Searching for a Stochastic Background of Gravitational Waves

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Outline

- Stochastic Gravitational Wave (GW) Backgrounds
  - Definitions and Conventions
  - Basic Data Analysis Technique (optimally filtered cross-correlation)
  - Overlap Reduction Function (observing geometry)

- Status of Ground-Based Observations
  - Upper Limits Set to Date
  - Status of Ongoing LIGO Research
  - Motivation for LIGO (Livingston)-ALLEGRO Observations

Types of Gravitational Wave Signals

Convenient classification for data analysis:

- Inspirals: "Chirp" signals (rapid decay of binary BH or NS orbit)
- Bursts: Unmodelled strong signals (e.g., Supernovae)
- Periodic: Continuous waves (e.g., rotating deformed NS)
- Stochastic: Random cosmological or astrophysical background

Stochastic Background of Gravitational Waves

- Random GW signal from superposition of unresolved sources

- Analogous to Cosmic Microwave Background, but
  - Spectrum unknown (compare CMB blackbody)
  - Component sources can be cosmological or astrophysical

- CMB comes from recombination of plasma to neutral atoms ionized plasma transparent to GWs ⟶ Cosmological GW BGs can tell us about earlier history of universe than CMB

Stochastic GW Spectrum

- Backgrounds in 10–1000 Hz frequency band likely extragalactic in origin, thus isotropic, unpolarized, gaussian, & stationary.
  $\longrightarrow$ defined entirely by spectrum

- Describe i.t.o. GW contribution to $\Omega = \frac{\rho}{\rho_{\mathrm{crit}}}$:

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{crit}}} \frac{d\rho_{\mathrm{GW}}}{d \ln f} = \frac{f}{\rho_{\mathrm{crit}}} \frac{d\rho_{\mathrm{GW}}}{df}$$

- Note $\rho_{\mathrm{crit}} \propto H_0^2$, so $h_{100}^2 \Omega_{\mathrm{GW}}(f)$ is independent of

$$h_{100} = \frac{H_0}{100\,\mathrm{km/\,s/\,Mpc}}$$

How to Tell Stochastic Signal from Random Noise

- Ground-based detectors noise-dominated
  & can't be pointed "off-source"
  → identifying a GW background in a single detector impractical

- Need correlations among detectors
  - Detector 1: $s_1 = h_1 + n_1$, Detector 2: $s_2 = h_2 + n_2$
  - h=stoch GW signal, n=noise (usu. much larger)

- Assume noise uncorrelated with signal & between detectors

- Cross-correlation:

$$\langle s_1 s_2 \rangle = \langle n_1 n_2 \rangle + \langle n_1 h_2 \rangle + \langle h_1 n_2 \rangle + \langle h_1 h_2 \rangle$$

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only surviving term is from stochastic GW signal

Statistics of Cross-Correlation

- Average cross-correlation:

$$\langle s_1 s_2 \rangle = \langle h_1 h_2 \rangle \propto T$$

- Variance of cross-correlation

$$\mathrm{var}(s_1 s_2) \approx \langle (s_1 s_2)^2 \rangle \approx \langle n_1^2 \rangle \langle n_2^2 \rangle \propto T$$

So standard deviation $\propto \sqrt{T}$ $\longrightarrow$ signal-to-noise $\propto \sqrt{T}$

Sensitivity to Stochastic GW Backgrounds

- Optimally filtered CC statistic

$$Y = \int df\, \tilde{s}_1^*(f)\, \tilde{Q}(f)\, \tilde{s}_2(f)$$

- Optimal filter $\tilde{Q}(f) \propto \frac{f^{-3}\Omega_{\mathrm{GW}}(f)\gamma_{12}(f)}{P_1(f)P_2(f)}$
  (Initial analyses assume $\Omega_{\mathrm{GW}}(f)$ constant across band)
- Optimally filtered cross-correlation method sensitive to

$$\Omega_{\mathrm{GW}} \propto \left(T \int \frac{df}{f^6} \frac{\gamma_{12}^2(f)}{P_1(f)P_2(f)}\right)^{-1/2}$$

