Can we use atom interferometers in searching for gravitational waves?

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Intuitive concept: $\delta\Phi \sim 1/\lambda$ $\Rightarrow$ lower λ as much as you can !!

$$\frac{\delta\Phi_{MW}}{\delta\Phi_{OW}} \approx \frac{\lambda_{OW}}{\lambda_{MW}} \approx \frac{mc^2}{\hbar\varpi} \approx 10^{10} \div 10^{12}$$

But only if any other thing is the same. Is it so? Is it easy?

- The Physics (e.g. : $m_p > 0$, $v_p < c$; $m_L = 0$, $v_L = c$ $\Rightarrow$ different dispersion equations $E = E(p)$)
- The Technology: very different state of evolution for OW and MW "optical components"

What kind of interference are we speaking about ? - 1

A coherent description of quantum particles exists, both for Dirac and K.G. scalar particles, from which the main contribution from gravity fields to the phase can be deduced. Neglecting coupling between spin and curvature we have:

$$-\frac{c^2}{2\hbar}\int_{t_0}^{t_1}\frac{dt}{E(p)}\,p^{\mu}h_{\mu\nu}p^{\nu}$$

- **scalar** (newtonian) $\displaystyle \frac{E}{2\hbar}\int_{t_0}^{t_1} h_{oo}\,dt$
- **vectorial** (e.g. Sagnac, Gale…..) $\displaystyle \frac{c}{\hbar}\int_{t_0}^{t_1} h_{io}\,p^i\,dt$ $\quad i = 1,2,3$
- **tensorial** (G.W.) $\displaystyle \frac{c^2}{2E\hbar}\int_{t_0}^{t_1} p^i h_{ij} p^j\,dt$ $\quad i,j = 1,2,3$

[C.J.Bordé et al, in “Gyros,Clocks,Interferometers…”, C.Lammerzahl, C.W.F.Everitt, F.W.Hehl (Eds), Springer, Berlin (2001)
C.J.Bordé, Gen.Rel.Grav., 36, (2004) in press]

What kind of interference are we speaking about ? - 2

Ramsey Interference

- Two level atoms (**g**round, **e**xcited)
- Two successive electromagnetic interactions
- No external momentum exchange (for the moment)

$\Rightarrow$ Only internal degrees of freedom are involved in the processes

e.m.1 (at t_1), e.m.2 (at t_2)

[w.p.] **g** $\longrightarrow$ **g, e** $\longrightarrow$ Overlap (**g, e**): **g** $\xrightarrow{\text{decay}}$ **e**

0 $\quad t_1 \quad t_2 \quad \longrightarrow t$

No possibility to know which are **g** and which are **e** along the (time) path: in the finale state there is interference between the two states: this is the **Ramsey Interference.**

It involves only <u>internal D.O.F</u>. : we look at the number of decays/s

|**g**> $\longrightarrow$ |**e**> , which shows interference fringes in the “time” space.

M.W. Evolution - 1

Standard quantum approach (in “rotating” system): first order perturbation theory for a dipole e.m./w.p. interaction. If for simplicity we assume zero detuning δ, the population of the **e** state after the interaction time τ with the e.m. field (by a laser of frequency ω_L) is

$$\left|c_e(\tau)\right|^2 = \frac{1}{2}(1 - \cos\Omega_{eg}\tau) \qquad (\blacklozenge)$$

where Ω_{eg} is the Rabi frequency ($\hbar\Omega_{eg} = -\prec e\left|\vec{d}\bullet\vec{E}\right|g\succ$), being d the dipole momentum of the atom and E the electric oscillating field. Putting $\omega_{eg} = \omega_e - \omega_g$, the detuning is defined by $\delta = \omega_L - \omega_{eg}$. Assumiming $\delta = 0$, the equation ($\blacklozenge$) describes the so called **resonant Rabi oscillations .**

Assume: $t = 0 \longrightarrow \left|c_g(0)\right|^2 = 1$; τ such that $\Omega_{eg}\tau = \pi/2$ ($\pi/2$ pulse). From ($\blacklozenge$) we have :

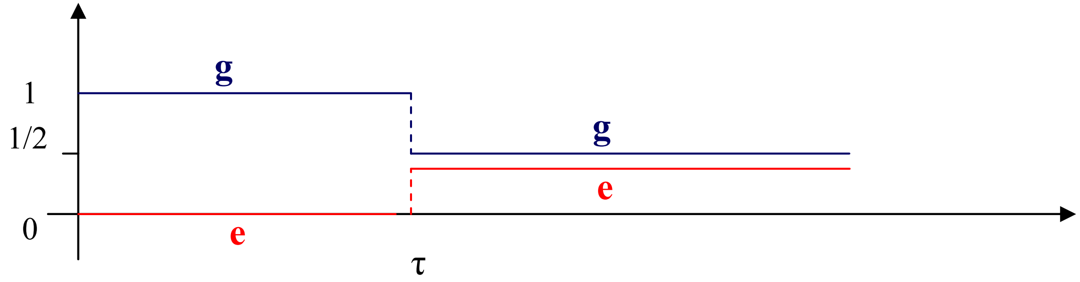
$$\left|c_e(\tau)\right|^2 = \frac{1}{2} \qquad\qquad \left|c_g(\tau)\right|^2 = \frac{1}{2}$$

