

Coherent Data Analysis Strategies Using a Network of Gravitational Wave Detectors

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Outline

- Overview
 - burst GW sources
- Motivation

- Our proposal
 - use detector network to construct null-stream (data with no GW signal)
 - application
- Examples
 - three detectors
 - two LIGO detectors
- Discussion

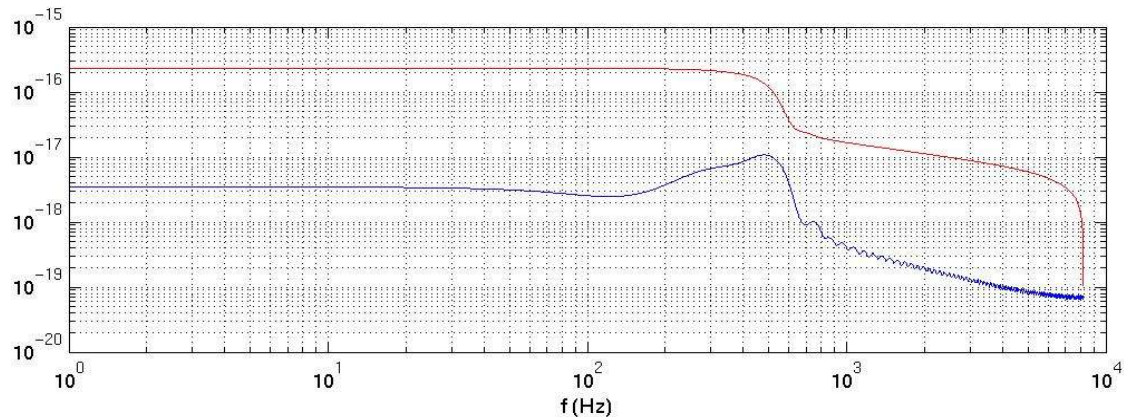
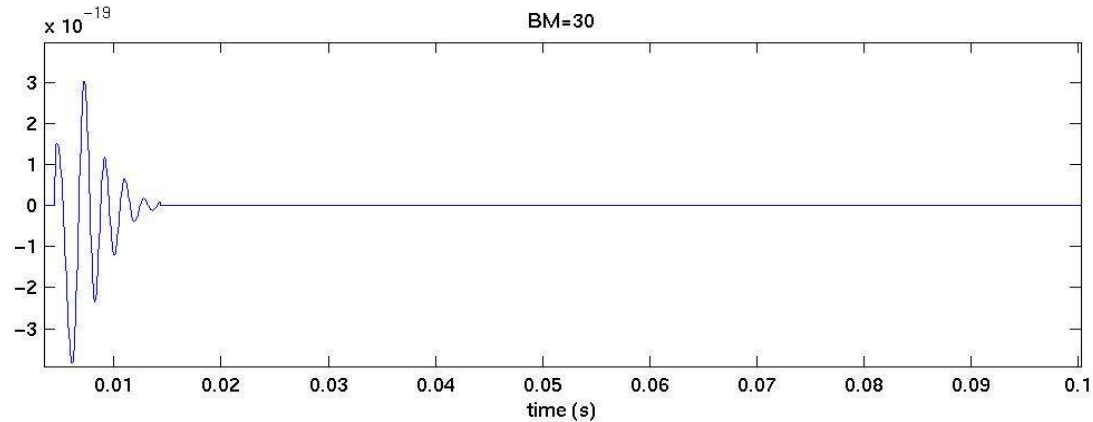
Burst GW Sources

- Burst=“unknown or only crudely modelled”
- “Popular” expectation
 - Short duration
 - $\sim 1 - 100$ ms
 - High frequency
 - $> \sim 500$ Hz

- Two numerical examples
 - rotational core collapse of massive stars
 - ~ 10 ms, $\sim 500-1000$ Hz
 - BH merger waveform (Lazarus project)
 - ~ 10 ms, ~ 500 Hz

Binary Black-Hole Merger Waveform

- $T \sim 10$ ms
- at 1 Mpc
- $f_c \sim 500$ Hz



(Baker, et al. 2002, PRD)

astrogravs.gsfc.nasa.gov/docs/catalog.html#bhbh

www.ligo.caltech.edu/~ajw/bursts/burstsim.htm

- Burst GW signals and noise can look alike
 - detector glitches
 - local disturbance (e.g., fly-by plane)
- Vetoing techniques are crucial
- Positional determination helps cross-check with observations in other wavelengths
- multiple detector analysis is a requirement

A single GW observatory
cannot convincingly detect a
GW event.

Method

For triggered coincidence events:

- Construct “null stream” as a particular linear combination of time-series data from multiple detectors
 - GW signals from a given direction will be cancelled out
- Construct a set of null streams as a function of presumed source positions
- Search the sky positions, look for the absence of the signal
 - if GW: verify the detection/localize source position
 - if noise: veto

- Our approach is equivalent to consistency check of data from multiple detectors
 - Amplitude
 - Phase/effective time delay
 - Coherent check of both

Principle of Null Stream Construction

Response of a detector to GW

$$h_i(t) = f_i^+ h_+(t) + f_i^\times h_\times(t)$$

$f_i^{+\times}$ is a function of ψ , α , δ , and detector orientation.

$$h_i(t) = A_i \sqrt{h_+^2(t) + h_\times^2(t)} \sin(2\psi + \xi_i + \xi^h(t))$$

- two effects of antenna pattern in observed signals
 - amplitude modulation
 - phase/effective arrival time modulation

Antenna Pattern Modulation

- A_i, ξ_i :
 - polarization angle independent
 - known for given source location

$$A_i = \sqrt{f_i^{+2} + f_i^{\times 2}}$$

$$\xi_i = \tan^{-1} \frac{f_{i,\psi=0}^+}{f_{i,\psi=0}^{\times}}$$

$$\xi^h(t) = \tan^{-1} \frac{h_{\times}(t)}{h_{+}(t)}$$

- Effective time delay error:
 - up to ~1 ms at 500 Hz
 - worse at smaller f

$$\delta\tau_{ij} \sim \frac{|\xi_i - \xi_j|}{2\pi f_0} \leq \frac{1}{2f_0}$$

Three-Detector Case

- Data

$$\begin{aligned}h_1(t) &= f_1^+ h_+(t) + f_1^\times h_\times(t) + n_1(t) \\h_2(t + \tau_{12}) &= f_2^+ h_+(t) + f_2^\times h_\times(t) + n_2(t) \\h_3(t + \tau_{13}) &= f_3^+ h_+(t) + f_3^\times h_\times(t) + n_3(t).\end{aligned}$$

- Null Stream=linear combination of data

- signal exactly cancelled out (e.g., Guersel & Tinto 1989)
- coefficients: polarization angle independent

$$A(\alpha, \delta, t) = A_{23}h_1(t) + A_{31}h_2(t + \tau_{12}) + A_{12}h_3(t + \tau_{13})$$

$$A_{ij} = (f_i^+ f_j^\times - f_j^+ f_i^\times).$$

Two -Detector Case

- Nearly perfectly aligned antenna beam pattern:

$$A_{12} = f_1^+ f_2^\times - f_2^+ f_1^\times \sim 0$$

- Null Stream (2-detector)
 - residual signal amplitude proportional to A_{12}
 - minimize residual signal variance at source direction

$$A(\alpha, \delta, t) = A_2 h_1(t) - \cos(\xi_1 - \xi_2) A_1 h_2(t + \tau_{12})$$

- Find minimum signal contribution in the null-stream
 - maximize the probability of data given noise model
 - minimize variance of normalized residual in f-domain
 - Compare it with expected noise distribution
 - ($P(\alpha, \delta)$ follows χ^2_{2N} distribution)