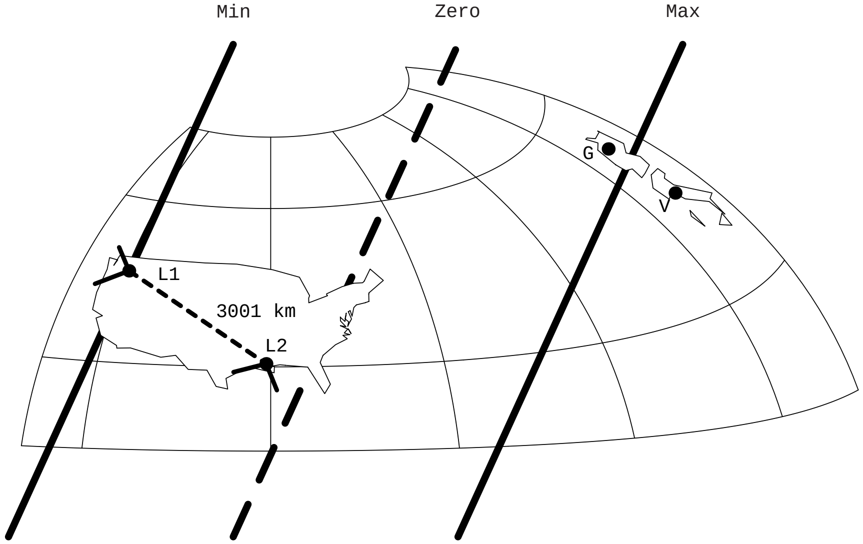
- Significant contributions when
  - detector noise power spectra $P_1(f)$, $P_2(f)$ small
  - overlap reduction function $\gamma_{12}(f)$ (geom correction) near ± 1

Overlap Reduction Function

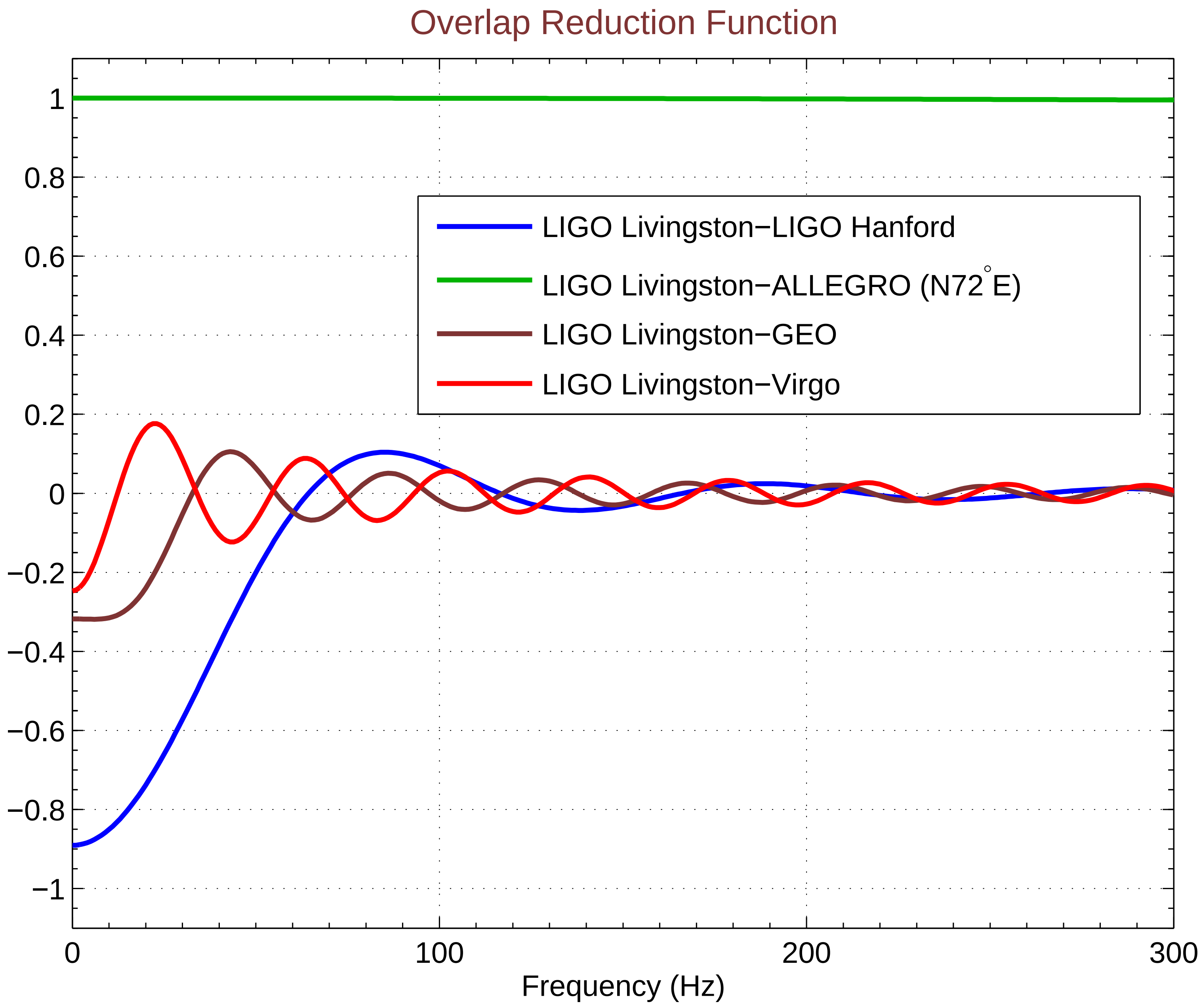
$$\gamma_{12}(f) = d_{1ab}\, d_2^{cd}\, \frac{5}{4\pi} \iint_{S^2} d^2\Omega\ P^{\mathrm{TT}}{}^{ab}_{cd}(\hat{\Omega})\, e^{i2\pi f \hat{\Omega}\cdot\Delta\vec{\mathbf{x}}/c}$$