M.W. Evolution - 2

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \longrightarrow\longrightarrow\longrightarrow\longrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$\pi/2$ pulse

This is a perfect Beam Splitter for internal states



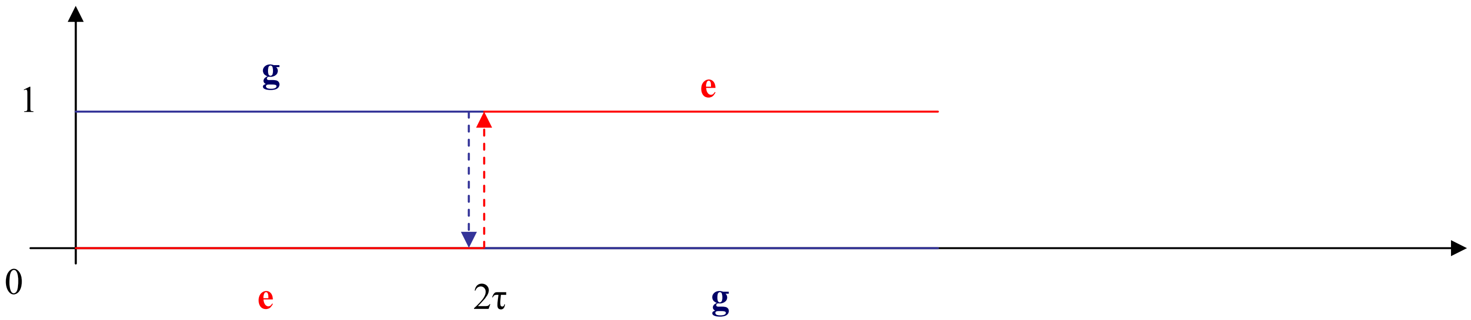
M.W. Evolution - 3

Use two $\pi/2$ pulses in order to have $2\Omega_{eg}\tau = \pi$ (π pulse): we have $\boxed{|c_e(2\tau)|^2 = 1}$

$$\boxed{\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow\rightarrow\rightarrow\rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}}$$

π pulse

This is a perfect Mirror for internal states



<u>Note</u>:

If $\delta \neq 0$, with two $\pi/2$ pulses (time interval T between them) same considerations as before with $\boxed{|c_e(2\tau + T)|^2 = \frac{1}{2}(1 + \cos\delta T)}$

M.W. Evolution - 4

Take into account external D.O.F. considering the momentum exchange :

$$|e, p_e> = |e> \otimes |p_e>$$

$$|g, p_g> = |g> \otimes |p_g>$$

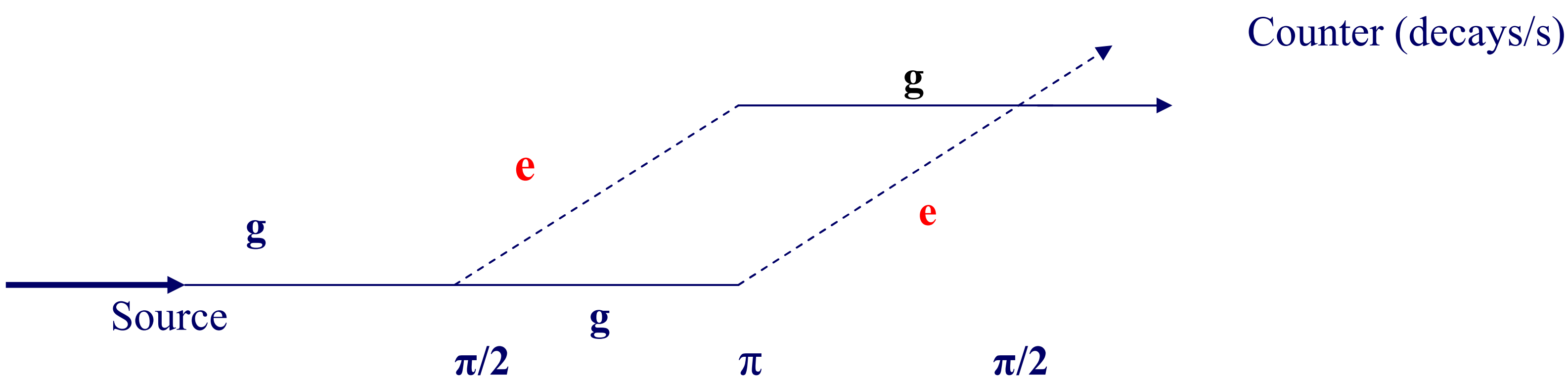
$$V = -\vec{d} \bullet \vec{E} \qquad \vec{E} = E_o \cos(\vec{k} \bullet \vec{r} - \omega_L t + \phi)$$

- kinetic term in the Hamiltonian (and in the phase too)
- recoil term in the detuning δ:

$$\delta = \omega_L - (\omega_{eg} + \frac{\vec{p} \bullet \vec{k}}{M} + \frac{\hbar k^2}{2M})$$

Even if the interference is on **internal d.o.f.**, the recoil opens the path: if we look for interference, we must close it spatially. Anyway, we are always looking at the fringes in the **number of decays/s**.

