Stochastic Gravitational-Wave Searches with Interferometers and Bars

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Outline

I Review Of Stochastic Background Searches

- Optimally-Filtered Cross-Correlation
- Overlap Reduction Function
- Notable Cross-Correlation Experiments

II LLO-ALLEGRO Cross-Correlations

- Overlap Modulation by Rotation of Bar
- Handling Different Sampling Rates & Heterodyning
- Working with Calibrated Data
- Status Reported at GR17

Cartoon courtesy of E. Coccia, NAUTILUS Group (Rome)

Stochastic Background

Backgrounds in 10–1000 Hz frequency band likely extragalactic in origin, thus isotropic, unpolarized, gaussian, & stationary.

Describe i.t.o. GW contribution to $\Omega = \frac{\rho}{\rho_{\mathrm{crit}}}$:

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{crit}}} \frac{d\rho_{\mathrm{GW}}}{d \ln f} = \frac{f}{\rho_{\mathrm{crit}}} \frac{d\rho_{\mathrm{GW}}}{df}$$

Note $\rho_{\mathrm{crit}} \propto H_0^2$, so $h_{100}^2 \Omega_{\mathrm{GW}}(f)$ is independent of

$$h_{100} = \frac{H_0}{100\,\mathrm{km/\,s/\,Mpc}}$$

How to Tell Stochastic Signal from Random Noise

- Need correlations among detectors
  - Detector 1: $s_1 = h_1 + n_1$, Detector 2: $s_2 = h_2 + n_2$
- Assume noise uncorrelated with signal & between detectors
- Cross-correlation:

$$\langle s_1 s_2 \rangle = \langle n_1 n_2 \rangle + \langle n_1 h_2 \rangle + \langle h_1 n_2 \rangle + \langle h_1 h_2 \rangle$$

only surviving term is from stochastic GW signal

Sensitivity to Stochastic GW Backgrounds

- Optimally filtered CC statistic

$$Y = \int df\, \tilde{s}_1^*(f)\, \tilde{Q}(f)\, \tilde{s}_2(f)$$

- Optimal filter $\tilde{Q}(f) \propto \frac{f^{-3}\Omega_{\mathrm{GW}}(f)\gamma_{12}(f)}{P_1(f)P_2(f)}$
  (Initial analyses assume $\Omega_{\mathrm{GW}}(f)$ constant across band)
- Optimally filtered cross-correlation method sensitive to

$$\Omega_{\mathrm{GW}} \propto \left(T \int \frac{df}{f^6} \frac{\gamma_{12}^2(f)}{P_1(f)P_2(f)}\right)^{-1/2}$$

- Significant contributions when
  - detector noise power spectra $P_1(f)$, $P_2(f)$ small
  - overlap reduction function $\gamma_{12}(f)$ (geom correction) near ± 1

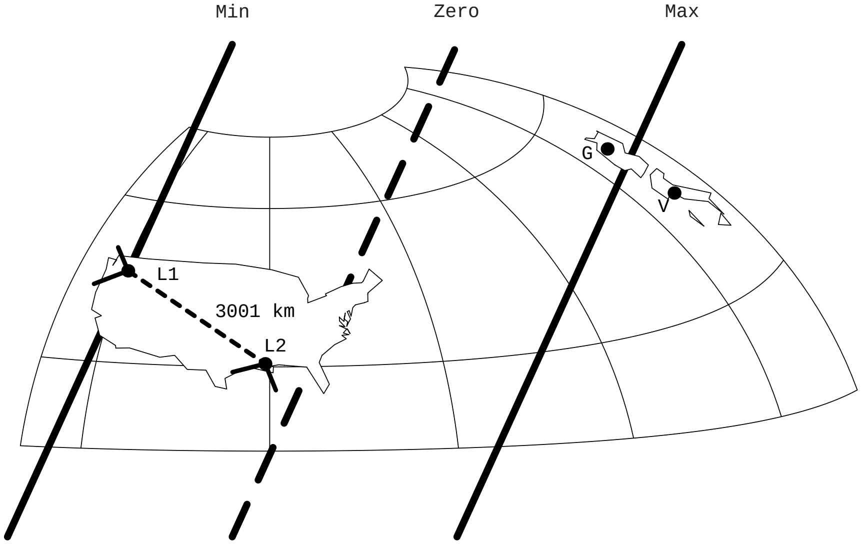
Overlap Reduction Function

$$\gamma_{12}(f) = d_{1ab} d_2^{cd} \frac{5}{4\pi} \iint_{S^2} d^2\Omega \; P^{\mathrm{TT}}{}^{ab}_{cd}(\widehat{\Omega}) e^{i2\pi f \widehat{\Omega}\cdot\Delta\vec{\mathbf{x}}/c}$$

