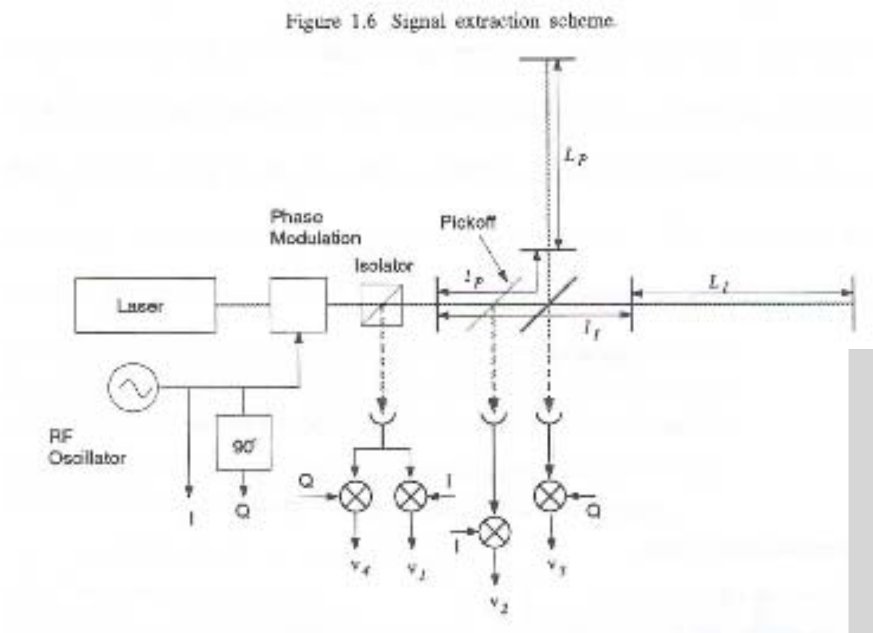
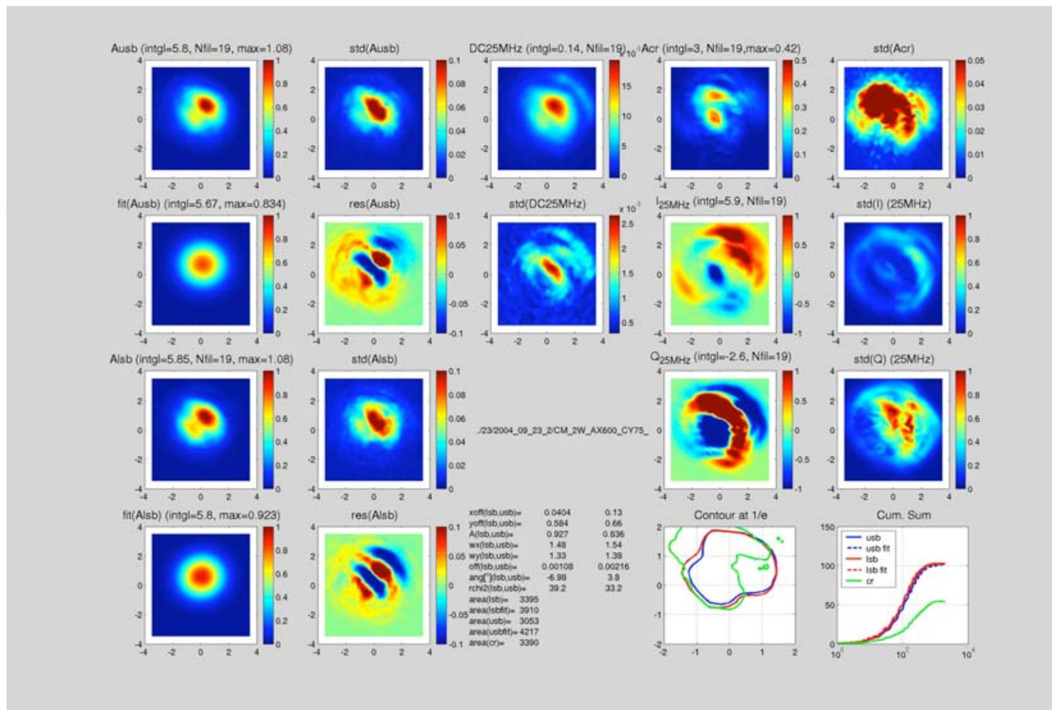
Next to lowest order approximation of LIGO I

Hiro Yamamoto



LIGO 1994

LIGO 2004



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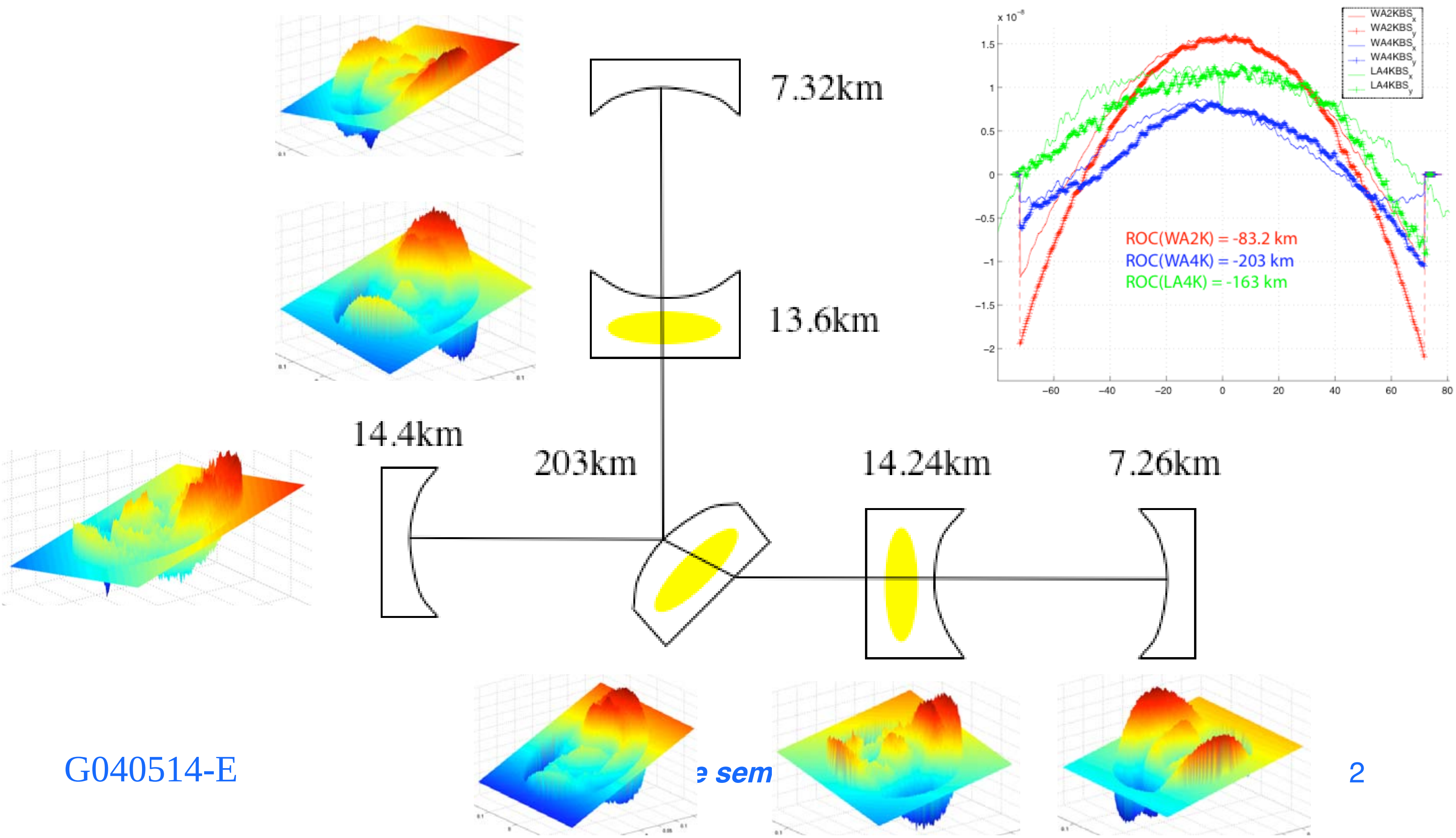
With a little bit of reality



A simplified picture of

- “What’s going on ?”
- Thermal lensing
- Sideband imbalance
- Effect of locking
- Output Mode Cleaner
- Optical gain, Spob, contrast defect, etc, etc
- Correlation between DC-picth/yaw and WFS signal

