# Effect of Thermal Lensing on WFS (FFT Study)

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# *WFS and Angular DOFs*

- ASY port:
  - WFS1Q ← DETM (differential ETM)

- Reflected port:
  - WFS3-I ← RM (recycling mirror)
  - WFS4-I ← CETM (common ETM)

- POX port
  - WFS2a (I) ← CITM (common ITM)
  - WFS2b (Q) ← DITM (differential ITM)

# The WFS Signal at any port

$$WFS(\eta, \Theta, \Gamma) \propto$$

$$P_{in}\Gamma \sum_{i=1}^{5} A_i \Theta_i \cos(\eta - \eta_i) \cos(\omega_m t - \phi_{Di})$$

$P_{in}$ : total power

$\Gamma$ : modulation index

$\Theta_i$ : Normalized angles wrt divergence angle

$A_i$ : Normalized signal amplitude for each dof

$\eta$ : Gouy phase between TEM00 & TEM01 at output port

$\eta_i$ : Gouy phase due to detector's position

$\phi_{Di}$ : Demodulation phase for i dof

Daniel T960115, Fritschel et al (1997)

# The Optimization

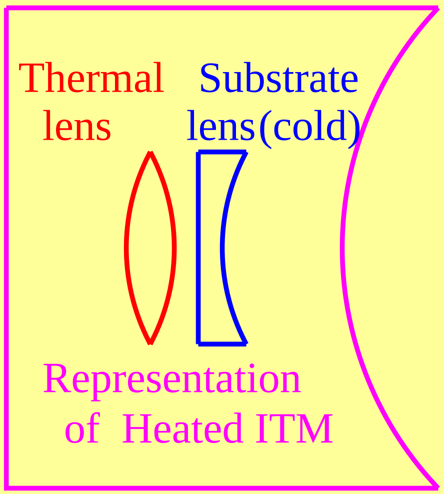
2 considerations for WFS Telescopes: [Nergis, T990130]

✓ The Gouy phase $\eta_i$ be optimized for max sensitivity of primary dof

✓The spot size is matched to the size of the quadrant diode (~3mm)

At WFS, the demodulation phase, $\phi_{Di}$ needs to be adjusted

# Representation of Thermal Lensing




Effective refractive index:

$$n_{\text{effective}} = 1.45 - \frac{R_{\text{HR}}}{f_{\text{thermal}}}$$

For a thermal state of ITM:

$$\frac{1}{f_{\text{effective}}} = \frac{1}{f_{\text{ITM}}} + \frac{1}{f_{\text{thermal}}}$$

$$\frac{1}{f_{\text{ITM}}} = \frac{-(1.45-1)}{R_{\text{HR}}}$$

For H1 (Common Heating):

$$0.967(\text{Hot}) \leq n_{\text{effective}} \leq 1.45(\text{Cold})$$

# Calculation Procedure

- From FFT IFO (under different thermal states) get 128X128 pixelized data for beams (CR & SBs) at BS, POX, Reflected port at RM.
- Propagate these beams using Matlab "FFTprop" tool (http://www.ligo.caltech.edu/~bbhawal/DOWNLOAD ) through telescope, distances and lens to WFS locations on output benches.
- Calculate signal in split-photodiode at WFS location
- Vary WFS location (so Gouy phase) to repeat "FFTprop" calculation

# What is that Gouy Phase, $\eta$ ?

4 dominant Parts of a WFS signal:

$$S_1 = \overline{\mathrm{CR}}_{00}\overline{\mathrm{SBP}}^*_{01}$$

$$S_2 = \overline{\mathrm{CR}}_{01}\overline{\mathrm{SBP}}^*_{00}$$

$$S_3 = \overline{\mathrm{SBM}}_{00}\overline{\mathrm{CR}}^*_{01}$$

$$S_4 = \overline{\mathrm{SBM}}_{01}\overline{\mathrm{CR}}^*_{00}$$

Amplitudes $\overline{\mathrm{CR}}_{00}$, $\overline{\mathrm{SBP}}_{01}$, etc are without Gouy phases

Quadrature and Inphase Signals:

$$Q = 2\mathrm{Re}\left[S_1 e^{-i\eta} + S_2 e^{+i\eta} + S_3 e^{-i\eta} + S_4 e^{+i\eta}\right]$$

$$I = -2\mathrm{Im}\left[S_1 e^{-i\eta} + S_2 e^{+i\eta} + S_3 e^{-i\eta} + S_4 e^{+i\eta}\right]$$

$\eta$ : Gouy phase between TEM00 & TEM01 at output port

All such terms contribute (ex: Pitch mode):