The simplest "closed" atom interferometer

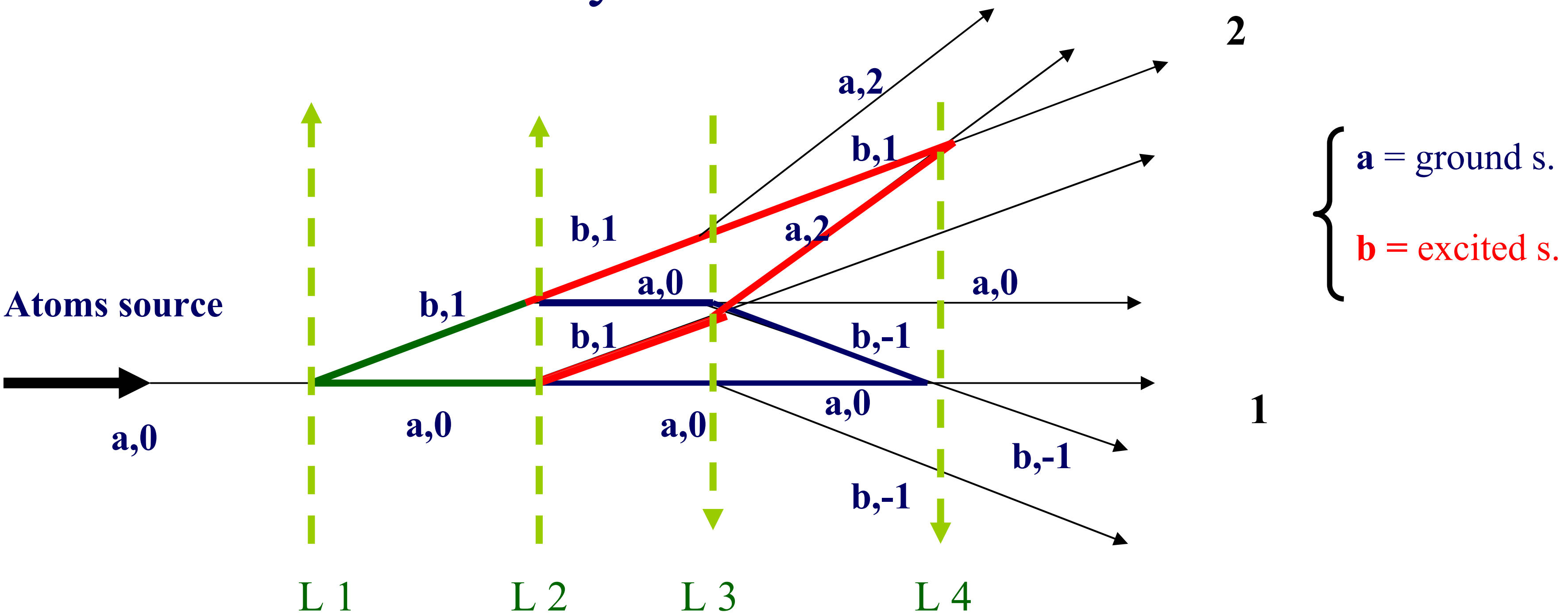


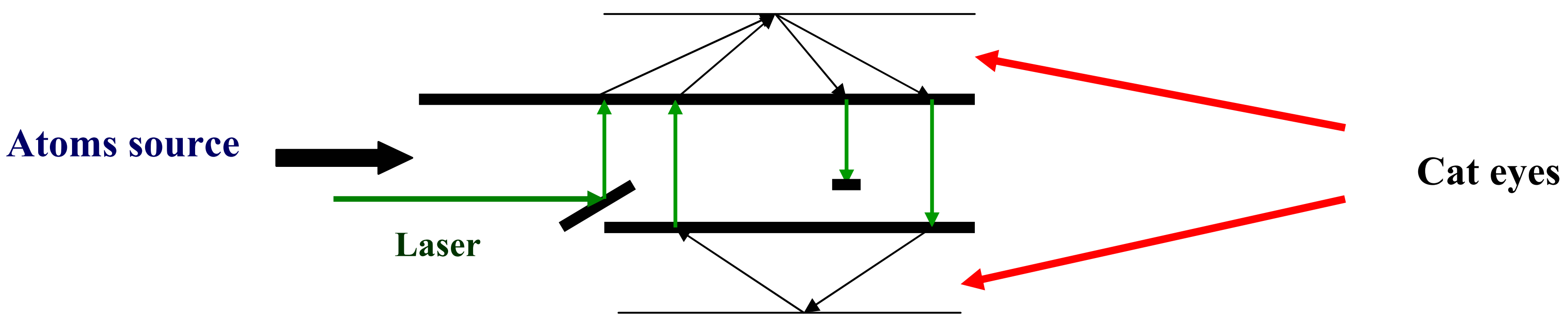
$$\left|c_{e,\vec{p}+\hbar\vec{k}}(2T+2\tau)\right|^2 = \frac{1}{2}\left(1 - \cos\left[\Delta\Phi - \frac{\delta\tau}{2}\right]\right)$$