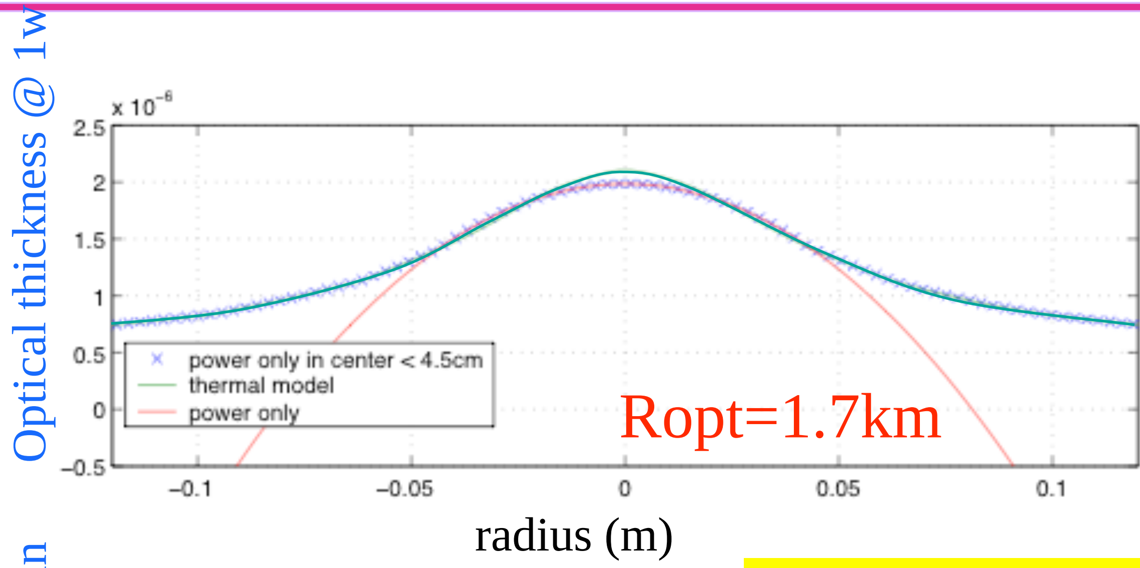
Tools

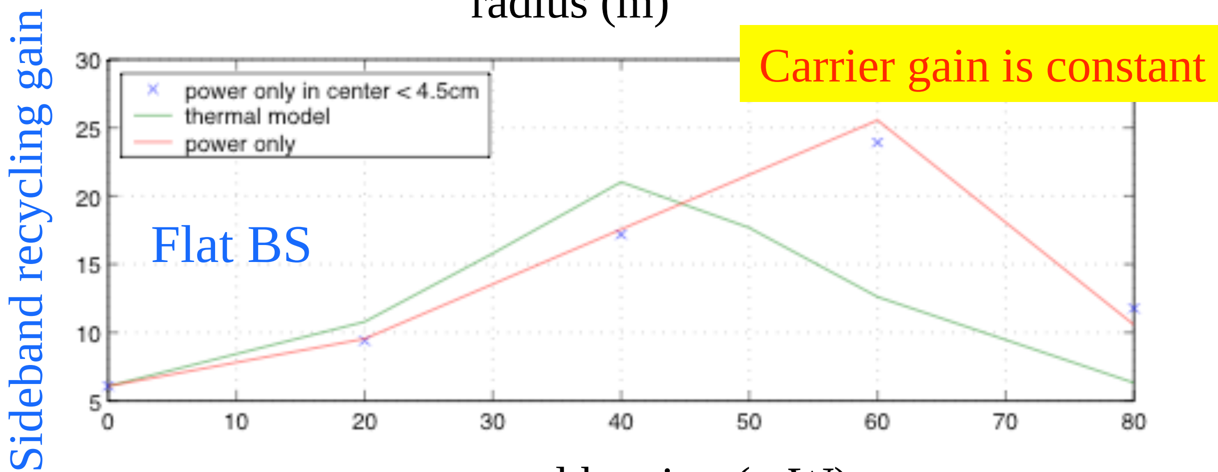
Tool	Pros	Cons
Analytic calculation	❑Underlying mechanism can be understood	❑Only simplified case can be analyzed
e2e	❑Time evolution can be traced ❑Semi-realistic sensing and controls can be included	❑Limited spatial profile ❑No details of optics
FFT	❑Details of optics can be included (Phase map, Thermal lens)	❑Only static

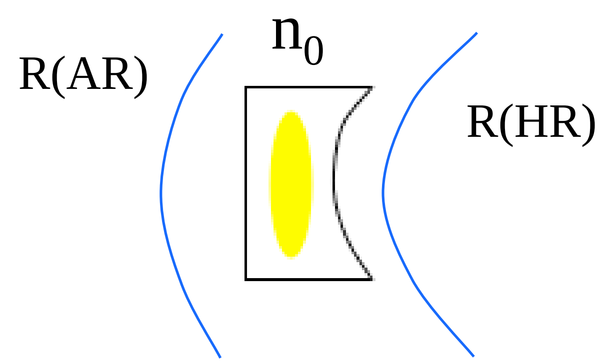
- Modal Model, not so bad
  - Modal Model can be used to study degenerate and/or unstable cavity
  - Valid only when perturbation is small
  - Field source mode is more important than cavity eigenstates

Thermal lensing and $n_{effective}$

- P. Willems calculated based on MIT model -







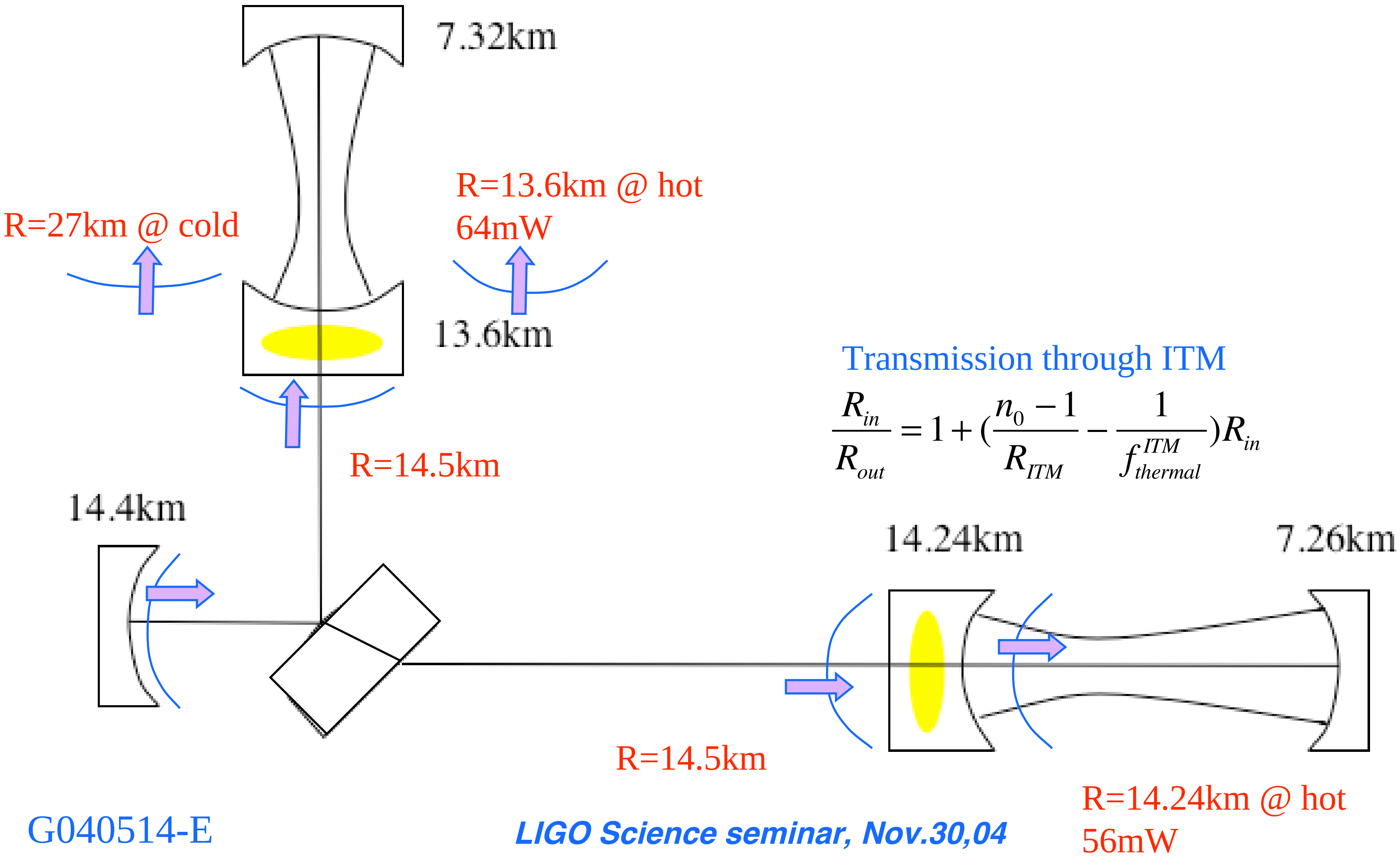
$$\frac{1}{f} = -\frac{n_0 - 1}{Rm} + \frac{Power}{Ropt}$$

