Modeling the Performance of Networks of Gravitational-Wave Detectors in Bursts Search

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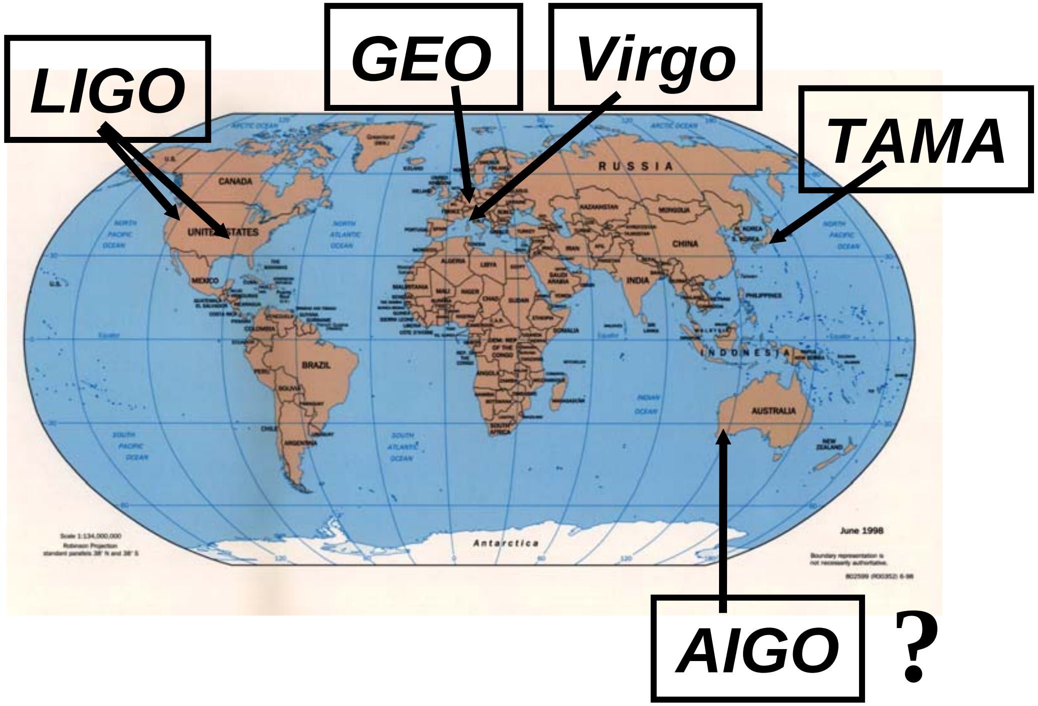
G050487-00-R

SURF Project

Multiple-Detector Searches

- Most confident detection and maximum exploitation of gravitational waves will require cooperative analyses by the various observatories:



  - » Decreased background.
  - » Better statistics on signal parameters.
  - » Better frequency coverage.
  - » Better sky coverage.
  - » Better sky location, polarization information.
  - » Independent hardware, software, and algorithms minimize chances of error.

l33 At present large scale interferometric GW detectors are operating or are being commissioned, as all of us know.Cooperative analysis by these obseravtories could be valuable for making more confident detection of GW and extracting maximal information from them. There are some of advantages of such analysis...

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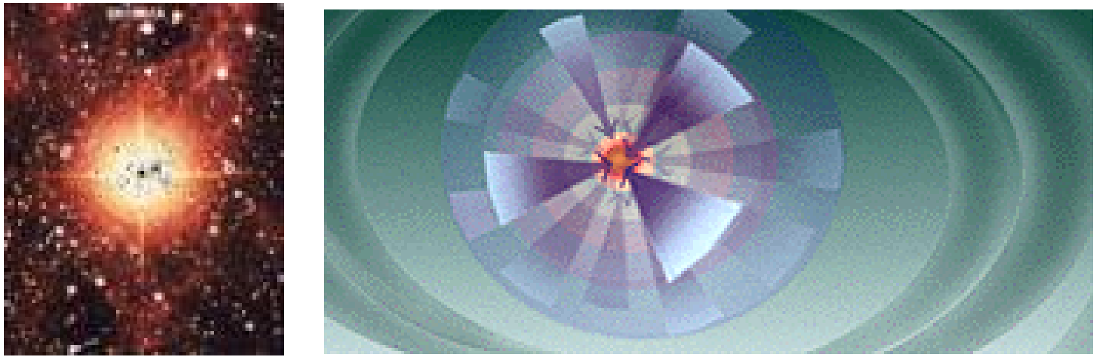
Multiple-Detector Searches

- This investigation is targeted towards ***Gravitational-Wave Bursts*** (GWBs)
- GWBs are generated by systems such as core-collapse supernovae, black-hole mergers and gamma-ray bursters
- Poor theoretical knowledge of the source and the resulting GW signal



Slide 3

I34 Such a cooperative analysis is particularly useful for GWBs detection for which our theoretical knowledge of the source and the resulting GW signal is quite limited.