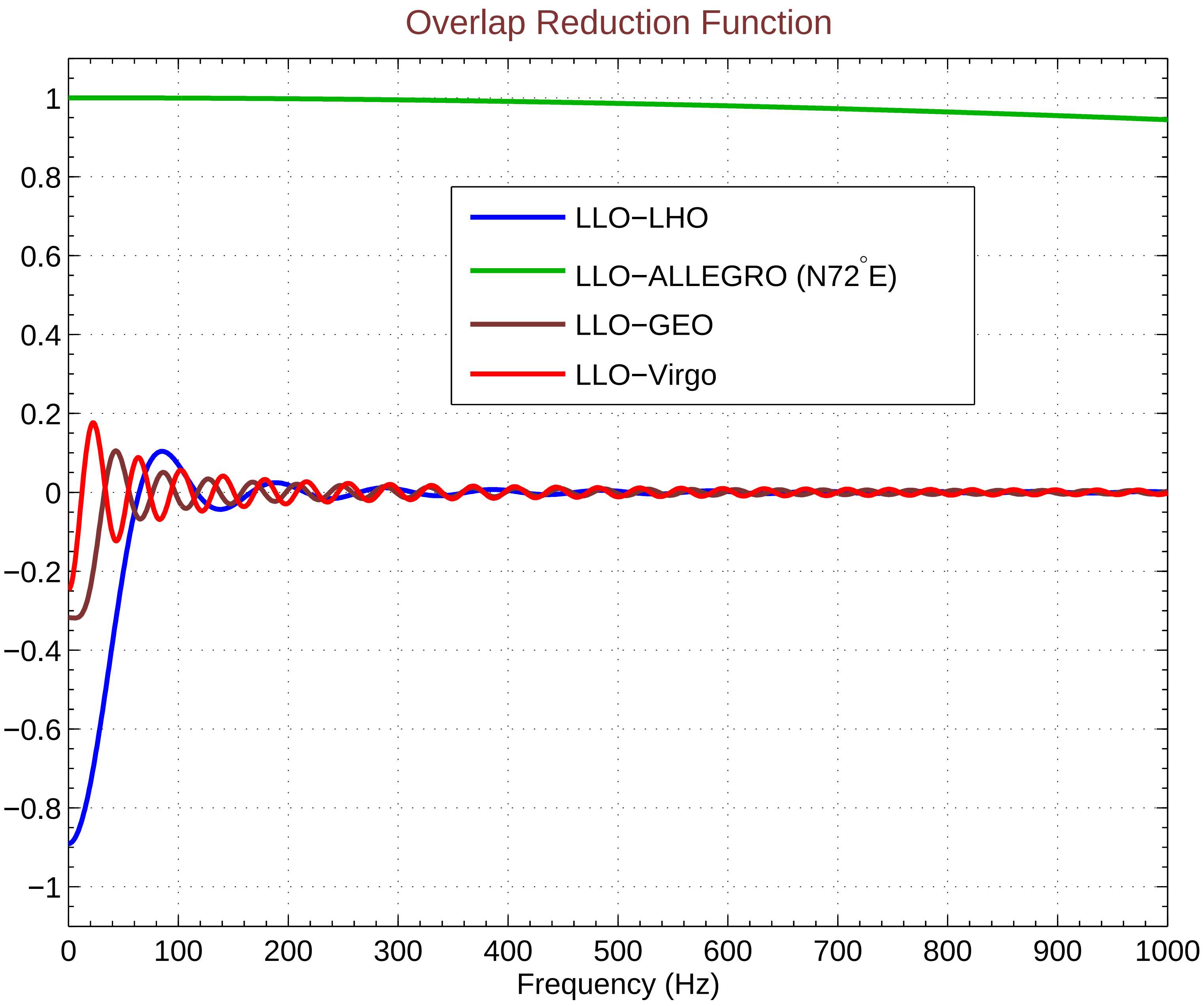
Depends on alignment of detectors (polarization sensitivity)