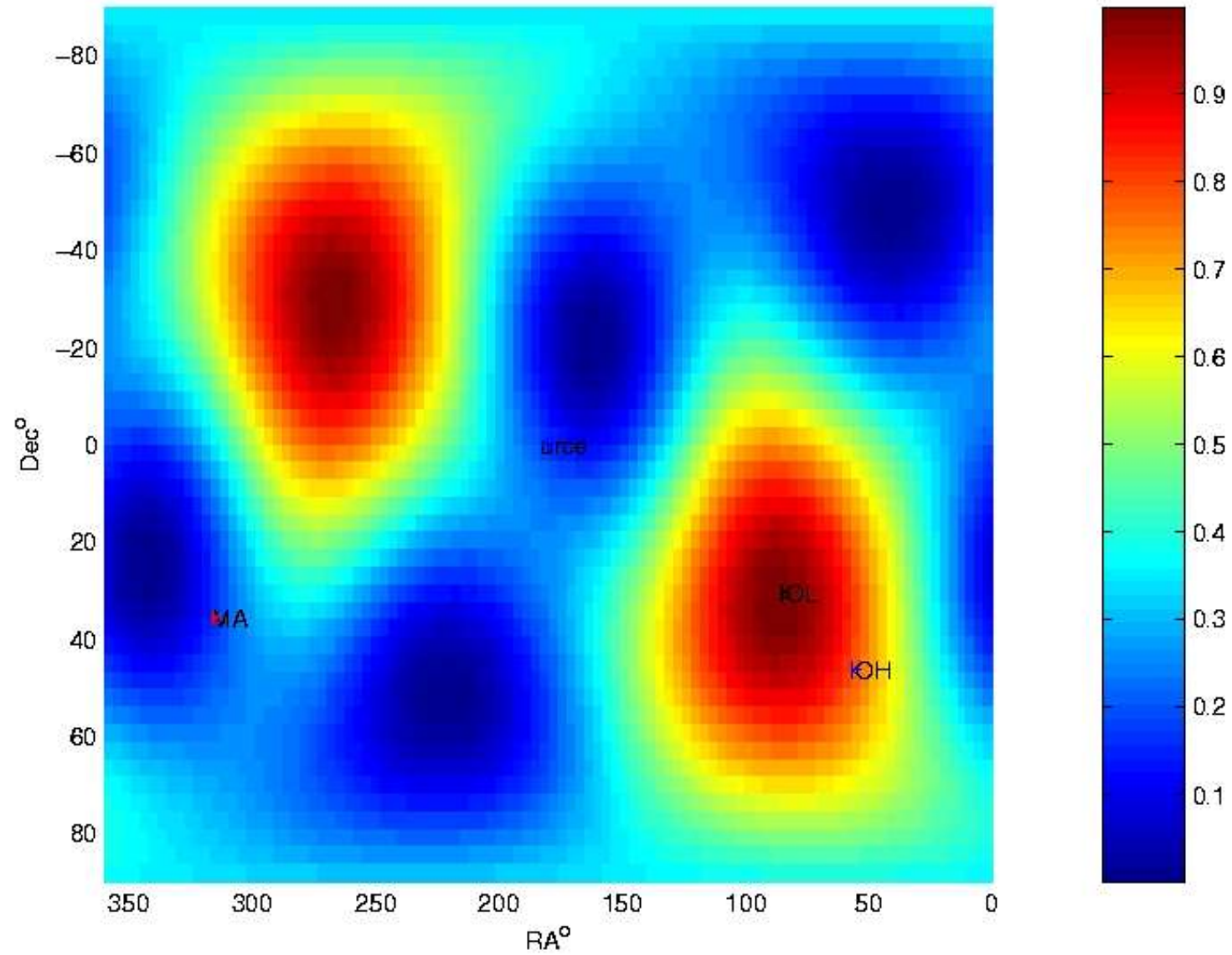
$$P(\alpha, \delta) = 2 \sum_{k=1}^N \frac{A_k^2(\alpha, \delta)}{\sigma_k^2}$$

$$\text{(3-detector:)} \quad \sigma_k^2 = A_{23}^2 \sigma_{1k}^2 + A_{31}^2 \sigma_{2k}^2 + A_{12}^2 \sigma_{3k}^2$$

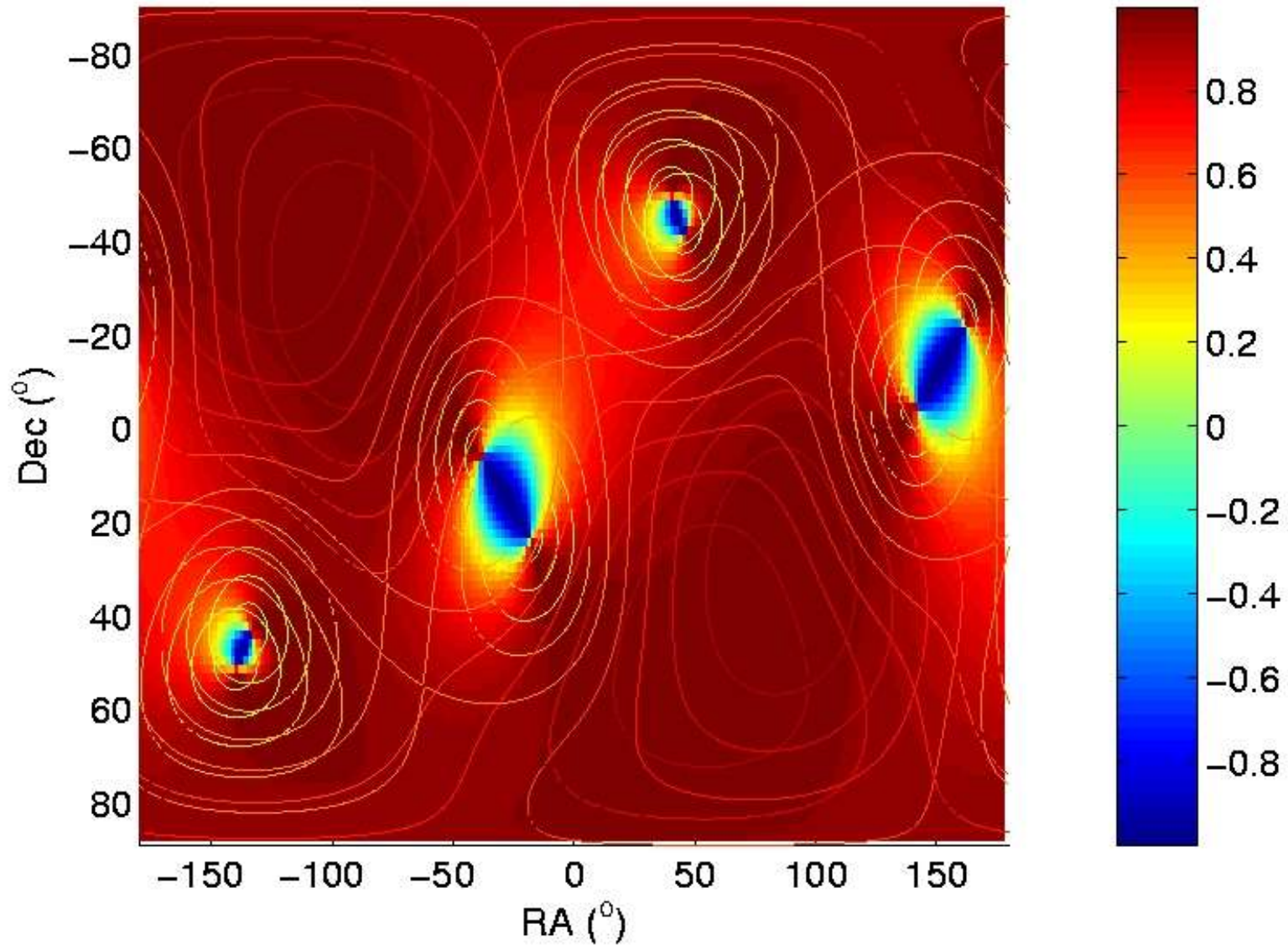
$$\text{(2-detector:)} \quad \sigma_k^2 = A_2^2 \sigma_{1k}^2 + A_1^2 \cos^2(\xi_1 - \xi_2) \sigma_{2k}^2$$

A_1

LLO



$$-\cos(\xi_1 - \xi_2)$$



Relevant Feature of LIGO, GEO and TAMA

- **LIGO (Livingston, Hanford):**
 - most sensitive, nearly perfectly aligned antenna pattern (H2 not considered yet)
 - h's out of phase for most part of the sky
- **GEO/VIRGO**
 - most sensitive around null of LIGO
 - GEO: long duty cycle
- **TAMA**
 - long time delay, i.e. good position/delay
 - long duty cycle

