## Pulsar kicks and a possible signal from a nearby supernova

- Pulsar kicks and dark matter may have a common explanation
- If so, peculiar signal may be detected LIGO and LISA
[AK, Segrè, Fuller, Pascoli, Mocioiu, Semikoz]


## Gravity waves from a supernova

- Supernovae would have made a much better source if the explosions were asymmetric
- Pulsars receive a kick at birth in a supernova. Origin unknown.
- Some explanations of the pulsar kicks predict a strong signal


## Pulsar velocities

Pulsars have large velocities, $\langle v\rangle \approx 250-450 \mathrm{~km} / \mathrm{s}$.
[Cordes et al.; Hansen, Phinney; Kulkarni et al.; Lyne et al. ]
A significant population with $v>700 \mathrm{~km} / \mathrm{s}$, about $15 \%$ have $v>1000 \mathrm{~km} / \mathrm{s}$, up to $1600 \mathrm{~km} / \mathrm{s}$. [Arzoumanian et al.; Thorsett et al.; ]
The high-velocity population is so large that some suggested the distribution is two-component, with average velocities $v_{1} \approx 90 \mathrm{~km} / \mathrm{s}$ and $v_{2} \approx 500 \mathrm{~km} / \mathrm{s}$ [Cordes, Chernoff]


## Proposed explanations:

- asymmetric collapse [Shklovskii] (small kick)
- evolution of close binaries [Gutt, Gunn, Ostriker] (not enough)
- acceleration by EM radiation [Harrison, Tademaru] (kick small, predicted polarization not observed)
- asymmetry in EW processes that produce neutrinos [Chugai; Dorofeev, Rodinov, Ternov] (asymmetry washed out)
- "cumulative" parity violation [Lai, Qian; Janka] (it's not cumulative )


## Asymmetric collapse


"...the most extreme asymmetric collapses do not produce final neutron star velocities above 200km/s" [Fryer '03]

## Supernova neutrinos

Nuclear reactions in stars lead to a formation of a heavy iron core. When it reaches $M \approx 1.4 M_{\odot}$, the pressure can no longer support gravity. $\Rightarrow$ collapse.

Energy released:

$$
\Delta E \sim \frac{G_{N} M_{\mathrm{Fe} \text { core }}^{2}}{R} \sim 10^{53} \mathrm{erg}
$$

$99 \%$ of this energy is emitted in neutrinos

## Pulsar kicks from neutrino emission?

Pulsar with $v \sim 500 \mathrm{~km} / \mathrm{s}$ has momentum

$$
M_{\odot} v \sim 10^{41} \mathrm{~g} \mathrm{~cm} / \mathrm{s}
$$

SN energy released: $10^{53} \mathrm{erg} \Rightarrow$ in neutrinos. Thus, the total neutrino momentum is

$$
P_{\nu ; \text { total }} \sim 10^{43} \mathrm{~g} \mathrm{~cm} / \mathrm{s}
$$

a $\mathbf{1 \%}$ asymmetry in the distribution of neutrinos
is sufficient to explain the pulsar kick velocities
But what can cause the asymmetry??

## Magnetic field?

Neutron stars have large magnetic fields. A typical pulsar has surface magnetic field $B \sim 10^{12}-10^{13} \mathrm{G}$.
Recent discovery of soft gamma repeaters and their identification as magnetars
$\Rightarrow$ some neutron stars have surface magnetic fields as high as $10^{15}-10^{16} \mathrm{G}$.
$\Rightarrow$ magnetic fields inside can be $10^{15}-10^{16} \mathrm{G}$.
Neutrino magnetic moments are negligible, but the scattering of neutrinos off polarized electrons and nucleons is affected by the magnetic field.

## Core collapse supernova

Onset of the collapse: $t=0$


## Core collapse supernova

Shock formation and "neutronization burst": $t=1-10 \mathrm{~ms}$


Protoneutron star formed. Neutrinos are trapped. The shock wave breaks up nuclei, and the initial neutrino come out (a few \%).

## Core collapse supernova

Thermal cooling: $t=10-15 \mathrm{~s}$


Most of the neutrinos emitted during the cooling stage.