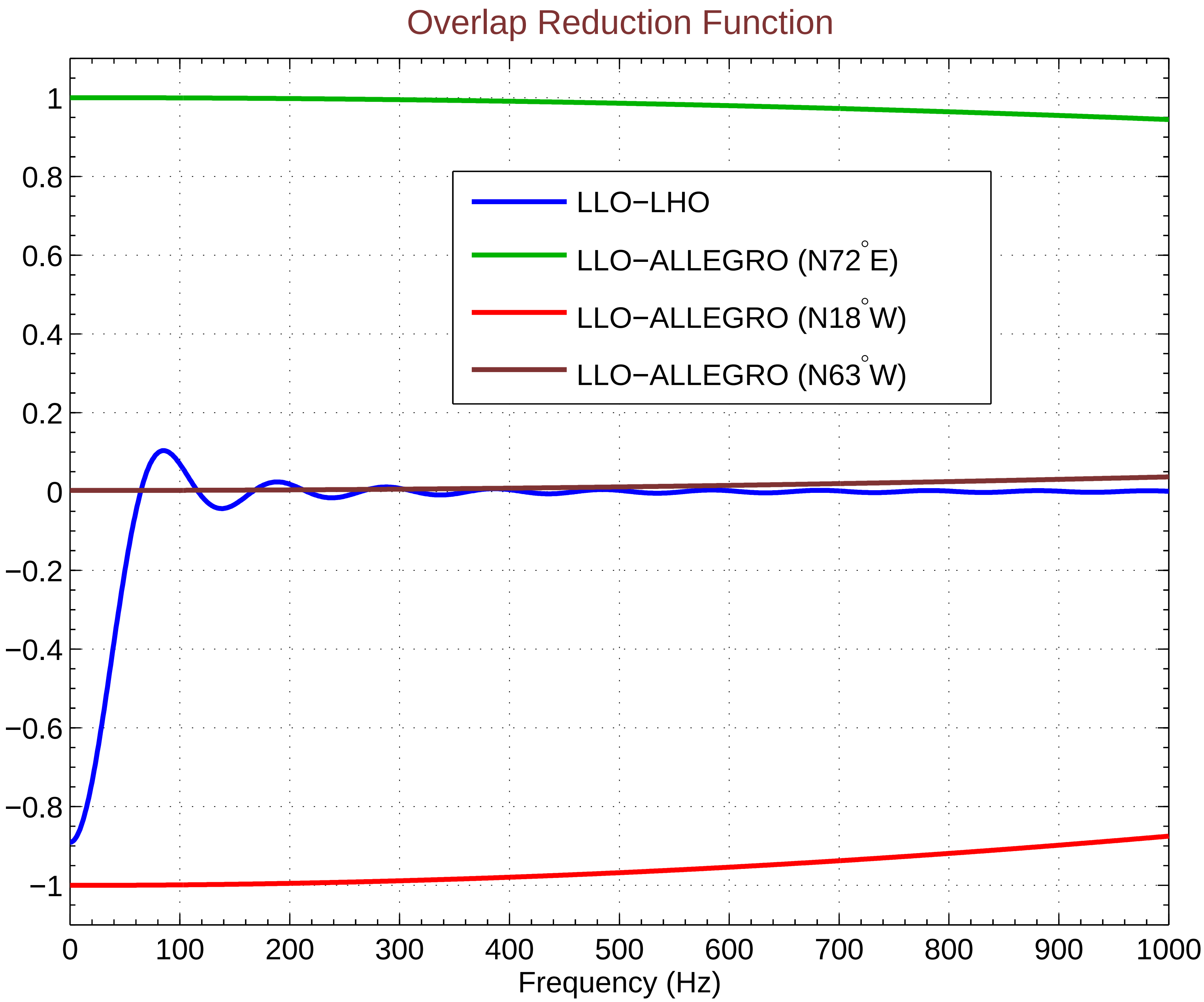
Frequency dependence from cancellations when $\lambda \lesssim$ distance

→ Widely separated detectors less sensitive at high frequencies



(figure from Allen & Romano PRD, gr-qc/9710117)


Overlap Reduction Function
LLO−LHO
LLO−ALLEGRO (N72°E)
LLO−GEO
LLO−Virgo
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
100
200
300
400
500
600
700
800
900
1000
Frequency (Hz)


Overlap Reduction Function
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
100
200
300
400
500
600
700
800
900
1000
Frequency (Hz)
LLO−LHO
LLO−ALLEGRO (N72°E)
LLO−ALLEGRO (N18°W)
LLO−ALLEGRO (N63°W)