$$= -\frac{n_{effective} - 1}{Rm}$$

$$\frac{1}{R_f(HR)} = \frac{1}{R_f(AR)} - \frac{1}{f}$$

Power = 58mW

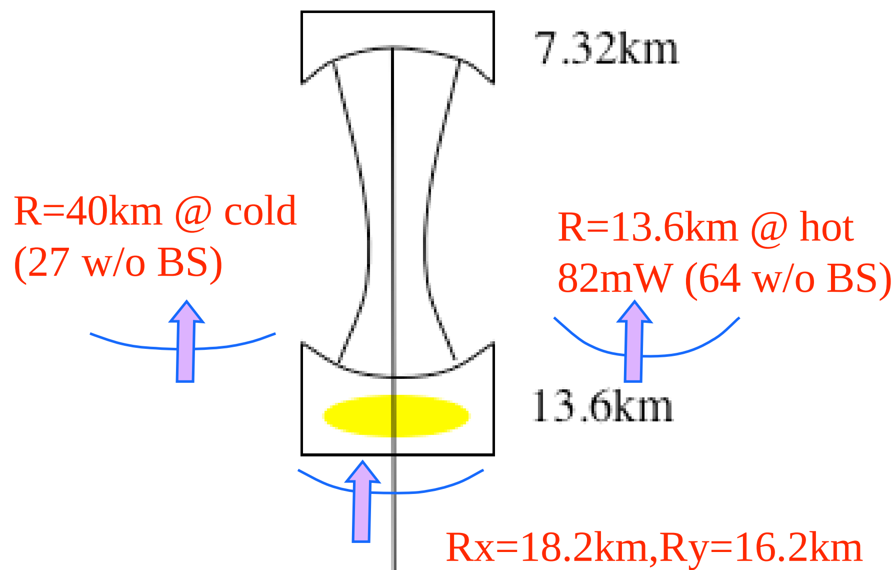
Mode matching



Transmission through ITM

$$\frac{R_{in}}{R_{out}} = 1 + \left(\frac{n_0 - 1}{R_{ITM}} - \frac{1}{f^{ITM}_{thermal}}\right) R_{in}$$

Mode matching with BS



Reflection by BS

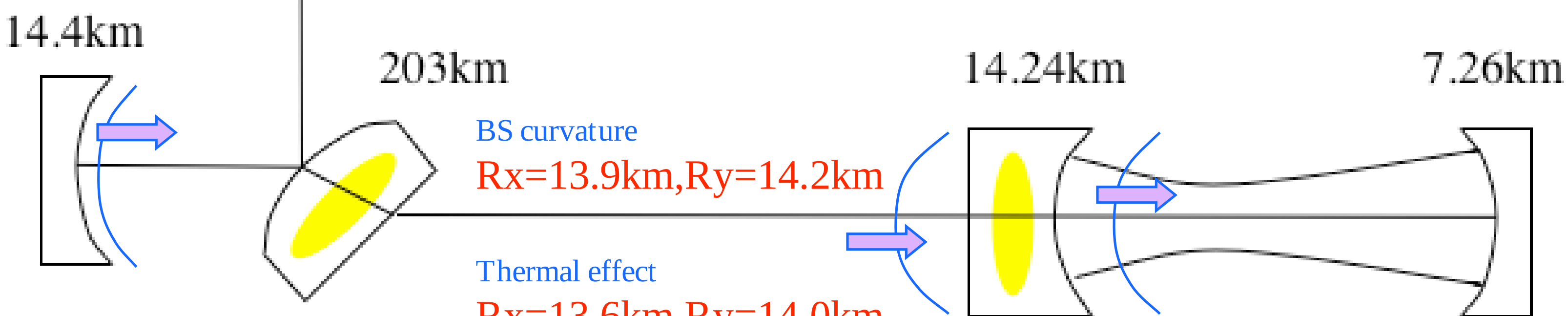
$$\frac{R_{in}}{R_{out}} = 1 - \frac{2}{R_{BS}} \frac{R_{in}}{C_{x/y}} \qquad C_x = 1/\sqrt{2},\ C_y = \sqrt{2}$$

Transmission through BS

$$\frac{R_{in}}{R_{out}} = 1 + \left(\frac{n_0 - 1}{R_{BS}} - \frac{1}{f^{BS}_{thermal}}\right)\frac{R_{in}}{C_{x/y}}$$

Transmission through ITM

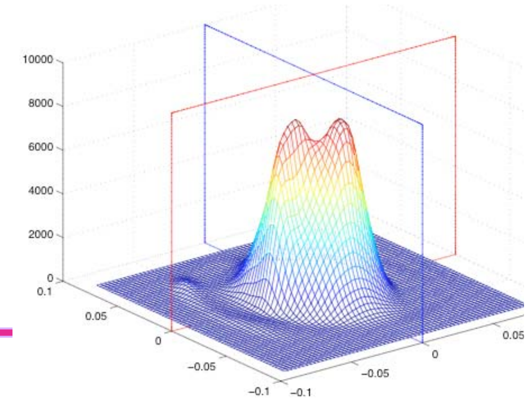
$$\frac{R_{in}}{R_{out}} = 1 + \left(\frac{n_0 - 1}{R_{ITM}} - \frac{1}{f^{ITM}_{thermal}}\right) R_{in}$$



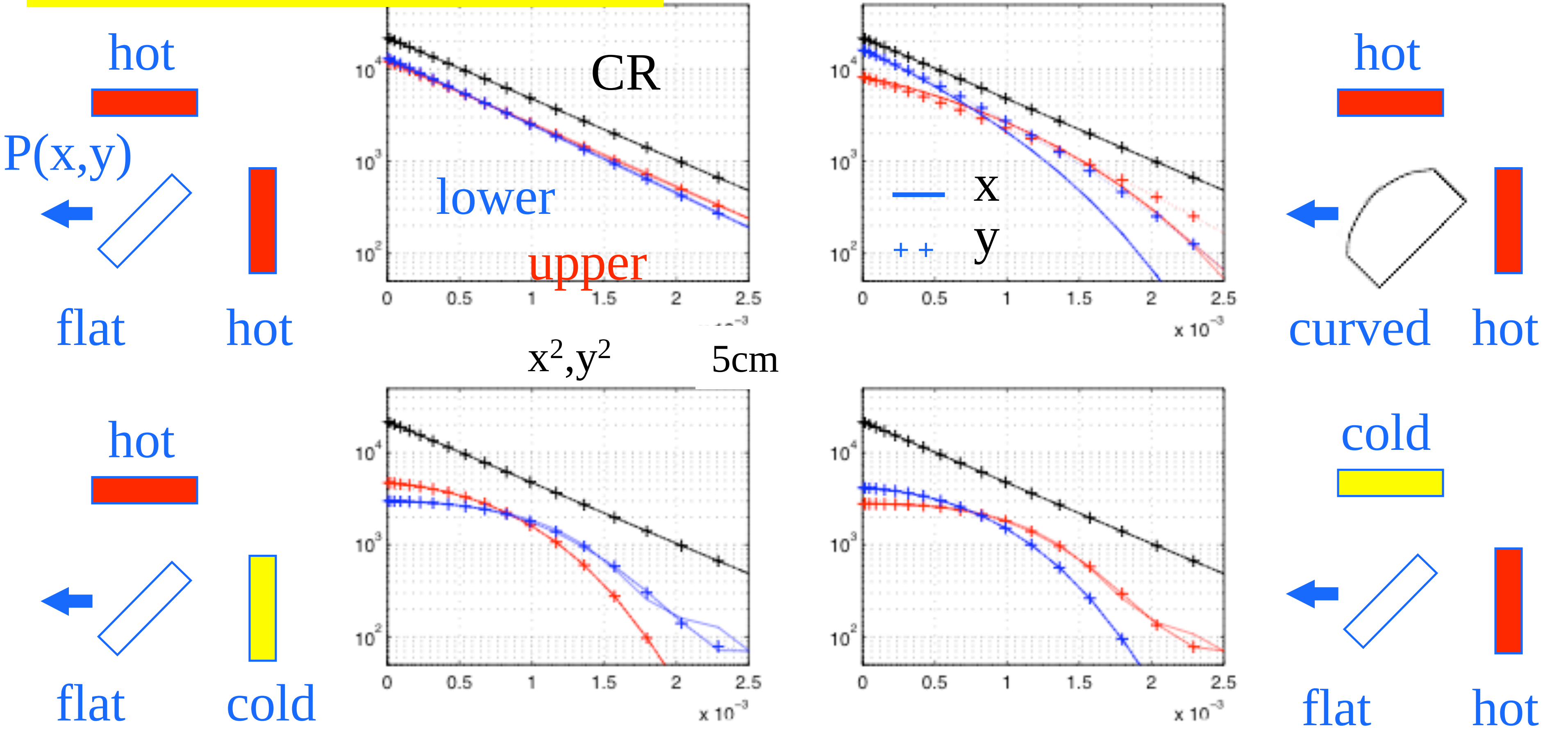
$$\frac{1}{f^{BS}_{thermal}} = \frac{1}{2}\frac{1}{2}\frac{4}{10}\frac{1}{f^{ITM}_{thermal}(hot)}$$

