CONSTRAINED EVOLUTION: CONCEPTS AND NEW RESULTS

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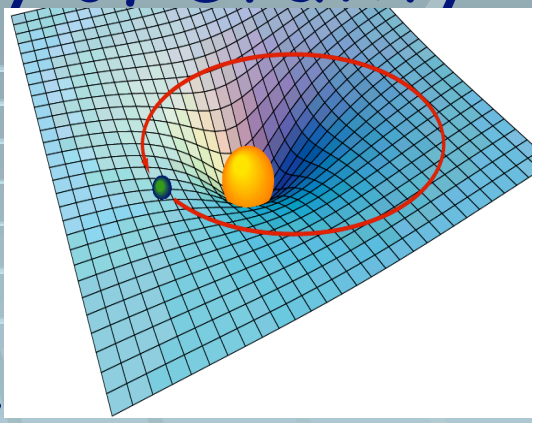
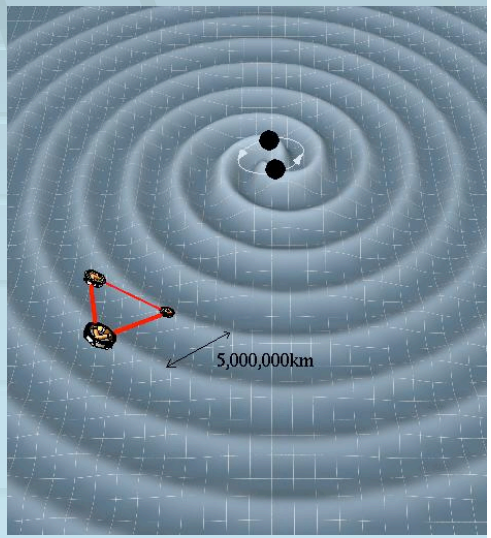
General Relativity

- General Relativity is *the* Theory of Gravity



  - GR descries Gravity as curvature

- GR "is" Newtonian Theory plus waves



Newtonian Gravity

- (Poisson:) $\nabla^2\Phi = \Phi^{,i}{}_{,i} = 4\pi\rho_{source}$

Waves: like E&M

- (divergence:) $E^{i}{}_{,i} = source$
- (wave equation)

$$\frac{\partial A_i}{\partial t} = E_i + \ldots.$$

$$\frac{\partial E_i}{\partial t} = [\nabla\times(\nabla\times A)]_i$$

$$= \nabla\nabla^2 A_i + \ldots.$$

General Relativity

- (Poisson) $\prod^2 \square = \frac{1}{8}[\tilde{R}\square + \frac{2}{3}\tilde{K}^2 \square^5$
  $+\square^7 (\tilde{K}_a{}^b - \frac{1}{3}\square_a{}^b \tilde{K})(\tilde{K}_b{}^a - \frac{1}{3}\square_b{}^a \tilde{K})] + src$

- (Divergence) $\prod_i (K^i{}_j - \frac{1}{3}\square_j K^b{}_b) = src'_j$

  (K_{ij} is the analog of E_i)

- (Wave Equation): $\frac{\partial}{\partial t} g_{ij} = -2K_{ij}$

  $\frac{\partial}{\partial t} K_{ij} = R_{ij} + K_i{}^c K_{jc} + K_{ij} K^a{}_a$

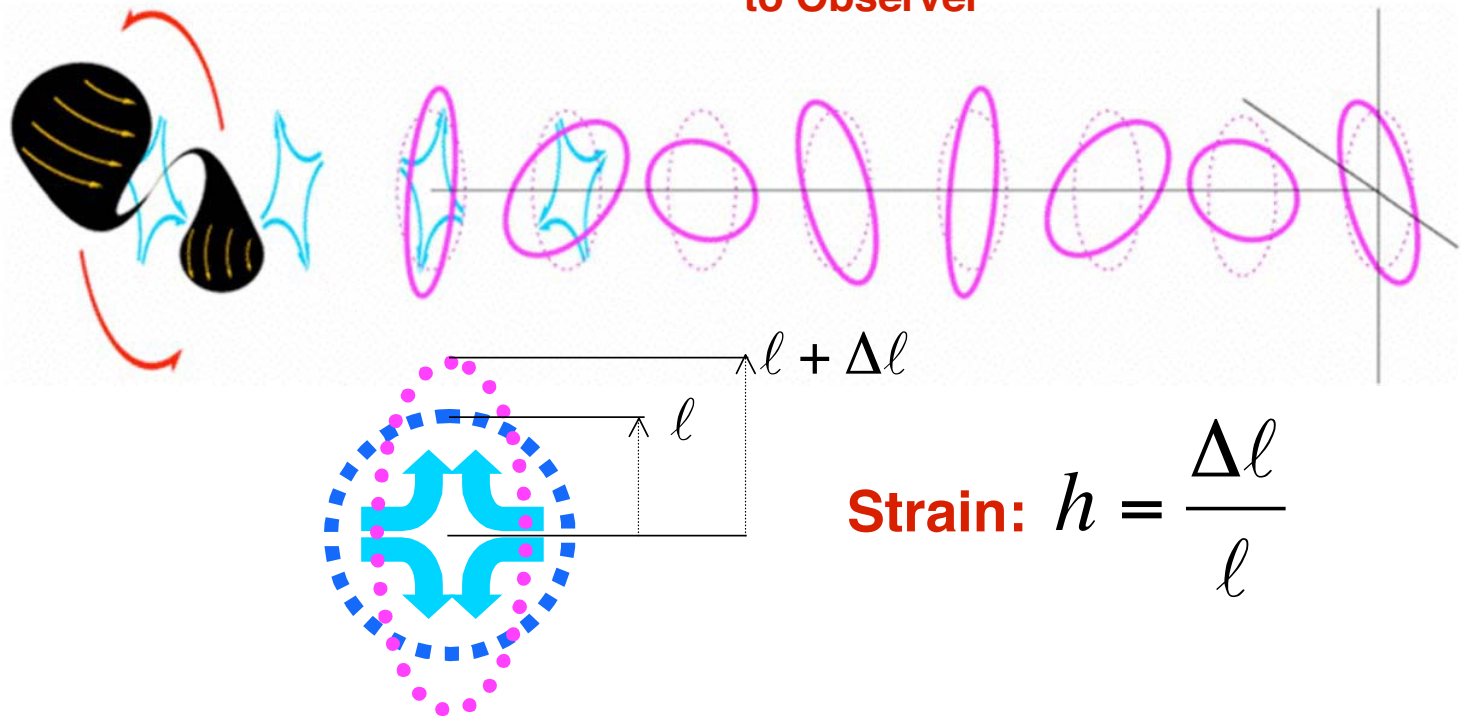
(special coordinates)

Gravitational Radiation and Detectors:
Gravitational Radiation

Source: Bulk Motion Produces Changing Tidal Field

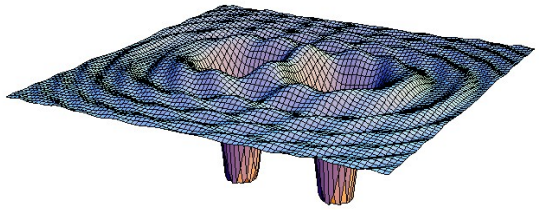
Oscillating Tidal Field Propagates (Unobstructed) to Observer

Observer Detects Distortion Strain



Strain: $h = \dfrac{\Delta\ell}{\ell}$

Binary black holes



From the GW detection point of view: the most promising source

From the theoretical point of view:

- Binary BH = the two body problem in General Relativity
- Extreme GR $\Longrightarrow$ probes the limit of GR (as weak field limit of string theory)

From the astrophysical point of view:

- Rate of binary black hole coalescence $\Longrightarrow$ massive star evolution
- Inspiral GW signal $\Longrightarrow$ precise measure of Hubble constant H_0
- GW observations of supermassive BH at high z $\Longrightarrow$ large structure formation

GR "is" Newtonian Gravity plus waves

3-metric is analog of A_i

$$g_{ij} = \Phi^4 \tilde{g}_{ij}$$

TRIAL METRIC

ANALOG OF NEWTONIAN Φ

GR "is" Newtonian Gravity plus waves

Φ modifies your "trial" $\tilde{g}_{ij}$.