Electroweak processes producing neutrinos (urca),

$$
p+e^{-} \rightleftharpoons n+\nu_{e} \text { and } n+e^{+} \rightleftharpoons p+\bar{\nu}_{e}
$$

have an asymmetry in the production cross section, depending on the spin orientation.

$$
\sigma\left(\uparrow e^{-}, \uparrow \nu\right) \neq \sigma\left(\uparrow e^{-}, \downarrow \nu\right)
$$

The asymmetry:

$$
\tilde{\epsilon}=\frac{g_{V}^{2}-g_{A}^{2}}{g_{V}^{2}+3 g_{A}^{2}} k_{0} \approx 0.4 k_{0}
$$

where $k_{0}$ is the fraction of electrons in the lowest Landau level.

In a strong magnetic field,

$k_{0}$ is the fraction of electrons in the lowest Landau level.
Pulsar kicks from the asymmetric production of neutrinos?
[Chugai; Dorofeev, Rodionov, Ternov]

Can the weak interactions asymmetry cause an anisotropy in the flux of neutrinos due to a large magnetic field?


Neutrinos are trapped at high density.

# Can the weak interactions asymmetry cause an anisotropy in the flux of neutrinos due to a large magnetic field? 


#### Abstract

No Rescattering washes out the asymmetry [Vilenkin; AK,Segrè, Vilenkin]. In approximate thermal equilibrium the asymmetries in scattering amplitudes do not lead to an anisotropic emission. Only the outer regions, near neutrinospheres, contribute (a negligible amount).

However, if a weaker-interacting sterile neutrino was produced in these processes, the asymmetry would, indeed, result in a pulsar kick!


Sterile neutrinos leave the star without scattering. Hence, they give the pulsar a kick.


## Sterile neutrinos with a small mixing to active neutrinos

$$
\left\{\begin{array}{l}
\left|\nu_{1}\right\rangle=\cos \theta\left|\nu_{e}\right\rangle-\sin \theta\left|\nu_{s}\right\rangle  \tag{1}\\
\left|\nu_{2}\right\rangle=\sin \theta\left|\nu_{e}\right\rangle+\cos \theta\left|\nu_{s}\right\rangle
\end{array}\right.
$$

The almost-sterile neutrino, $\left|\nu_{2}\right\rangle$ was never in equilibrium. Production of $\nu_{2}$ could take place through oscillations.
The coupling of $\nu_{2}$ to weak currents is also suppressed, and $\sigma \propto \sin ^{2} \theta$.
The probability of $\nu_{e} \rightarrow \nu_{s}$ conversion in presence of matter is

$$
\begin{equation*}
\left\langle P_{\mathrm{m}}\right\rangle=\frac{1}{2}\left[1+\left(\frac{\lambda_{\mathrm{osc}}}{2 \lambda_{\mathrm{s}}}\right)^{2}\right]^{-1} \sin ^{2} 2 \theta_{m} \tag{2}
\end{equation*}
$$

where $\lambda_{\text {osc }}$ is the oscillatino length, and $\lambda_{\mathrm{s}}$ is the scattering length.

## Sterile neutrinos in cosmology: dark matter

Sterile neutrinos are produced in primordial plasma through oscillations.
The resulting density of relic sterile neutrinos:

$$
\Omega_{\nu_{2}} \sim 0.3\left(\frac{\sin ^{2} 2 \theta}{10^{-8}}\right)\left(\frac{m_{s}}{\mathrm{keV}}\right)^{2}
$$

[Dodelson, Widrow; Dolgov, Hansen; Fuller, Shi; Abazajian, Fuller, Patel]


A sterile neutrino in this range, consistent with dark matter, can also explain the observed velocities of pulsars though $\nu_{e} \rightarrow \nu_{s}$ oscillations in a supernova.