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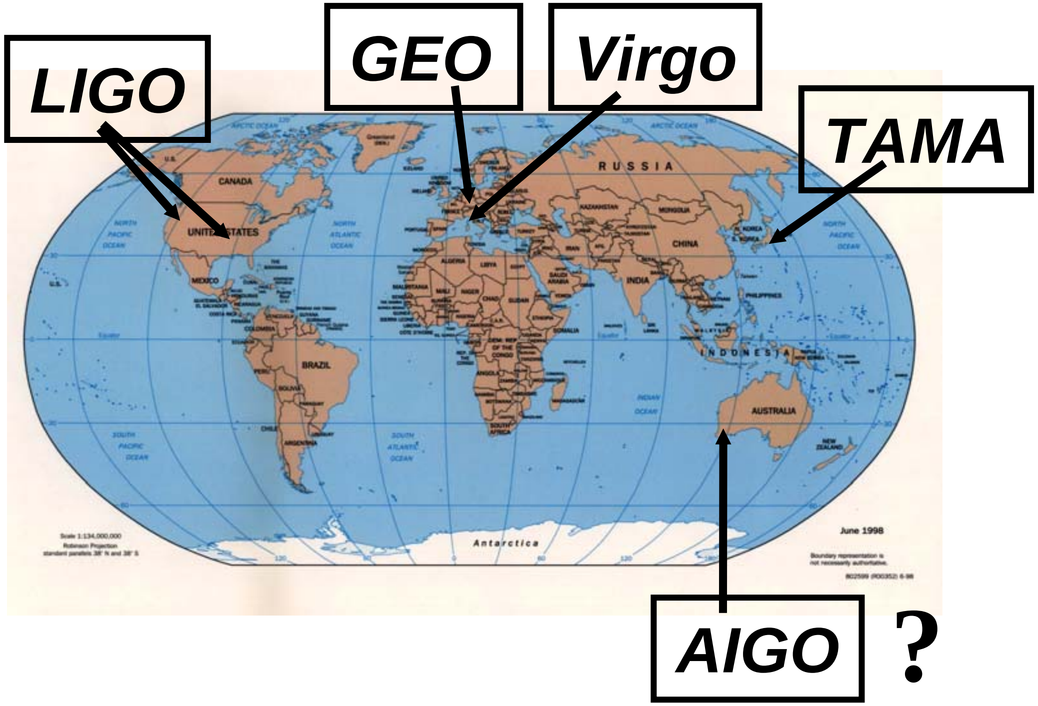
Multiple-Detector Searches

- Unfortunately, these benefits don't come without hard work. Physical and technical challenges abound.



Detectors see:

- » … different frequency bands.
- » … different parts of the sky.
- » … different polarization combinations.

- » Different search algorithms, file formats, sampling frequencies, etc.

l35 Of course, this analysis pipeline presents several disadvantage requiring hard work, because of different nature of detectors. On the one hand differences in detectors decrease the possibility of error or bias, on the other hand they make collaborative analysis thecnically challenging

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Multiple-Detector Searches

- GW search codes have a single power threshold which is varied to tune the analyses. I2

- Multi-detector GWB searches are tuning according ***Neyman-Pearson criterion***

Achieve maximum probability of detection while not allowing the probability of false alarm to exceed a certain value I1

$$\max\{P_D\},\quad \textit{so that } P_F \le \alpha$$

Slide 5

l1 The NP cr states that we should construct our decision rule in order to have maximim probability of detection while not allowing the probability of false alarm to exceed a certain value alfa.

The maximization is over all decision rules.

So in our case we should choose the best threshold set, which allow for the best detection probability while keeping FAR below specified threshold

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l2 This threshold is used to perform a selection of trigger list: all triggers with SNR below specified threshold are nelected and remaining triggers are possible candidate GWB. Obviously higher thresholds result in lower FAR in but poorer detection efficiency for low amplitude GW signal, and lower thresholds allow weak signlas to be seen but they increase false event rate.

