Gravitational Waves from Hyper-Accretion onto Nascent Black Holes

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Will discuss

- GRB engine: hyper-accreting black holes
- Magneto-Rotational (Instability) dynamics:
  - toroidal, non-axisymmetric MRI modes
  - ν cooling ("alpha disk")
  - (large-scale) relativistic disk dynamics
- Quasi-Normal Ringing modes (a.k.a. Ring Down modes)
- Resonant Driving of QNR modes
- Strain Signal Strength ($|h_{ij}^{TT}|$)
- Future

Hyper-Accreting Black Holes

- Hyper-novae scenario, basic ingredients
  - death of a massive star $M_* \gtrsim 30\text{-}40\ M_\odot$ ($M_{He} > 12\text{-}16\ M_\odot$)
  - secularly growing black hole: $3M_\odot \leq M < \sim 15\ M_\odot$
  - high accretion rate: $0.1 M_\odot\ \mathrm{sec}^{-1} \leq M_{dot} \leq 1\ M_\odot\ \mathrm{sec}^{-1}$
  - photo-disintegration + neutronization $r < 70\ [GM_{hole}/c^2]$

    (onset of strong cooling) (Popham, Woosley & Fryer '99)
  - local ν cooling (i.e., negligible advection) (Di Matteo et al '02)

"slim" α–disk: $h/r \approx 0.4$ ($M_\varphi^{-1} = c_s / v_\varphi$)

Keplerian rotation rate : $\Omega_\pm = (r^{3/2} \pm a)^{-1}$

even for $r < r_{ms}$

(numerical GRMHD, Hirose et al 2003)

h: height, r: radius, $M\varphi$: azimuthal Mach number a: spin parameter, $\Omega_\pm$: angular velocity

Magneto-Rotational Instability

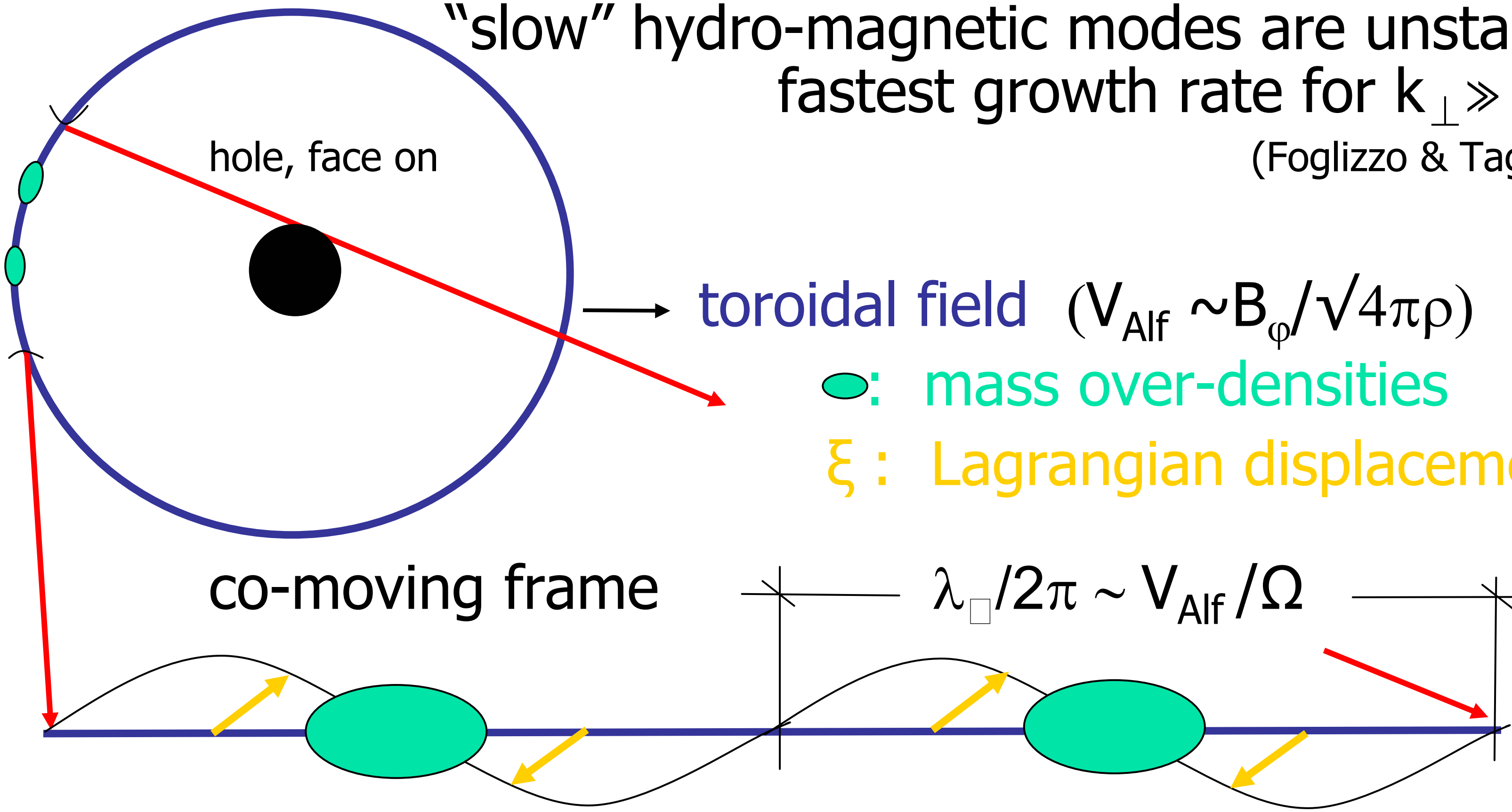
culprit of angular momentum transport and energy deposition in accretion disks