Choose well, and $\Phi \sim 1$.

$$g_{ij} = \Phi^4 \tilde{g}_{ij}$$

TRIAL METRIC

$\Phi \sim 1$: physically motivated guess close to actual data

KERR-SCHILD FORM

Schwarzschild: M/r

- Metric g_{ab} = *flat space + pole*

- **EXACT BLACK HOLE SOLUTION!**

  --Kerr, or Schwarzschild-

KERR-SCHILD MULTI-BH DATA SETTING

Approximate (g_{ab})

M_1/r_1 M_2/r_2

- Metric $\tilde{g}_{ab}$ = *flat space* + $pole_1$ + $pole_2$

  - **APPROXIMATE 2-BLACK HOLE SOLUTION-**

    --Kerr, or Schwarzschild

    --arbitrary mass, spin, location, momentum

KERR-SCHILD MULTI-BH DATA SETTING

Approximate (g_{ab})

$g_{ab} = \Omega^4 \tilde{g}_{ab}$; find Ω | |1

-differences less than 1%

GOOD WAY TO SET DATA!

-K_{ab} changes are <u>small</u> also

Getting Started: INITIAL DATA

- INITIAL DATA (CONSTRAINT EQUATIONS)
- Solve them first-
- Then evolution ("wave") equation preserves the constraints *analytically.*

EVOLUTION in Full GR (e.g. *vacuum* Binary B H)

- Solve initial data problem

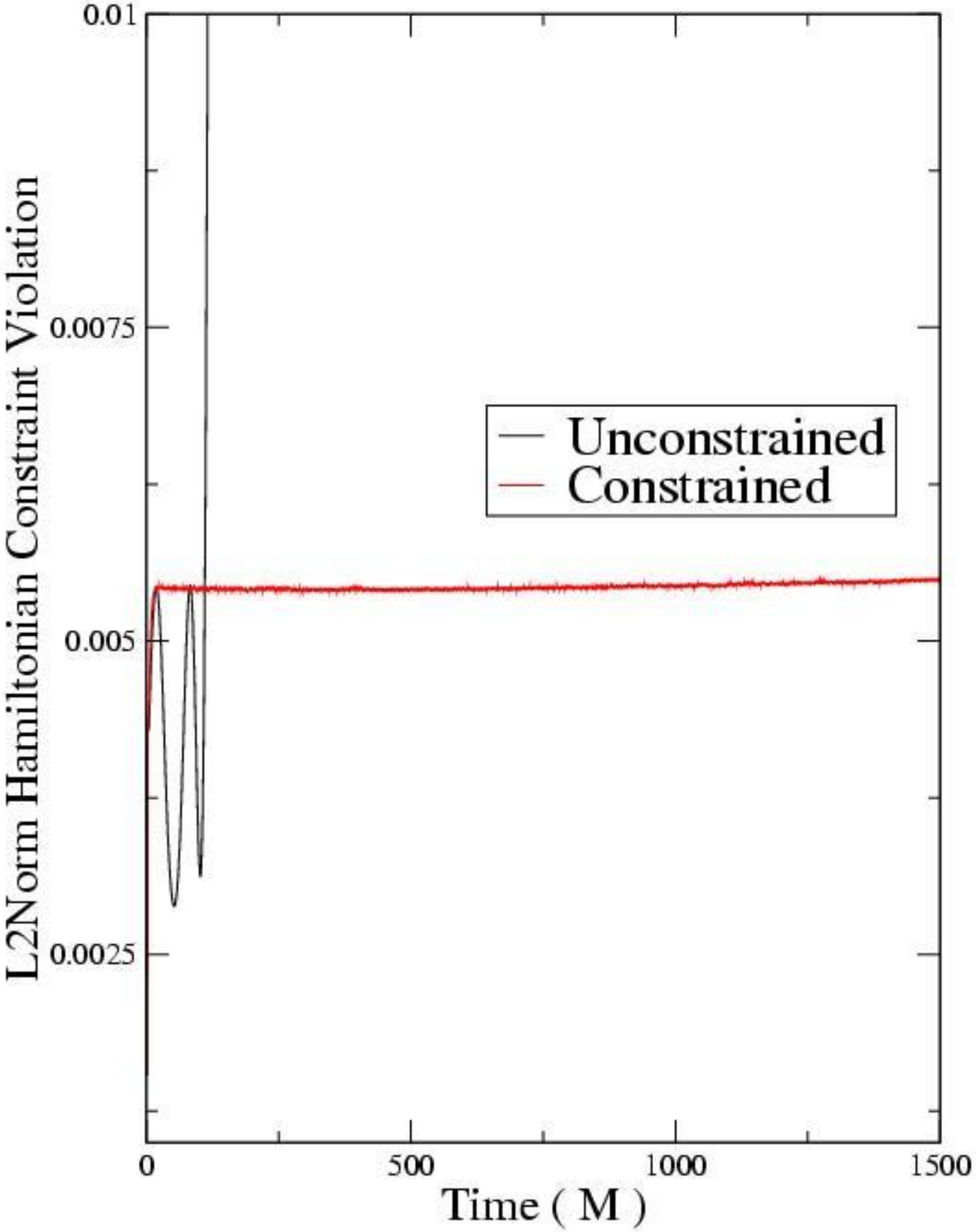
- Evolve time equations one step,

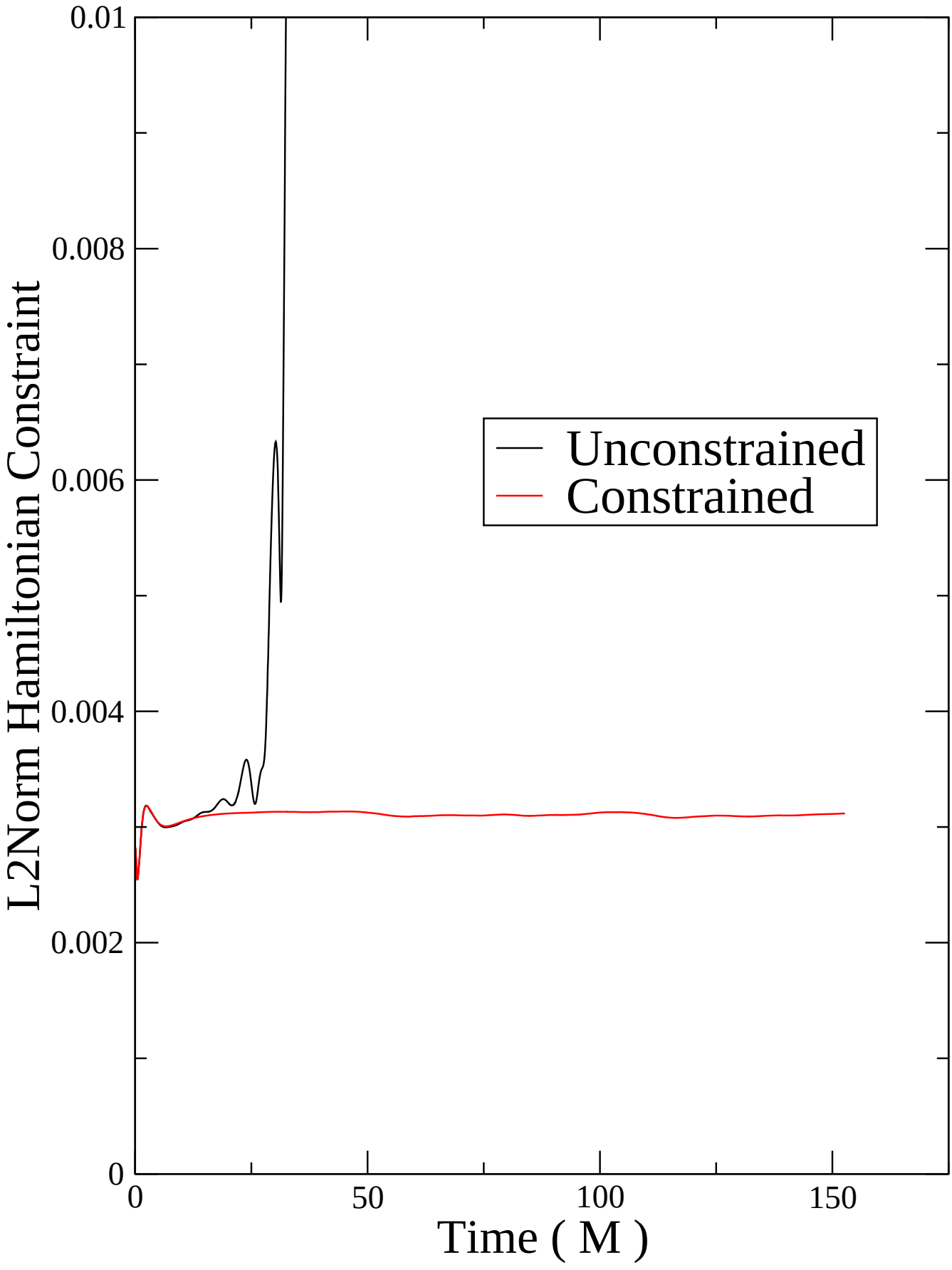
  get new g_{ab}, K_{ab}.
- Use this new g_{ab} as $\tilde{g}_{ab}$ in a new initial data solve:

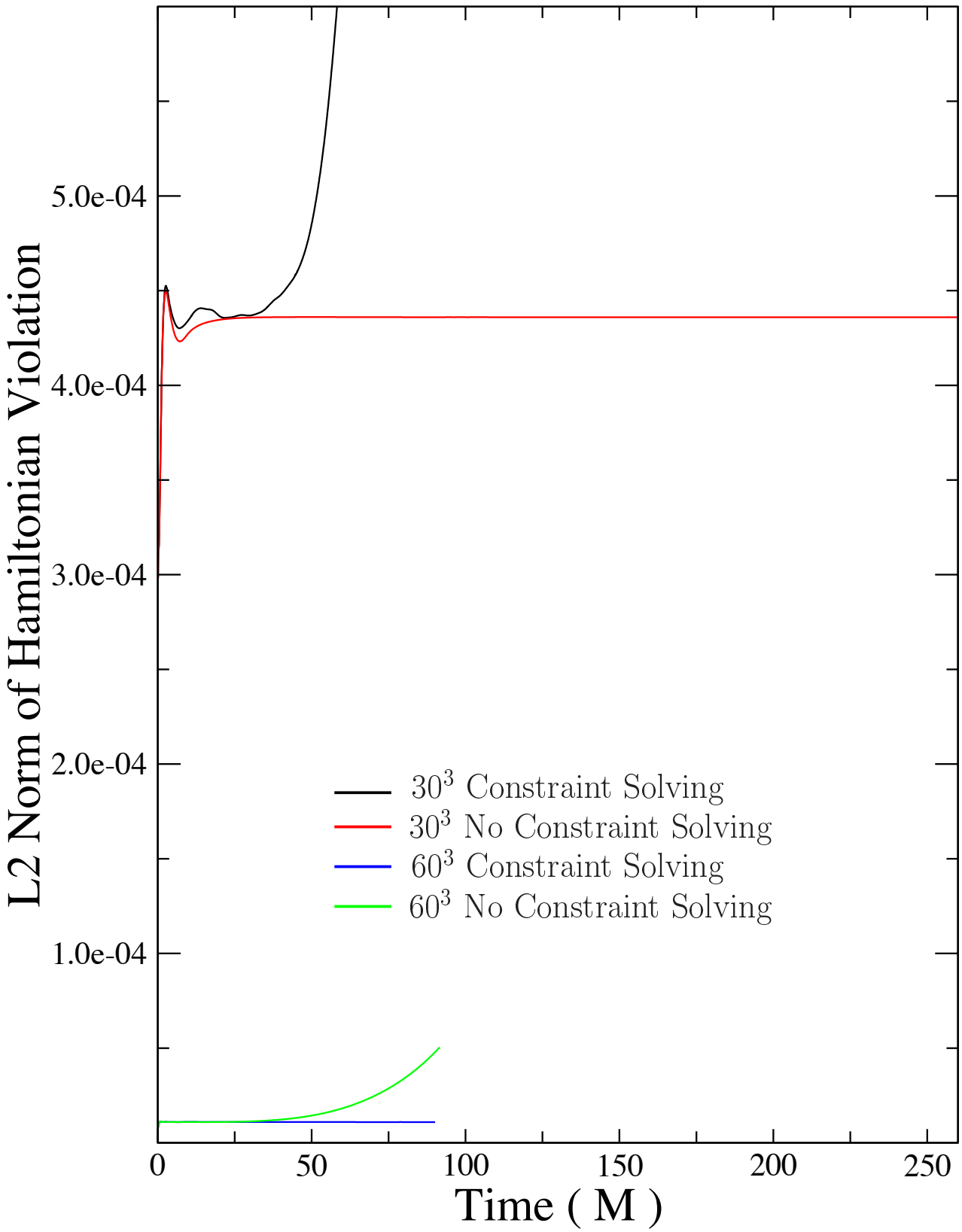
  $\square = 1 + \square$, $|\square| \ll 1$; change in $K_{ab} \ll 1$.

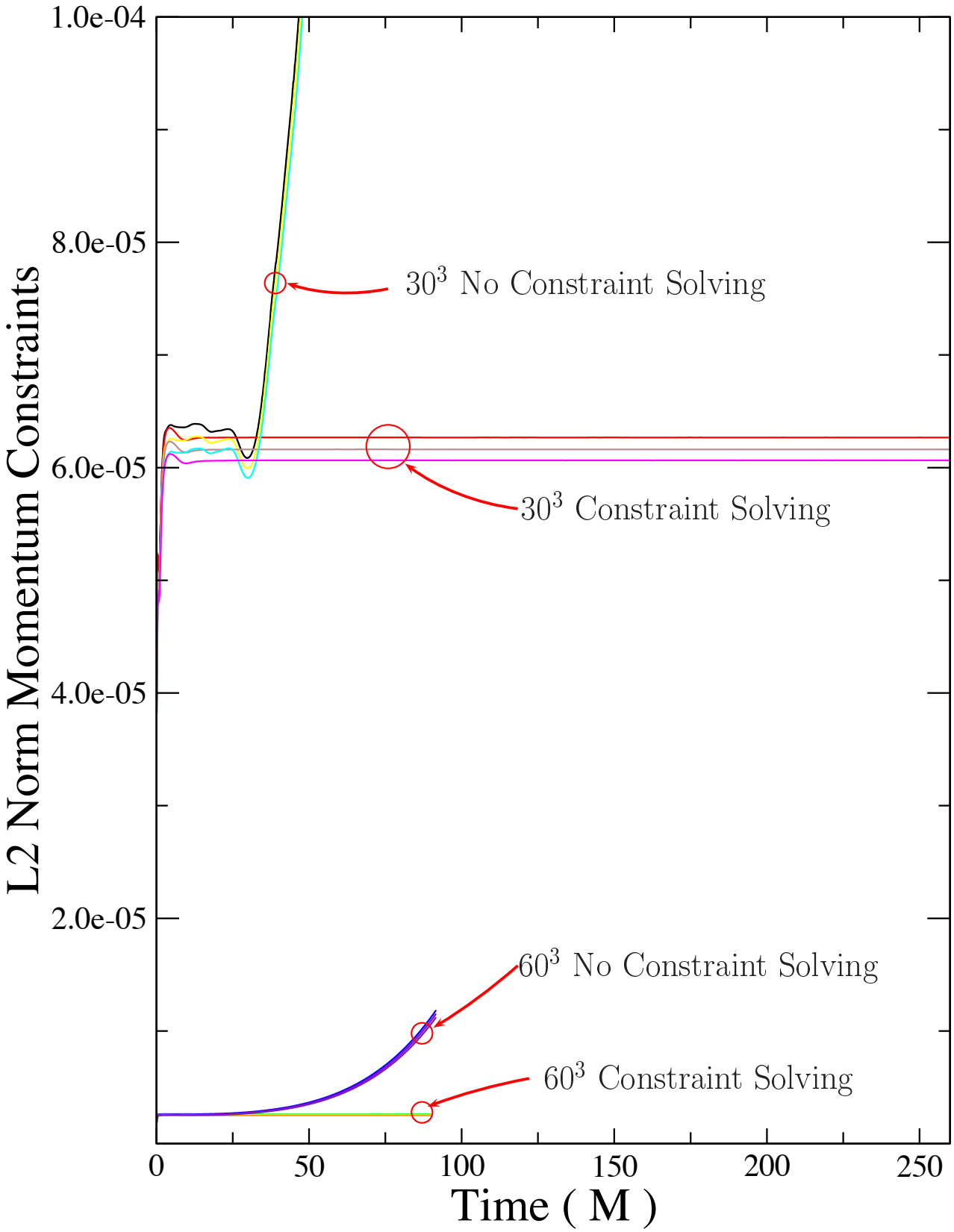
  ----this gives definitive g_{ab} K_{ab}
- REPEAT ("linearization")