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I36

Target of the project

- Develop a software tool in Matlab to *find the optimal tuning of analyses in actual network GWB search*

- Such a tool could be also useful
  - » to simulate the behavior of GW detectors in trigger-based searches for GW bursts (GWBs)
  - » for independent validation of the search analysis I4
  - » to estimate sensitivity to populations of signals other than those directly tested in the search I3
  - » to estimate the effect of uncertainties in the properties of the individual detectors (calibration,..)

l3	estimate sensitivity for simulated signals, not yet detected, but that are likely to happen ligo, 9/18/2005
l4	that means for a single detector ligo, 9/18/2005
l36	Goal of my project is developing a software network simulator, which must be a quantitative model for GW detectors network, to find the best tuning of network analyses, which means to find experimantally the best power threshold set to satisfy NP-cr. It can be also useful to simulate..., to quantify the effects of uncertainties or possible errors in the description od detectors, such as uncertainty in calibration,... ligo, 9/21/2005

l37

Procedure

- Single-IFO Event Generation:
  - ETGs: Excess Power, TFClusters, BlockNormal for LIGO, Excess Power for TAMA l7
  - Tune single-IFO power threshold.
- Single-Detector Efficiency:
  - Optimally oriented curve, for chosen threshold
- Network Efficiency:
  - Measure based on known single-detector efficiencies
- Single-Detector False Alarm Rate:
  - Estimate for fixed power threshold
- Network False Alarm Rate:
  - Estimate after
    - Temporal Coincidence test in all IFOs. l8
    - Frequency, amplitude comparisons. l9
- Best power threshold set, satisfying *N-P criterion*:
  - Find among sets generating FAR below desired value and the best network efficiency

Slide 7

l7 Event Trigger generator
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l8 Coincidence tests allow network FAR to reduce
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l9 it's hard to compare frequency and amplitude for detectors with different noise curve sensitivity and not-aligned. For LIGO-TAMA search was choosen as frequency range of analysis the one where all the interferometers have approximately the same efficiency.Otherwise nw efficiency would be affected by the least sensitive detector. Amplitude coparisons are complicated for not-aligned detector, because they are sensitive to different combinations of hte 2 polarization coponents og a GW.So, a simple amplitude comparison can only be applied to aligned detectors.
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l37 The procedure I followed in my project is outlined here.
First step is collecting trigger lists generated from detectors and do SNR threshold test. Next step is estimating detector efficiency for chosen threshold. Su`bsequentially, nw efficiency is computed based on the known single-detector efficiency curves. Afterwards, single-det FAR is estimated and nw FAR, after temporal coincidence test, frequency and ampitude comparison.
After having developed functions to do all of this, a master function was done to find the best threshold set to satisfy NP cr.
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I38

LIGO

Procedure

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- Data used:
  - » S2 LIGO-TAMA analysis for GWBs search
    - – Run 14 (playground data) and 17 (full data set) with simulated GWBs added
    - – Run 15 (playground data) with no injections
- Injected simulated signals: I10
  - » linearly polarized Gaussian-modulated sinusoids

$$h_+(t) \propto h_{rss} \sin\left[2\pi f_0 \left(t - t_0\right)\right] e^{-\frac{(t-t_0)}{\tau^2}}$$

I17

$$h_x(t) = 0$$

  - » milliseconds duration
  - » narrow band
  - » central frequency spanning the frequency range of interest in LIGO-TAMA search analysis (700 – 2000 Hz) I11

Slide 8

l10 This analysis requires to choose a target population, inculding waveform and the distribution of sources over the sky.This family was selected. These signals (tot 16800) were added to the data stream before passing through TFCluster or Excess Power algorithm.

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l11 It's the most suitable range because LIGO and tAMA have approximately the same noise level and comparable sensitivity. It's not the lowest noise level for LIGO but it is for TAMA; in fact LIGO carries out an independent GWB analysis of the S2 ata concentrating on the band 100 - 1100 Hz, which is the best range for LIGO.

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l17 hrss is the root sum square amplitude of the plus polarization and it is found to be a convenient measure of the signal strength

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l38 Data used are those of S2 run...