- compressible, non-axisymmetric, toroidal MRI modes

"slow" hydro-magnetic modes are unstable

fastest growth rate for $k_{\perp} \gg k_{\parallel}$

(Foglizzo & Tagger '95)



toroidal field ($V_{Alf} \sim B_{\varphi}/\sqrt{4\pi\rho}$)

: mass over-densities

ξ : Lagrangian displacement

Will show:

Hyper-accretion → few large clumps near the marginally bound radius. Optimal for nearly monochromatic GW!

- Model involves
  - Magneto-rotationally induced disk dynamics
  - General relativistic effects on the MRI
  - Neutrino stress effects (diffusive pressure support)

- Model yields:
  - About 2π massive clumps inside $r_{ms} > r > r_{mb}$
  - Free-fall from r_{mb} on a timescale $\delta t_{\text{free fall}} \sim \Omega^{-1}_{+} (r_{mb})$
  - Coherence in arrival times to r_{mb} for large clumps

→ Resonant Driving of Quasi-Normal Ringing Kerr Modes

MRI + Hyper-accretion:

ν cooling and relativistic disk dynamics

ν stress: diffusive pressure support

- radiative heat conduction
- non-elastic fluid properties (Araya-Góchez & Vishniac '03)

⇒ "long-lived" mass over-densities (Turner et al '03)

general relativity:

- MRI: coriolis ⇄ shear (+ elastic coupling) magnetic
- as $r \longrightarrow r_{ph}$

  shear wins: ⇒ large-scale modes

(Araya-Góchez '02)

Kerr Geometry Particle dynamics:
Circular Geodesic Radii

- Order of radii: $[GM/c^2 = 1]$

$D^{-1} = g^{rr}$, inv. prop radial component of background metric

$C^{-1}_{\pm} \propto \gamma_g^{\,2}$, inv. prop. gravitational red shift squared

$X^2_{\pm} \propto 1 - 3D/4C_{\pm}$, "epicycle" frequency, radial oscillations

r_{ms} : marginally stable orbit radius $X_{\pm}(r) \rightarrow 0$

r_{ph} : photon radius $C_{\pm}(r) \rightarrow 0$

r_{+} : event horizon $D\ (r) \rightarrow 0$

r_{mb} : min. radius of "cusp" in effective potential
akin to L_1 point in close binaries; (Kozlowski et al 1978)