$\Delta\Phi$ takes into account all that happens to the atoms (GW too)

- the absorption (emission) of momenta modifies both internal and external states simultaneously
- this Mach-Zender atom interferometer is too symmmetric: no hope for effects from GWs on the interference fringes on the number of decays/s

The Ramsey-Bordé Interferometer





Towards the optics of M.W. - 1

- Atom Interferometers exist (and do work)
- Coherent approaches to quantum particle in curved space (weak field) has been developed

$\Downarrow$

- Find general tools for phase calculation in A.I. in presence of G.W.
- Design a "good" interferometer: optimal configuration, source, detection…..
- Find all possible noise sources and lower them to the best you can

We discuss only the first point with a simple look at the second one

Towards the optics of M.W. - 2

- In a limited interval of velocity the dispersion function E = E(p) can be "simplified" through a power series expansion $\longrightarrow$ a Schroedinger type equation is obtained

- If we look at the equation of a (laser) beam in a cavity with some (posibly) non-linear susceptivity, indicating by U the shape of the mode and z the propagation axis, we have:

$$\boxed{i\frac{\partial U}{\partial z} = \frac{1}{2k}\nabla_t^2 U + \frac{k_o^2}{2k}\sum_n \chi_n r^n U}$$

which is a Schroedinger-like equation, provided the exchanges $t \longleftrightarrow z$; $M/\hbar \longleftrightarrow k$

- Gaussian modes are a good basis for light; but gaussian wave packets are a basis too for particle "beams"

[C.J.Bordé in "*Fundamental systems in quantum optics*", LXIII Les Houches Session, J.Dalibard,J.M.Raimond,J.Zinn-Justin (Eds), Elsevier, Amsterdam (1992)
C.J.Bordé, Gen.Rel.Grav, <u>36</u> (2004) in press]

Towards the optics of M.W. - 3

Look at the gaussian (lowest) modes for light and for a particle:

$$U = i\frac{w_o}{\sqrt{X}}\exp\left[-i\frac{Y}{X}\frac{k_o w_o^2}{2}r^2\right]$$

$$\frac{i}{2}\frac{Y}{X}k_o w_o^2 = \frac{w_o^2}{w^2} + i\frac{k_o w_o^2}{2R}; \qquad \frac{Y}{X} = \frac{1}{Q}$$

$$\Psi(q,t) = \frac{1}{(2\pi)^{\frac{1}{4}}(\Delta q)^{\frac{1}{2}}\left[1+\frac{i\hbar t}{2M(\Delta q)^2}\right]^{\frac{1}{2}}}$$

$$\exp\left\{-\frac{q^2}{4(\Delta q)^2\left[1+\frac{i\hbar t}{2M(\Delta q)^2}\right]}\right\}$$

Note : $M/\hbar \longleftrightarrow k$; $t \longleftrightarrow z$; $\Delta q \longleftrightarrow w_o/2$;

Towards the optics of M.W. - 4

- similar equations
- similar basis

Can we use the tools of gaussian (light) optics in gaussian (m.w.) optics?
And especially the ABCD approach?

ABCD matrices approach for light

Gaussian mode(P_1) ⟶ medium [ABCD matrices] ⟶ gaussian mode(P_2)

$$Q_2 = \frac{AQ_1 + B}{CQ_1 + D}$$

ABCD law for gaussian (light) optics

Yes, we can; **but**:

- remember the correspondences
- most important: how can we write the ABCD matrices for M.W.s propagation?