$$\mathrm{CR}_{ij}\mathrm{SBP}^*_{ik} + \ldots$$

where $abs(j-k)$ = an odd integer

Define

$$S_j = R_j + i * I_j$$

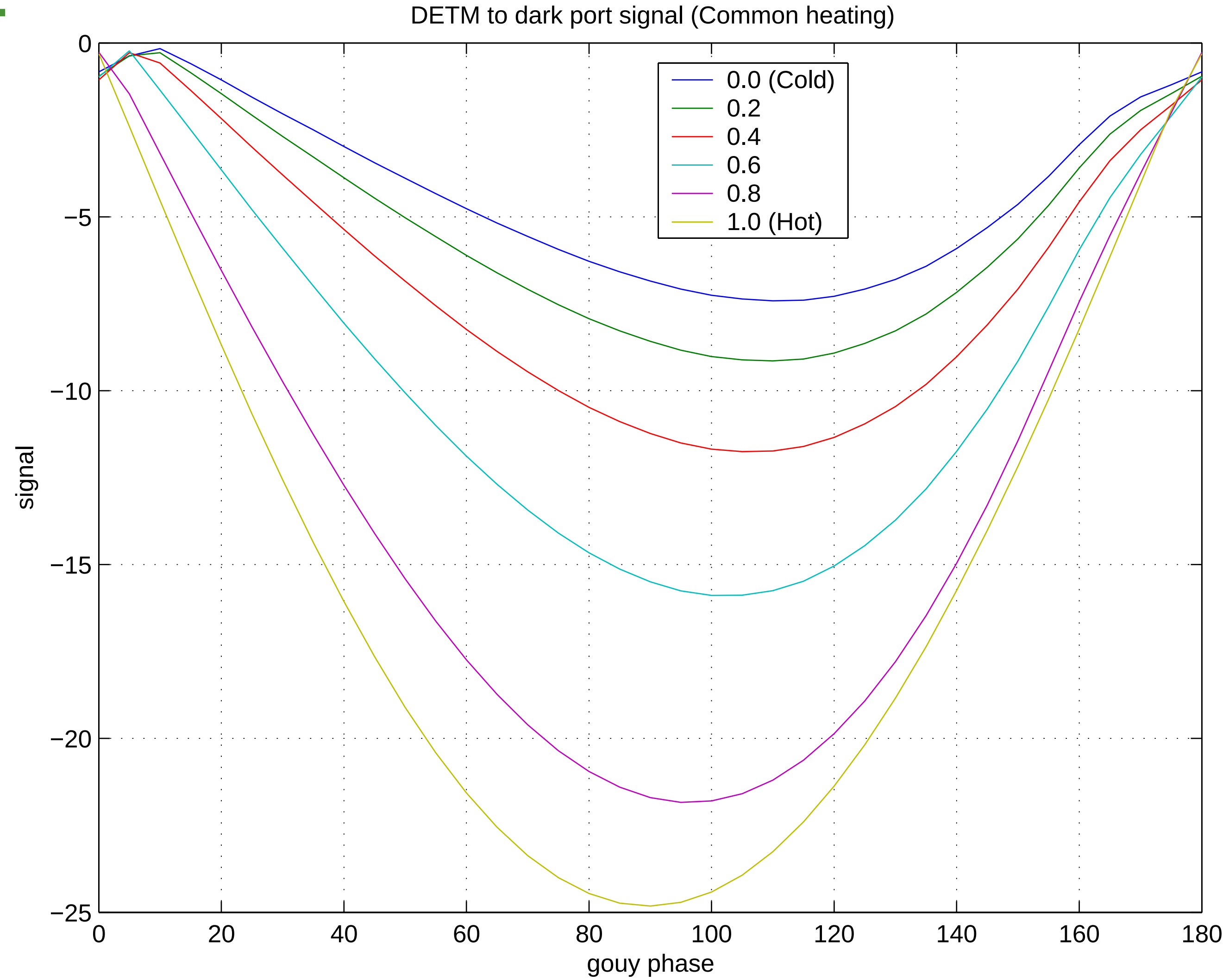
$$Q = 2[(R_1+R_2+R_3+R_4)^2+(I_1-I_2+I_3-I_4)^2]^{1/2}$$

$$\eta_Q = tan^{-1}\left[\frac{(I_1 - I_2 + I_3 - I_4)}{(R_1 + R_2 + R_3 + R_4)}\right]$$

$$I = -2[(I_1+I_2+I_3+I_4)^2+(R_2-R_1+R_4-R_3)^2]^{1/2}$$

$$\eta_I = tan^{-1}\left[\frac{(R_2 - R_1 + R_4 - R_3)}{(I_1 + I_2 + I_3 + I_4)}\right]$$

# WFS1Q (← DETM) Signals



# Reflected Port Signals: WFS 3I & 4I

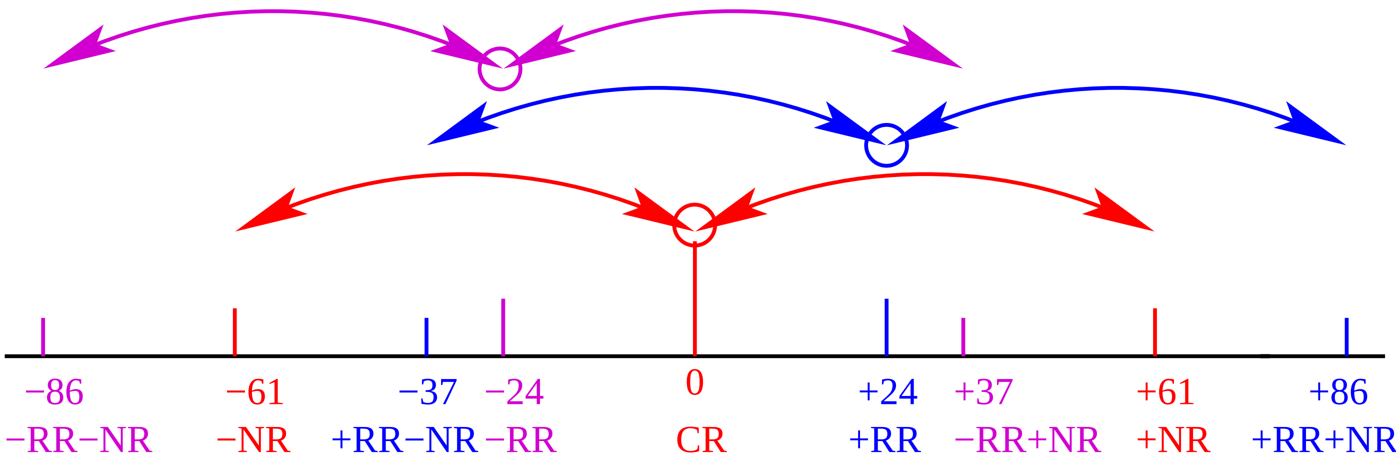
- NRSB demodulation: (RM → WFS3I,CETM→WFS4I)