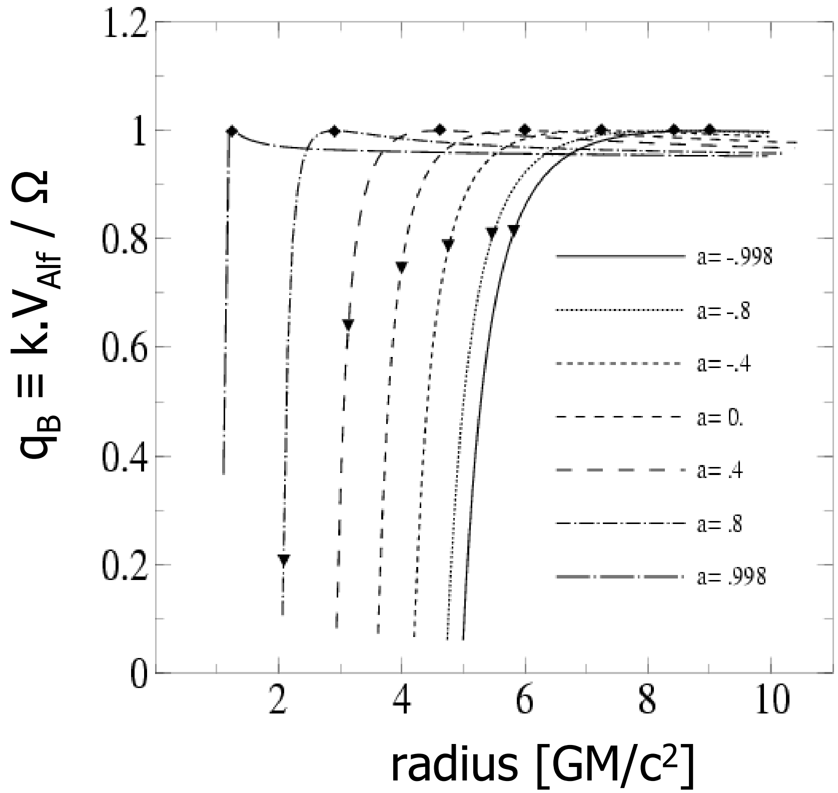
$r_{ms} > r_{mb} > r_{ph} > r_{+}$ (for a=0, 6 > 4 > 3 > 2)

Relativistic Wave-numbers of Fastest Growth

Dimensionless,
spin dependent wave-number:
$q_B \equiv k.V_{Alf}/\Omega \;\; (\propto k_{\|})$

For Keplerian flow:
$q^2_B \rightarrow 1 - X^4/16$
$\rightarrow 1 - (1 \pm a/r^{3/2})^2 \{1\text{-}3D/4C_{\pm}\}^4$