R=14.24km @ hot
52mW (54 w/o BS)
only BS effect

LIGO ITM differential heating and beam splitter curvature



- Linear line : gaussian
- Blue vs red : sideband imbalance
- --- vs + + + : astigmatism



Effects of mode matching in PRM

- Carrier field is insensitive to thermal state of ITMs and BS curvature
  - » CR reflected by arm does not have higher mode excited
  - » SB reflected has higher order mode excited due to curvature mismatch
- Michelson cavity can induce imbalance of upper and lower sidebands
  - » Oscillation phase part change sign, but Gouy phase and terms due to mode mixing are SB sign independent.
  - » Sideband imbalance in PRM is observable when two cavities in PRM are different

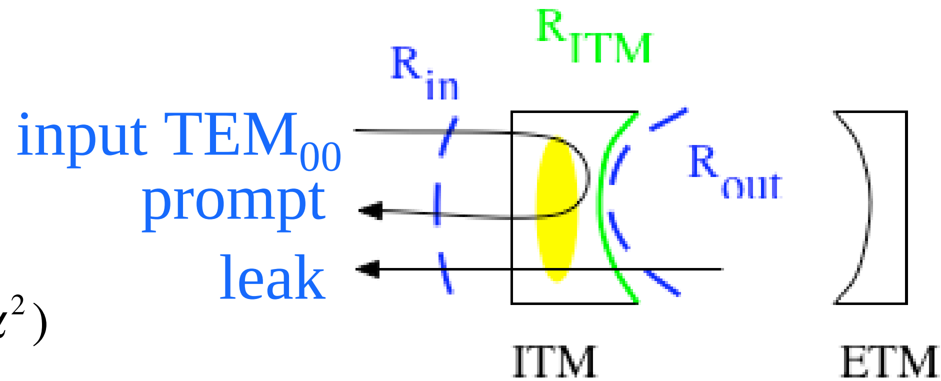
$$\mathrm{Propagator} = 1 - r_{RM} \cdot r_{ITM} \cdot \exp(-i\phi)$$

$$\phi = k_{SB} \cdot d\ell_{snp} - c_1 \cdot \eta - c_2 \cdot \alpha^2$$

Reflection by arm cavity with curvature mismatch

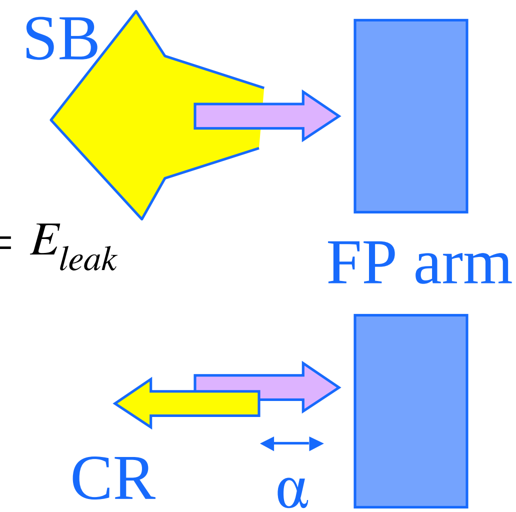
$$\alpha = \frac{z}{z_0}\left(1 - \frac{R_{ITM}}{R_{out}}\right)$$

E_{mn} is the mode in PRC which couples to the arm when α=0

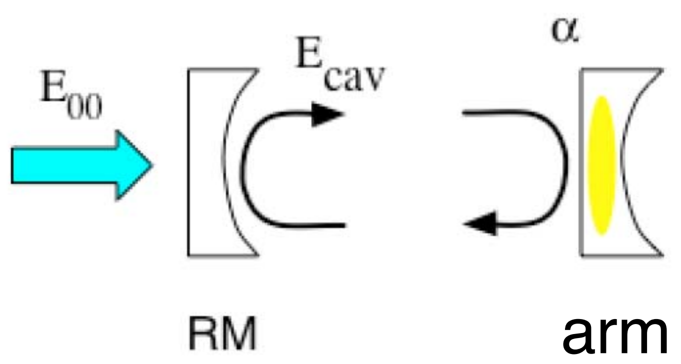


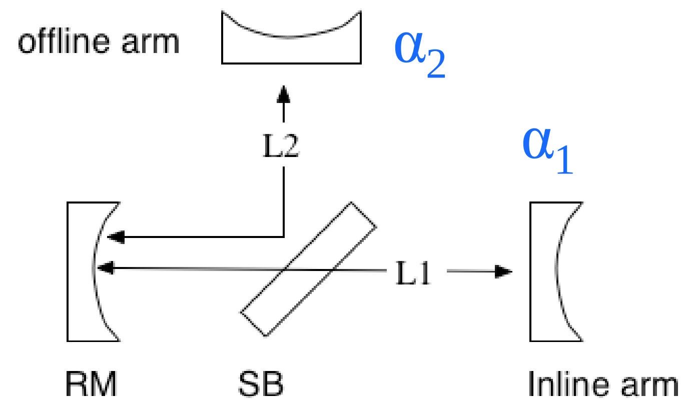
$$E_{SB} = \frac{1}{1+i\alpha}E_{00} - \frac{i\alpha/\sqrt{2}}{(1+i\alpha)^3}(E_{02}+E_{20}) + O(\alpha^2)$$

$$\begin{aligned} E_{CR} &= \frac{1}{1+i\alpha}E_{00} - \frac{i\alpha/\sqrt{2}}{(1+i\alpha)^3}(E_{02}+E_{20}) + O(\alpha^2) \quad \Leftarrow E_{prompt} \\ &\quad -2\frac{1}{1+i\alpha/2}\left(\frac{1}{1+i\alpha/2}E_{00} - \frac{i\alpha/2/\sqrt{2}}{(1+i\alpha/2)^3}(E_{02}+E_{20}) + O(\alpha^2)\right) \quad \Leftarrow E_{leak} \\ &= -\frac{1}{1+i\alpha}E_{00} + O(\alpha^2) \end{aligned}$$



Sideband imbalance in curvature mismatched FPs





$$E_{cav} = \sum_{m,n} \frac{C_{mn}(\alpha)\cdot HG_{mn}}{1 - r_{RM}\cdot \exp(i\phi)}$$