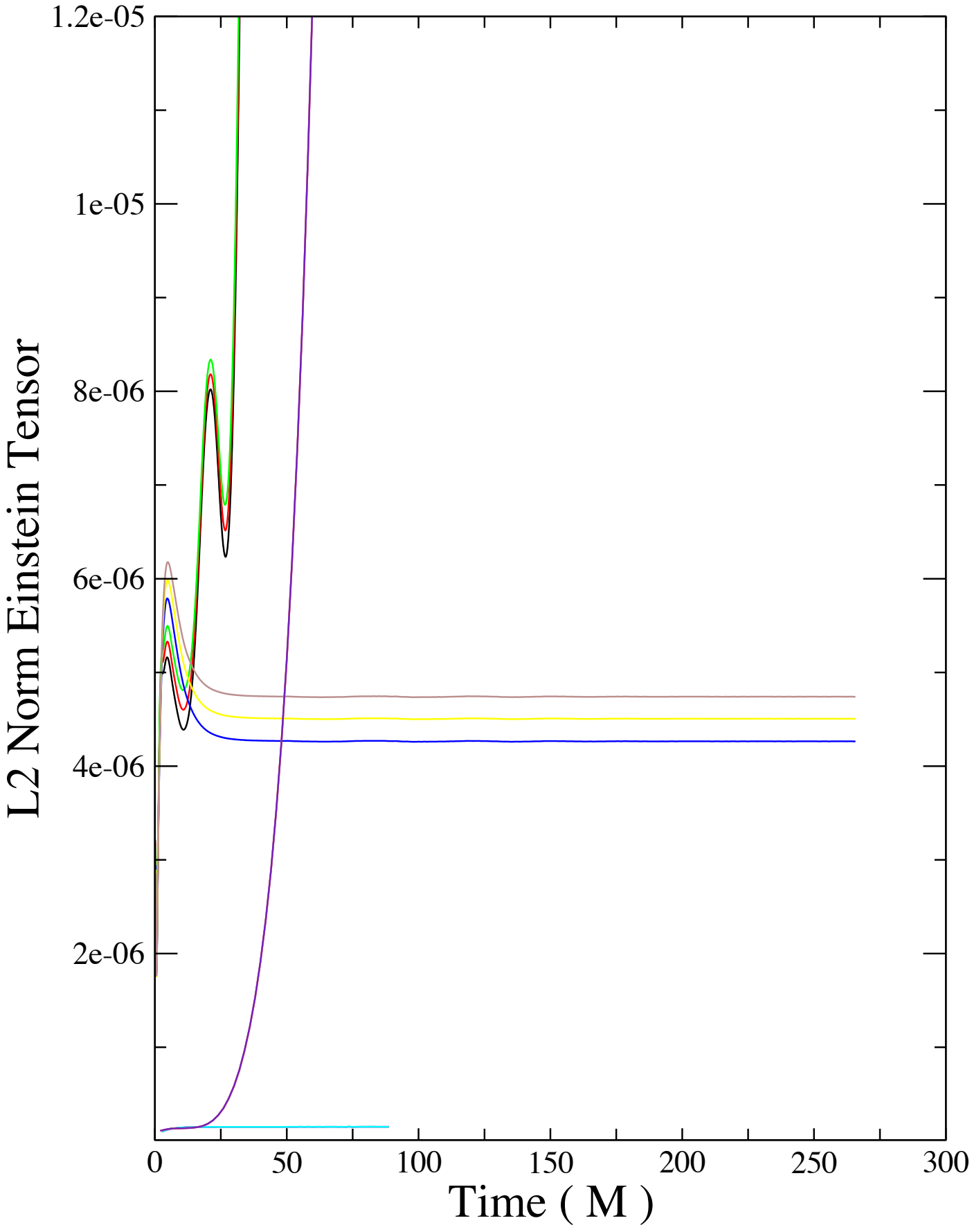
1-D Constrained Evolution

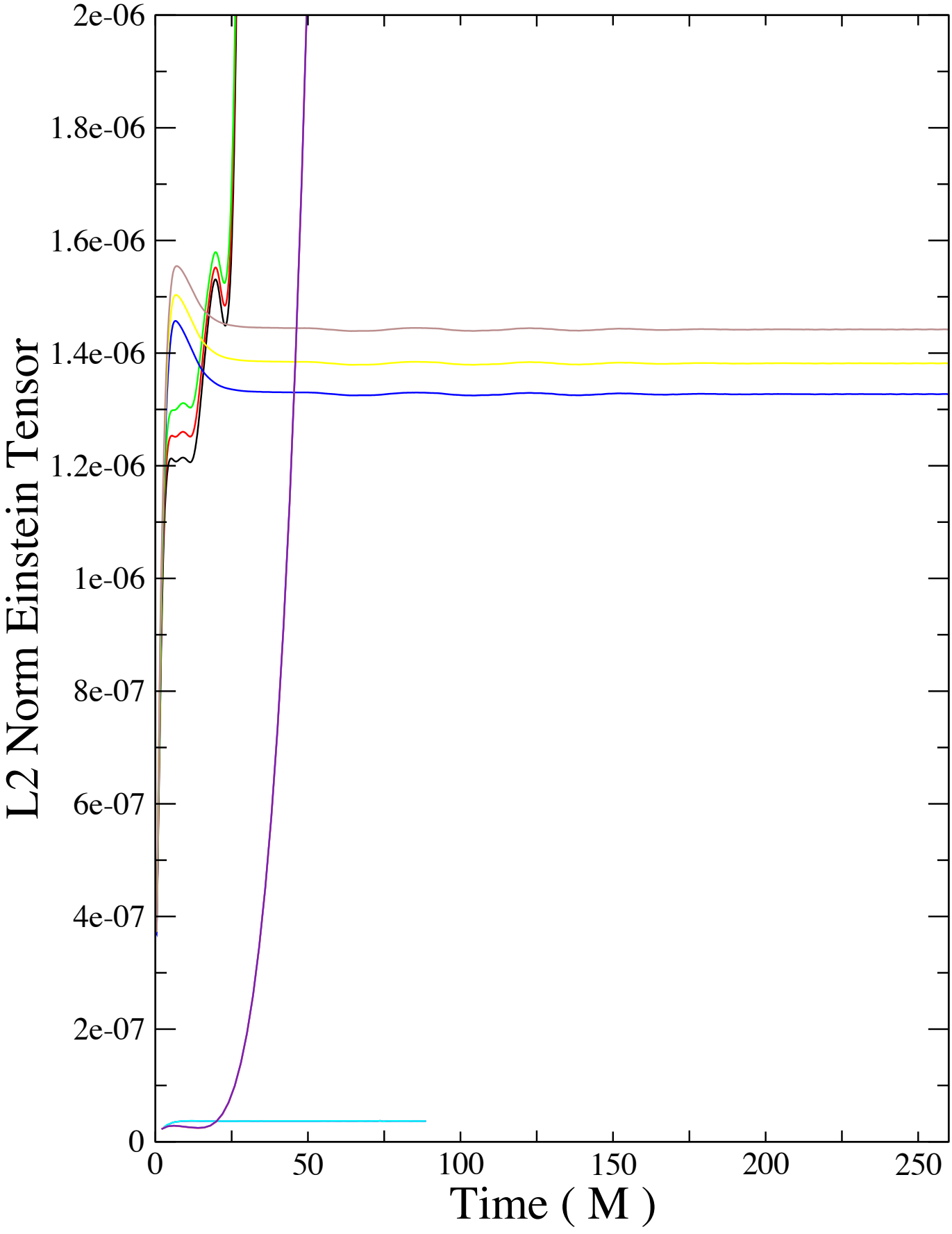


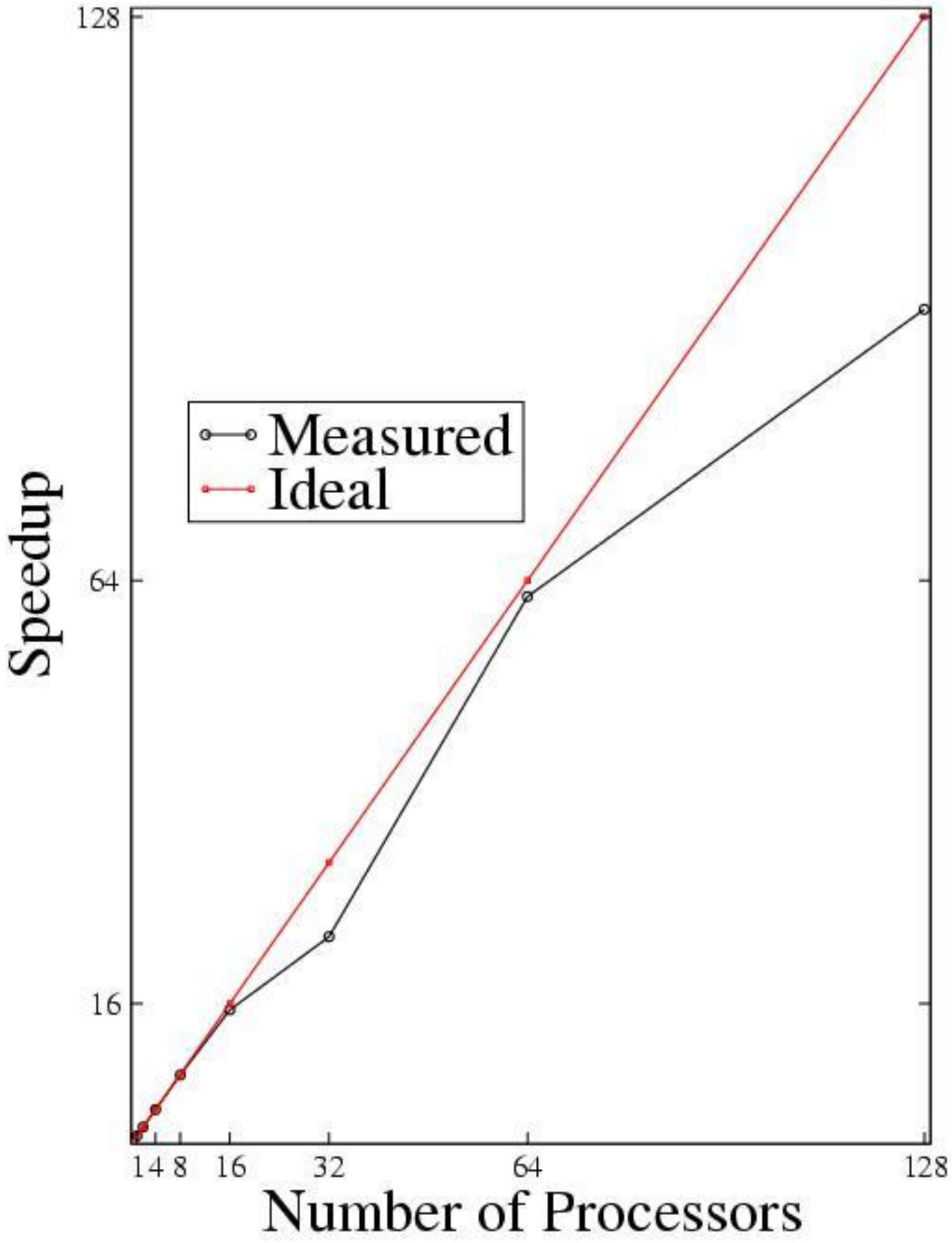

3-D Constrained Evolution
L2Norm Hamiltonian Constraint
Time (M)
Unconstrained
Constrained
0.01
0.008
0.006
0.004
0.002
0
0
50
100
150


L2 Norm of Hamiltonian Violation
5.0e-04
4.0e-04
3.0e-04
2.0e-04
1.0e-04
0
50
100
150
200
250
Time (M)
30^3 Constraint Solving
30^3 No Constraint Solving
60^3 Constraint Solving
60^3 No Constraint Solving


1.0e-04
8.0e-05
6.0e-05
4.0e-05
2.0e-05
L2 Norm Momentum Constraints
0
50
100
150
200
250
Time (M)
30^3 No Constraint Solving
30^3 Constraint Solving
60^3 No Constraint Solving
60^3 Constraint Solving


1.2e-05
1e-05
8e-06
6e-06
4e-06
2e-06
L2 Norm Einstein Tensor
0
50
100
150
200
250
300
Time (M)


L2 Norm Einstein Tensor