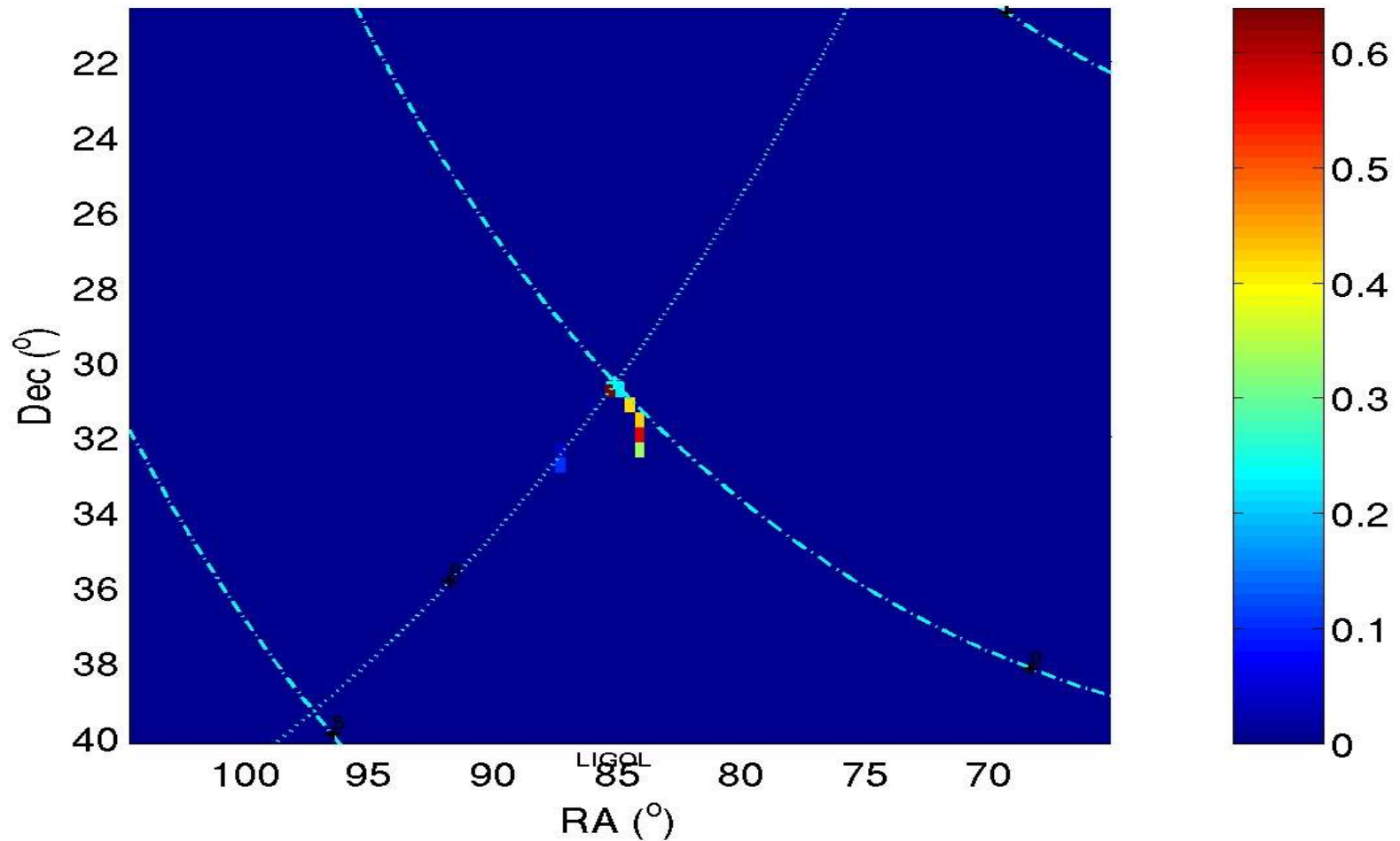
Monte-Carlo Simulation in Progress

- Detectors:
 - 2 LIGO+GEO and 2 LIGO+TAMA
 - 2 LIGOs+VIRGO
- Noise: use projected $S(f)$:
 - Noise: stationary Gaussian. Initial LIGO
- Signal: use numerical BH merger waveform from Lazarus project
 - $M_{total} = 20 M_{sun}$
- Assuming time dependence of antenna pattern negligible compared with signals
 - when calculating expected noise

Examples

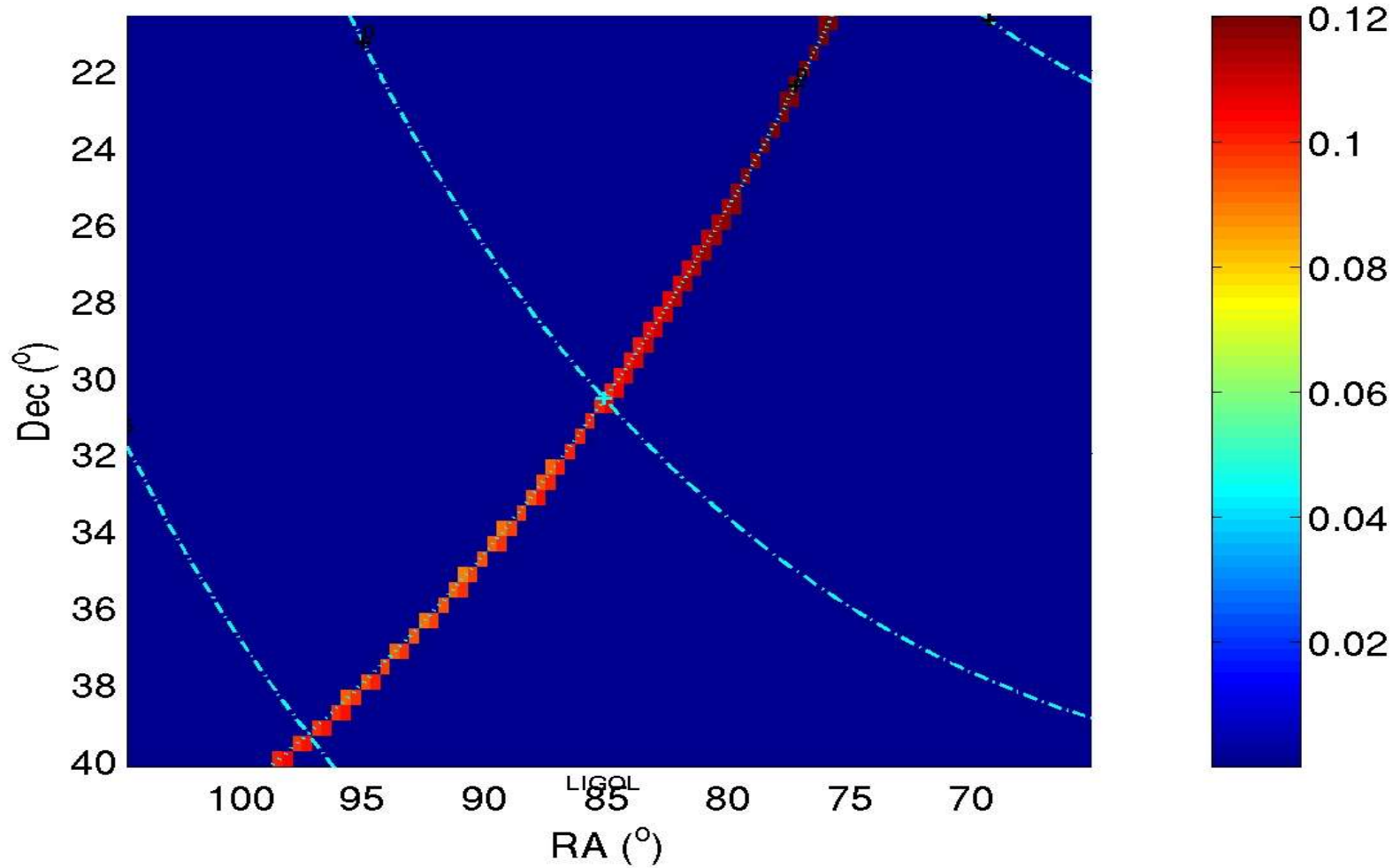
χ^2 test: LLO-LHO-GEO

10-10 Ms merger, $d=1$ Mpc, 3-detector $\text{snr}=206$,
near maximum sensitivity of LIGO
2-deg resolution, contour $\delta\tau_{ij} = 0, 5$ ms