The path to reach ABCD matrices - 1

$$\text{w.p.}(P_1) \xrightarrow[\text{[ABCD]}]{?} \text{w.p.}(P_2)$$

Suppose the Hamiltonian quadratic at most:

$$H = \vec{p} \bullet \overleftrightarrow{\alpha} \bullet \vec{q} + \frac{\vec{p} \bullet \overleftrightarrow{\beta} \bullet \vec{p}}{2M} - \frac{M}{2} \vec{q} \bullet \overleftrightarrow{\gamma} \bullet q$$

- write the classical action $S_{cl}(1,2)$
- write the quantum propagator for w.p.(1) $\longrightarrow$ w.p.(2) (e.g. applying the principle of correspondence to S_{cl})
- Apply the quantum propagator to a basis of gaussian w.p.s from (1) to (2)
- Write formally the ABCD law and require it be identically satisfied

The path to reach ABCD matrices - 2

We obtain:

$$\mathrm{w.p.}(t_2, p_2, q_2) = \exp\left[\frac{iS_{cl}(t_1,t_2)}{\hbar}\right] \mathrm{w.p.}(t_1, p_1, q_1)$$

where:

$$\begin{pmatrix} q_2 \\ \frac{p_2}{M} \end{pmatrix} = \begin{pmatrix} A_{21} & B_{21} \\ C_{21} & D_{21} \end{pmatrix} \begin{pmatrix} q_1 \\ \frac{p_1}{M} \end{pmatrix} + \begin{pmatrix} \xi \\ M\,\dot{\xi} \end{pmatrix}$$

$$\ddot{\xi} + \beta\,\dot{\beta}^{-1} - \beta\gamma\xi = 0$$

$$\begin{pmatrix} X_2 \\ Y_2 \end{pmatrix} = \begin{pmatrix} A_{21} & B_{21} \\ C_{21} & D_{21} \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}$$

and:

$$\begin{pmatrix} A_{21} & B_{21} \\ C_{21} & D_{21} \end{pmatrix} = \tau \exp\left[\int_{t_1}^{t_2} \begin{pmatrix} \alpha & \beta \\ \gamma & \alpha \end{pmatrix} dt\right]$$

τ being a time ordering operator.