## Active-sterile conversions in a neutron star

In matter, there is a potential $V_{m}$ for $\nu_{e}$, but not for $\nu_{s}$ :

$$
\begin{aligned}
V\left(\nu_{s}\right) & =0 \\
V\left(\nu_{e}\right) & =-V\left(\bar{\nu}_{e}\right)=V_{0}\left(3 Y_{e}-1+4 Y_{\nu_{e}}\right) \\
V\left(\nu_{\mu, \tau}\right) & =-V\left(\bar{\nu}_{\mu, \tau}\right)=V_{0}\left(Y_{e}-1+2 Y_{\nu_{e}}\right)
\end{aligned}
$$

The difference $V_{m} \equiv V\left(\nu_{e}\right)-V\left(\nu_{s}\right)$

Mixing angle in matter is different from vacuum:

$$
\begin{equation*}
\sin ^{2} 2 \theta_{m}=\frac{\left(\Delta m^{2} / 2 p\right)^{2} \sin ^{2} 2 \theta}{\left(\Delta m^{2} / 2 p\right)^{2} \sin ^{2} 2 \theta+\left(\Delta m^{2} / 2 p \cos 2 \theta-V_{m}\right)^{2}} \tag{3}
\end{equation*}
$$

$$
\begin{align*}
V_{m}=\frac{G_{F} \rho}{\sqrt{2} m_{n}}\left(3 Y_{e}-1+\right. & \left.4 Y_{\nu_{e}}+2 Y_{\nu_{\mu}}+2 Y_{\nu_{\tau}}\right)  \tag{4}\\
& \simeq(-0.2 \ldots+0.5) V_{0} \tag{5}
\end{align*}
$$

where $V_{0}=G_{F} \rho / \sqrt{2} m_{n} \simeq 3.8 \mathrm{eV}\left(\rho / 10^{14} \mathrm{gcm}^{-3}\right)$
Mixing is suppressed when $V_{m} \gg\left(\Delta m^{2} / 2 k\right)$.

$$
\left\{\begin{array}{l}
\left|\nu_{1}\right\rangle=\cos \theta_{m}\left|\nu_{e}\right\rangle-\sin \theta_{m}\left|\nu_{s}\right\rangle  \tag{6}\\
\left|\nu_{2}\right\rangle=\sin \theta_{m}\left|\nu_{e}\right\rangle+\cos \theta_{m}\left|\nu_{s}\right\rangle
\end{array}\right.
$$

The coupling of $\nu_{2}$ to weak currents is also suppressed, and $\sigma \propto \sin ^{2} \theta$. The probability of $\nu_{e} \rightarrow \nu_{s}$ conversion in presence of matter is

$$
\begin{equation*}
\left\langle P_{\mathrm{m}}\right\rangle=\frac{1}{2}\left[1+\left(\frac{\lambda_{\mathrm{osc}}}{2 \lambda_{\mathrm{s}}}\right)^{2}\right]^{-1} \sin ^{2} 2 \theta_{m} \tag{7}
\end{equation*}
$$

where $\lambda_{\text {osc }}$ is the oscillatino length, and $\lambda_{\mathrm{s}}$ is the scattering length.

However, the matter potential can evolve on short time scales.

$$
\begin{align*}
& V_{m}=\frac{G_{F} \rho}{\sqrt{2} m_{n}}\left(3 Y_{e}-1+4 Y_{\nu_{e}}+2 Y_{\nu_{\mu}}+2 Y_{\nu_{\tau}}\right)  \tag{8}\\
& V_{m}>0 \Rightarrow \text { Transitions } \nu_{e} \rightarrow \nu_{s} \Rightarrow V_{m} \text { decreases } \\
& V_{m}<0 \Rightarrow \text { Transitions } \bar{\nu}_{e} \rightarrow \nu_{s} \Rightarrow V_{m} \text { increases }
\end{align*}
$$

Therefore,
[Abazajian, Fuller, Patel]

$$
\begin{gathered}
\qquad V_{m} \rightarrow 0 \\
\sin \theta_{m} \rightarrow \sin \theta_{0} \\
\text { production of } \nu_{s} \text { is unsuppressed }
\end{gathered}
$$

Electroweak processes (urca) producing neurtrinos, including sterile neutrinos,

$$
p+e^{-} \rightleftharpoons n+\nu_{e} \text { and } n+e^{+} \rightleftharpoons p+\bar{\nu}_{e}
$$

have asymmetry in the production cross section, depending on the spin orientation. In polarized medium, the asymmetry is of the order $0.4 \times k_{0}$ :


The asymmetry in sterile neutrinos is not affected by rescattering. Sterile neutrinos escape

Sterile neutrinos leave the star without scattering. Hence, they give the pulsar a kick.