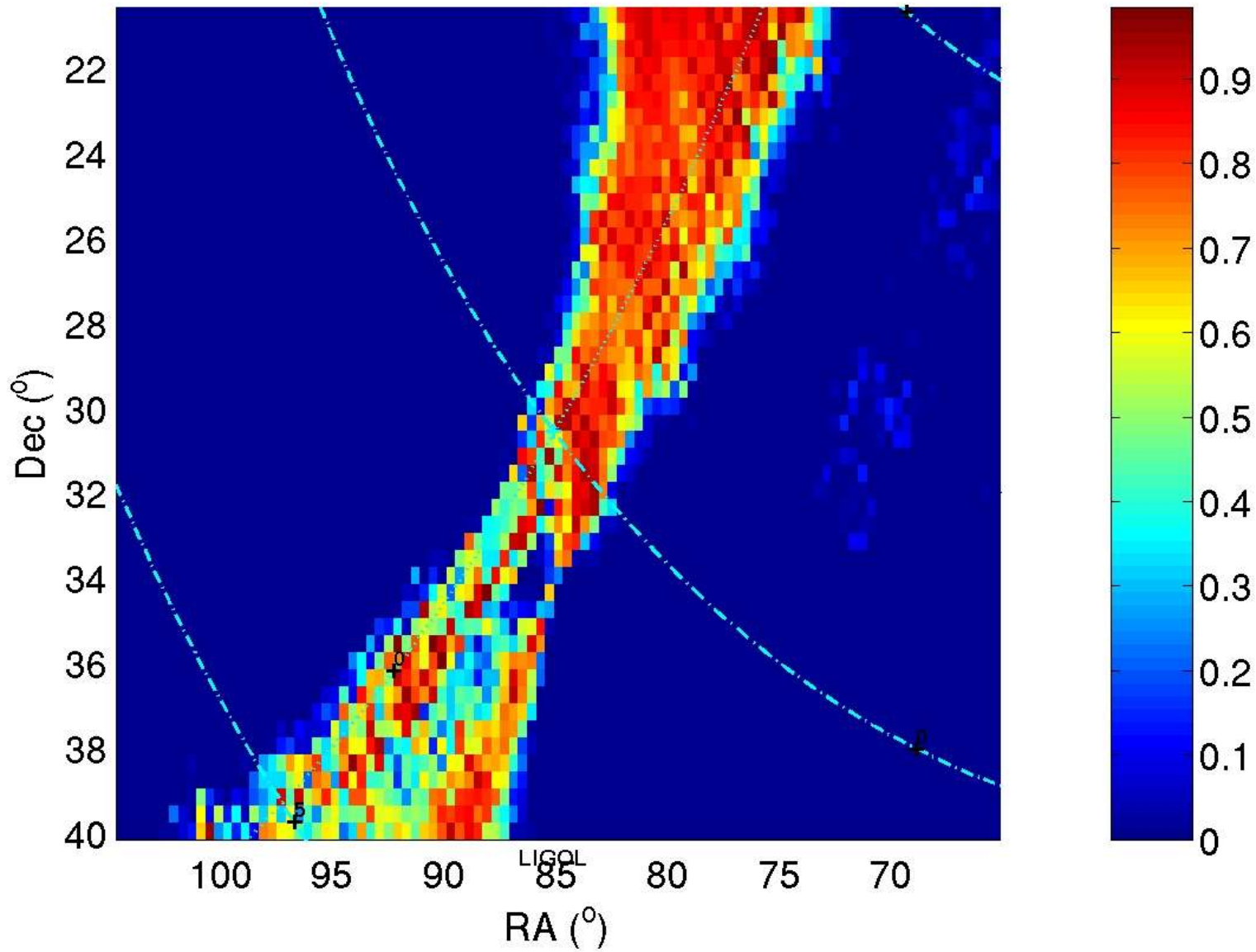


2-detector: LLO-LHO

same data, $d=1$ Mpc, 2-detector $\text{snr}=203$

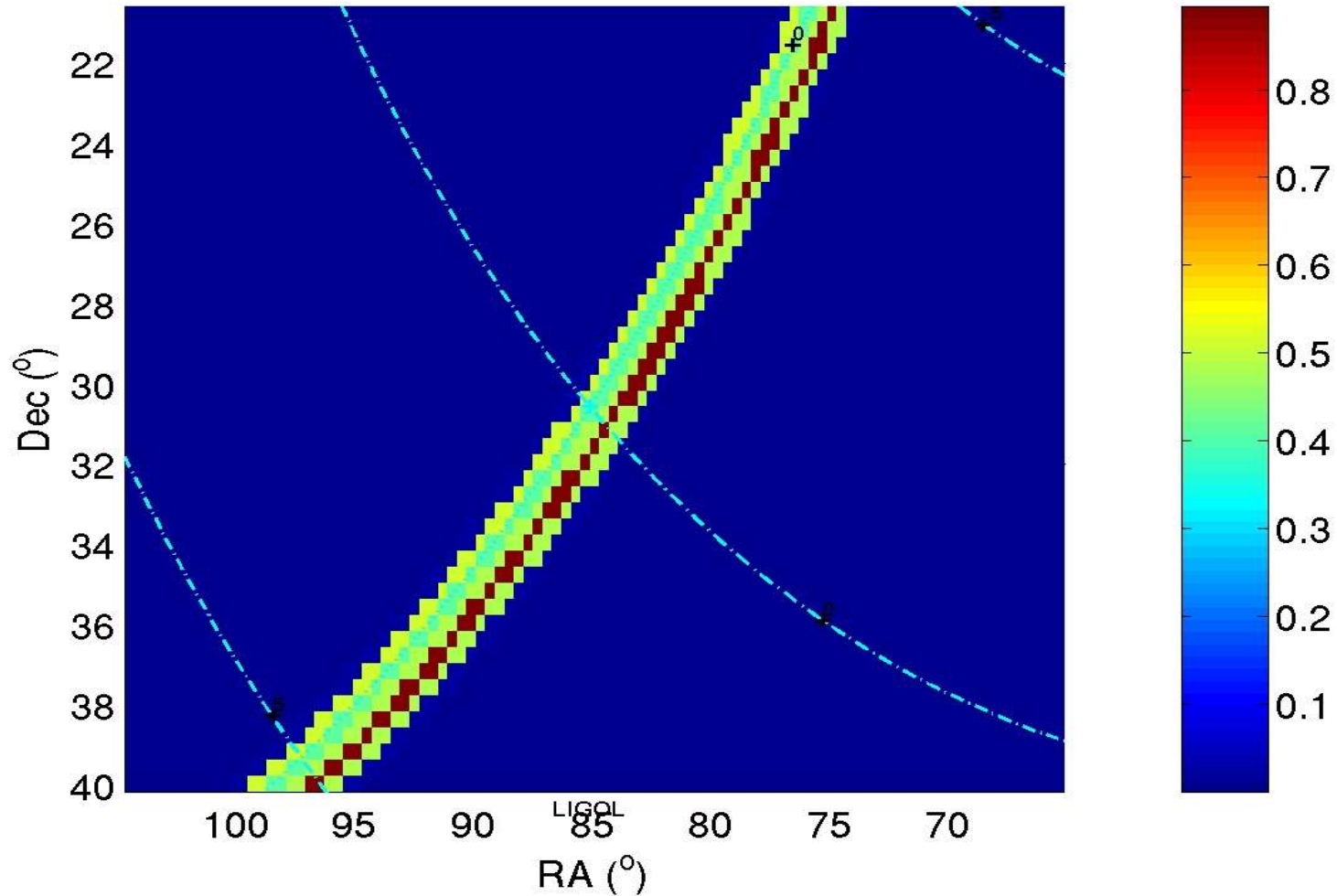


3-detector: LLO-LHO-GEO
d=10 Mpc, 3-detector snr=21

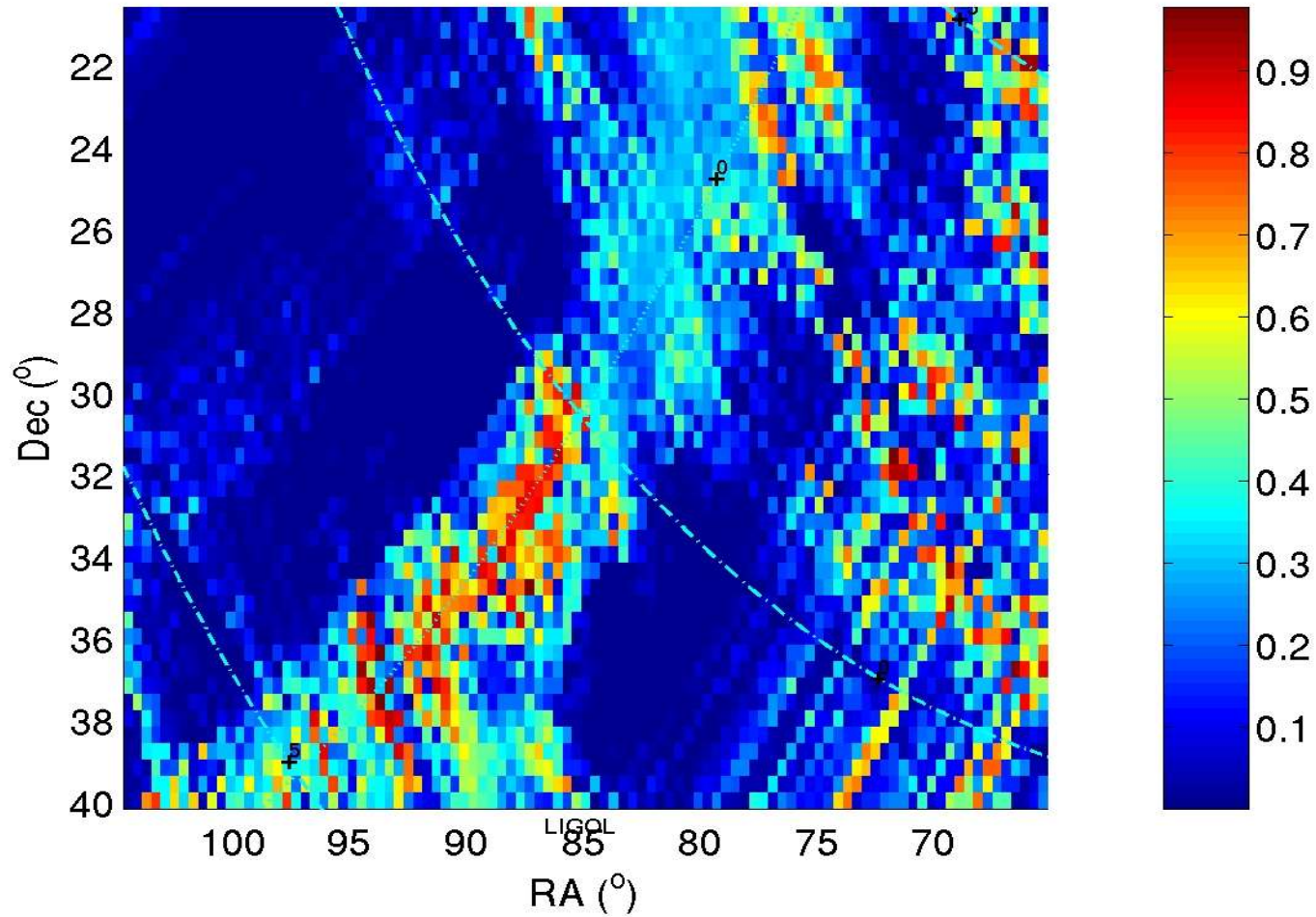


2-detector: LLO-LHO

d=10 Mpc, same data, 2-detector snr=20