The Beam Splitter influence

Standard 1st order perturbation approach for weak dipole interaction

↓

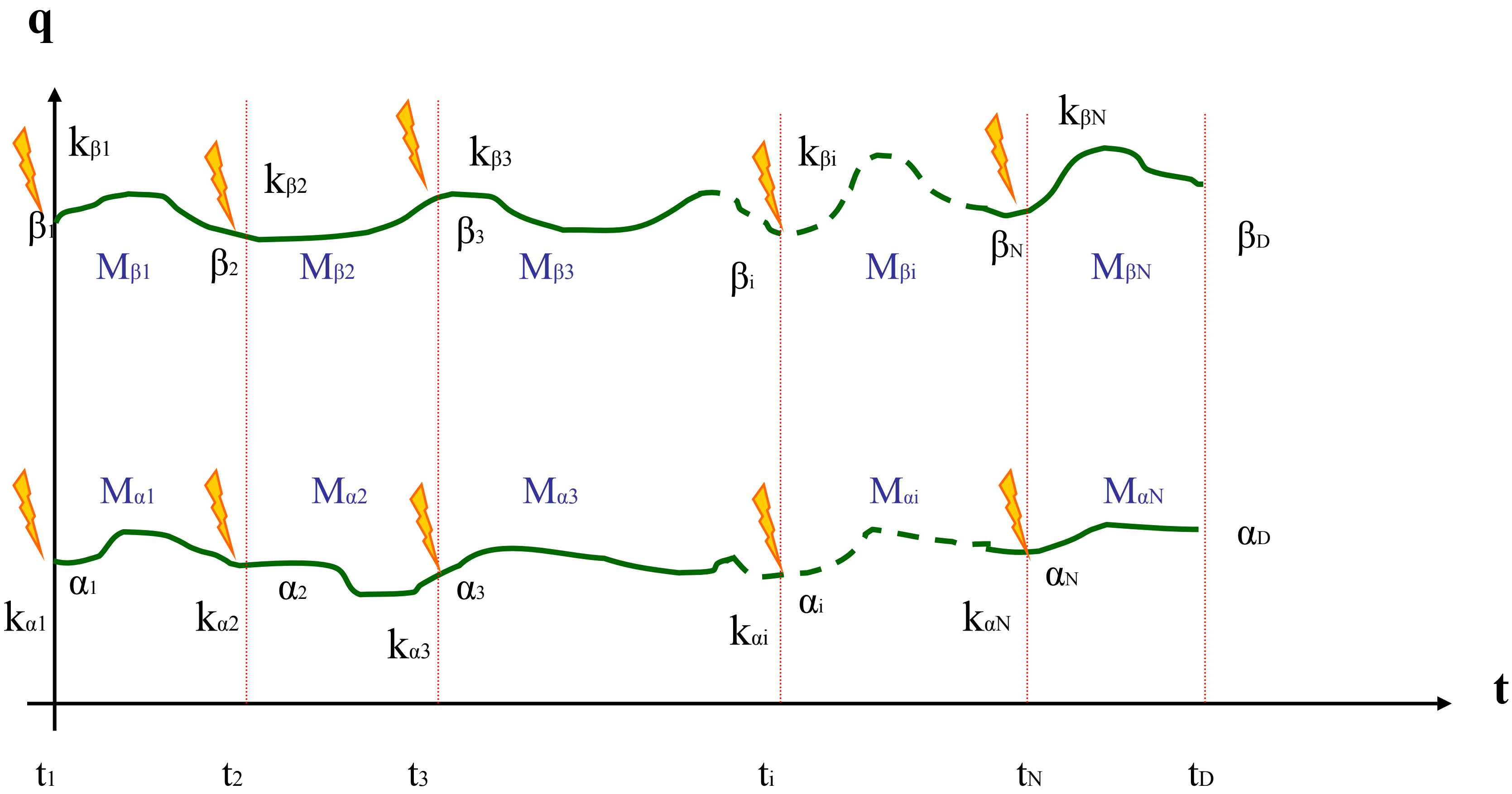
ttt theorem

The B.S. introduces a multiplicative amplitude and phase factor simply related to the laser beam quantities (indicated by a star):

$$\boxed{M_{bs} \exp\left(\omega^* t^* - k^* q^* + \phi^*\right)}$$

[C.Antoine, C.J.Bordé, *Phys.Lett.A,* 306 , 277-284 (2003) and references therein]

Phase shift for a sequence of pairs of homologous paths (an interferometer geometry) - 1



Phase shift for a sequence of pairs of homologous paths (an interferometer geometry) - 2

From ABCD law, ttt theorem, and properties of classical action $S_{cl}(P_1,P_2)$ for N beam-splitters :

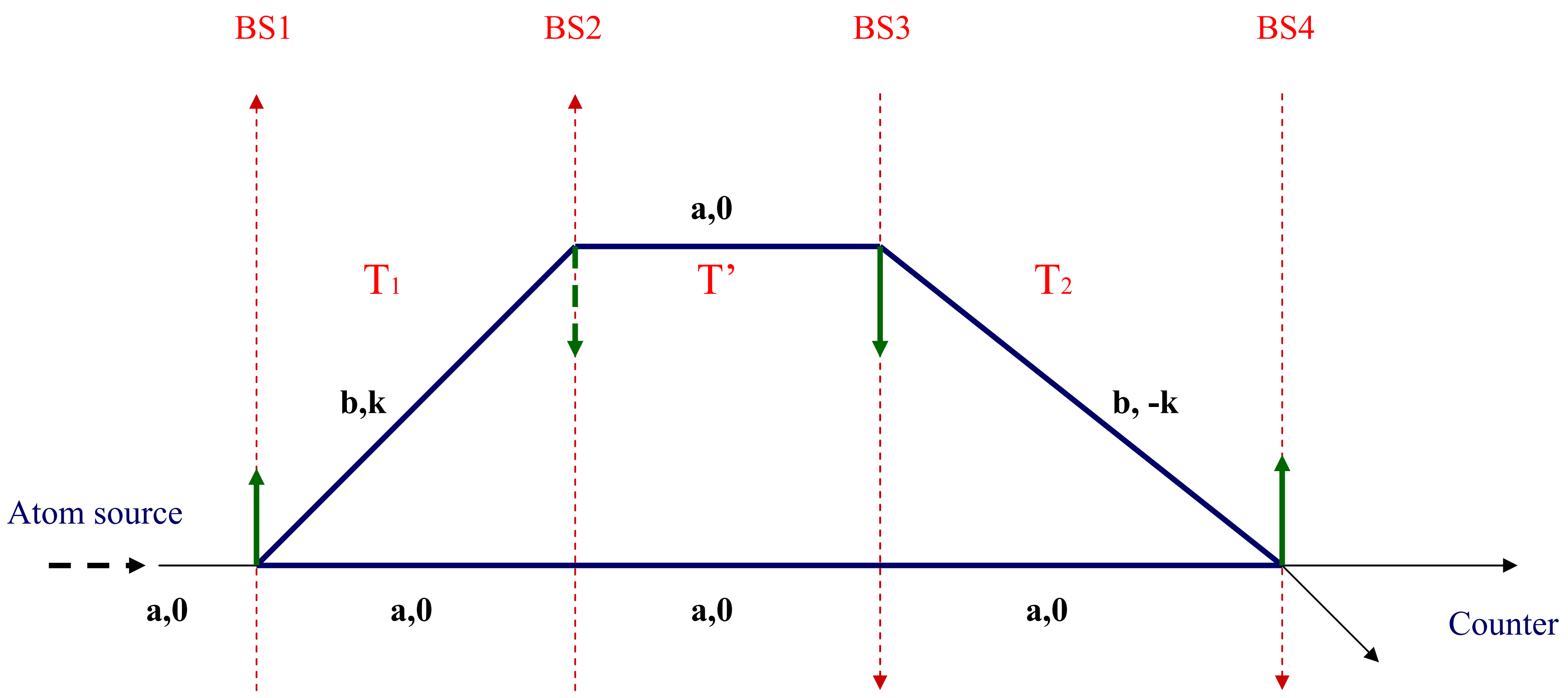
$$\Delta\Phi = -\frac{p_{\alpha 1}+p_{\beta 1}}{2\hbar}\left(q_{\beta 1}-q_{\alpha 1}\right)+\sum_{i=1}^{N}\left[\left(k_{\beta i}-k_{\alpha i}\right)\frac{q_{\alpha i}+q_{\beta i}}{2}-\left(\varpi_{\beta i}-\varpi_{\alpha i}\right)t_i+\left(\varphi_{\beta i}-\phi_{\alpha i}\right)\right]+$$