$q_B \rightarrow 0^+$ as r_{ph} is approached

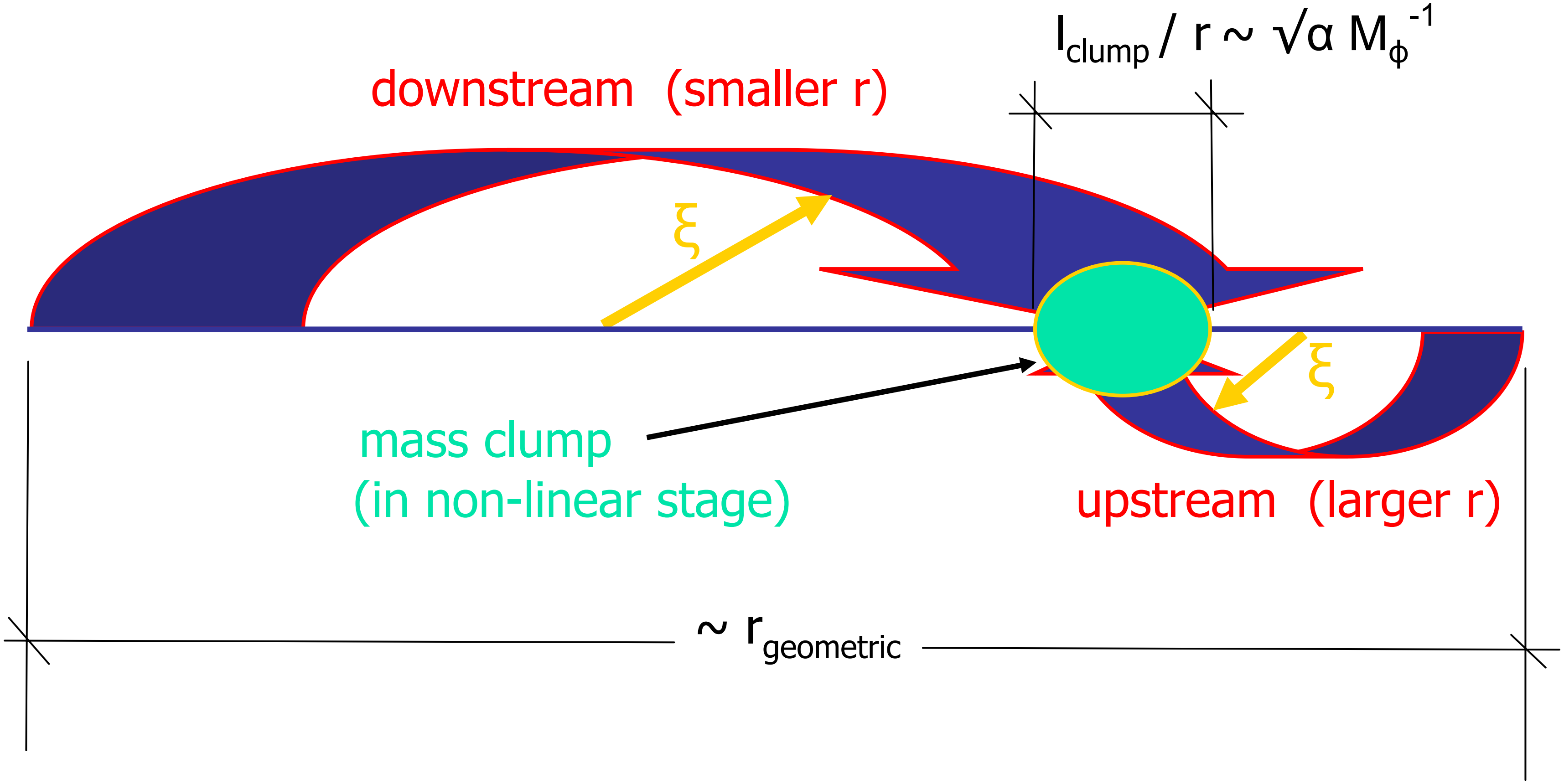


(Araya-Góchez '02)

Relativistic annulus: $r_{ms} > r > r_{mb}$

delayed "free-fall" stage, WKB inspiral modes

- Inside $r < r_{ms}$ only need $\delta t \sim \Omega^{-1}$ for MRI dynamics to take place



QNR modes of Oscillation
- single excitation event

- Head-on collisions: point particle limit $M_{clump} \gg M_{hole}$
  - Gravitational Bremsstrahlung (weak field)
  - Quasi-Normal Ringing (strong field) (DRPP '76, Lousto & Price '97)
- Quick estimate: one orbit @ r_{mb}, single clump:
  - $h = |h_{ij}^{TT}| \sim [G/c^2]\ \delta M/D_L \sim 3_{-25}$, ($\delta M \sim 3.0_{-4}\ M_{\odot}$ @ 27 Mpc)
- QNR modes: Kerr geometry's damped oscillations
  - Bar-like (l,m = 2,2) mode frequency and waveform

    $\omega_{22} = 1 - .63\,(1-a)^{.3}\ [2\pi/M_{hole}]$ (Echeverria '89)

    $h(t) = H_0/d_L\ S_{22}(\varphi,\theta,a)\ \exp i(\omega_{22}\, t - \phi)\ e^{-\Gamma t}$

Damping implies relatively small energy deposition into GW

$H_0 \sim \sqrt{\varepsilon}\ \ \delta M/M,\ \ \varepsilon_0 \lesssim 3\%$ (DRPP '76, Flanagan & Huges '98)

Collective effects
- resonant driving of QNR modes

- Premises
  - Clump formation in relativistic annulus $r_{ms} > r > r_{mb}$
  - Free-fall from $r \approx r_{mb}$ (from the cusp in effective potential)

  ⇒ $\delta t_{\text{free fall}} \sim \Omega^{-1}_{+}(r_{mb})$

  drives quadrupole oscillations at twice $\Omega_{+,mb}$:

  $$\omega_{dr} = 2\Omega_{+,mb}$$

  With natural frequency ω_{QNR} and damping rate Γ_{QNR}:

  ⇒ $|2\Omega_{mb} - \omega_{QNR}| \leq \frac{1}{2}\Gamma_{QNR}$ for $a \geq .95$!