If the fraction of energy emitted in sterile neutrinos is

$$
\begin{equation*}
r_{\mathcal{E}}=\left(\frac{\mathcal{E}_{\mathrm{s}}}{\mathcal{E}_{\mathrm{tot}}}\right) \sim 0.05-0.7 \tag{9}
\end{equation*}
$$

(as it can easily be), then the resulting momentum asymmetry is

$$
\begin{equation*}
\epsilon \sim 0.02\left(\frac{k_{0}}{0.3}\right)\left(\frac{r_{\varepsilon}}{0.5}\right) \tag{10}
\end{equation*}
$$

which is sufficient to explain the pulsar kick velocities.

Parameter range: need the equilibration of $V_{m} \rightarrow 0$ to occur faster than $\sim 1 \mathrm{~s}$.

$$
\begin{align*}
\tau_{V} \simeq & \frac{V_{m}^{(0)} m_{n}}{\sqrt{2} G_{F} \rho}\left(\int d \Pi \frac{\sigma_{\nu}^{\mathrm{urca}}}{e^{\left(\epsilon_{\nu}-\mu_{\nu}\right) / T}+1}\left\langle\boldsymbol{P}_{m}\left(\nu_{e} \rightarrow \nu_{s}\right)\right\rangle-\right. \\
& \left.\int d \Pi \frac{\sigma_{\bar{\nu}}^{\mathrm{urca}}}{e^{\left(\epsilon_{\bar{\nu}}-\mu_{\bar{\nu}}\right) / T}+1}\left\langle\boldsymbol{P}_{m}\left(\bar{\nu}_{e} \rightarrow \bar{\nu}_{s}\right)\right\rangle\right)^{-1} \tag{11}
\end{align*}
$$

where $d \Pi=\left(2 \pi^{2}\right)^{-1} \epsilon_{\nu}^{2} d \epsilon_{\nu}$, and $V_{m}^{(0)}$ is the initial value of the matter potential $V_{m}$.
[Abazajian, Fuller, Patel]

$$
\begin{aligned}
\tau_{V}^{\text {on-res }} & \simeq \frac{2^{5} \sqrt{2} \pi^{2} m_{n}}{G_{F}^{3} \rho} \frac{\left(V_{m}^{(0)}\right)^{6}}{\left(\Delta m^{2}\right)^{5} \sin 2 \theta}\left(e^{\frac{\Delta m^{2} / 2 V_{m}^{(0)}-\mu}{T}}+1\right) \\
& \sim\left(\frac{2 \times 10^{-9} \mathrm{~S}}{\sin 2 \theta}\right)\left(\frac{10^{14} \frac{g}{c m^{3}}}{\rho}\right)\left(\frac{20 \mathrm{MeV}}{T}\right)^{6}\left(\frac{\Delta m^{2}}{10 \mathrm{keV}^{2}} .\right) \\
\tau_{V}^{\text {off-res }} & \simeq \frac{4 \sqrt{2} \pi^{2} m_{n}}{G_{F}^{3} \rho} \frac{\left(V_{m}^{(0)}\right)^{3}}{\left(\Delta m^{2}\right)^{2} \sin ^{2} 2 \theta} \frac{1}{\mu^{3}} \\
& \sim\left(\frac{6 \times 10^{-9} \mathrm{~S}}{\sin ^{2} 2 \theta}\right)\left(\frac{V_{m}^{(0)}}{0.1 \mathrm{eV}}\right)^{3}\left(\frac{50 \mathrm{MeV}}{\mu}\right)^{3}\left(\frac{10 \mathrm{keV}^{2}}{\Delta m^{2}}\right)^{2}
\end{aligned}
$$

[Fuller, AK, Mocioiu, Pascoli]

Allowed range of parameters (time scales, fraction of total energy emitted):

[Fuller, AK, Mocioiu,Pascoli]