$$\phi_{CR,00} = -2k_{CR}L + 2\eta_{00} - \arctan(\alpha)$$

$$\phi_{SB}(k_{SB}L) = \phi_{CR,00} - 2k_{SB}L + \delta\phi(\alpha,\eta)$$

$$\phi_{SB}(k_{SB}L,\alpha) \neq \phi_{SB}(-k_{SB}L,\alpha)$$

$$P(k_{SB}L,\alpha) \neq P(-k_{SB}L,\alpha)$$

$$k_{SB}L_1 = -k_{SB}L_2 = \frac{\pi\cdot\delta L_{syn}}{\lambda_{SB}}$$

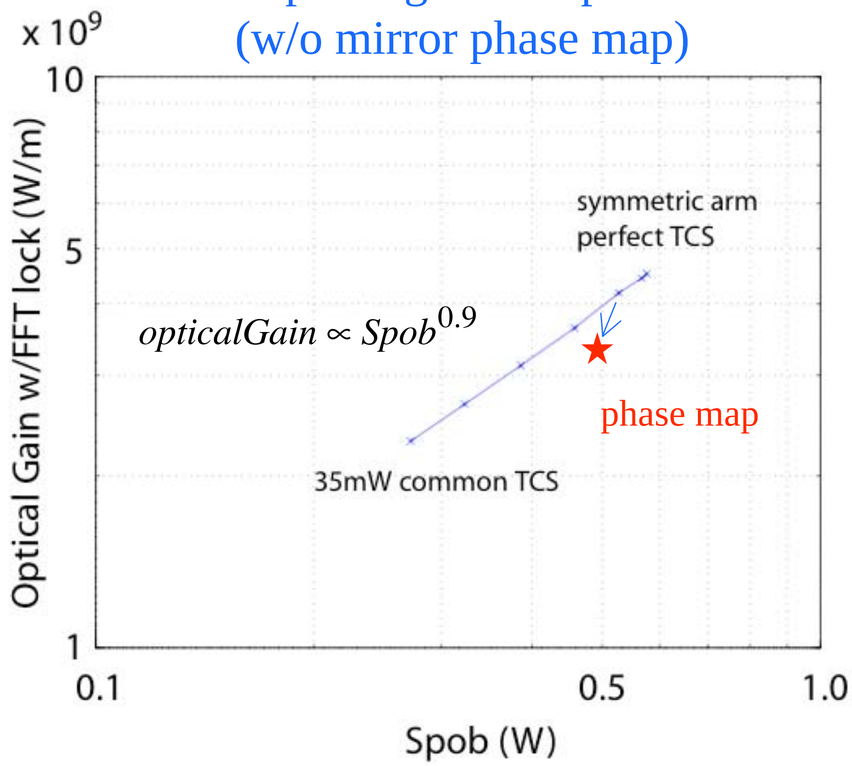
$$P(k_{SB}L_1,\alpha_1) = P(-k_{SB}L_2,\alpha_1)$$

$$P(k_{SB}L_1,\alpha_1) + P(k_{SB}L_2,\alpha_2)$$
$$= P(-k_{SB}L_1,\alpha_2) + P(-k_{SB}L_2,\alpha_1)$$

IFO performance

	Lock	CR gain	Upper SB gain	Lower SB gain	Contrast Defect $\times 10^{-6}$
symmetric arm, best TCS	FFT	46	23.7	23.7	58
As built arm, best TCS	FFT	46	21.7	25.3	220
As built arm, com.TCS	FFT	46	19.6	24.6	233
	LSC	46	22.3	22.3	244
As built arm, com.TCS w/ phase map	FFT	36	18.3	23.3	429
	LSC	36	21.5	20.2	439

optical gain vs Spob
(w/o mirror phase map)



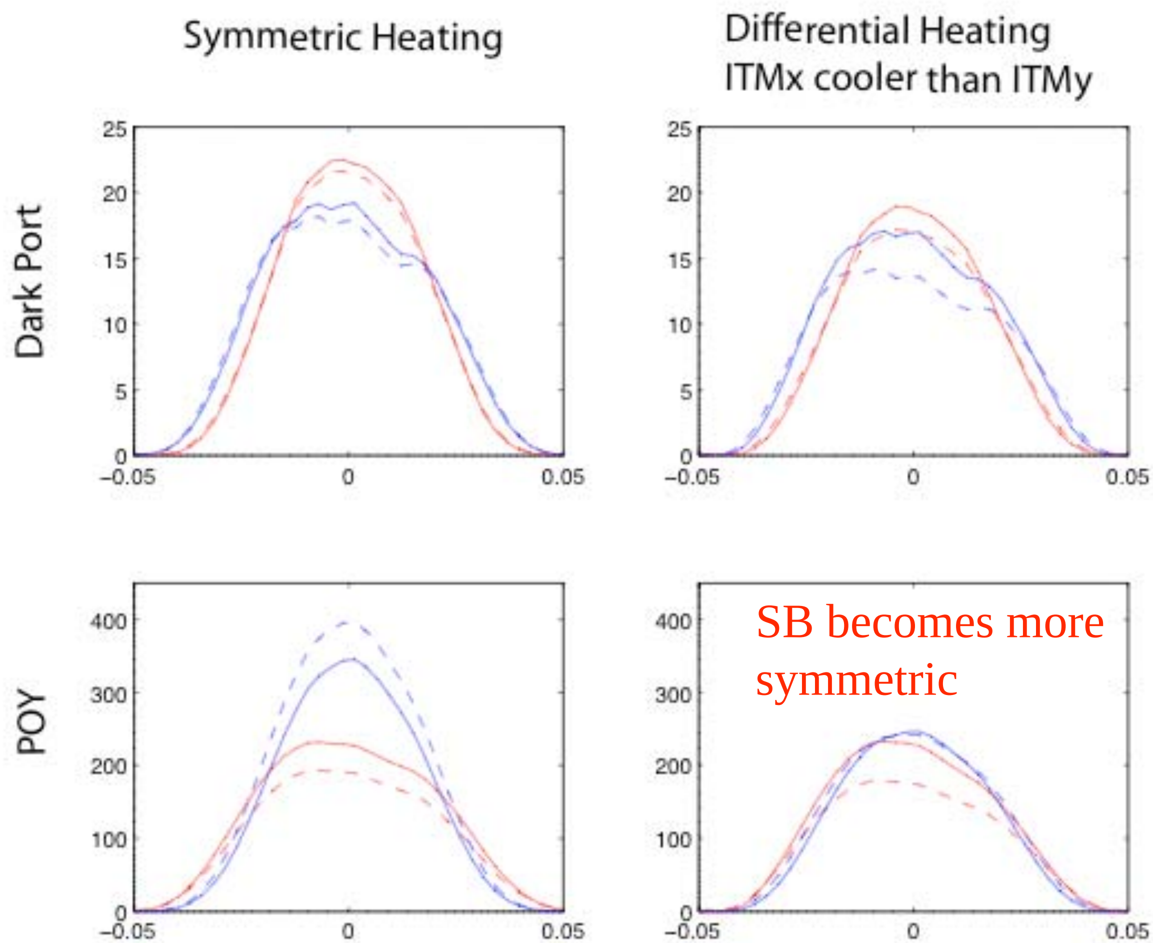
SB imbalance comes from R_{ITM} and R_{BS}

LIGO

FFT vs LSC lock

n(ITMx)-n(ITMy)

