- Speed Meters
  - Michelson
  - Sagnac
- Feedback or Cancellation of Radiation Pressure Noise