## Resonant active-sterile neutrino conversions in matter

Matter potential:

$$
\begin{aligned}
V\left(\nu_{s}\right)= & 0 \\
V\left(\nu_{e}\right)= & -V\left(\bar{\nu}_{e}\right)=V_{0}\left(3 Y_{e}-1+4 Y_{\nu_{e}}\right) \\
V\left(\nu_{\mu, \tau}\right)= & -V\left(\bar{\nu}_{\mu, \tau}\right)=V_{0}\left(Y_{e}-1+2 Y_{\nu_{e}}\right)+c_{L}^{Z} \frac{\vec{k} \cdot \vec{B}}{k} \\
& c_{L}^{Z}=\frac{e G_{F}}{\sqrt{2}}\left(\frac{3 N_{e}}{\pi^{4}}\right)^{1 / 3}
\end{aligned}
$$

## Mikheev-Smirnov-Wolfenstein (MSW) effect



The resonance condition is

$$
\begin{equation*}
\frac{m_{i}^{2}}{2 k} \cos 2 \theta_{i j}+V\left(\nu_{i}\right)=\frac{m_{j}^{2}}{2 k} \cos 2 \theta_{i j}+V\left(\nu_{j}\right) \tag{12}
\end{equation*}
$$

The resonance is affected by the magnetic field and occurs at different density depending on $\vec{k} \cdot \vec{B}$, that is depending on direction.

As a result, the active neutrinos convert to sterile neutrinos at different depths on different sides of the start.
Temperature is a function of $r$. The energy of an escaping sterile neutrino depends on the temperature of at the point it was produced.

The magnetic field shifts the position of the resonance because of the $\frac{\vec{k} \cdot \vec{B}}{k}$ term in the potential:


In the absebce of magnetic field, $\nu_{s}$ escape isotropically

The magnetic field shifts the position of the resonance because of the $\frac{\vec{k} \cdot \vec{B}}{k}$ term in the potential:


## A crude estimate of the kick

The mean energy of emitted sterile neutrinos is proportional to the temperature at the point of production. The point of resonant conversion depends on direction:

$$
\begin{equation*}
r(\phi)=r_{0}+\delta \cos \phi \tag{13}
\end{equation*}
$$

where $\cos \phi=(\vec{k} \cdot \vec{B}) / k$ and $\delta$ is determined by the equation:

$$
\begin{equation*}
2 \frac{d N_{n}(r)}{d r} \delta \approx e\left(\frac{3 N_{e}}{\pi^{4}}\right)^{1 / 3} B \tag{14}
\end{equation*}
$$

This yields

$$
\begin{equation*}
\delta=\left(\frac{3 N_{e}}{\pi^{4}}\right)^{1 / 3} \frac{e}{2} B / \frac{d N_{n}(r)}{d r}=\frac{e \mu_{e}}{2 \pi^{2}} B / \frac{d N_{n}(r)}{d r} \tag{15}
\end{equation*}
$$

where $\mu_{e} \approx\left(3 \pi^{2} N_{e}\right)^{1 / 3}$ is the chemical potential of the degenerate (relativistic) electron gas.
Asymmetry in the outgoing momentum (assuming Stefan-Boltzmann):

$$
\begin{align*}
\frac{\Delta k}{k}=\frac{1}{3} \frac{T^{4}\left(r_{0}-\delta\right)-T^{4}\left(r_{0}+\delta\right)}{T^{4}\left(r_{0}\right)} & \approx \frac{8}{3} \frac{1}{T} \frac{d T}{d r} \delta  \tag{16}\\
& \approx \frac{4 e}{3 \pi^{2}}\left(\frac{\mu_{e}}{T} \frac{d T}{d N_{n}}\right) B \tag{17}
\end{align*}
$$

Estimate the derivative $\frac{d T}{d N_{n}}$ using $N_{n}=\frac{2\left(m_{n} T\right)^{3 / 2}}{\sqrt{2} \pi^{2}} \int \frac{\sqrt{z} d z}{e^{\left(z-\mu_{n}\right) / T}+1}$.