0.96-0.96 | 1.10-0.96

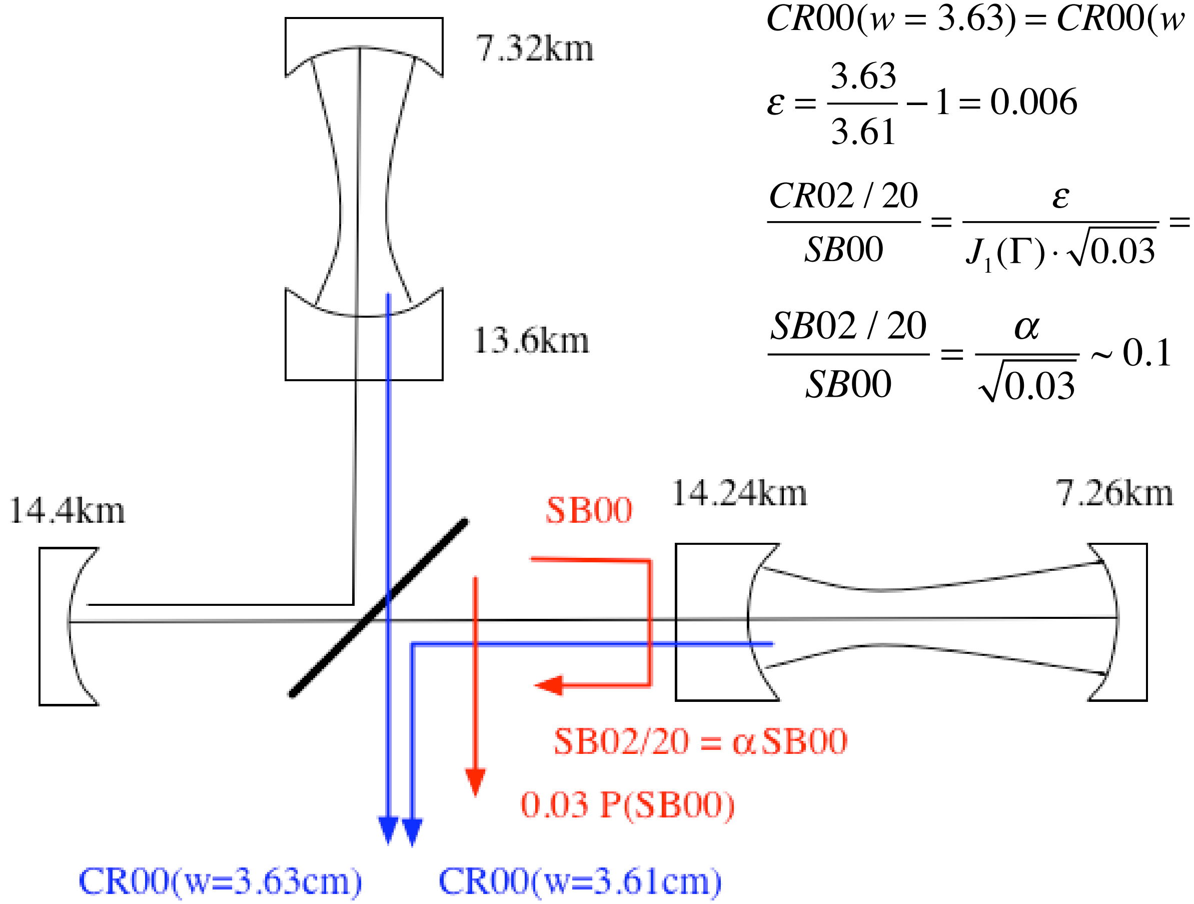


—— lower SB —— upper SB

- - - - FFT lock —— LSC lock

		FFT	LSC
symmetric	θCR	0.3	-1.9
	θSB+	-0.6	-2.3
	θSB-	7.2	5.1
	Spob	-0.57i	-0.57i
differential	θCR	0.2	-8
	θSB+	4.9	-1.2
	θSB-	11.8	5.1
	Spob	-0.48i	-0.50i

modes in the dark port
- back on the envelope -



$$CR00(w=3.63) = CR00(w=3.61) + \varepsilon \cdot CR02/20(w=3.63)$$

$$\varepsilon = \frac{3.63}{3.61} - 1 = 0.006$$

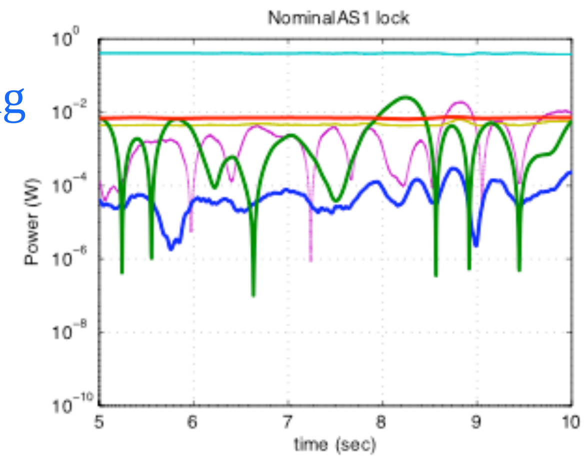
$$\frac{CR02/20}{SB00} = \frac{\varepsilon}{J_1(\Gamma)\cdot\sqrt{0.03}} = 0.15$$

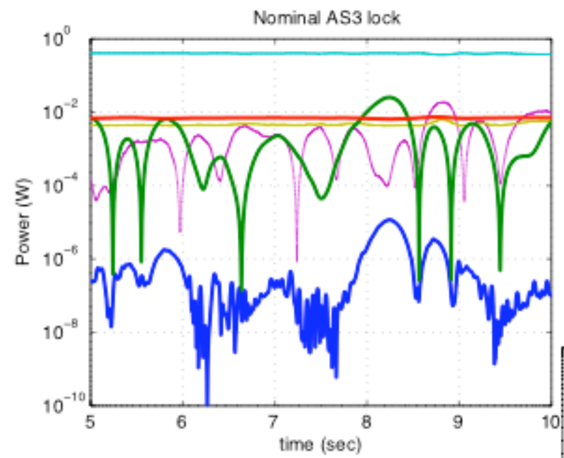
$$\frac{SB02/20}{SB00} = \frac{\alpha}{\sqrt{0.03}} \sim 0.1$$

modes in the dark port

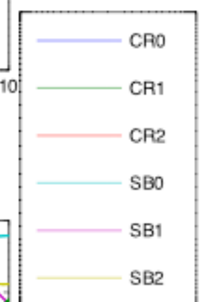
- e2e simulation -

common heating
α~0.01
use dark port
signal to lock



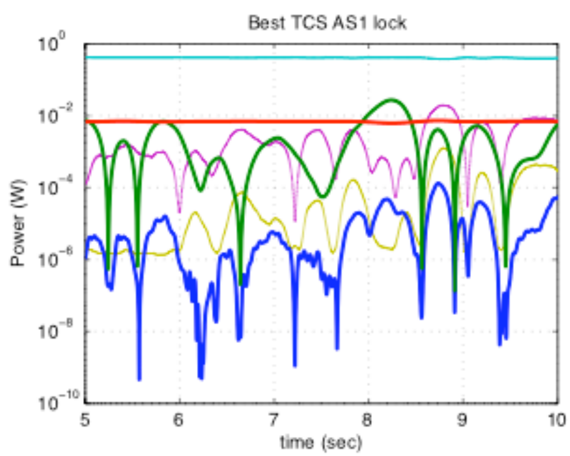


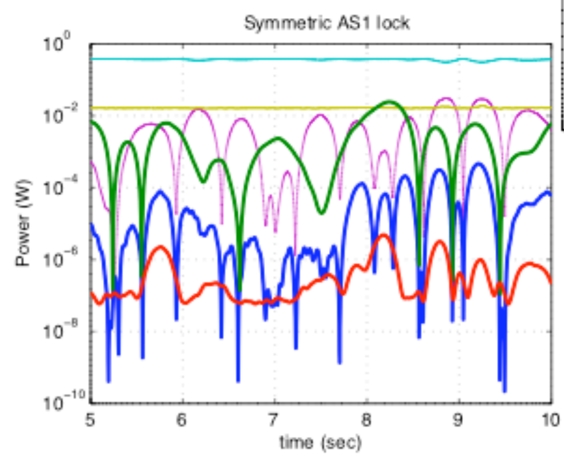
common heating
α~0.01
use OMC output
signal to lock