2e-06
1.8e-06
1.6e-06
1.4e-06
1.2e-06
1e-06
8e-07
6e-07
4e-07
2e-07
0
0
50
100
150
200
250
Time (M)


128
64
16
Speedup
Measured
Ideal
1 4 8 16 32 64 128
Number of Processors

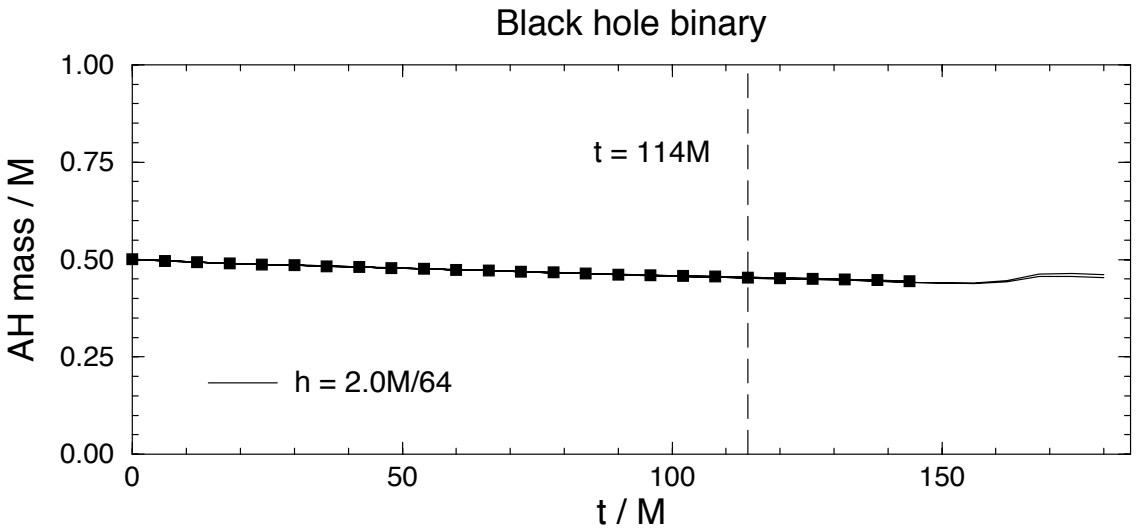


Figure 1: Evolution of the AH mass for the black hole binary with $\rho_0 = 3.0M$. The evolution lasts longer than one orbital period of $114M$ defined by the initial data. The squares mark a run with 7 nested levels with coarsest resolution $2M$ and finest resolution $h = 0.03125M$, and with the spherical outer boundary at about $48M$, which crashes around $145M$. Also plotted are results from seven control runs with the outer boundary at $24M$ and $96M$, with a cubical outer boundary, and with the AH extracted on a coarser grid to check its convergence. (Bruegmann, Tichy, Jansen, gr-qc/0312112).