- Waveforms: un-damped sinusoids @ ω_{dr}
  - Saturation amplitude

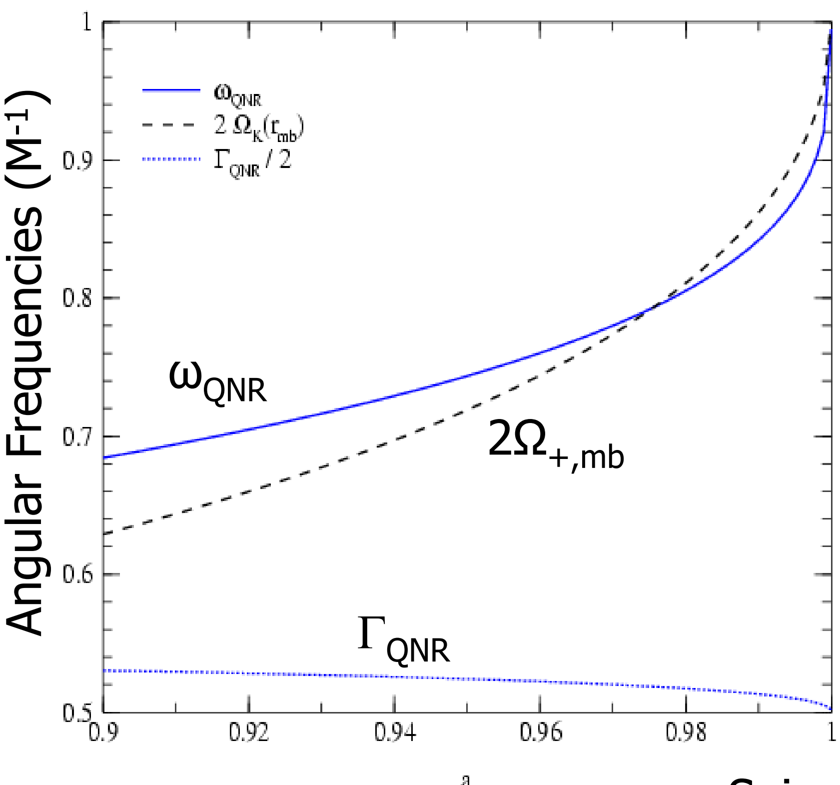
  $$H_{sat}/H_0 \propto \{\omega_{dr}^2 - \omega_{QNR}^2 - i\,\omega_{dr}\Gamma_{QNR}\}^{-1}$$

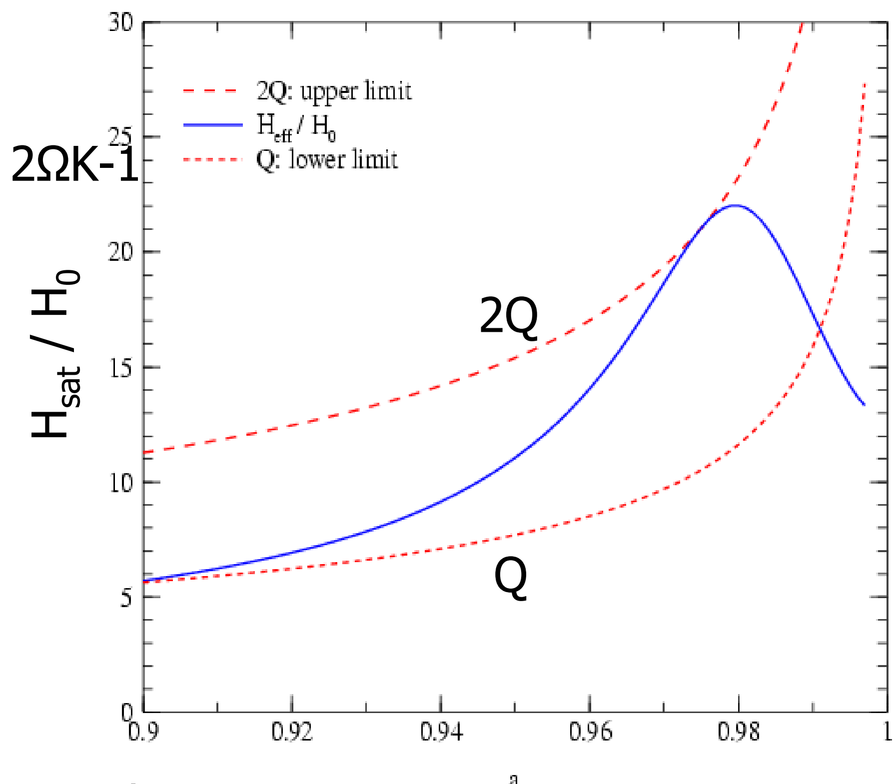
  (at resonance) $\propto \omega_{QNR}/\Gamma_{QNR} = 2Q$