1) CR with ± NRSB $(\propto \Gamma_{\mathrm{NRSB}})$

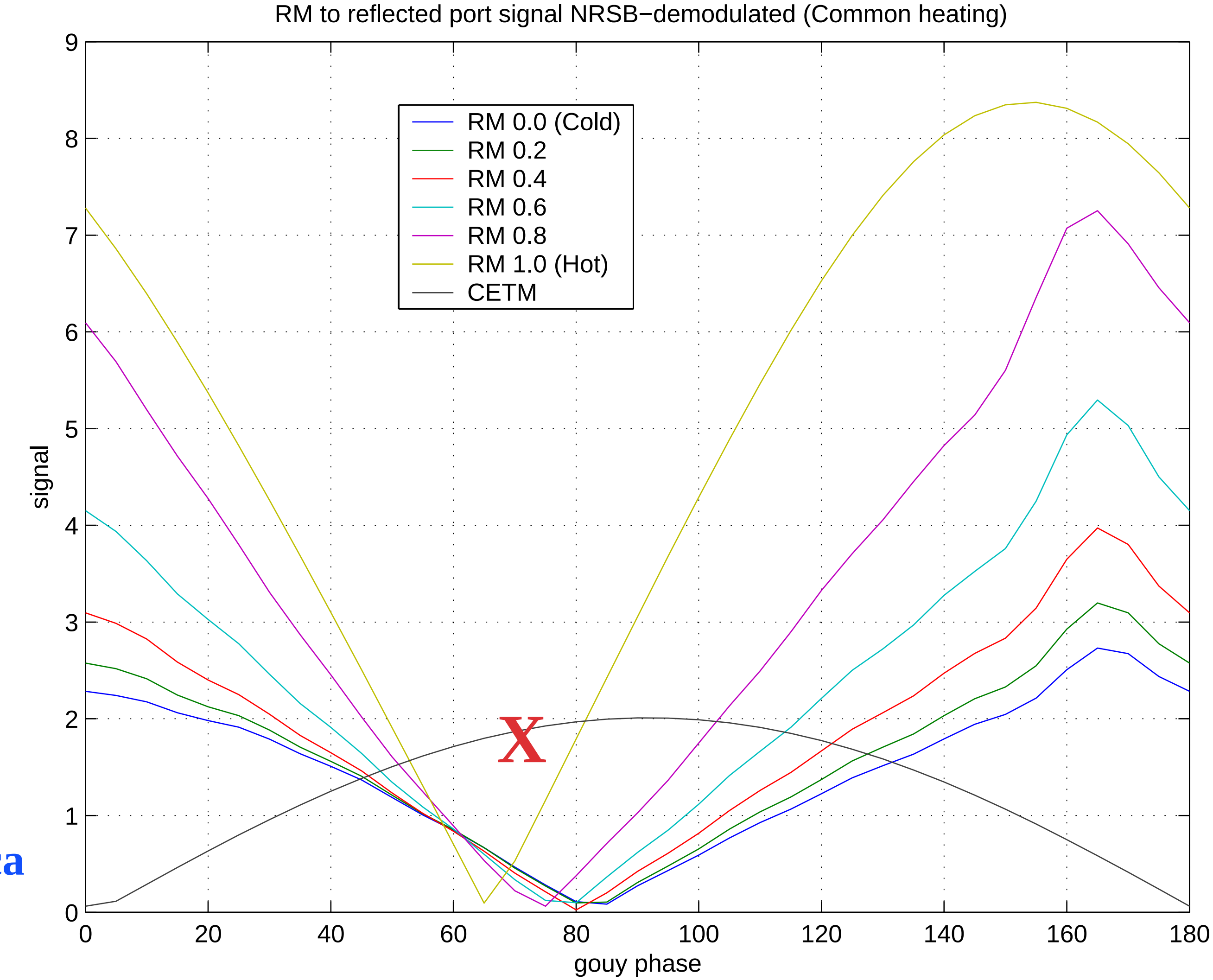
- (only for RM→WFS3-I)

2) Upper RRSB with [RRSB ± NRSB] $(\propto \Gamma^2_{\mathrm{RRSB}}\Gamma_{\mathrm{NRSB}})$

3) Lower RRSB with [-RRSB ± NRSB] $(\propto \Gamma^2_{\mathrm{RRSB}}\Gamma_{\mathrm{NRSB}})$