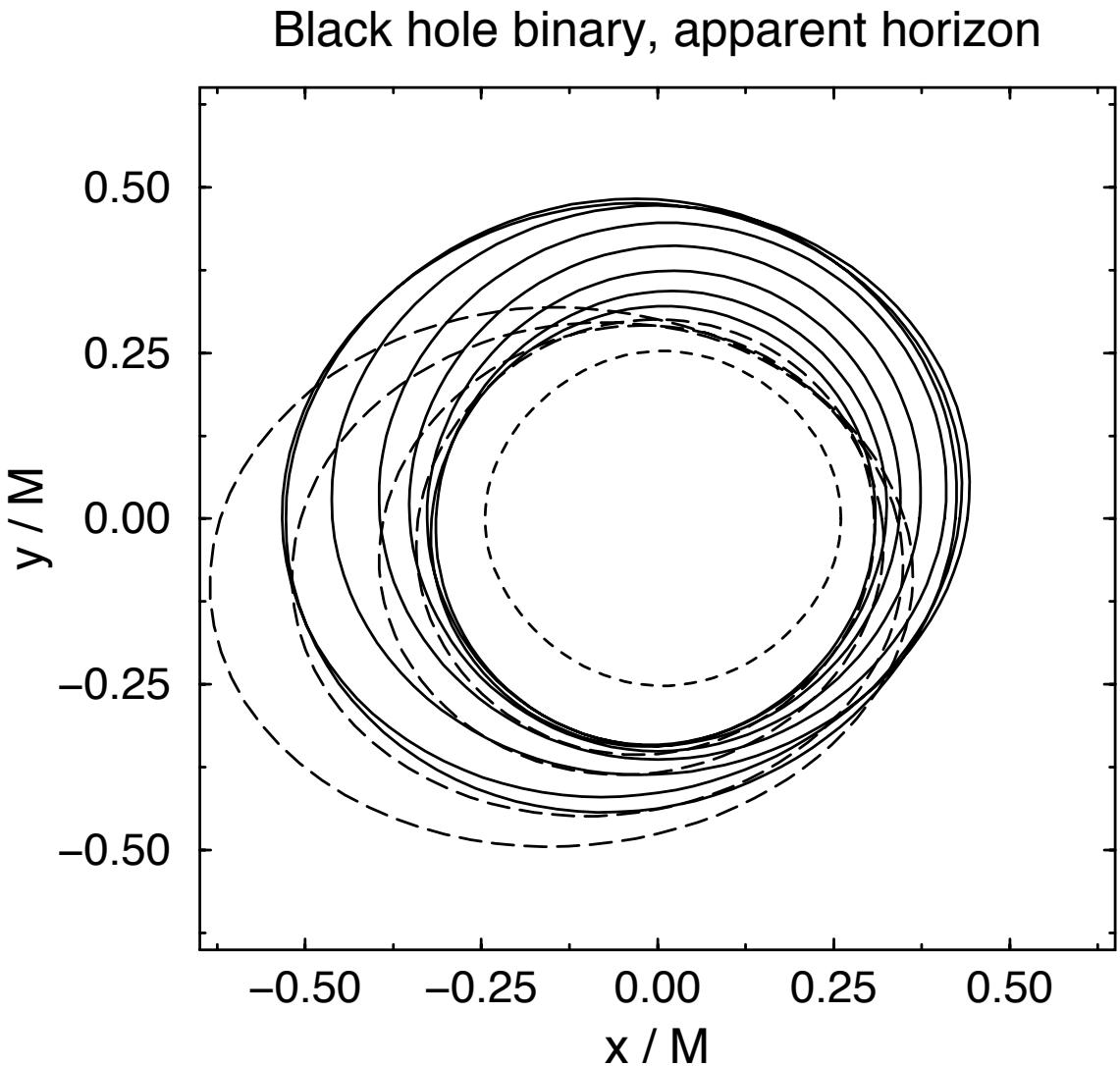


Figure 2: The coordinate location of the AH around one of the black holes in the x-y-plane is shown every $12M$ for a typical run. The AH starts as the smallest circle (dashed) in the center, quickly moves out to the largest circle within $5M$ of evolution while the gauge adjusts itself near the black hole, slowly shrinks toward the center, until eventually (long dashed lines) it drifts out of shape before the run fails around $145M$. (Bruegmann, Tichy, Jansen, gr-qc/0312112).

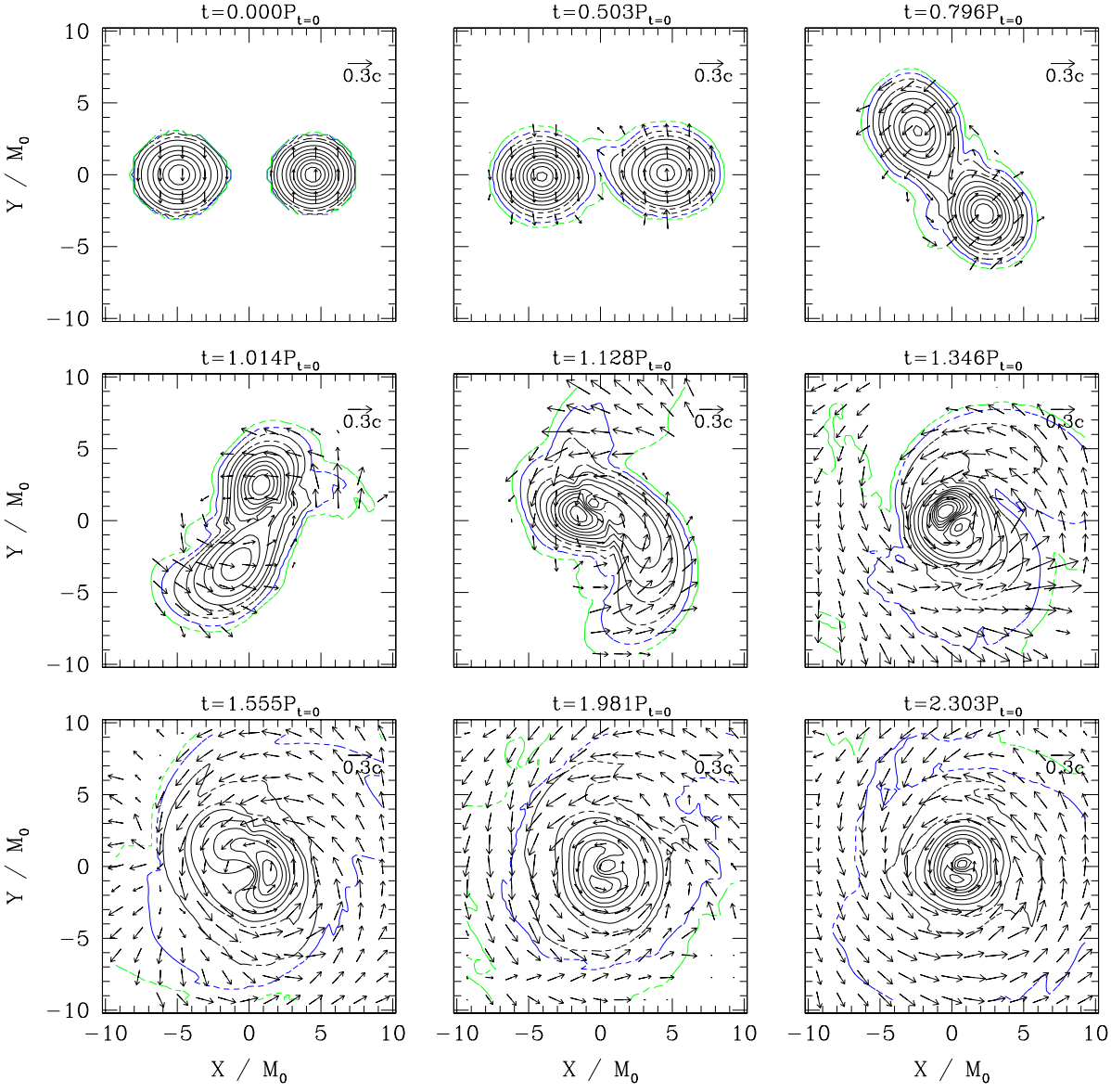


FIG. 1. Snapshots of the density contour curves for ρ in the equatorial plane for model M1315. The solid contour curves are drawn for $\rho/0.15 = 1 - 0.1j$ for $j = 0, 1, 2, \cdots, 9$, and the dashed-solid curves are for $\rho/0.15 = 0.05$, 0.01, 10^{-3} and 10^{-4}. Vectors indicate the local velocity field (v^x, v^y), and the scale is shown in the upper right-hand corner. $P_{t=0}$ denotes the orbital period of the quasiequilibrium configuration given at $t = 0$. The length scale is shown in units of GM_0/c^2, where M_0 is the gravitational mass computed at $t = 0$. (Shibata, Taniguchi, Uryu, gr-qc/0310030.)