Resonant Driving of QNR Modes

$2\Omega_+(r_{mb})$, ω_{QNR} & Γ_{QNR} (l.m=2,2) as functions of spin parameter a

Quality factor Q and saturation strain amplitude H_{eff} as functions of a





Spin parameter: a

Signal Strength & Energy Deposition

- Augmented efficiency of energy deposition (one clump)

  $\varepsilon = (H_{sat}/H_{0})^2\, \varepsilon_0$ (recall $\varepsilon_0 \lesssim 3\%$)

- Total energy deposition ($\Delta T = 2\pi N_{cycles}/\, \omega_{dr}$)

  $\Delta E_{GW} = [c^2]\, \varepsilon\, (\delta M)_{geo} \ln (M_{final}/M_{initial})$

- $<[S/N]^2> \int d \ln\omega \;\; h^2_{char} / h^2_{noise}$

  With $h \equiv |h_{ij}^{TT}|$ (Flanagan & Huges '98)

  $h^2_{char}(\omega) = 2(1+z)^2/\pi^2 D_L^2 \;\; d_\omega E\, [(1+z)\omega]$

  $|h_{char}|^2_{\omega = \omega_{dr}} \longrightarrow (1+z)^2 / 2\pi^2 \;\; (|H_{sat}| / D_L)^2 \;\; N^2$

$h \longrightarrow [G/2\pi c]\, (2(1+z)/\pi\, D_L)^2\, \varepsilon\, (\delta M)_{geo}\, \Delta T \ln (M_{final}/M_{initial})$

(Araya-Góchez '03)

Strain Amplitude Estimates

$h^2_{char} = [G/2\pi c]\ \varepsilon\ (2(1+z)/\pi\ D_L)^2\ (\delta M)_{geo}\ \Delta T\ \ln(M_{final}/M_{initial})$

- For GRB030329,
  plug in $z = .1685$, $D_L = 810$ Mpc, $M_{initial} = 15\ M_\odot$, $a = .98$,
  $\Delta T = 1$ sec, and $(\delta M)_{geo} = M_{dot}\ \Omega^{-1}_{+}\ (r_{mb}) = 1.83_{-4}\ M_\odot$