$$+\sum_{i=1}^{N}\frac{M_{\beta i}-M_{\alpha i}}{2\hbar}\left\{\left[\frac{S_{\alpha i}}{M_{\alpha i}}+\frac{p_{\alpha i+1}}{2M_{\alpha i}}\left(q_{\beta i+1}-q_{\alpha i+1}\right)-\frac{p_{\alpha i}+\hbar k_{\alpha i}}{2M_{\alpha i}}\left(q_{\beta i}-q_{\alpha i}\right)\right]\right.$$

$$\left.+\left[\frac{S_{\beta i}}{M_{\beta i}}+\frac{p_{\beta i+1}}{2M_{\beta i}}\left(q_{\beta i}-q_{\alpha i}\right)-\frac{p_{\beta i}+\hbar k_{\beta i}}{2M_{\beta i}}\left(q_{\alpha i}-q_{\beta i}\right)\right]\right\}$$

[C.Antoine, C.Bordé, *J.Opt. B: Quantum Semiclass.Opt.*, 5, 199-207 (2003)]

Build the simplest A.I. for G.W. - 1



Put $T_1 = T_2$, $T' \longrightarrow 0$ ➡ the simplest unsymmetrical A.I.

Build the simplest A.I. for G.W. - 2

The machine

- Choose FNC (It's better!!!)
- Calculate ABCD matrices in presence of GW at the 1_{st} order in the strain amplitude h (e.g.: a wave with polarization "+" impinging perpendicularly on the plane of the interferometer)
- Apply $\Delta\Phi$ expression (slide 19) to the interferometer
- Use ABCD law to substitute all q_j coordinates
- Fully simplify
- Print $\Delta\Phi$
- End

Note : the job should be worked in the frequency space: use Fourier transform, please!

Build the simplest A.I. for G.W. -3

The phase shift:

$$\Delta[\Phi(\omega)] = -\frac{k}{m} p_1 \, h(\omega)\, \omega^2 \, T^2 \, f(\omega)$$

where:

$2/m = 1/M_a + 1/M_b$; $k\hbar$ = transversal momentum (by BS);

p_1 = initial (longitudinal) momentum;

$f(\omega)$ = complex frequency response of the interferometer

$$f(\omega) = \frac{2}{\omega}\left\{\left[\sin\omega T\left(\frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}\right)^2 + \frac{\cos 2\omega T - \cos\omega T}{\omega T}\right] + i\left[\frac{\sin 2\omega T - \sin\omega T}{\omega T} - \cos\omega T\left(\frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}\right)^2\right]\right\}$$

Build the simplest A.I. for G.W. - 4

The sensitivity (1):

Suppose the A.I. "shot noise" limited, with η a kind of efficiency of the decay process we use.

$$\Delta[\Phi(\omega)]_{\text{shot noise}} = \frac{\eta}{\sqrt{\dot{N}}}$$

At the level of S.N.R. = 1 we have:

$$h(\omega) = \frac{2\eta\hbar}{\sqrt{\dot{N}}\; p_T L \omega^2\; T} \frac{1}{|f(\omega)|}$$

$$|f(\omega)| = \frac{2}{\omega}\left|\frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}\right| \sqrt{1 - 2\,\frac{\sin \omega T}{\omega T} + \left(\frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}\right)^2}$$

Note: $|f(\omega)| \sim T \quad (\omega \to 0)$; $\quad |f(\omega)| \sim (2/\omega)\,|\sin(\omega T/2)/(\omega T/2)| \to 0 \quad (\omega \to \infty)$