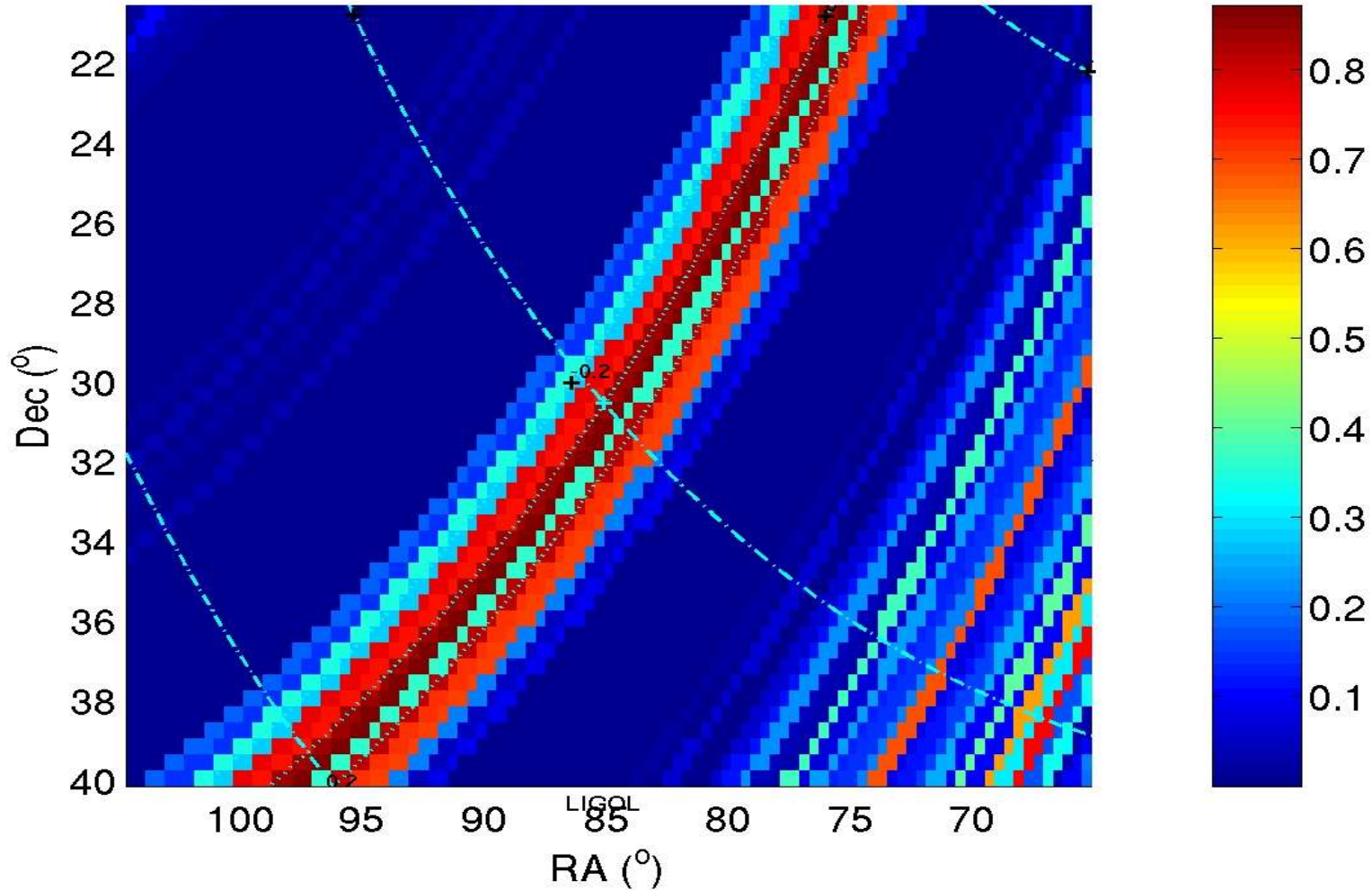


3-detector: LLO-LHO-GEO
d=20 Mpc, 3-detector snr=10



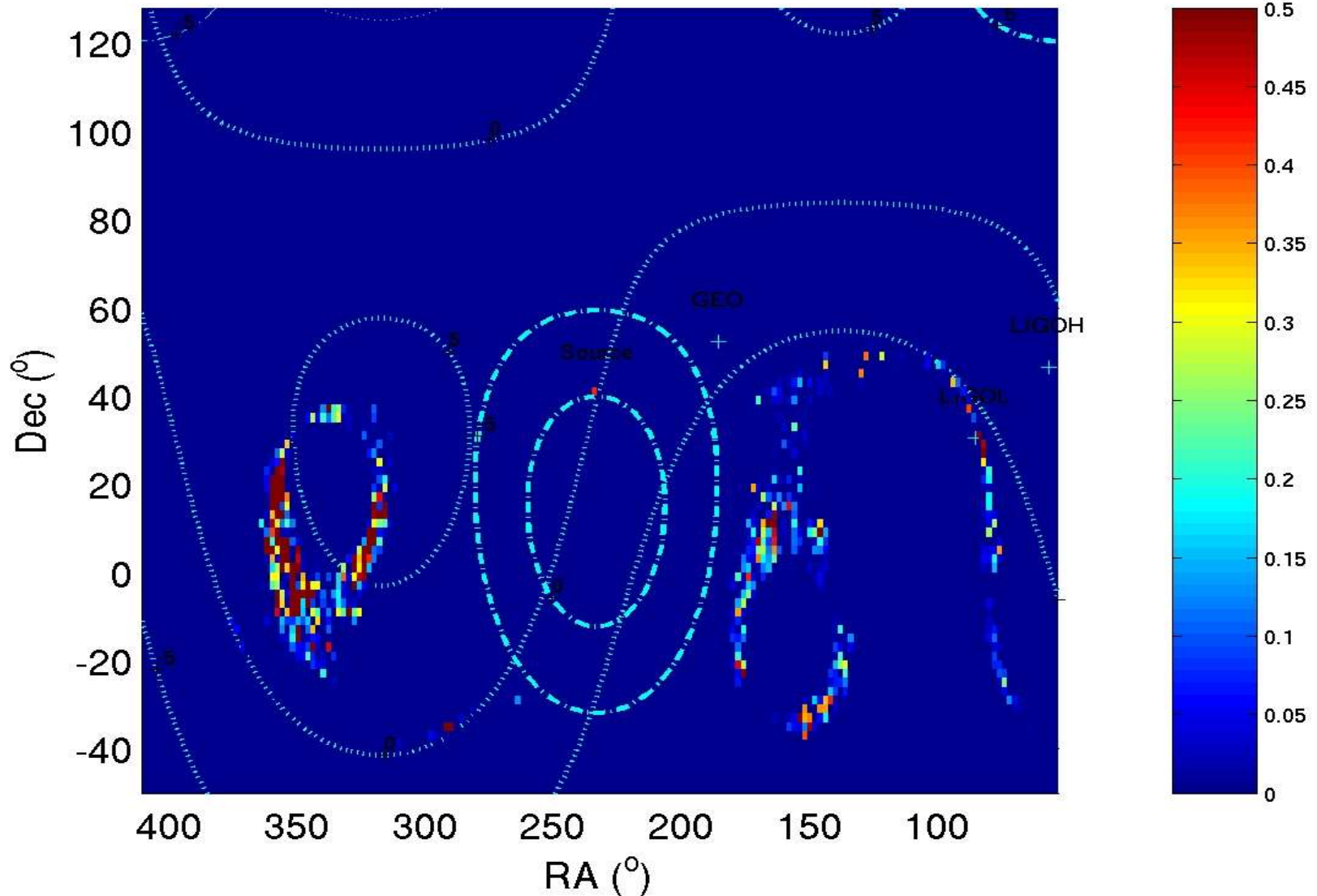
2-detector: LLO-LHO

d=20 Mpc, same data, 2-detector snr=10

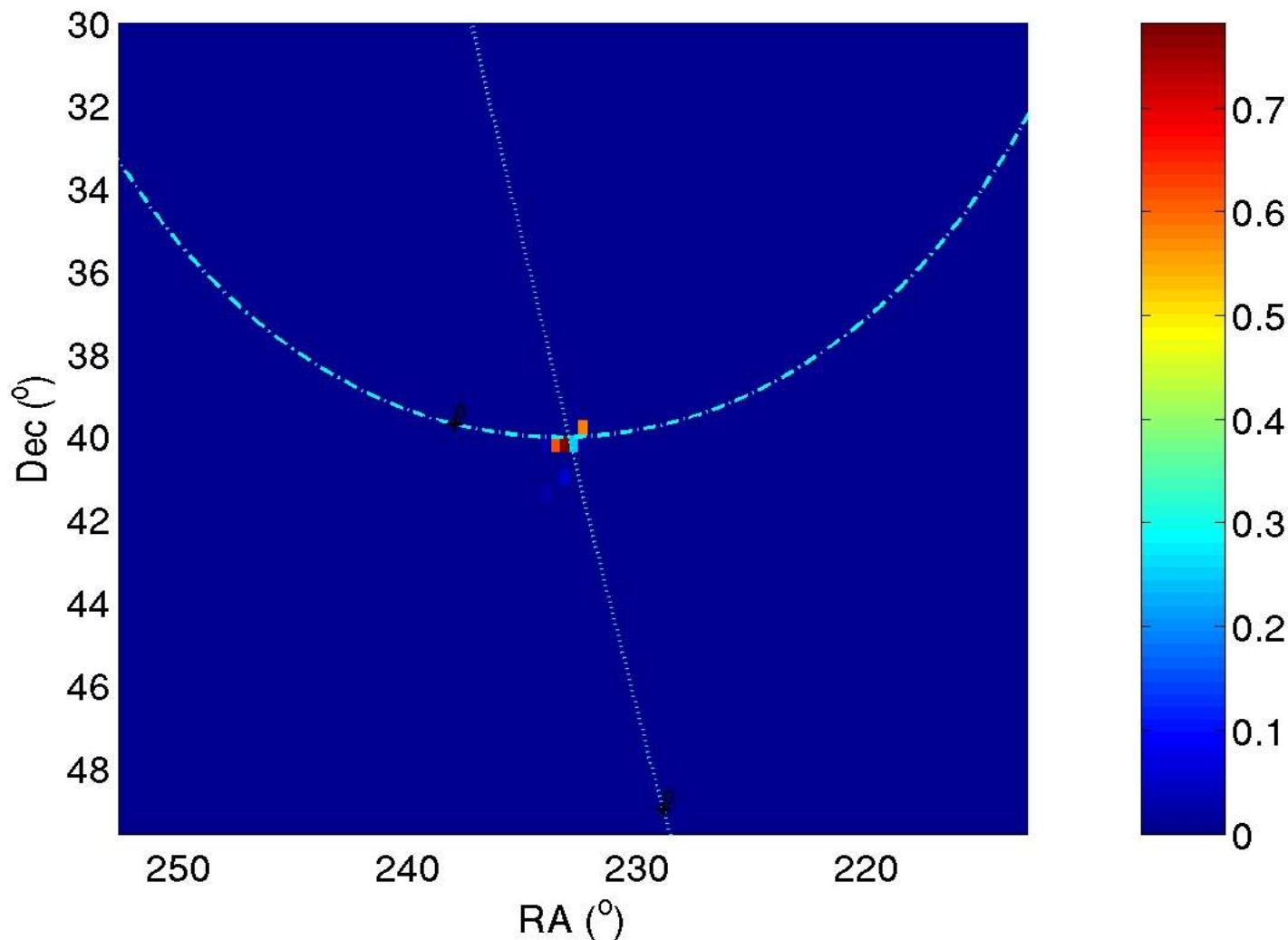


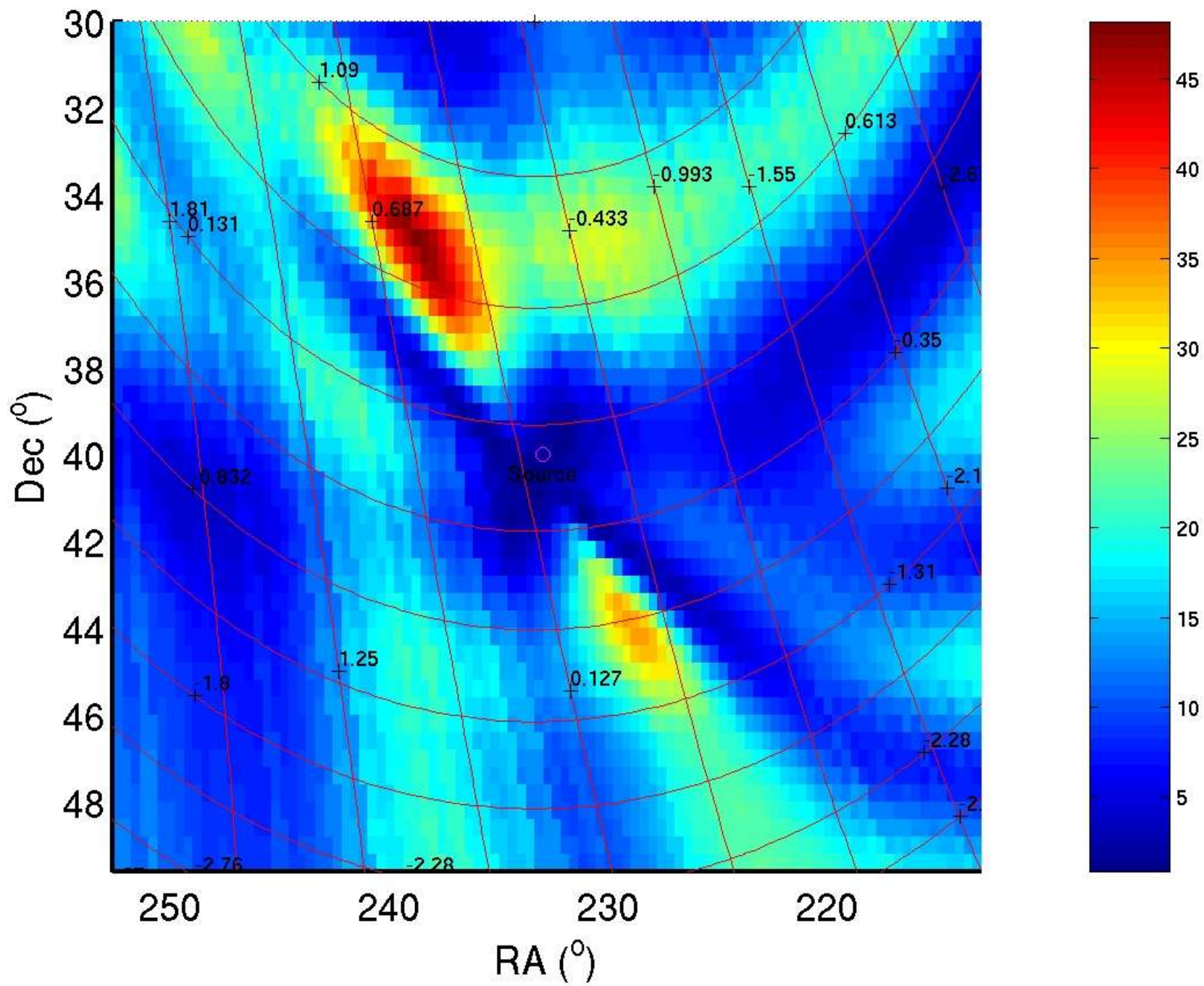