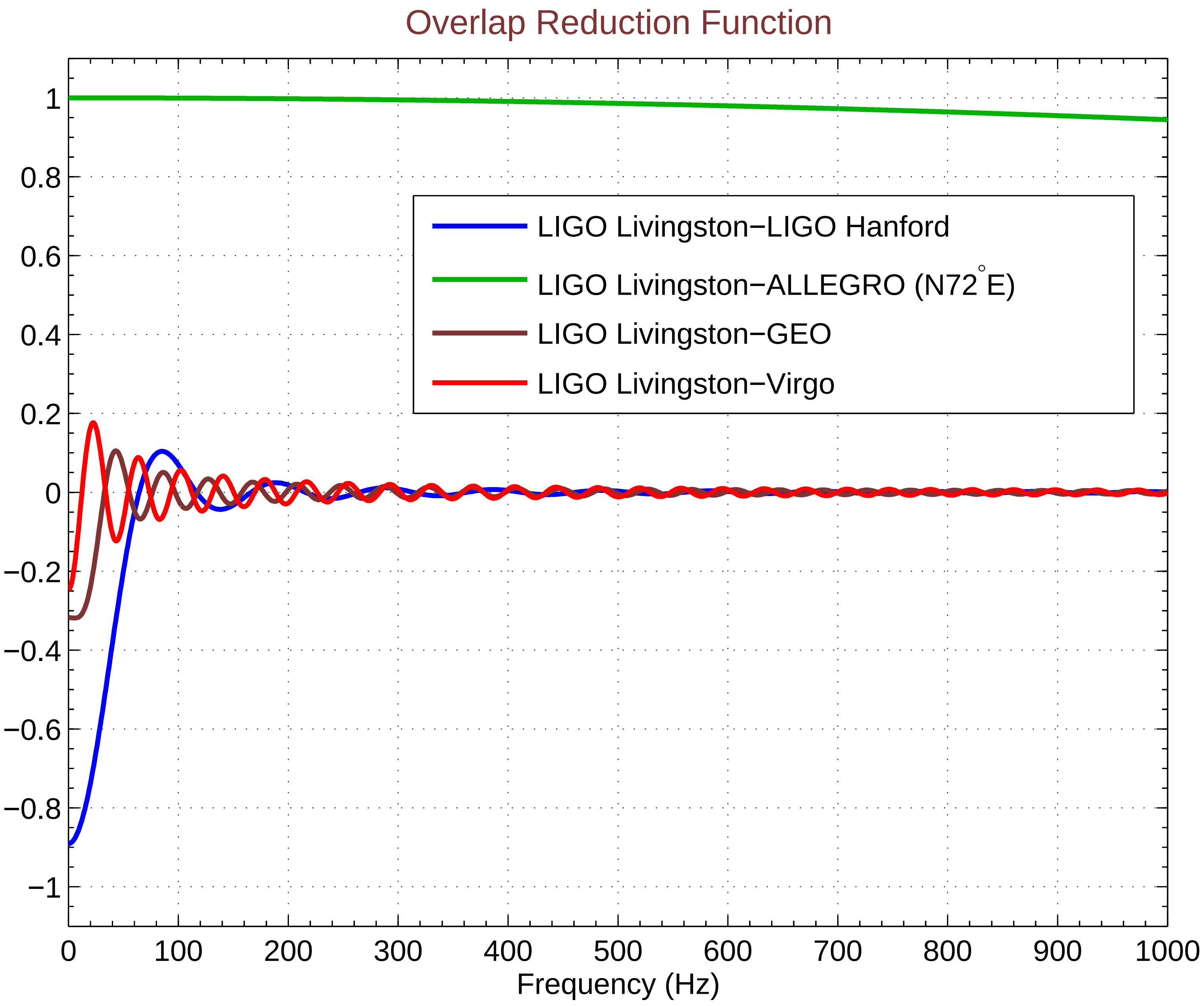
- Depends on alignment of detectors (polarization sensitivity)
- Frequency dependence from cancellations when $\lambda \lesssim$ distance