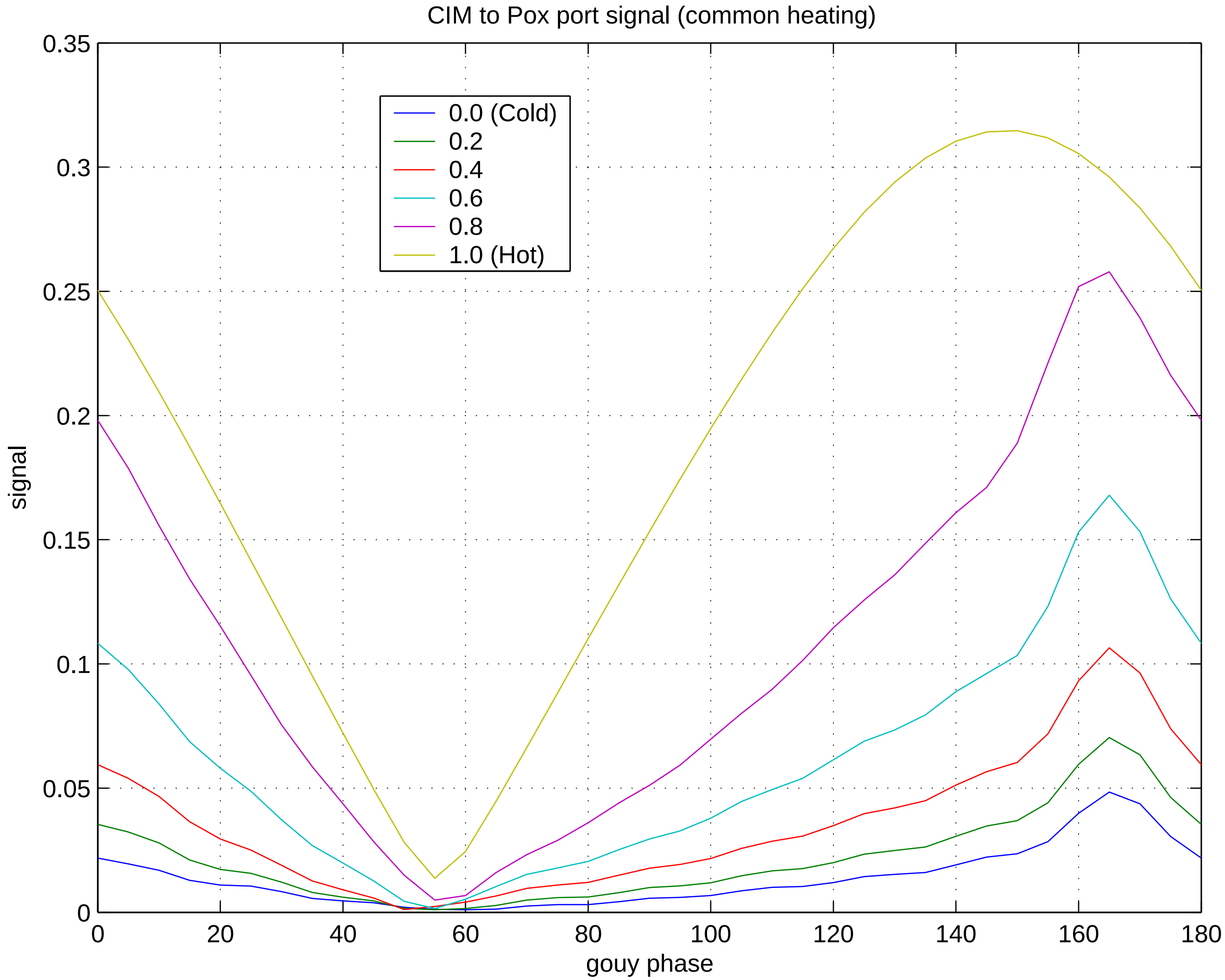


# WFS 3-I (← RM) & 4-I (← CETM) Signals

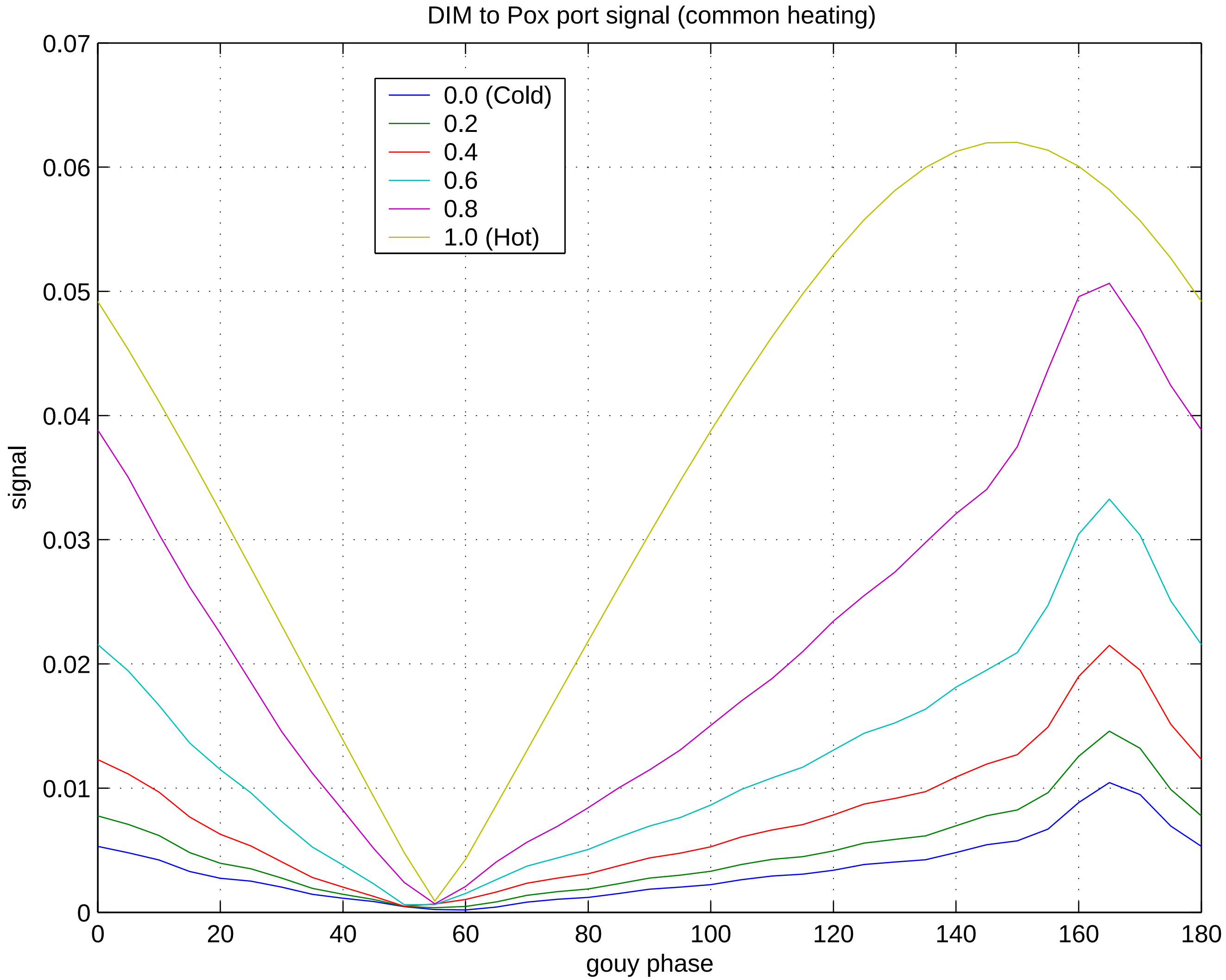