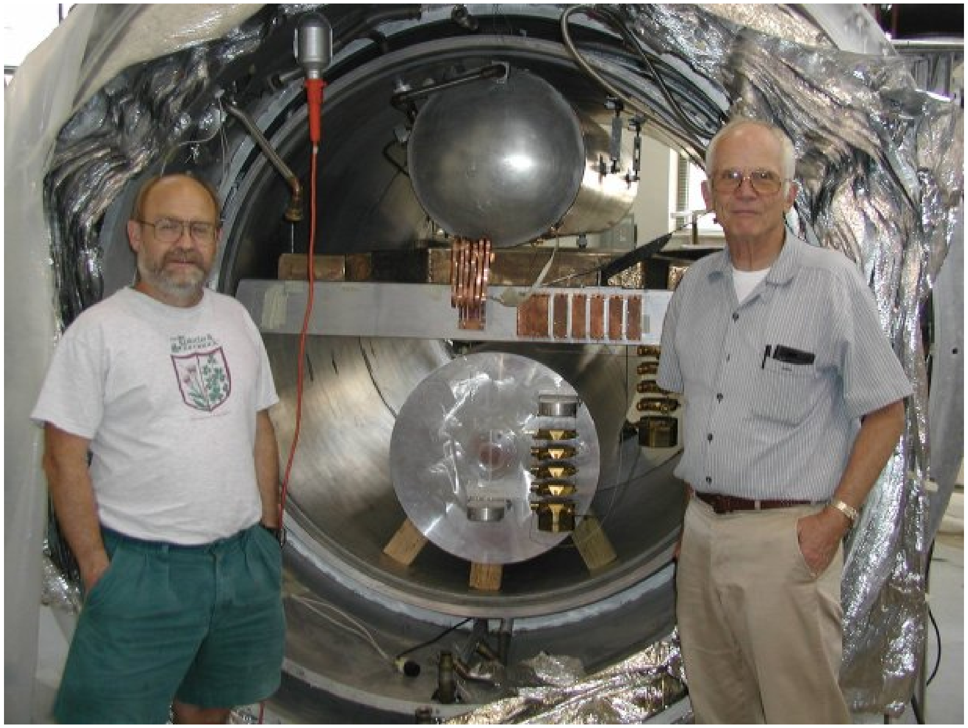
Upper Limits

- Correlation between EXPLORER & NAUTILUS bars (Astone et al, 1999): $h_{100}^2\Omega_{\mathrm{GW}}(907\,\mathrm{Hz}) \leq 60$
- Correlation between LIGO Hanford & Livingston S1 data (LSC, Abbott et al, 2004): $h_{100}^2\Omega_{\mathrm{GW}}(f) \leq 23$ at $64 < f < 265$
- Correlation between LIGO Hanford & Livingston S2/S3 Data Analysis Ongoing; Preliminary S2 result reported at GR17: $h_{100}^2\Omega_{\mathrm{GW}}(f) \leq 0.018^{+0.007}_{-0.003}$ at $50 < f < 300$
- Correlations between LIGO Livingston & ALLEGRO data

This talk focusses on methods

Status was reported last week at GR17

ALLEGRO Detector (Baton Rouge, LA)