  → Widely separated detectors less sensitive at high frequencies



(figure from Allen & Romano PRD, gr-qc/9710117)


Overlap Reduction Function
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
100
200
300
Frequency (Hz)
LIGO Livingston−LIGO Hanford
LIGO Livingston−ALLEGRO (N72°E)
LIGO Livingston−GEO
LIGO Livingston−Virgo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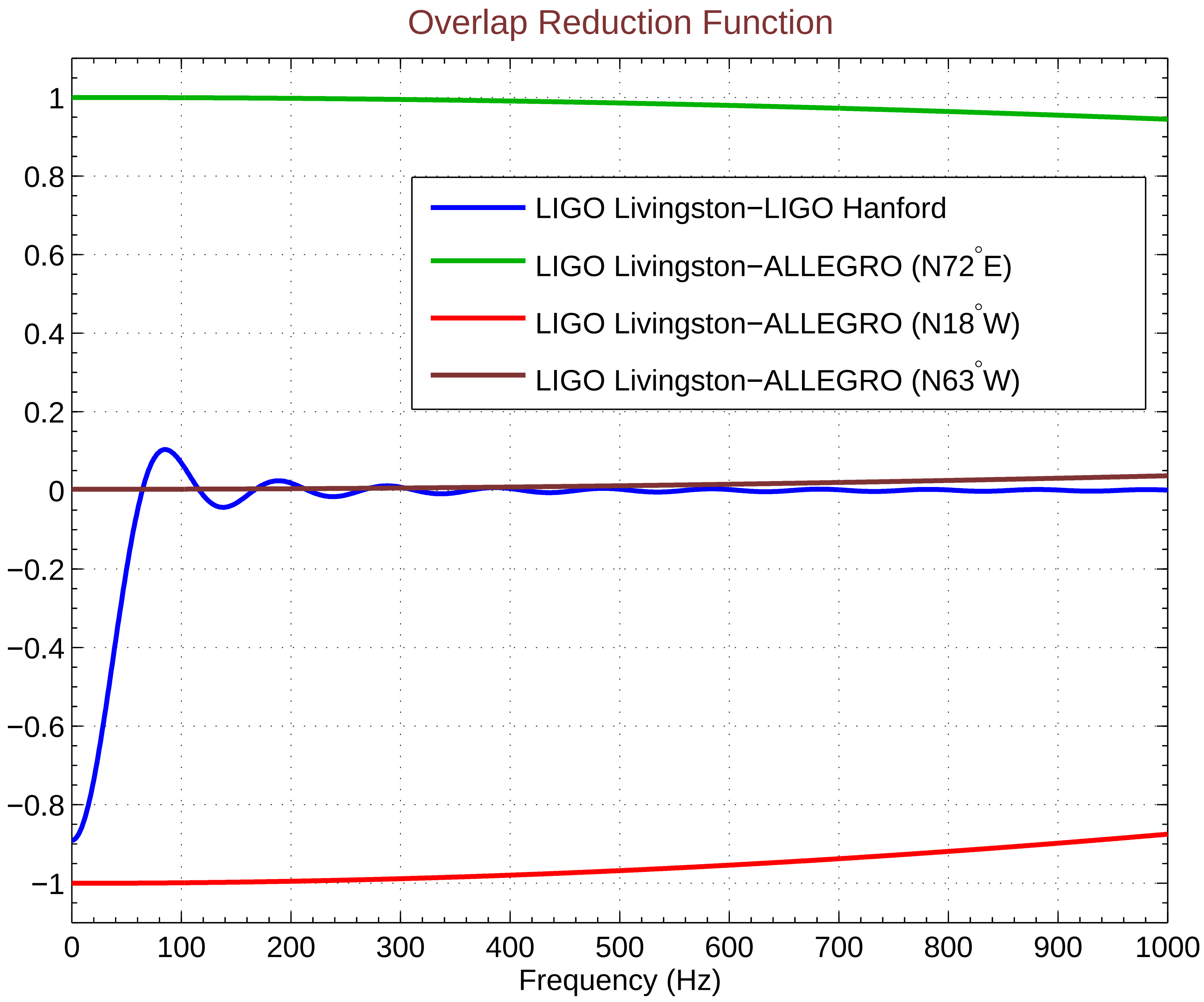
Frequency (Hz)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Frequency (Hz)
LIGO Livingston−LIGO Hanford
LIGO Livingston−ALLEGRO (N72°E)
LIGO Livingston−ALLEGRO (N18°W)
LIGO Livingston−ALLEGRO (N63°W)

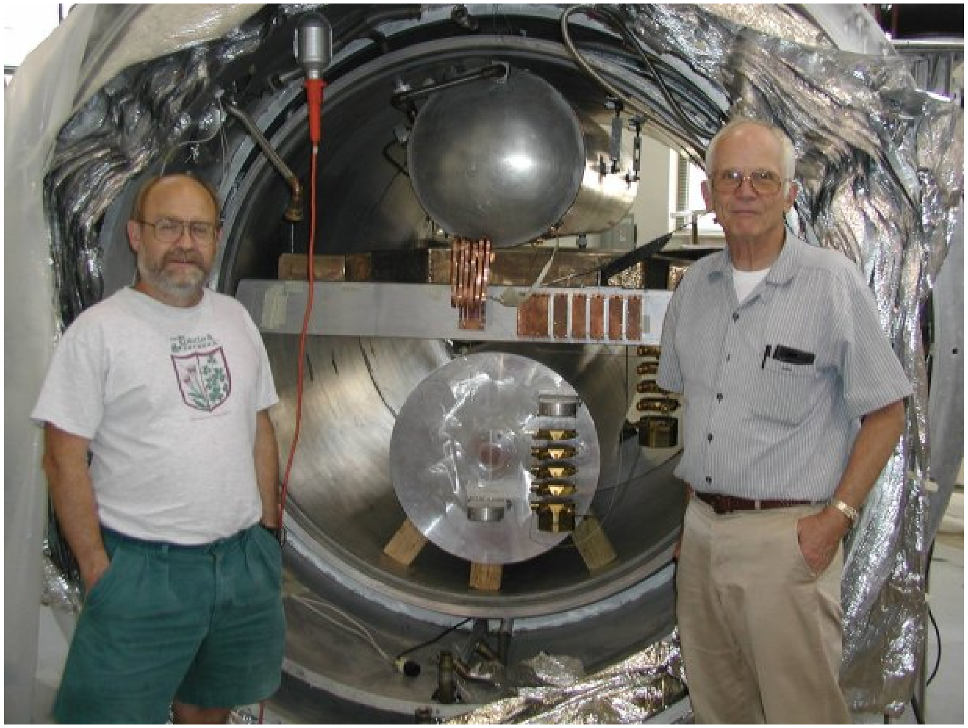
Cartoon courtesy of E. Coccia, NAUTILUS Group (Rome)