time evolution of powers of various modes

differential
heating
α~0
use dark port
signal to lock

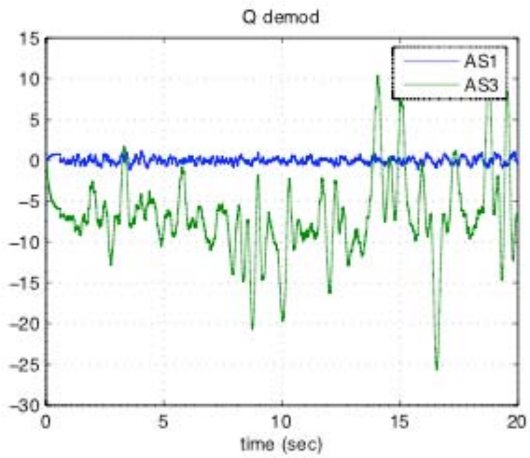


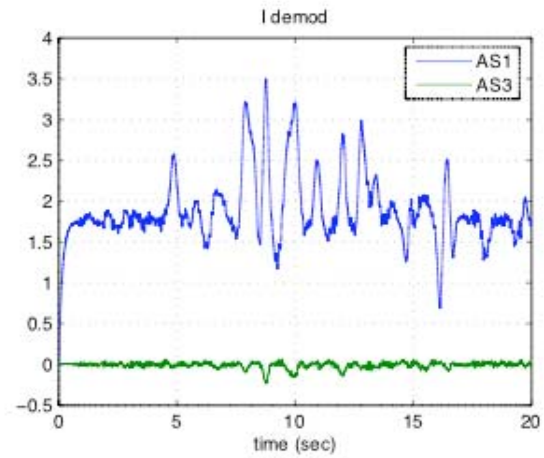


Symmetric arm
common heating
α~0.01
dark port signal

OMC and ASQ and ASI
(1) nominal matching

use dark port signal to lock common heating $\alpha \sim 0.01$



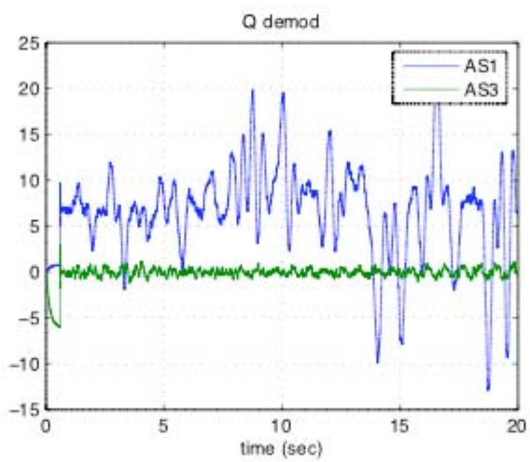


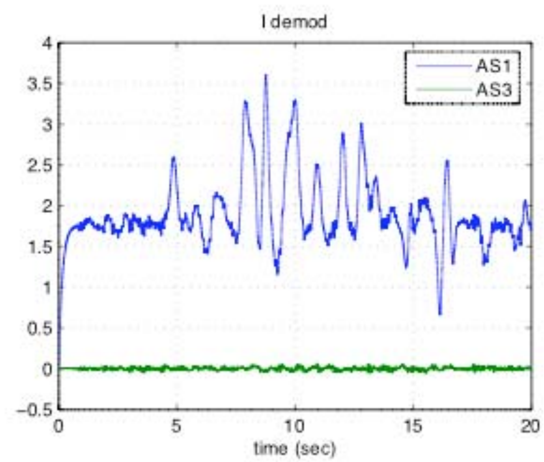
Q-demod

I-demod

AS1: dark port
AS3: OMC output

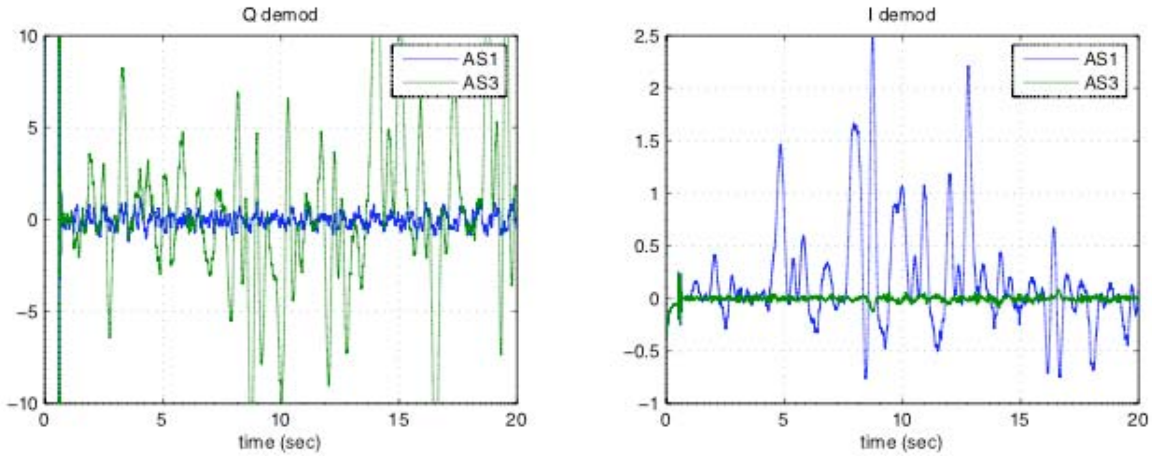
use OMC output signal to lock common heating $\alpha \sim 0.01$



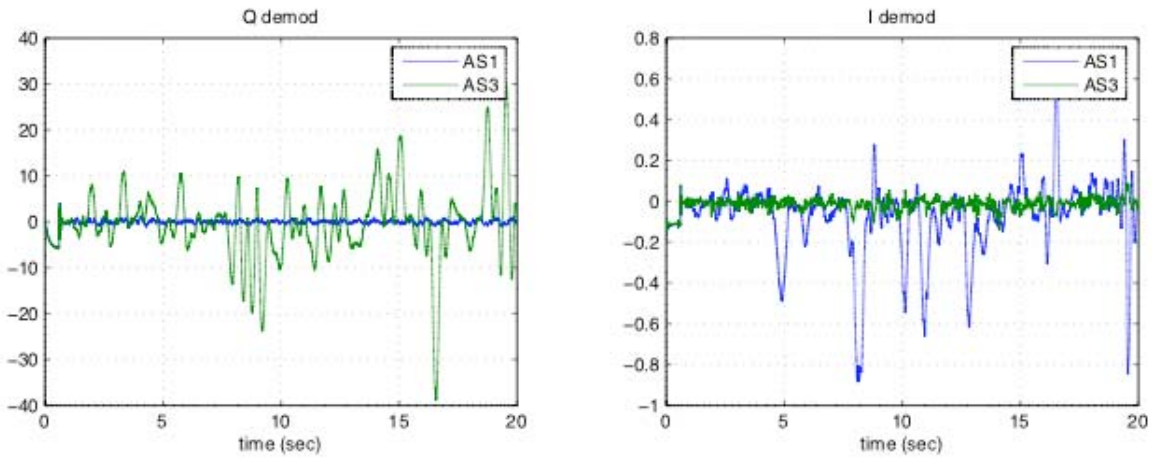


OMC and ASQ and ASI
(2) better matching

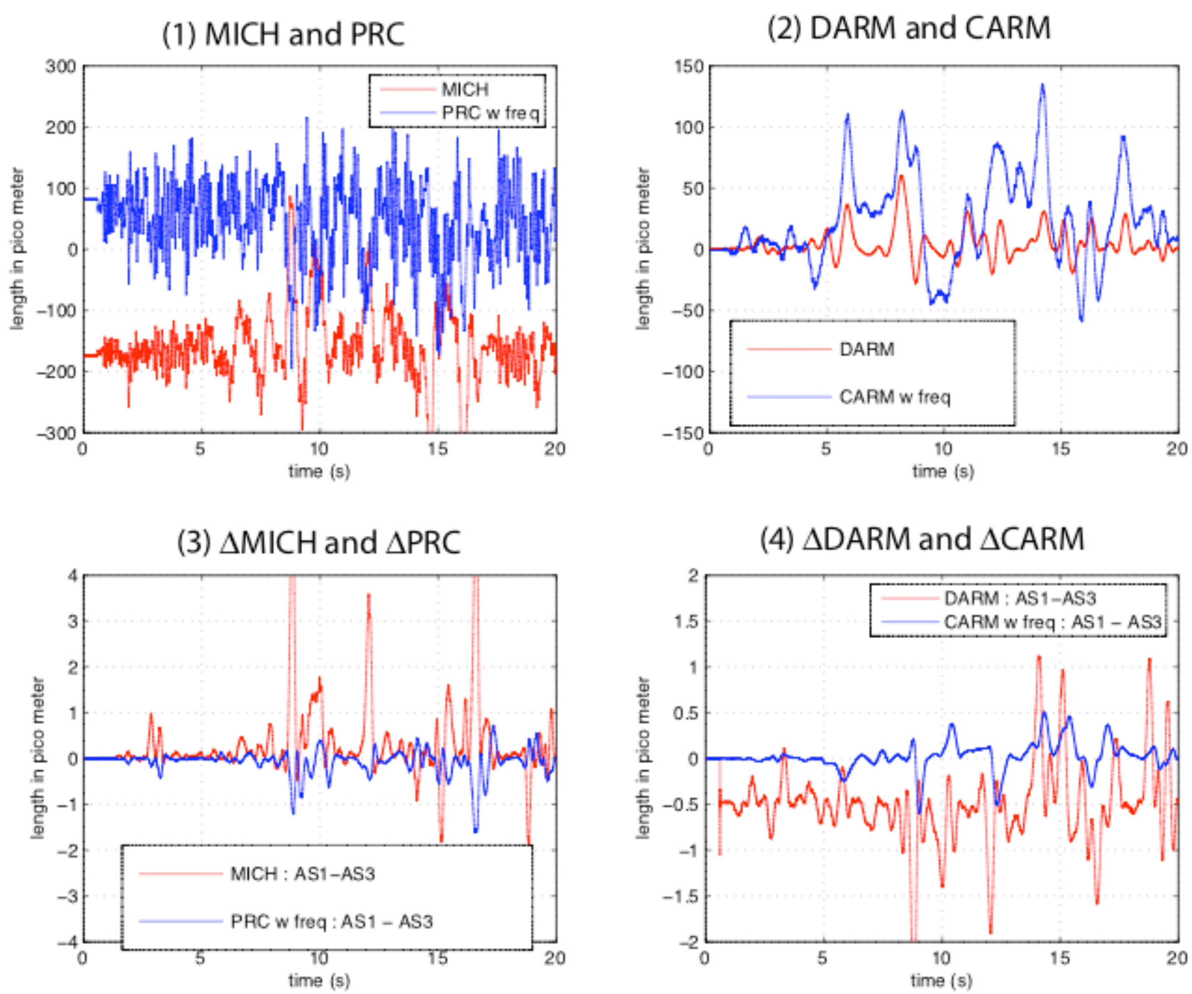
differential
heating
α~0
use dark port
signal to lock