**Also see H1 elog Feb6,24 (2004) by Luca**

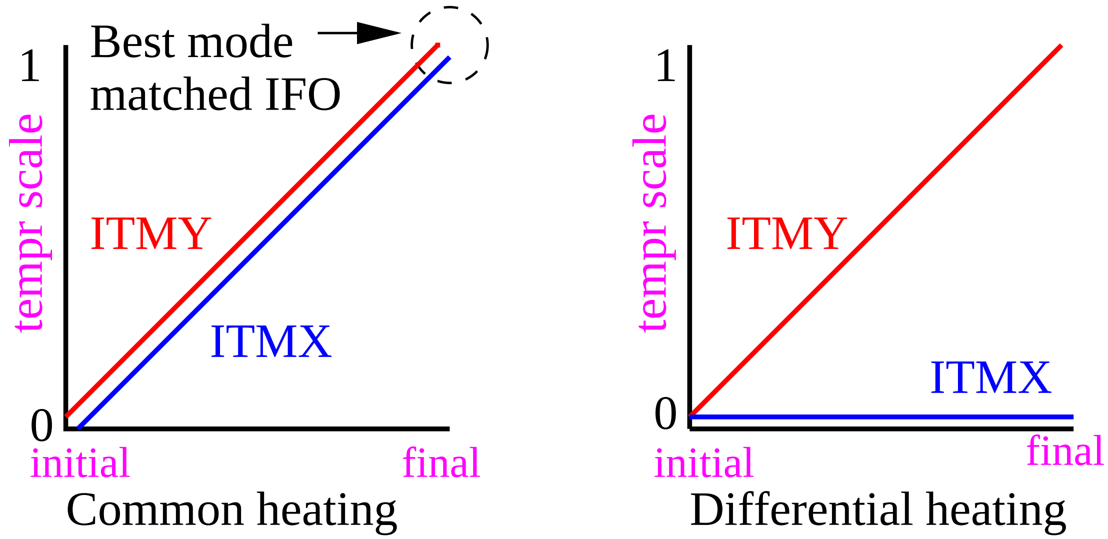
# WFS2a (← CITM) Signals from POX



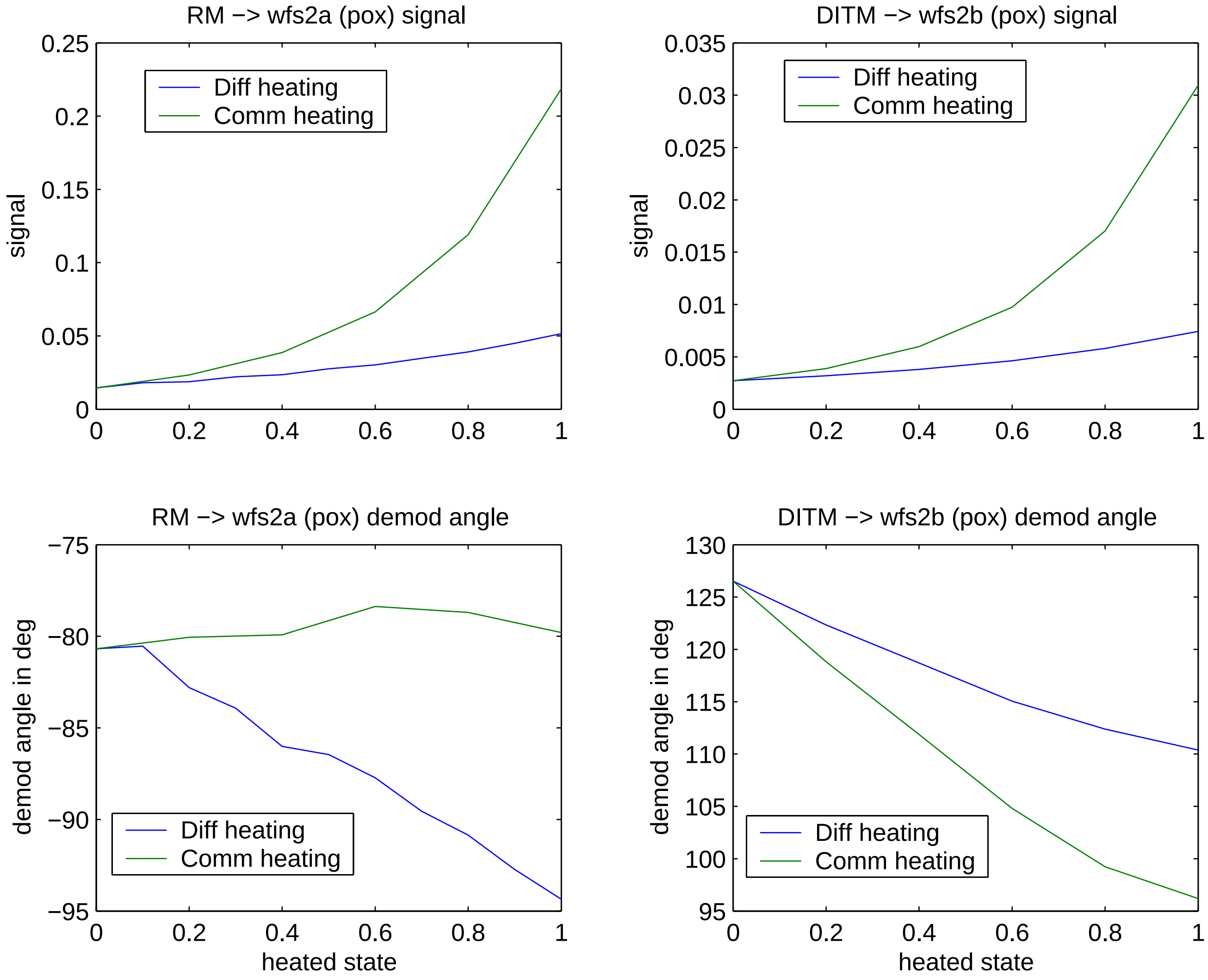