Stochastic BG Searches

- Upper Limits so Far:
  - Correlation between Garching & Glasgow prototype IFOs [Compton et al, MG7 proceedings, 1994]: $h_{100}^2 \Omega_{\mathrm{GW}}(f) \lesssim 3 \times 10^5$
  - Correlation between EXPLORER & NAUTILUS bars [Astone et al, A&A **351**, 811 (1999)]: $h_{100}^2 \Omega_{\mathrm{GW}}(907\,\mathrm{Hz}) \le 60$
  - Correlation between LIGO Hanford & Livingston S1 data [LSC, Abbott et al, PRD **69**, 122004 (2004)]: $h_{100}^2 \Omega_{\mathrm{GW}}(f) \le 23$ at $64 < f < 265$

- Ongoing Analyses:
  - Correlations between LIGO Hanford & Livingston
  - Correlations between LIGO Livingston & ALLEGRO
  - Correlations between EXPLORER & NAUTILUS bars

ALLEGRO Detector (Baton Rouge, LA)



W. Johnson, ALLEGRO & W. Hamilton from LSU Website

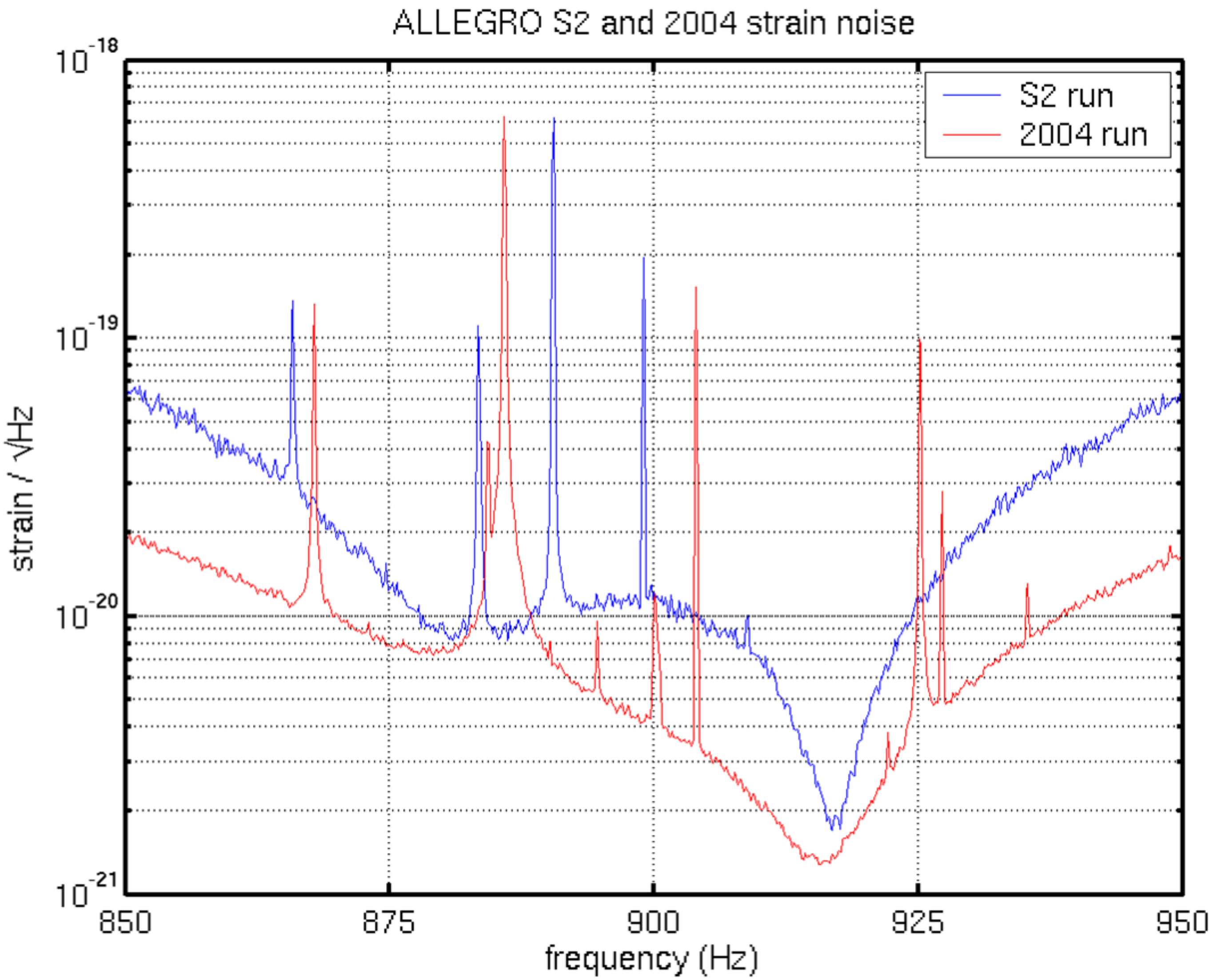


Figure from McHugh GR17 Presentation

LIGO (Livingston)-ALLEGRO Correlations

- Only ~40 km apart → $\gamma(900\,\text{Hz}) \approx 95\%$ for best alignment
- Sensitive in diff freq band from LIGO Livingston/Hanford pair 900 Hz vs 50–300 Hz
- New experimental technique: rotate ALLEGRO to calibrate cross-correlated noise [Finn & Lazzarini, PRD **64**, 082002 (2001)]
  - Aligned & Anti-aligned orientations have opposite GW sign $\longrightarrow$ can "cancel" out CC noise by subtracting results