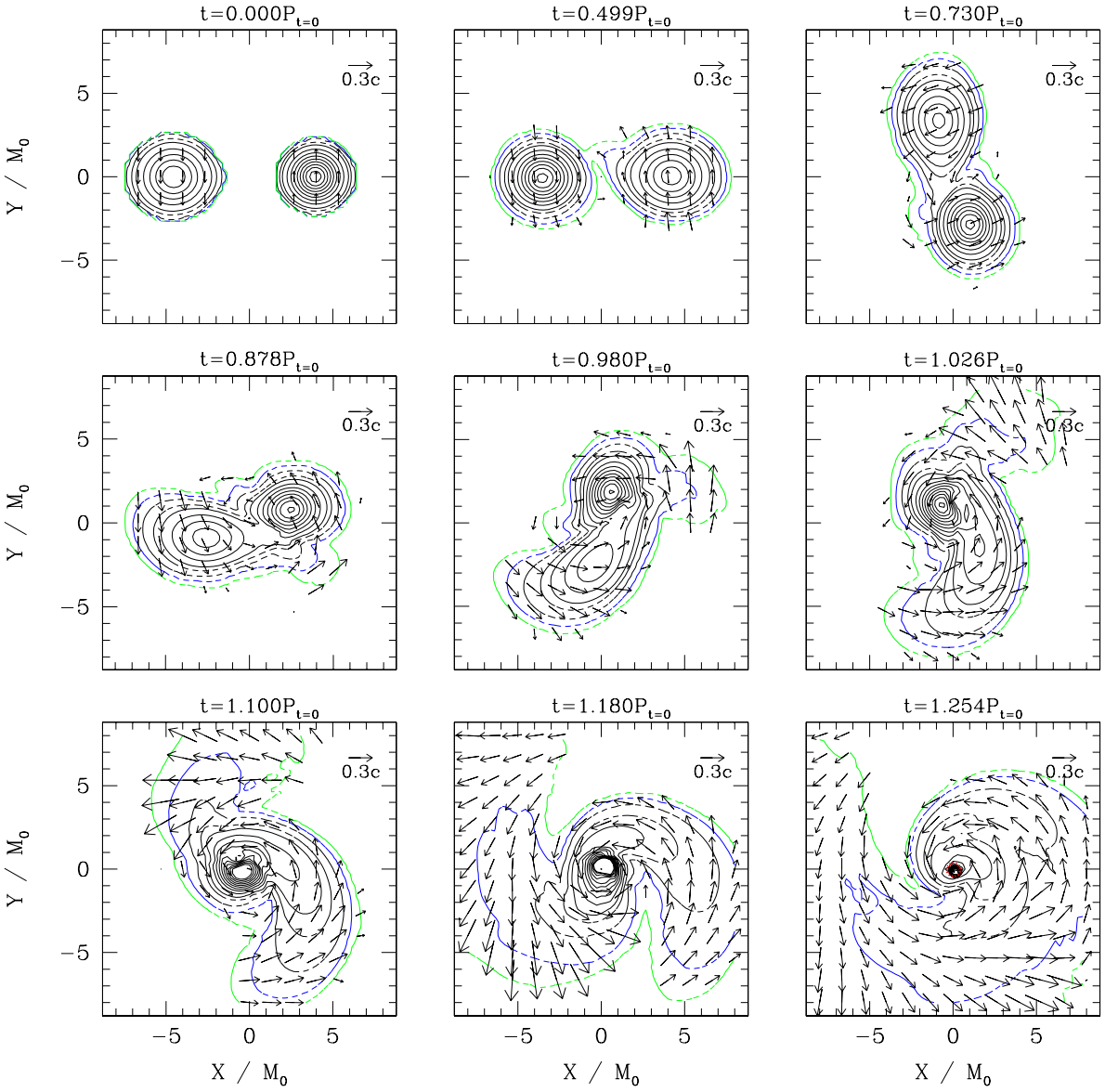


FIG. 2. The same as Fig. 1 but for model M1418. Here, the solid contour curves are drawn for $\rho/0.20 = 1 - 0.1j$ for $j = 0, 1, 2, \cdots, 9$, and the dashed-solid curves are for $\rho/0.20 = 0.05$, 0.01, 10^{-3} and 10^{-4}. The thick dotted circle in the last panel of radius $r \sim 0.5M_0$ denotes the location of the apparent horizon. (Shibata, Taniguchi, Uryu, gr-qc/0310030.)

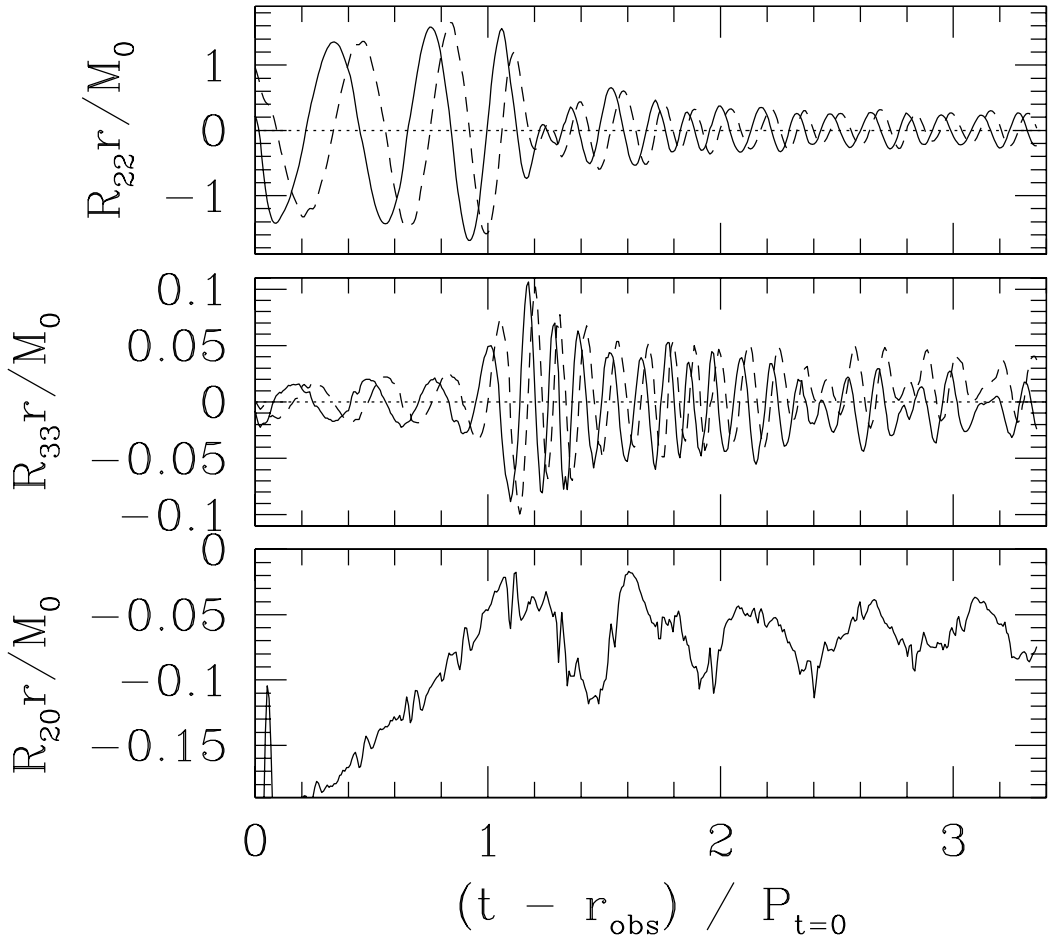


FIG. 3. $R_{22\pm}r$, $R_{33\pm}r$, and $R_{20}r$ as a function of the retarded time for model M1315. The solid and dashed curves for R_{22} and R_{33} denote R_{lm+} and R_{lm-}. (Shibata, Taniguchi, Uryu, gr-qc/0310030.)