# WFS2b (← DITM) Signals from POX



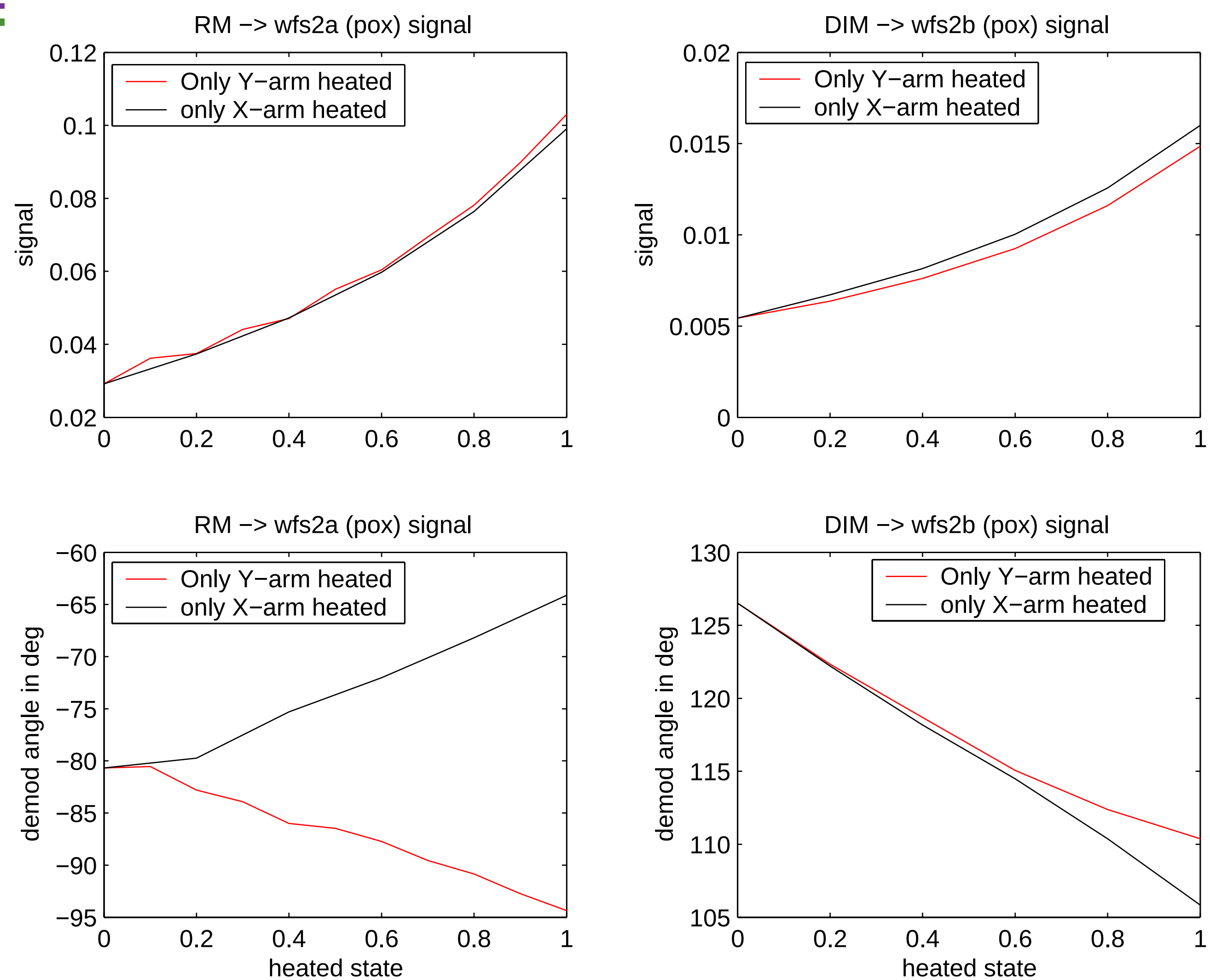
# Common and Extreme Differential Heating