Finally,

$$
\begin{equation*}
\frac{\Delta k}{k}=\frac{4 e \sqrt{2}}{\pi^{2}} \frac{\mu_{e} \mu_{n}^{1 / 2}}{m_{n}^{3 / 2} T^{2}} B \tag{18}
\end{equation*}
$$

At the core density $\rho \sim 10^{14} \mathrm{~g} / \mathrm{cm}^{3}$, one gets the asymmetry

$$
\begin{equation*}
\frac{\Delta k}{k}=\frac{4 e \sqrt{2}}{\pi^{2}} \frac{\mu_{e} \mu_{n}^{1 / 2}}{m_{n}^{3 / 2} T^{2}} B \sim 0.01\left(\frac{B}{10^{15} \mathrm{G}}\right) \tag{19}
\end{equation*}
$$

[AK,Segrè]
A more careful calculation gives the same order of magnitude [Barkovich et al., PR D66, 123005 (2002); AK, Segrè, PR D D59 061302 (1999)].

The core density $\rho \sim 10^{14} \mathrm{~g} / \mathrm{cm}^{3}$ determines the

$$
\Delta m^{2} \sim(10 \mathrm{keV})^{2}
$$

Adiabaticity: the oscillation length

$$
\lambda_{\mathrm{osc}} \approx\left(\frac{1}{2 \pi} \frac{\Delta m^{2}}{2 k} \sin 2 \theta\right)^{-1} \sim \frac{1 \mathrm{~mm}}{\sin 2 \theta}
$$

must be smaller than (1) the scale height of density (2) the mean free path of neutrinos. $\Rightarrow$

$$
\sin ^{2} \theta \gtrsim 10^{-10}
$$

The range of parameters [AK, Segrè; Fuller,AK,Mocioiu,Pascoli]:


## Resonant (1) \& off-resonant (2) emissions combined:


the pulsar kick regions overlap with the dark matter region


How "natural" is the mixing $\sin ^{2} \theta \sim 10^{-8}$ ?
Models of neutrino masses commonly predict:

$$
\sin ^{2} \theta \sim \frac{m_{1}}{m_{2}} \quad[\text { e.g, Kaus and Meshkov }]
$$

for a heavy neutrnio with a $10 \mathrm{keV}=10^{5} \mathrm{eV}$ mass and a light one with a $10^{-3} \mathrm{eV}$ mass, this ratio is about right.

## Pulsar kicks: why sterile neutrinos?

Why not ordinary active neutrinos?
To get a pulsar kick out of $\nu_{\mu, \tau} \leftrightarrow \nu_{e}$ oscillations, one would require the resonant neutrino conversion to take place between the electron and $\tau$ neutrinospheres, at density $\rho \sim 10^{11}-10^{12} \mathrm{~g} / \mathrm{cm}^{3}$. This density corresponds to

$$
\left(\Delta m^{2}\right)^{1 / 2} \sim 10^{2} \mathrm{eV}
$$

This is inconsistent with experimental/cosmological limits.

## Chandra, XMM-Newton can see keV photons.



Virgo cluster image from XMM-Newton

Chandra, XMM-Newton can see photons: $\nu_{s} \rightarrow \nu_{e} \gamma$


Chandra, XMM-Newton can see photons: $\nu_{s} \rightarrow \nu_{e} \gamma$


## Chandra, XMM-Newton can see photons: $\nu_{s} \rightarrow \nu_{e} \gamma$


non-zero lepton asymmetry changes the dark matter range [Abazajian, Fuller, Tucker]

## Gravity waves



Artist's conception by Roulet [Summer School lectures in Trieste]
Rotating "beam" of neutrinos is the source of GW


## Gravity waves



Artist's conception by Roulet [Summer School lectures in Trieste]
Rotating "beam" of neutrinos is the source of GW


## Gravity waves at LIGO and LISA



[Loveridge, Phys. Rev. D69 024008 (2004)]

## Conclusions

- Sterile neutrinos in the $1-20 \mathrm{keV}$ range can explain the observed pulsar kicks
- The same neutrino could be the dark matter
- Two puzzles from a single new particle
- Minimal extension of the Standard Model that is consistent with cosmology
- Can verify this mechanism through observations of X-rays from nearby clusters, or from gravity waves in the event of a nearby supernova
- A gravity wave signature not expected form a supernova


## Resonant (1) \& off-resonant (2) emissions combined:


[AK, Segrè, PL B396, 197 (1997) ]
[Fuller,AK,Mocioiu,Pascoli, Phys. Rev. D 68, 103002 (2003)]