χ^2 test: LLO-LHO-GEO

10-10 Ms merger, $d=1$ Mpc, M.F. 3-detector $\text{snr}=93$,
near null of LIGO, 2-deg resolution, contour $\delta\tau_{ij}=0$, 5 ms



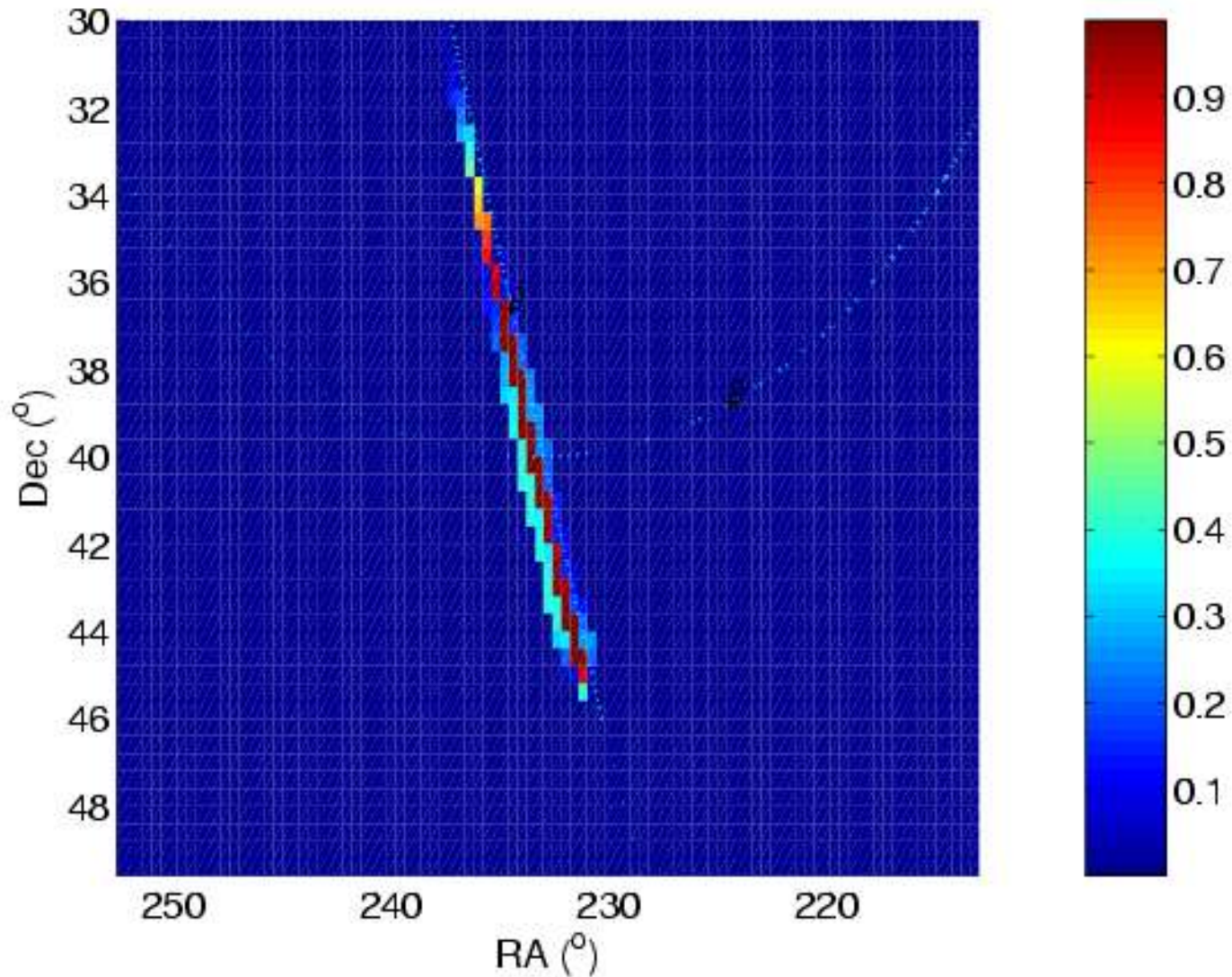
d=1 Mpc close-up:
positional determination $\sim 10'$
(snr=93, searched with $15'$ resolution)