Symmetric arm
common heating
α~0.01
dark port signal



Effect on length DOF

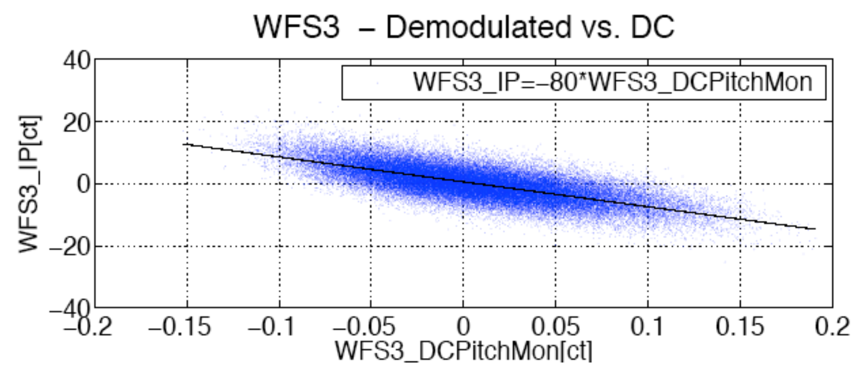


Beam positioning on WFS head

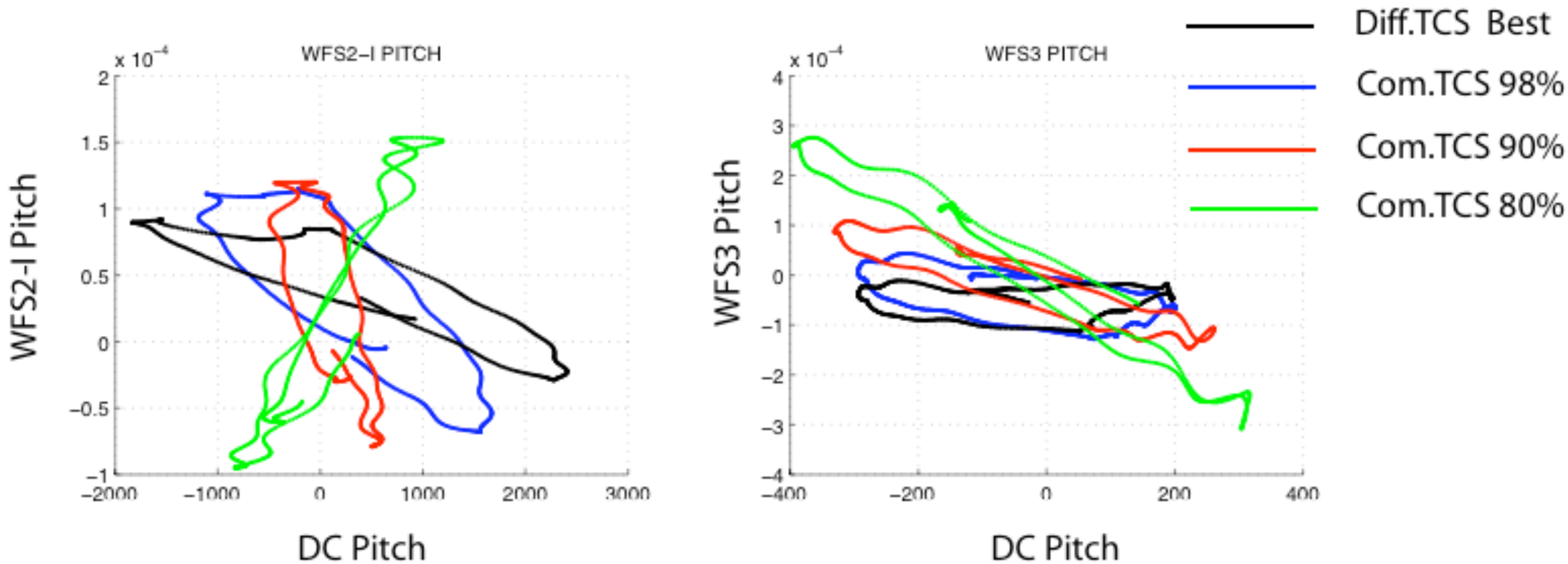
T030290(Luca,etc), eLog(Keita on Nov.18,04)

DCpitch = DC(N) - DC(S)

WFS = Q/I(N) - Q/I(S)

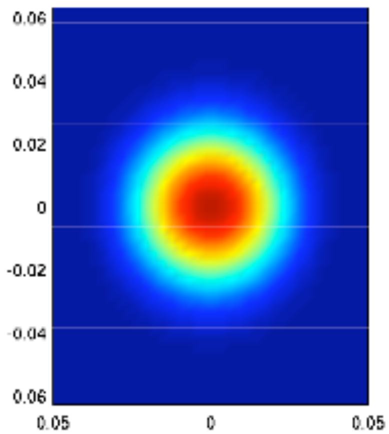


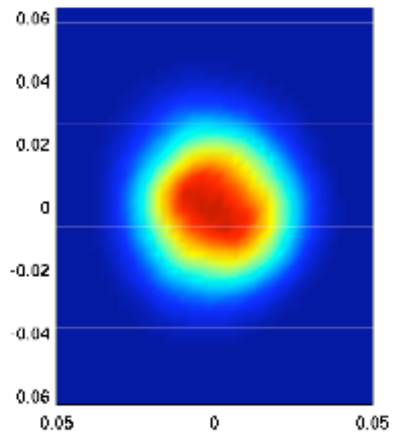
e2e simulation for 1.45 seconds at different thermal states

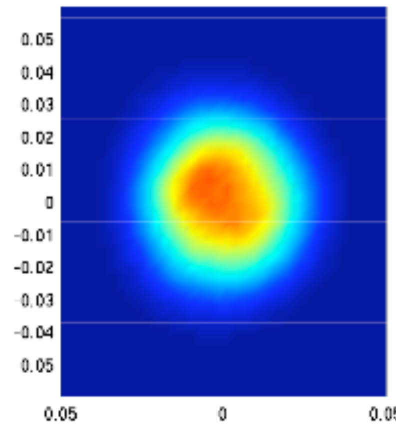


Dark Port beam profile by FFT
- ideal vs reality-prime -

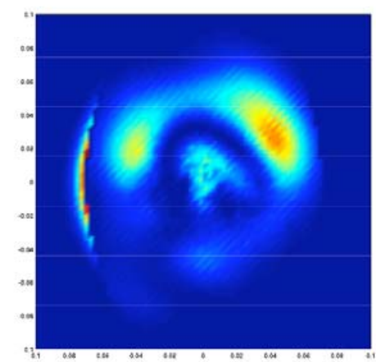
upper SB







and CR



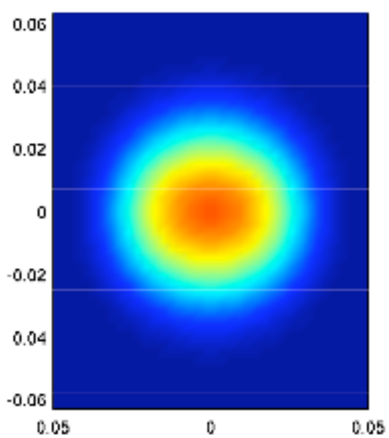
200k BS curvature

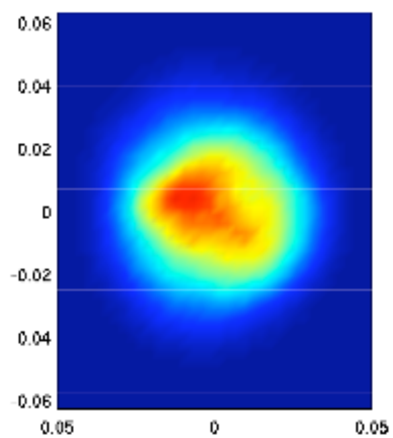
No phase map
Symmetric heating

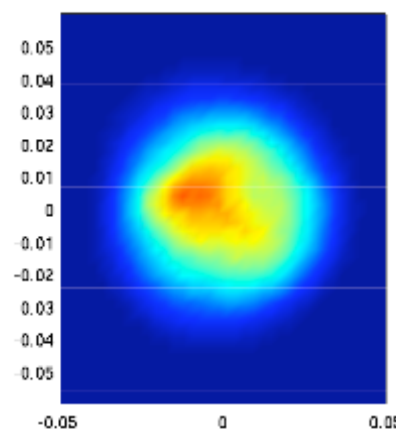
With phase map
Symmetric heating

With phase map
Differential heati

lower SB







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