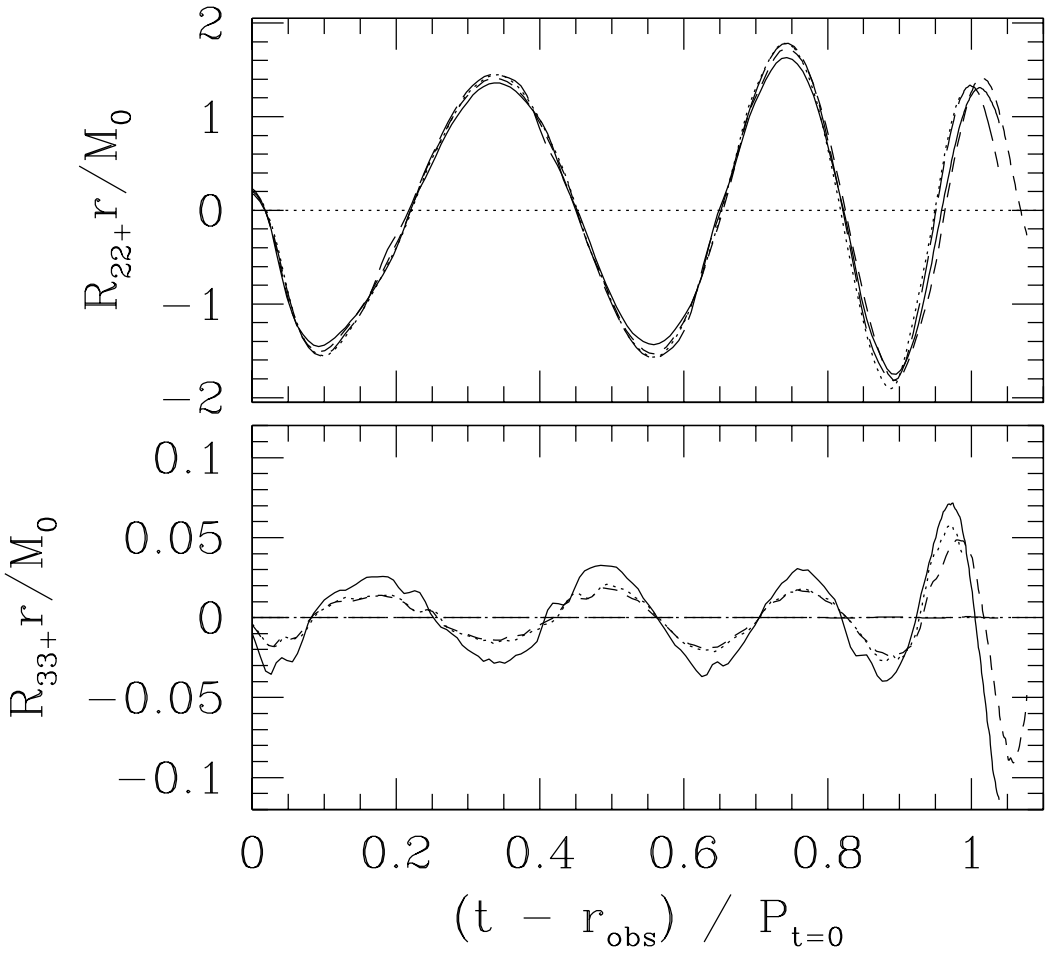
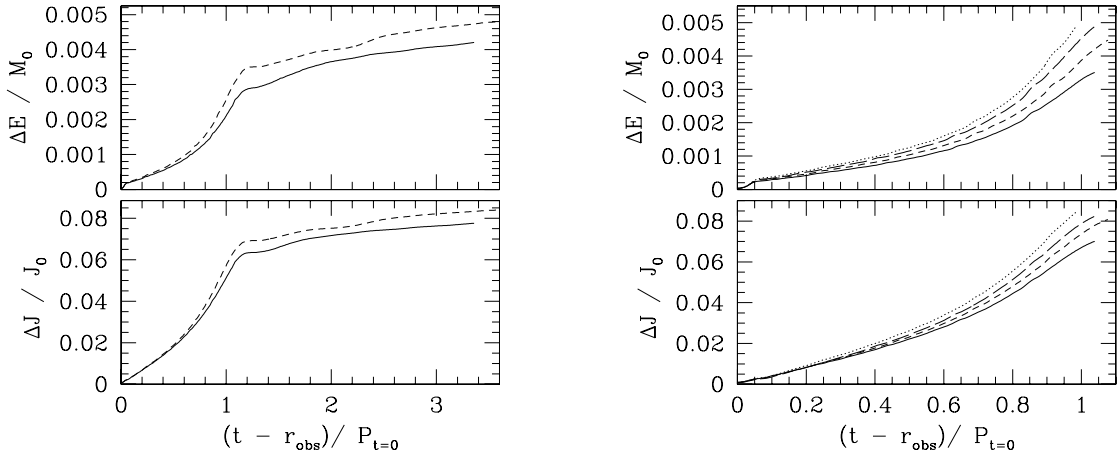
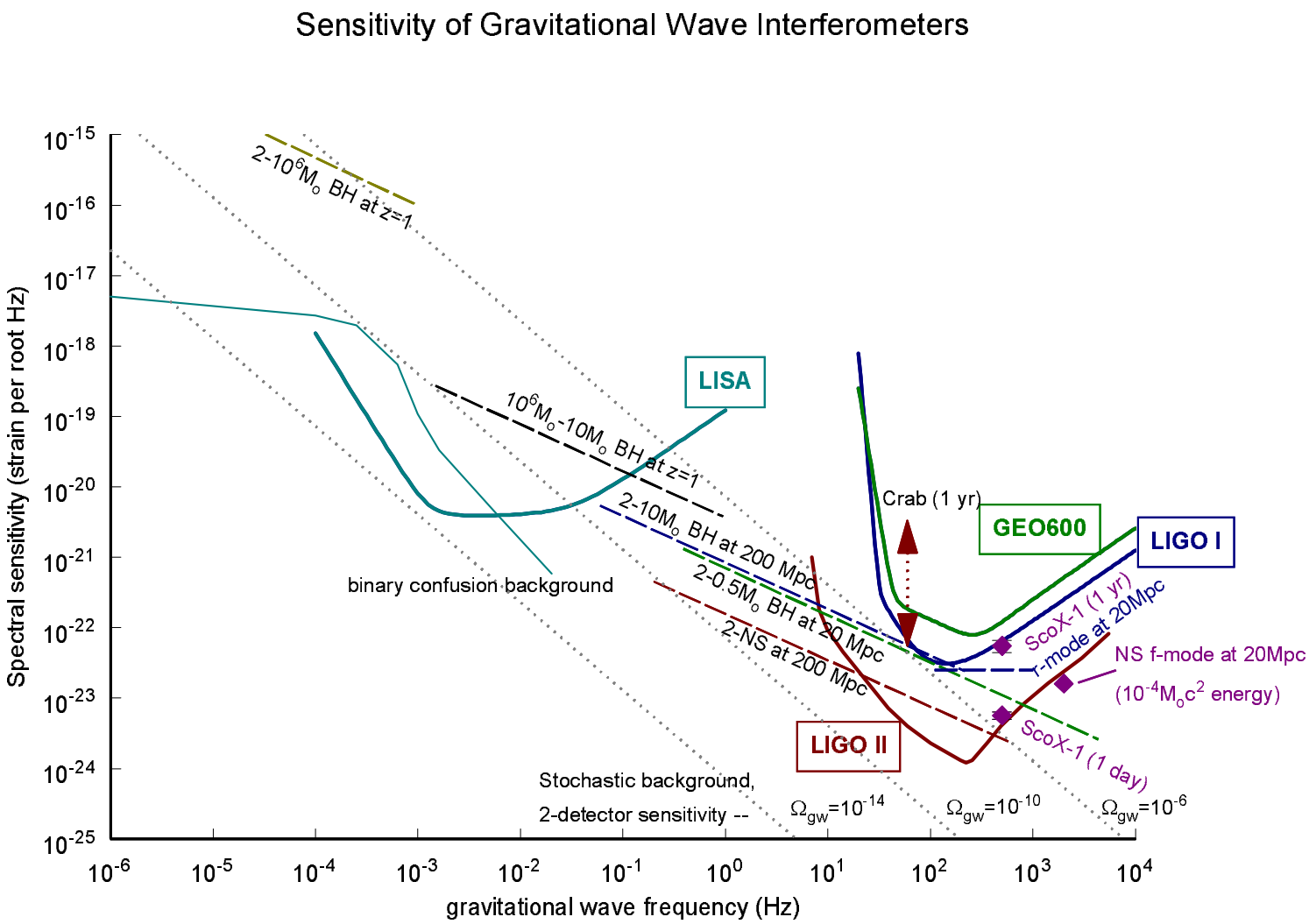


FIG. 4. $R_{22+}r$ and $R_{33+}r$ as a function of the retarded time for models M1616 (long-dashed curves), M1517 (dashed curves), M1418 (solid curves), and M159183 (dotted curves). (Shibata, Taniguchi, Uryu, gr-qc/0310030.)



(a) (b)

FIG. 5. The accumulated energy and angular momentum loss by gravitational radiation as a function of the retarded time (a) for models M1315 (solid curves) and M1414 (dashed curves), and (b) for models M1418 (solid curves), M1517 (dashed curves), M1616 (long-dashed curves), and M159183 (dotted curves). (Shibata, Taniguchi, Uryu, gr-qc/0310030.)



[Schutz, CQG 16, A131 (1999)]