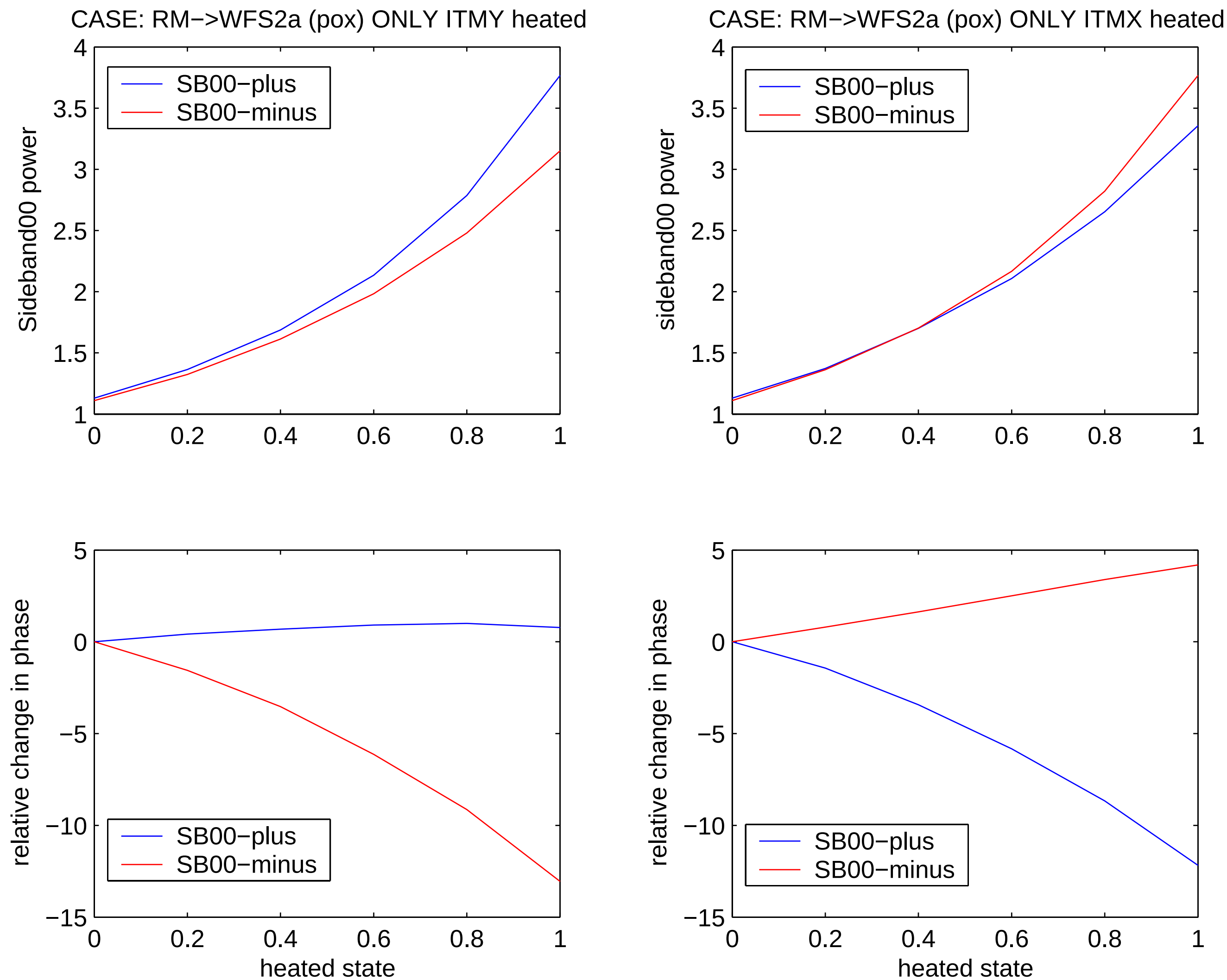
# Effect of Thermal Lens on Demod. Phase of WFS2



# *Demodulation Phase & Extreme Differential Heating*



# Sidebands and Extreme Differential Heating




Also see Hiro (G040442)