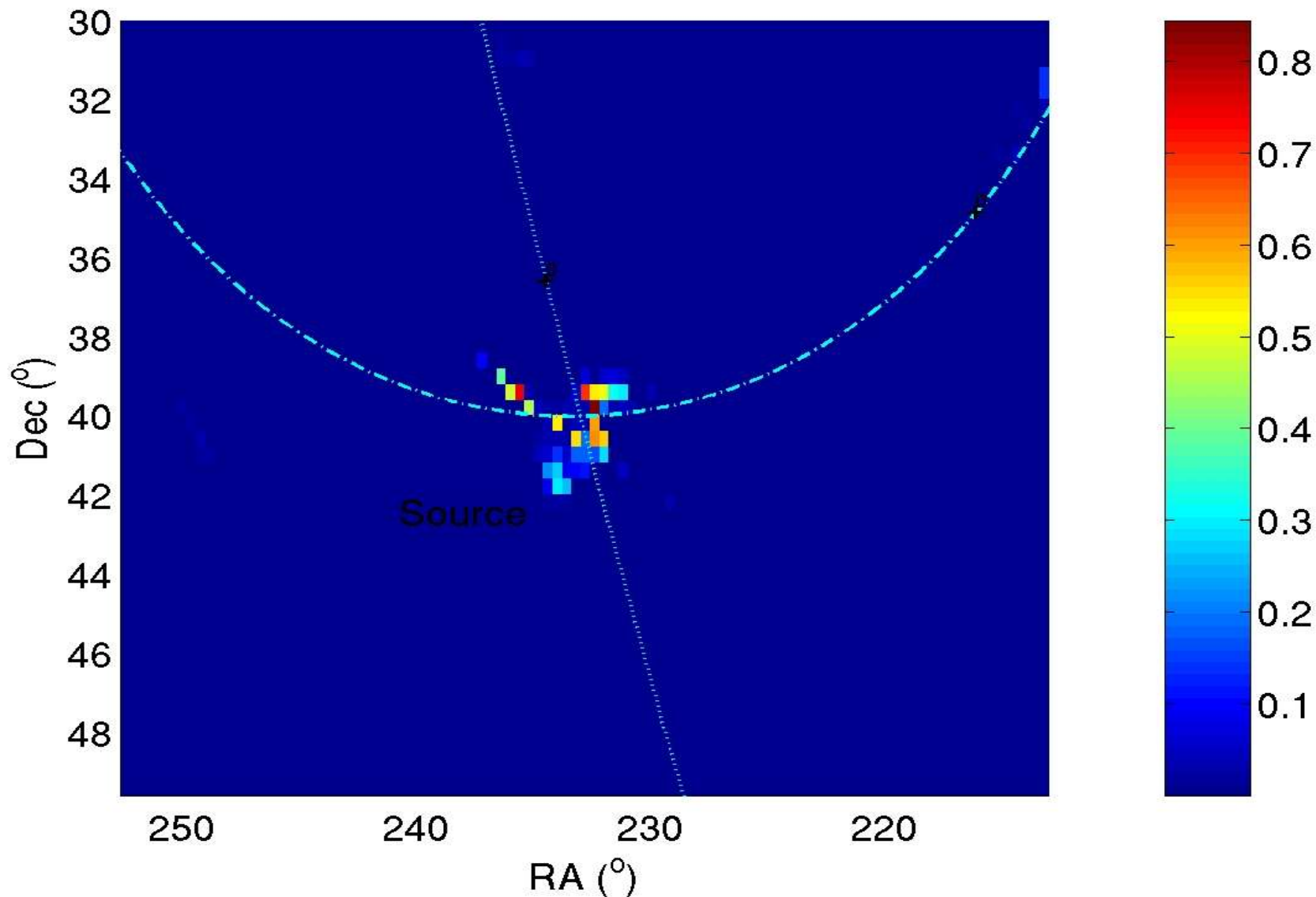


2-detector (LLO-LHO)

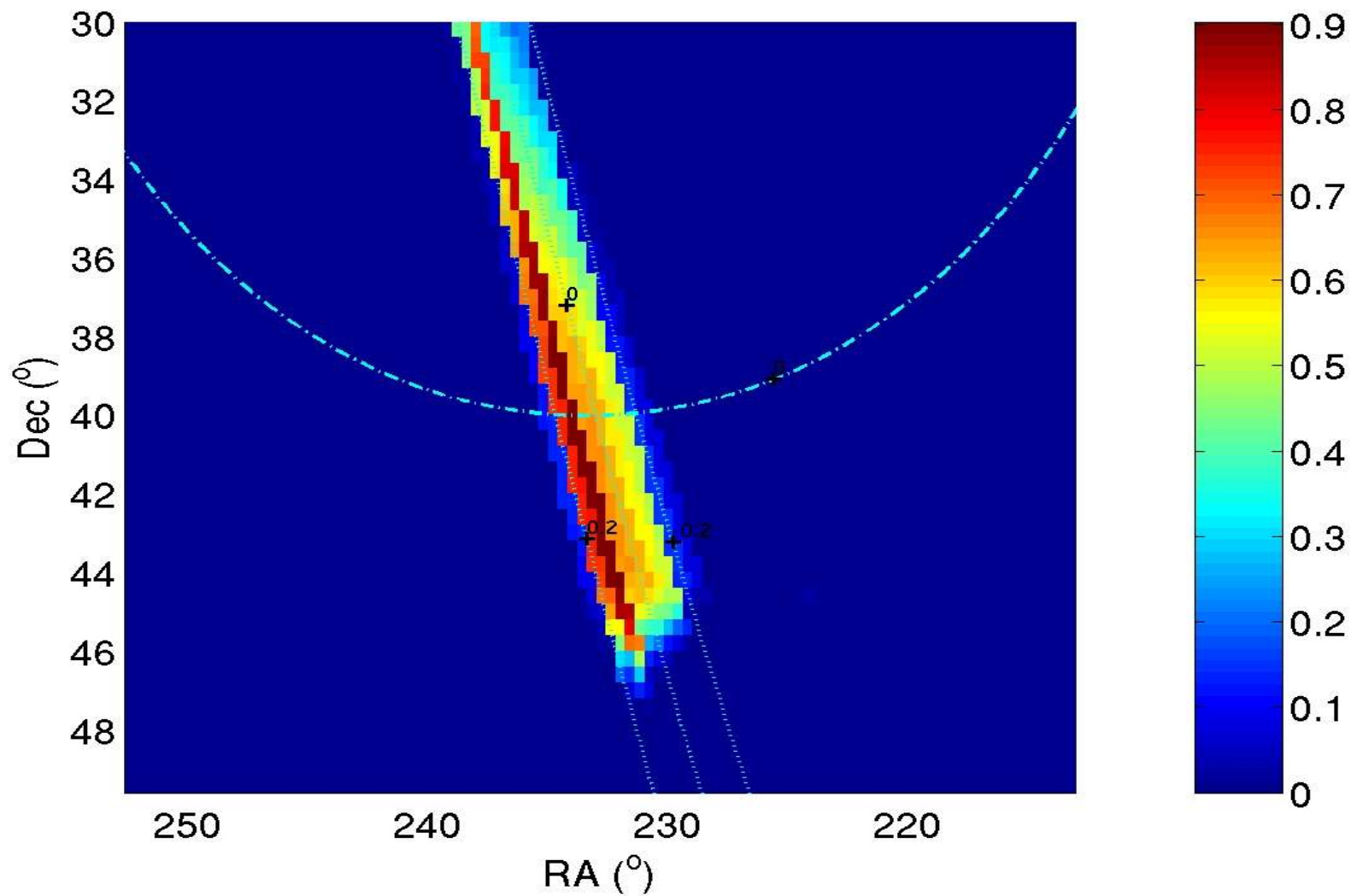
$d=1$ Mpc, $\text{snr}=54$, same data



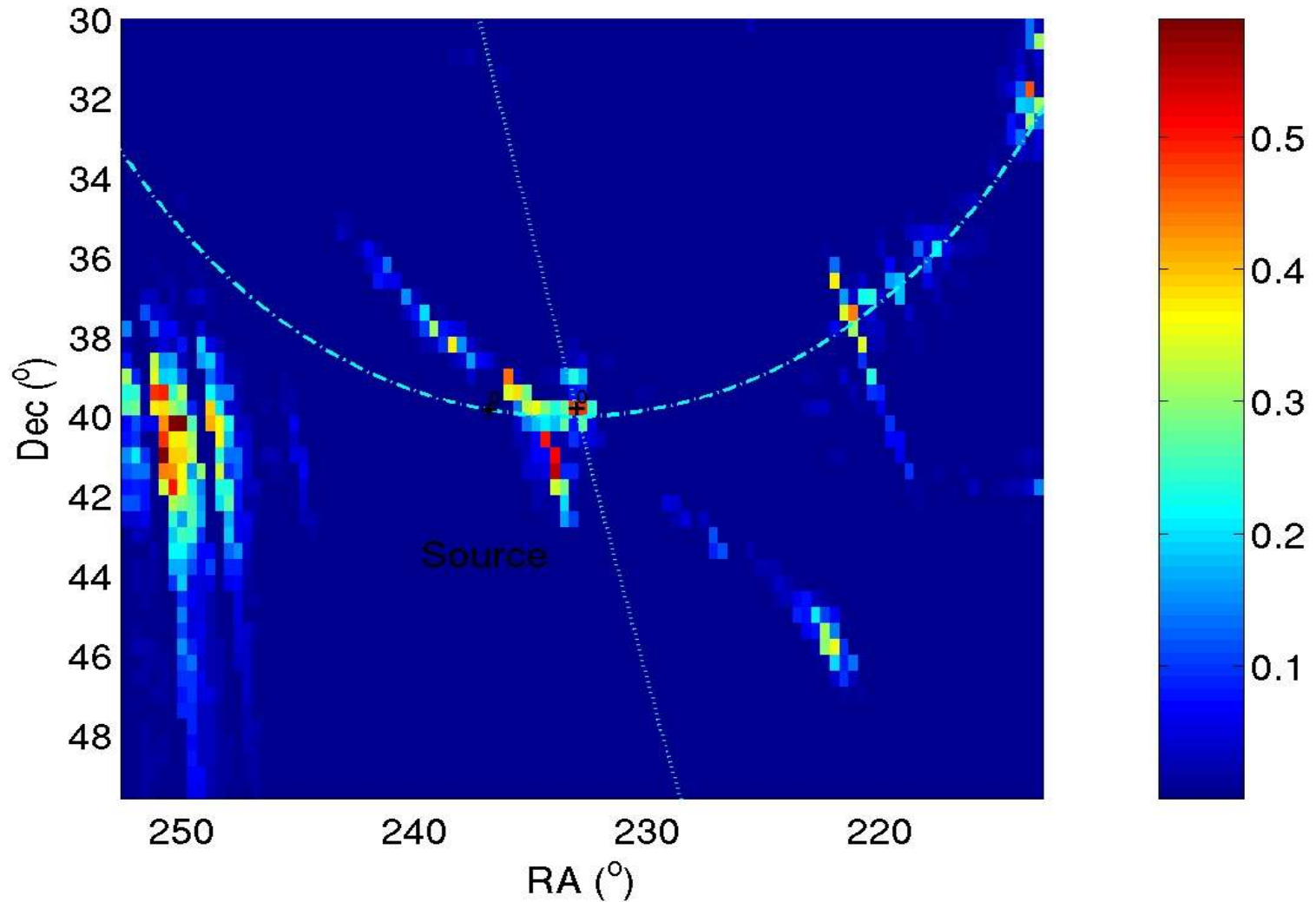
d=3 Mpc, snr=31 (3-detector: LLO-LHO-GEO)



d=3 Mpc, snr=18
(same data. 2-detector: LLO-LHO)