to yield

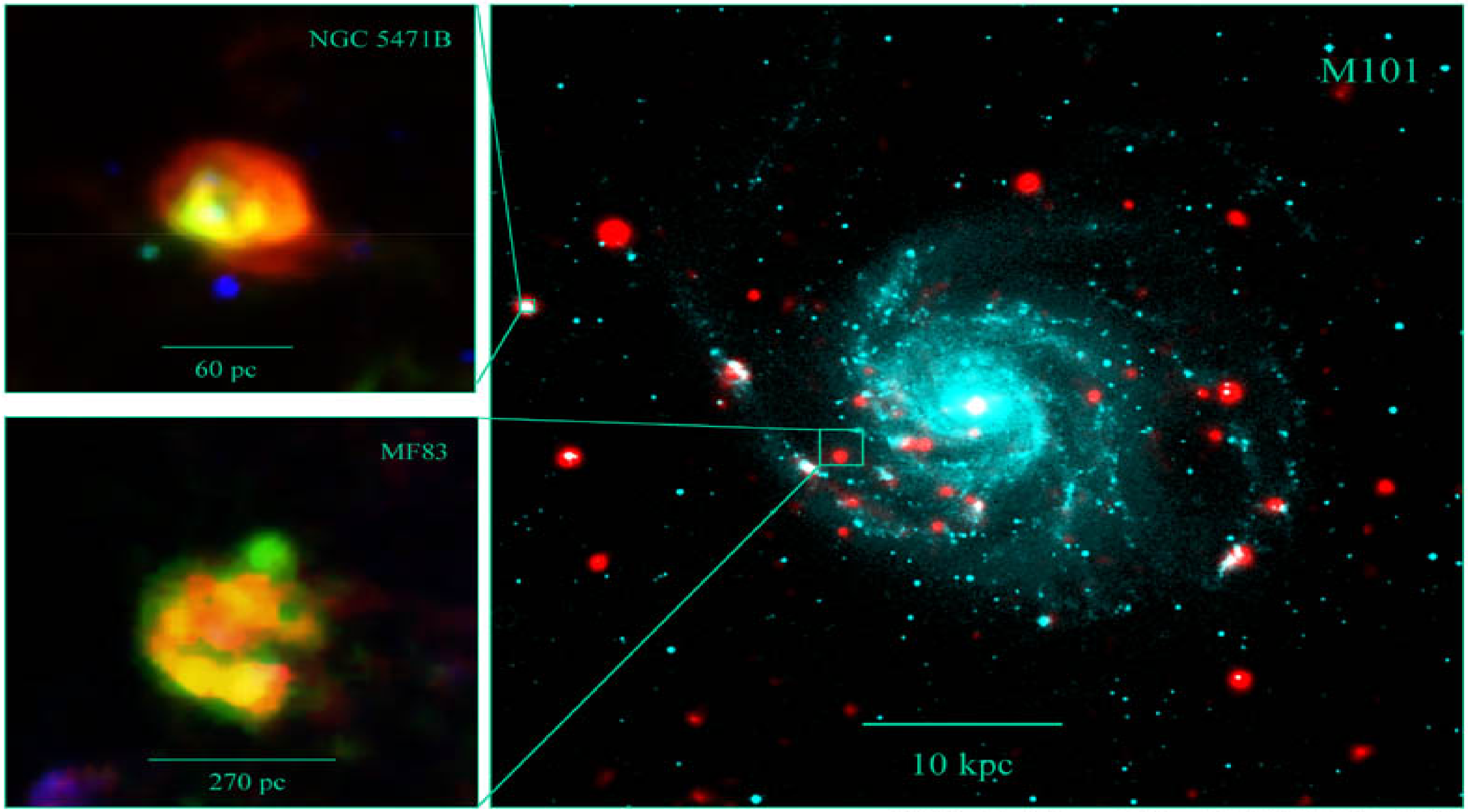
$h_{char} = 8.4_{-23}$ @ $f = 1490$ Hz

- For GRB980425,
  plug in $D_L = 27$ Mpc, $M_{initial} = 15\ M_\odot$, $a = .98$, $\Delta T = 1$ sec
  and $(\delta M)_{geo} = M_{dot}\ \Omega^{-1}_{+}\ (r_{mb}) = 1.83_{-4}\ M_\odot$

yields

$h_{char} = 2.16_{-21}$ @ $f = 1741$ Hz

Nearby Hyper-Novae?



Hyper-novae remnants in M101 (7.1 Mpc) ? (Y. Chu et al '99)

In Summary

- MHD + GR + ν–cooling in hype-accreting black holes may drive QNR modes in resonant fashion for $.99 \geq a \geq .9$
- GW amplitude may reach $\approx$ 22 times DRPP estimate from a single clump in-fall!, depending on hole spin a

Enhanced energy deposition : $(H_{sat}/H_0)^2 = 484$
(optimal for energy deposition into gravitational waves)

- Hyper-accreting holes $\neq$ magnetized torus-hole systems

(van Putten '03)

  - MRI vs Papaloizou-Pringle instability
  - Hyper-accretion vs suspended accretion ("magnetic wall")
  - MHD vs force-free magnetosphere (dissipation?)
  - Typical frequencies: 1500 Hz vs 500 Hz

Searching for driven QNR waveforms with LIGO

- Next: template families
  - secular black hole growth
  - going into and out of resonance:
    - Gaussian envelopes for the QNR amplitudes
  - varying $M_{initial}$ (increasing) & M_{dot} (decreasing)

- Future
  - GRB/Hyper-novae are very promising sources of GW
  - Searching for GW from driven hole ringing is very feasible:

clean nearly monochromatic signals with very large amplitudes!