# Change of Beam-width in LLO:WFS1 path

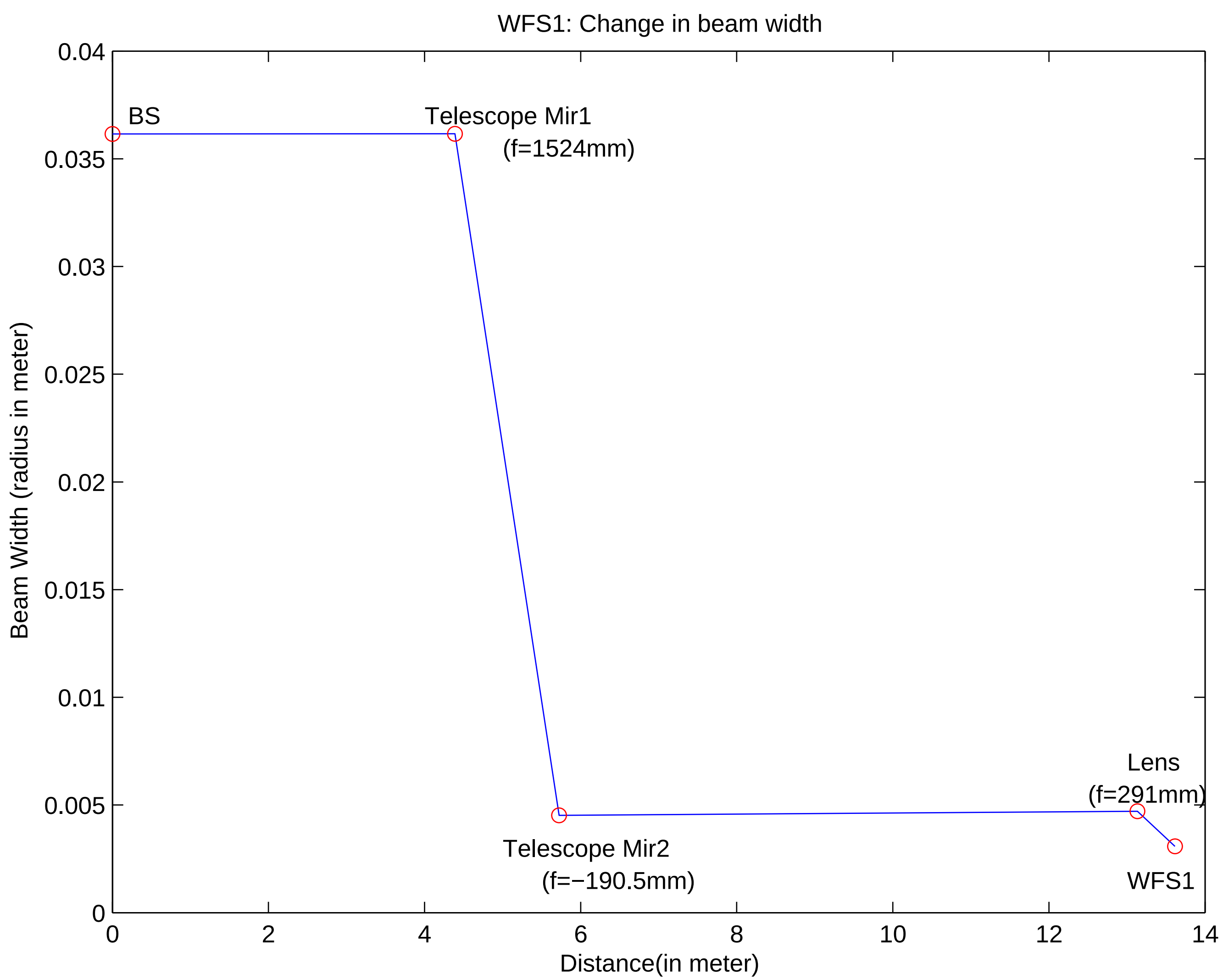



Figure for other WFSs:

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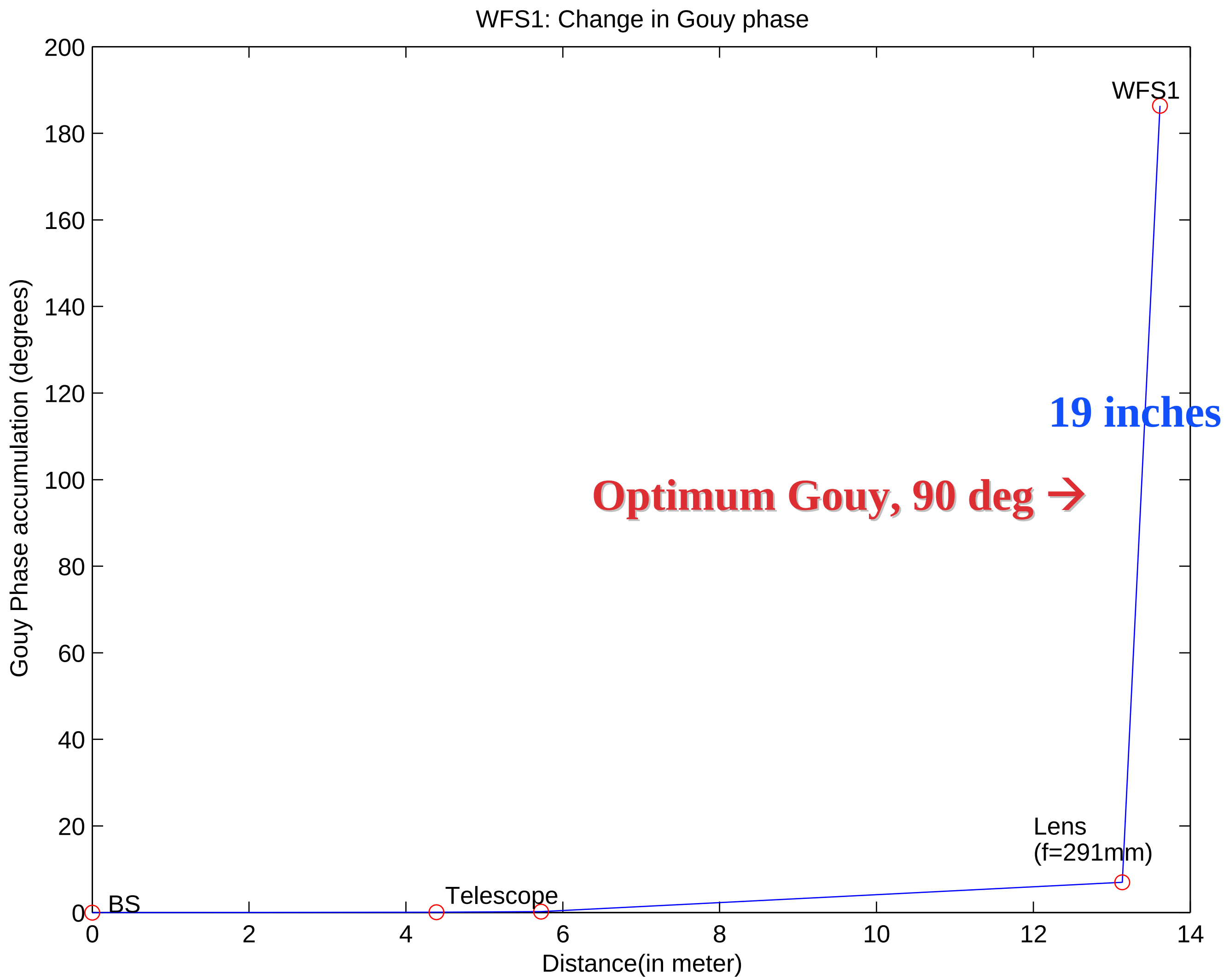
# Change of Gouy Phase in LLO: WFS1 path




**Mystery**

**@**

**LLO**

Figures for other WFSs:

LLO elog: 01/27/2005

# *Last Slide: WFS Mysteries …*

[Related to this Study only]

- Why is LLO WFS  working even though some of its Gouy phase (empirically optimized) locations are not the right places to get strong signals?
- Why NO changes in demodulation phase is observed in WFS2 at LLO?
- …