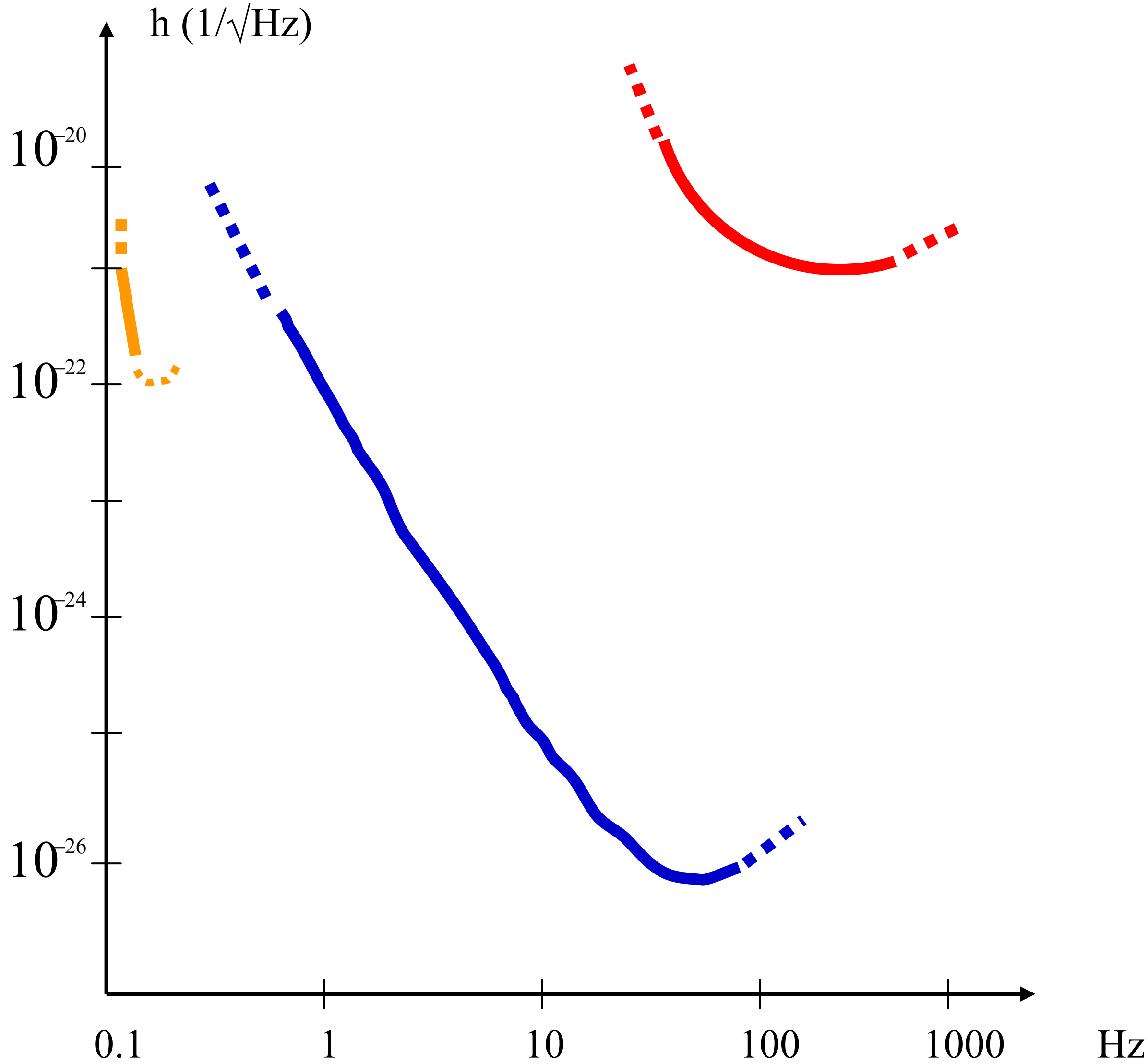
Build the simplest A.I. for G.W. - 5

The sensitivity (2):

$v_L = 10^6$ m/s; $\dot{N} = 10^{18}$ atoms(H)/s;
$T = 10^{-3}$ s; $L = 10^3$ m; $v_T = 10$ m/s

$v_L = 10^7$ m/s; $L = 10^5$ m
$T = 10^{-2}$

$v_L = 10$ m/s; $v_T = 5$ m/s; $L = 50$ m;
$\dot{N} = 10^{18}$ atoms(Cs)/s



Final Remarks

- comprehensive approach to the problem (A.I. + G.W.)
- “automatic” tool for solving atom interferometers
- realistic values of physical parameters (someone on the borderline…)
- interesting value of the sensitivity even for a “minimal” atom interferometer
- don’t forget BEC
- noise budget ?
- from the idea to the experiment

Looking at a future (not too far from now, hopefully) and at a very hard work, bearing in mind the title of this talk, in my opinion we can say……………

Conclusion

….be optimistic: we can!!!

Some general references:

- P.Berman (Ed), *Atom Interferometry*, Ac.Press, N.Y. (1997)
- S.Chu, in *"Coherent atomic matter waves"*, 72° Les Houches Session, R.Kaiser, C.Westbrook, F.David (Eds), Springer Verlag, N.Y. (2001)
- C.J. Bordé, *CR Acad.Sc.Paris,* t2-SIV, 509-530 (2001)
- C.J.Bordé, *Metrologia*, 39, 435-463 (2002)

Please Note:

Not all of the Authors have had the possibility of revising the final form of this talk; they share obviously the scientific content, but mistakes, misunderstandings, misprints are totally under my own responsibility alone (F.V.).