W. Johnson, ALLEGRO & W. Hamilton from LSU Website

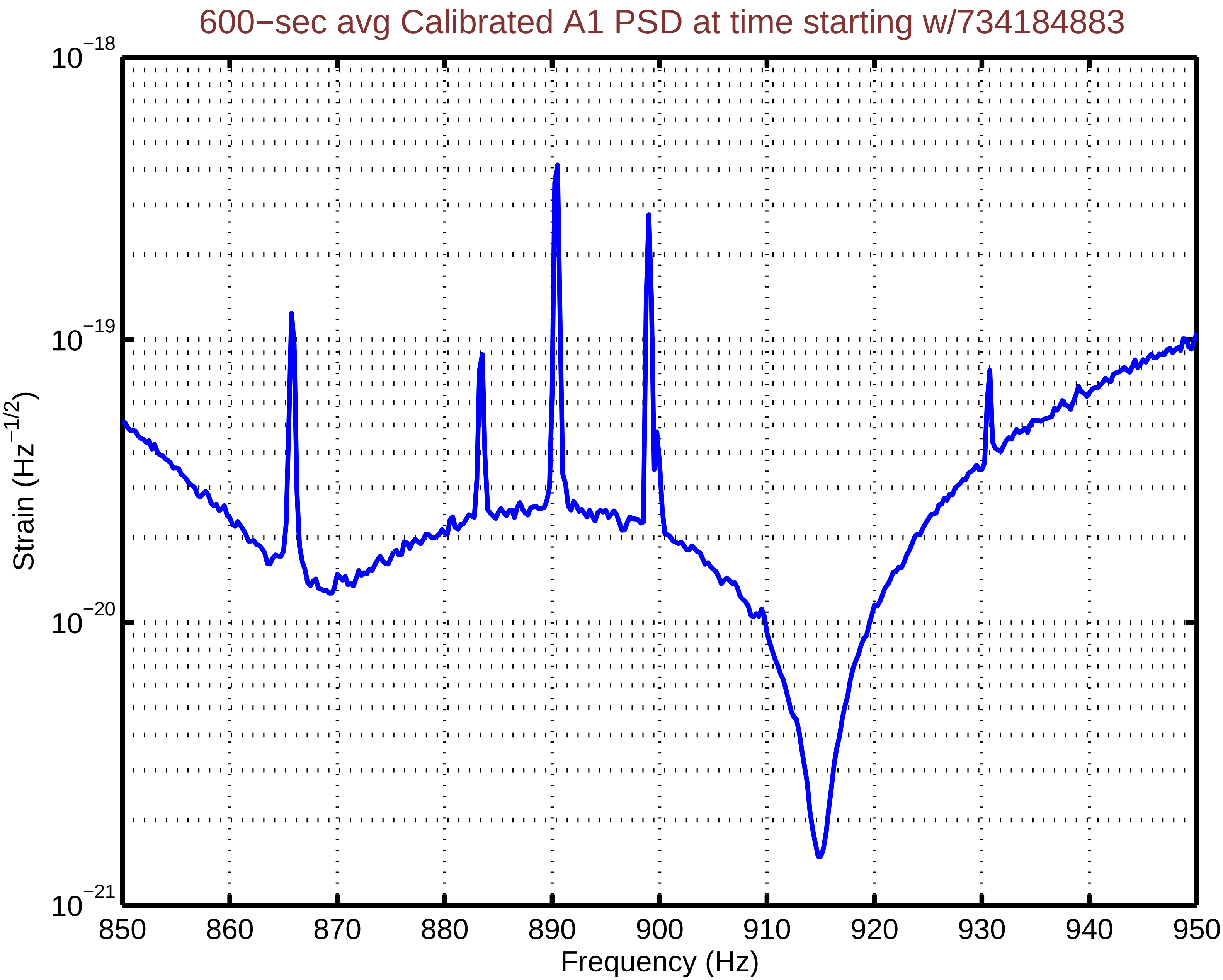
LLO-ALLEGRO Correlations

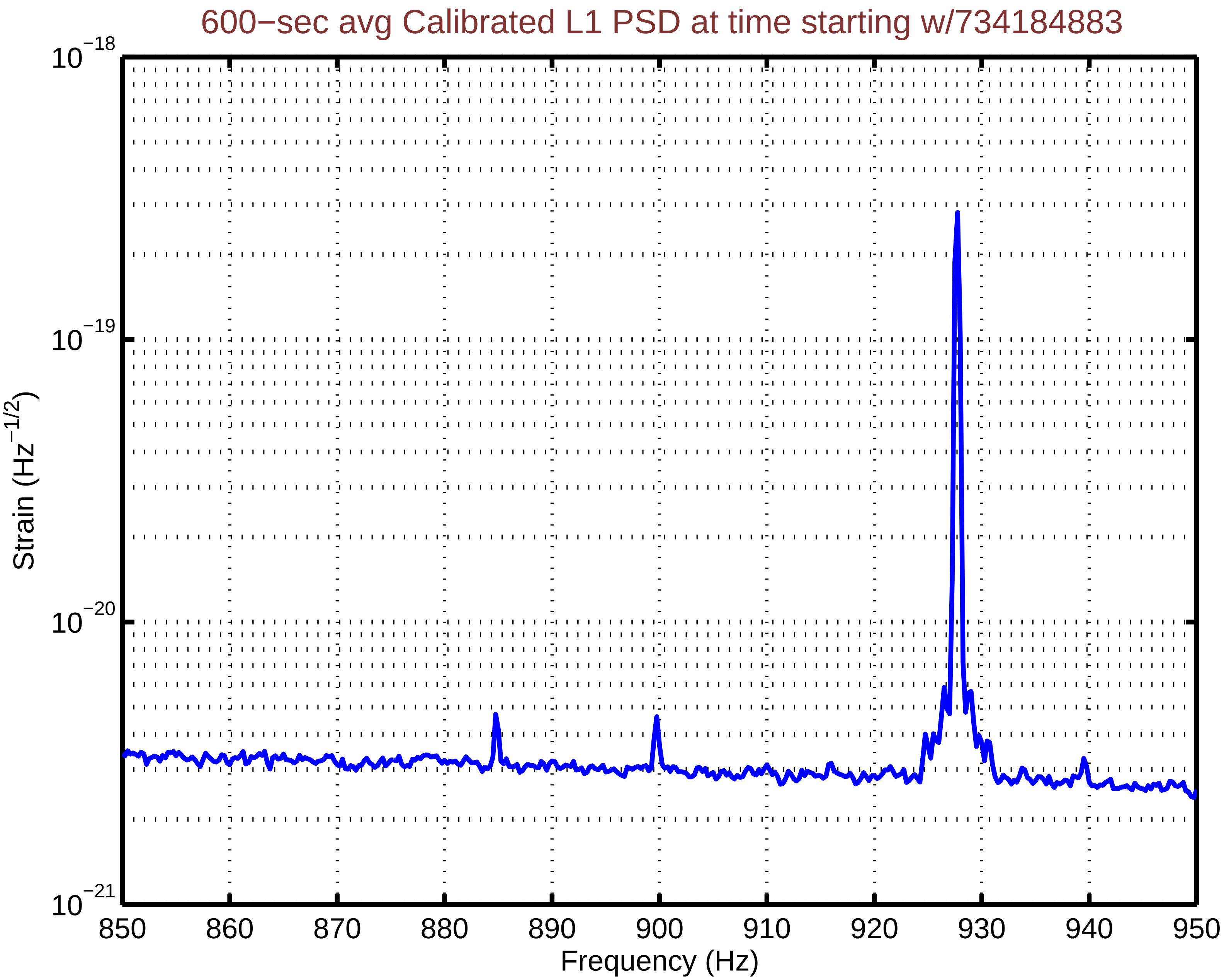
- Only $\sim$40 km apart → $\gamma(900\,\mathrm{Hz}) \approx 95\%$ for best alignment