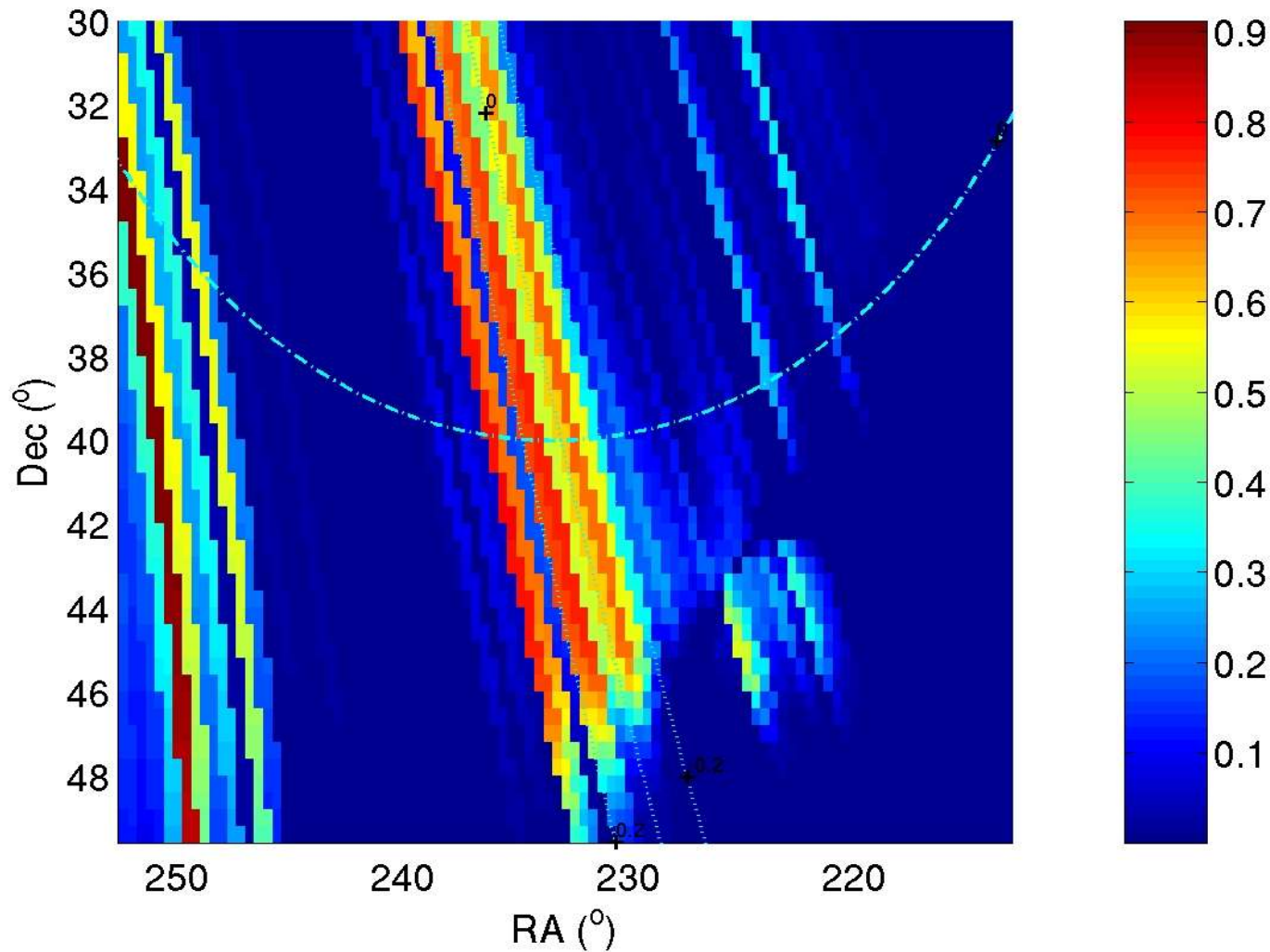


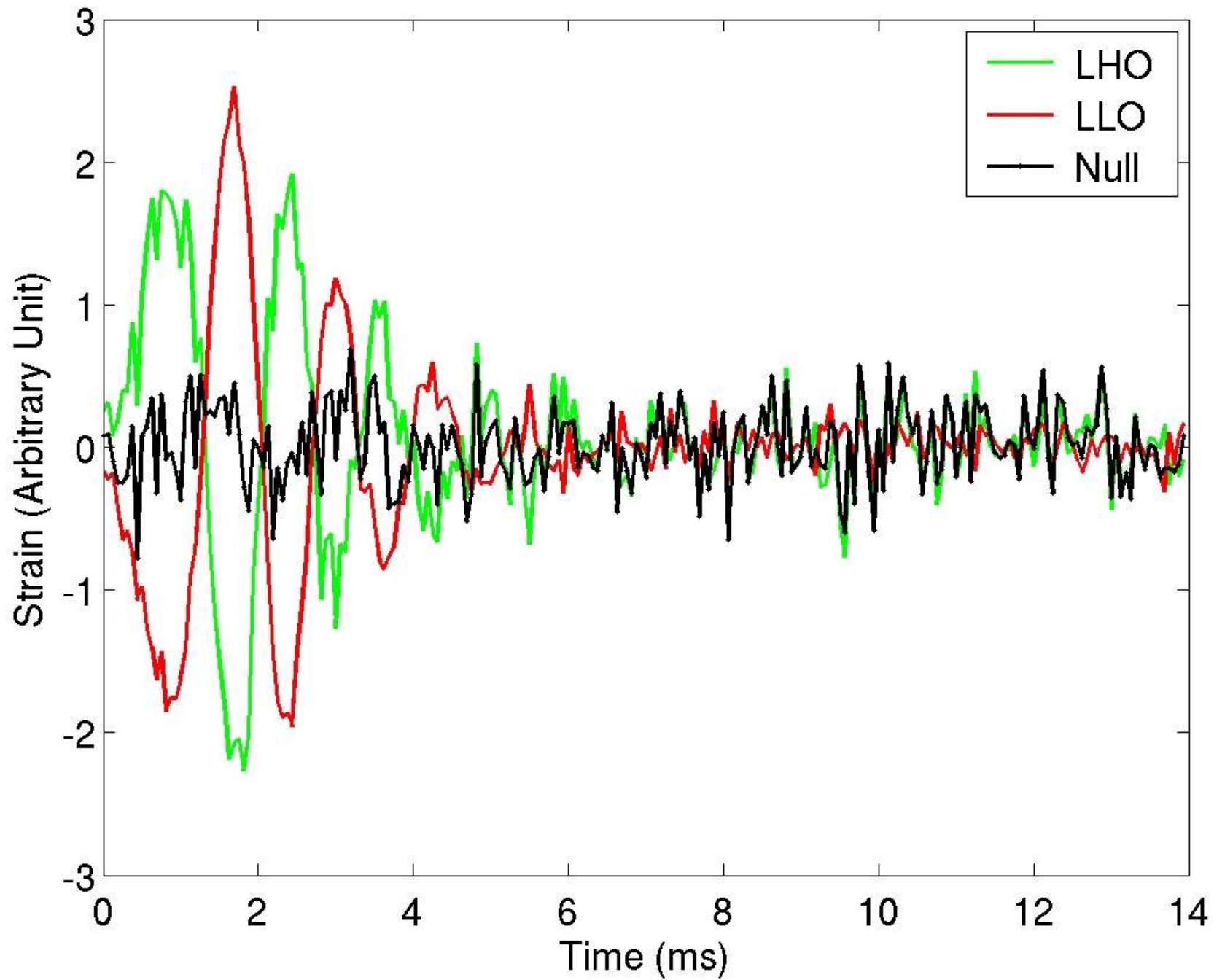
d=5 Mpc (snr=18, maybe)



2-detector (LLO-LHO)

d=5 Mpc, snr=11, same data







Monte-Carlo Simulation (in progress)

Summary

- Discussion of null-stream construction for 2,3 detectors
 - Work on a list of triggered “coincident” events from multiple detectors
 - application to positional determination
 - could possibly localize BH-BH to <a few degrees up to $\text{snr} \sim \text{few}$ for each detector
 - » $\sim \text{deg}$ in 1 dim, possibly to 20 Mpc with merger waveform only
 - implication to veto
 - works for any waveforms
 - ignore arrival time change due to Earth rotation
 - Antenna pattern : time-dependence negligible when calculating expected noise

Future work

- Implication to Veto
- Monte-Carlo Simulation
 - Dependence on snr of each detector, source direction, observable signal properties
- include bars, realistic noise, other type of waves (e.g. inspiral waves), etc.