  - Null orientation has no expected GW signal $\longrightarrow$ "off-source" measurement of CC noise
- Currently analyzing S2 (2003 Feb 14-Apr 14) data; ALLEGRO was offline for S3 (2003 Oct 31-2004 Jan 9), now running again; Further work planned for S4 & beyond

Status of Ongoing LIGO Stochastic BG Searches

- LIGO Livingston-LIGO Hanford ($50\,\text{Hz} \lesssim f \lesssim 300\,\text{Hz}$)
  - S1 (2002 Aug 23-Sep 9): $h^2_{100}\Omega_{\text{GW}}(f) \leq 23$
    Published in PRD **69**, 122004 (2004)
  - S2 (2003 Feb 14-Apr 14) preliminary result reported at GR17
    $h^2_{100}\Omega_{\text{GW}}(f) \leq 0.018^{+0.007}_{-0.003}$
  - S3 (2003 Oct 31-2004 Jan 9) being analyzed;
    Expected sensitivity $\Omega_{\text{GW}}(f) \sim 5 \times 10^{-4}$
  - Also correlating 2km & 4km IFOs @ Hanford

- LIGO Livingston-ALLEGRO ($f \sim 900\,\text{Hz}$)
  - S2 (2003 Feb 14-Apr 14) being analyzed;
    Expected sensitivity of $\Omega_{\text{GW}}(f) \sim 10$
  - Project $\gtrsim 100\times$ improvement in sensitivity for S4

Summary

- Stochastic GW backgrounds can tell us about early-universe cosmology or astrophysical populations

- Basic analysis technique: optimally-filtered cross-correlation between detectors

- Observing geometry (via overlap reduction function) favors detector pairs which are close and similarly oriented

- Research underway w/both interferometers & bars