- Sensitive in different freq band from LLO/LHO pair

- New experimental technique: rotate ALLEGRO to callibrate cross-correlated noise (Finn & Lazzarini)
  - Aligned & Anti-aligned orientations have opposite GW sign
    $\longrightarrow$ can "cancel" out CC noise by subtracting results
  - Null orientation has no expected GW signal
    $\longrightarrow$ "off-source" measurement of CC noise

- Currently analyzing S2 (2003 Feb 14-Apr 14) data; ALLEGRO was offline for S3 (2003 Oct 31-2004 Jan 9), now running again; Further work planned for S4 & beyond


600−sec avg Calibrated A1 PSD at time starting w/734184883
10^{-18}
10^{-19}
10^{-20}
10^{-21}
Strain ($Hz^{-1/2}$)
850
860
870
880
890
900
910
920
930
940
950
Frequency (Hz)


600−sec avg Calibrated L1 PSD at time starting w/734184883
10^{-18}
10^{-19}
10^{-20}
10^{-21}
Strain ($Hz^{-1/2}$)
850
860
870
880
890
900
910
920
930
940
950
Frequency (Hz)

LLO-ALLEGRO: Technical Considerations

- ALLEGRO data heterodyned at 899 Hz & sampled at 250 Hz
  LIGO data digitally downsampled 16384 Hz → 2048 Hz
  Time domain resampling undesirable: $2^{10}/5^3$ sampling ratio
  → work in freq domain w/overlapping frequencies

- Uncalibrated ALLEGRO data have sharper spectral features
  → Work w/calibrated heterodyned strain "$h(t)$" for ALLEGRO

- Calibrating ALLEGRO data is major undertaking
  (McHugh + Johnson & LSU)
  (Coherent analysis requires more precise calibration than before)
  See McHugh GR17 talk for more details

Crash Course on Heterodyning (base-banding)

Think in terms of continuous Fourier transform

$$\widetilde{G}(f) = \int_{-\infty}^{\infty} dt\, e^{-i2\pi f(t-t_0)} G(t)$$

Analogue heterodyne: multiply by exp oscillating @ base freq f_b:
$G_h(t) = e^{-i2\pi f_b(t-t_0)} G(t)$ so that Fourier transform is

$$\widetilde{G}_h(f) = \widetilde{G}(f_b + f)$$

Low-pass anti-aliasing filter on G_h is then band-pass filter on G;

$$\widetilde{g}_h(f) = \begin{cases} \widetilde{G}_h(f) & |f| \le \frac{1}{2\,\delta t} \\ 0 & |f| > \frac{1}{2\,\delta t} \end{cases}$$

$g_h(t)$ then sampled @ $\frac{1}{\delta t}$ so $f_{\mathrm{Ny}} = \frac{1}{2\,\delta t}$; range of physical freqs

$$f_b - f_{\mathrm{Ny}} \le f_{\mathrm{phys}} \le f_b + f_{\mathrm{Ny}}$$

Working in Frequency Domain

- LLO & ALLEGRO data are FFTed to produce freq series (normalized to approximate CFT)
  $\widetilde{s}^L[f]: \quad 0 \leq f \leq f^L_{\mathsf{Ny}}$
  $\widetilde{s}^A_h[f]: \quad -f^A_{\mathsf{Ny}} \leq f < f^A_{\mathsf{Ny}}$
  If duration is T, zero-padded to $2T$, each has freq res $\delta f = \frac{1}{2T}$

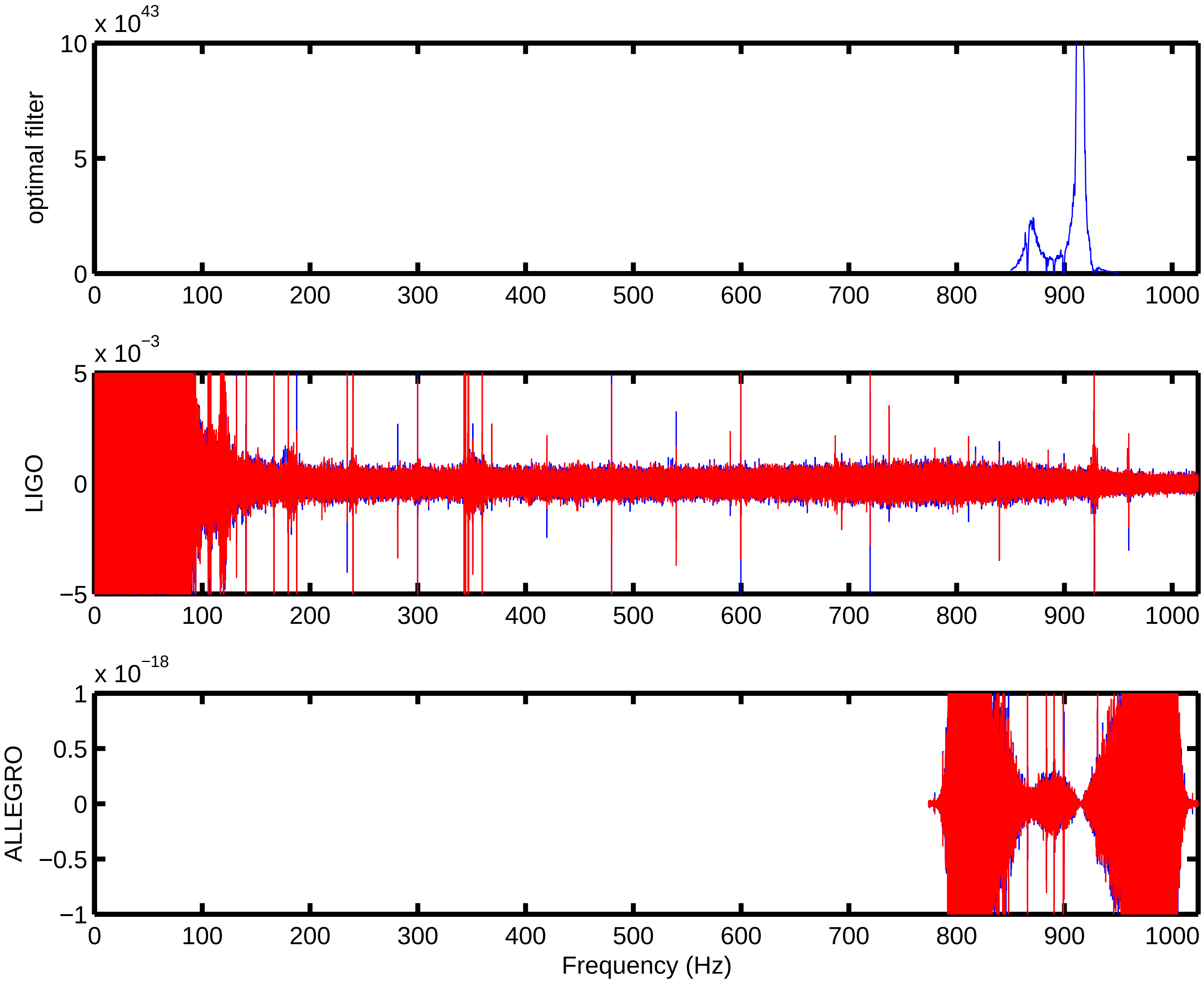
- Optimal filter created in freq domain w/same freq resolution

$$\widetilde{Q}[f]: \quad f_{\mathsf{min}} \leq f \leq f_{\mathsf{max}}$$

- Cross-correlation statistic is

$$Y = \sum_{f=f_{\mathsf{min}}}^{f_{\mathsf{max}}} \delta f \left(\widetilde{s}^L[f]\right)^* \widetilde{Q}[f]\, \widetilde{s}^A_h[f - f_b] \approx \int_{f_{\mathsf{min}}}^{f_{\mathsf{max}}} df \left[\widetilde{s}^L(f)\right]^* \widetilde{Q}(f)\, \widetilde{s}^A(f)$$

  So long as $[f_{\mathsf{min}}, f_{\mathsf{max}}]$ a subset of LLO & ALLEGRO freq ranges & $\frac{f_b}{\delta f} \in \mathbb{Z}$, freq bins “line up”


x 10^43
10
5
0
optimal filter
0
100
200
300
400
500
600
700
800
900
1000
x 10^−3
5
0
−5
LIGO
0
100
200
300
400
500
600
700
800
900
1000
x 10^−18
1
0.5
0
−0.5
−1
ALLEGRO
0
100
200
300
400
500
600
700
800
900
1000
Frequency (Hz)

Example of Frequency Domain Method

- Assume $T = 50\,\mathrm{sec}$;
  after zero-padding $\delta f = .01\,\mathrm{Hz}$ for both ALLEGRO & LLO

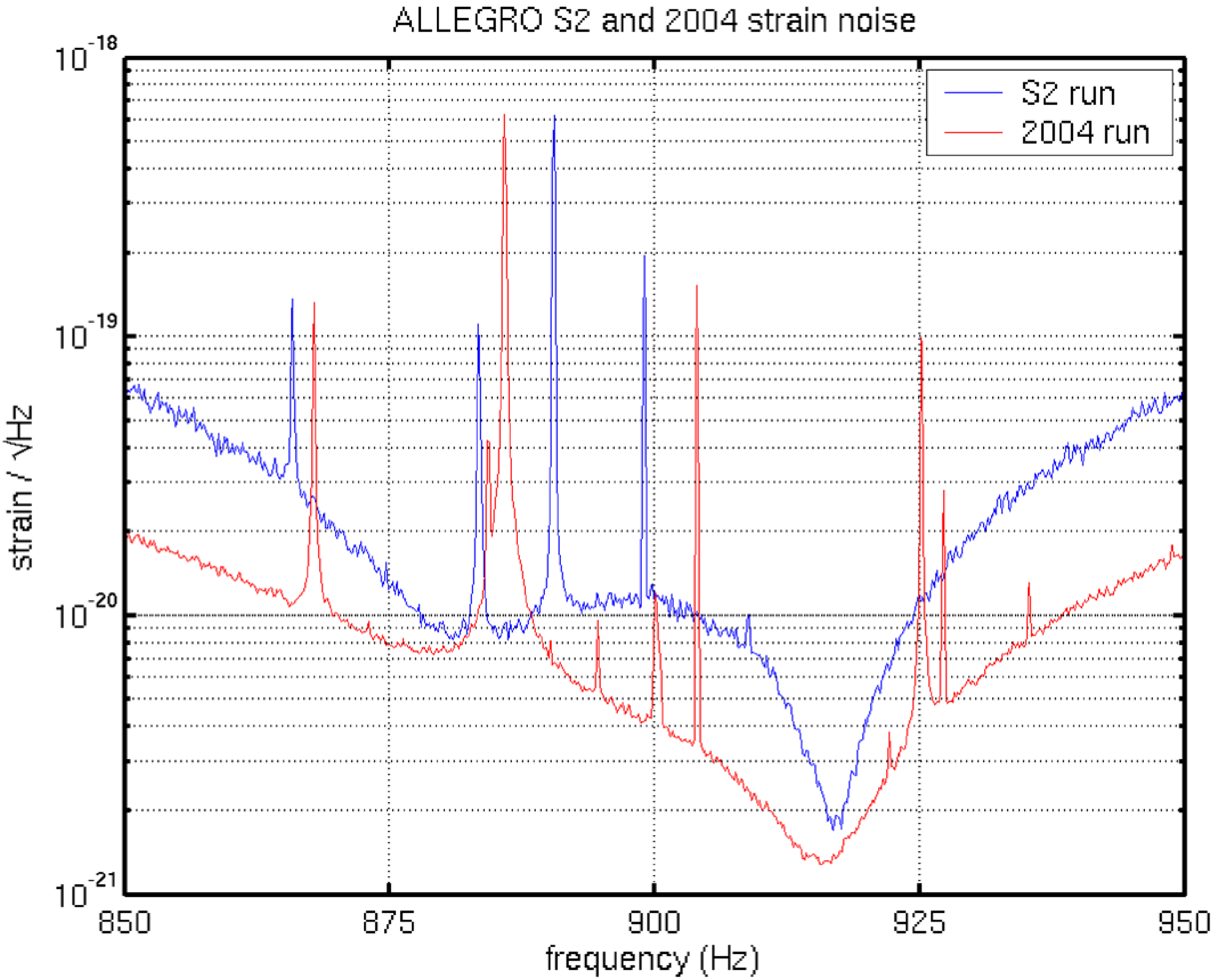
- FFT real LLO data, sampled at $2048\,\mathrm{Hz}$
  102401 bins: DC to $1024\,\mathrm{Hz}$ (Nyquist)

- FFT cmplx heterodyned ALLEGRO data, sampled at $250\,\mathrm{Hz}$
  25000 bins:
  $774\,\mathrm{Hz}$ $(f_b - f_{\mathrm{Ny}}^A)$ to $1023.99\,\mathrm{Hz}$ $(f_b + f_{\mathrm{Ny}}^A - \delta f)$

- Correlate only the bins from (say) $850\,\mathrm{Hz}$ to $950\,\mathrm{Hz}$
  ALLEGRO & LLO bins “line up”

LLO-ALLEGRO: Summary

- Probes higher frequency band: $\sim 850 - 950\,\mathrm{Hz}$
- Rotate ALLEGRO to modulate stochastic response
  (data taken in 3 orientations during S2)
- Freq-domain method seems to solve sampling rate problems
  ∃ more careful analytic demonstration
- Analyzing S2 data; next coïncident run is S4
- Reported at GR17:
  Expected S2 sensitivity of $\Omega_{\mathrm{GW}}(f) \sim 10$
  Projected S4 sensitivity of $\Omega_{\mathrm{GW}}(f) \sim 0.07$


ALLEGRO S2 and 2004 strain noise
S2 run
2004 run
strain / √Hz
10^{-18}
10^{-19}
10^{-20}
10^{-21}
850
875
900
925
950
frequency (Hz)

References

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   ICTP lecture: gr-qc/0008027
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5. P. Astone et al, A&A **351**, 811 (1999)
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7. LIGO S1 Paper: B. Abbott et al, PRD: gr-qc/0312088
8. Finn & Lazzarini, PRD: gr-qc/0104040;
   Poster: LIGO graphical presentation LIGO-G010246-00-E
9. LSC Stochastic BG Page: `http://www.ligo.org/sgwb/`
10. JTW GWDAW talk: LIGO-G030692-00-Z
11. LHO-LLO GR17 talk: LIGO-G040312-00-Z
12. LLO-ALLEGRO GR17 talk: LIGO-G040304-00-Z