To estimate sensitivity of each detector and of the nw simulated GWB are added to the data stream from individual detectors before passing thorough ETG algorithm (TFCluster or Exess Power) and new data are re-analyzed in the same manner as is done in the actual GW search.

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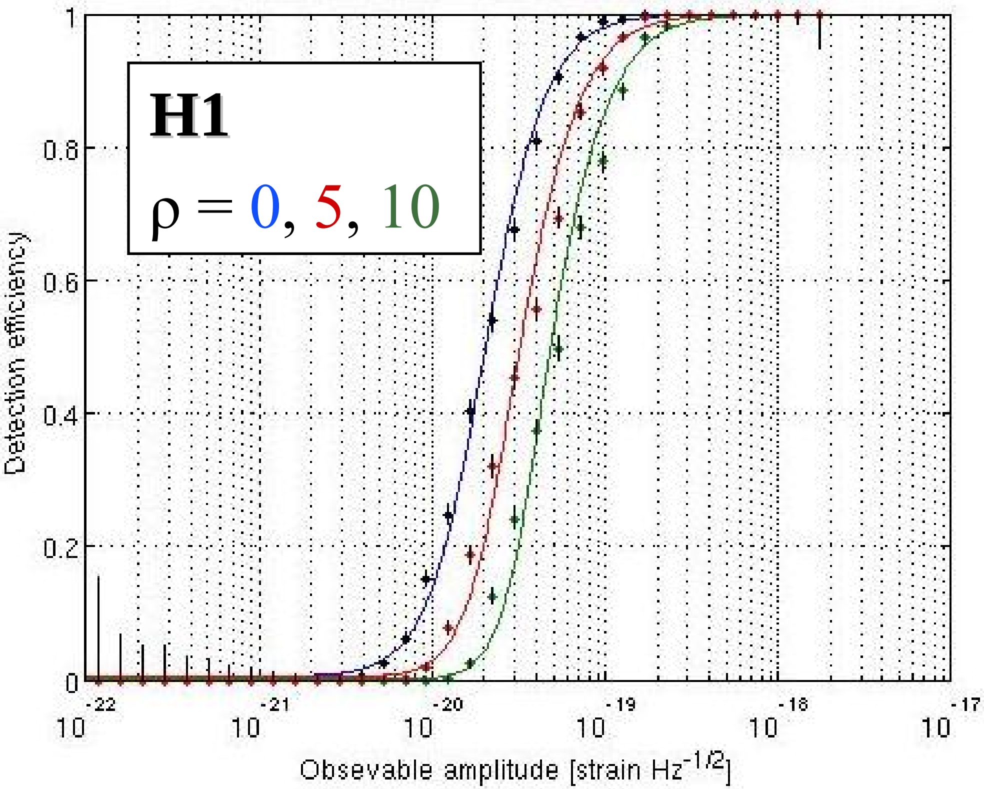
I39

I43

Single-Detector Efficiency

- Consider triggers with SNR > ρ
- Compute time coincidences between triggers and analyzed injections I12
- Tolerance for timing errors (~10 ms) I13
- Use sigmoid fitting function
- Compute ‘*optimally oriented*’ efficiency curve, as function of $h_{obs} = h_{rss}|F+|$



…as expected, efficiency gets worse increasing SNR threshold

l43 SNR is not meant as the usual concept. It's meant here as a general measure each ETG algorithm has its own measure of signal strenght. For TFCluster f.e. SNR^2 = sum_{f bins in-band} |h|^2/S(f).
is the root square of sum over in-band frequencies of the ratio of estimated signal power to the background noise. This approximation sometimes breaks down, especially if the noise is fluctuating. F.e. sometimes miximum likelihood estimator, used in the case of event occurred to estimate how much of the power is due to the signal, gives this power zero and SNR is zero, consequently
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l39 After generation of trigger list from detectors and knowing parameters of injected signals, single-detector efficiency can be estimated at fixed SNR threshold. To do this is required to compute time coincidneces between triggers and analyzed injections, allowing for tolerance for timing errors, of the order of magnitude of 10 ms (10 or 20 ms).
Curves obtained are optimally oriented curves, that are efficiency versus observalble amplitude, that is true injected amplitude times antenna resopnse factor. In this case only F+ factor because injected signals are supposed to have only h+ polarization component.
Detector efficiency for different SNR thresholds is shown and as we expect, increasing the value, efficiency makes worse.
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l13 introduced as an effective duration of injection
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l12 First coincidencs between injections and segments
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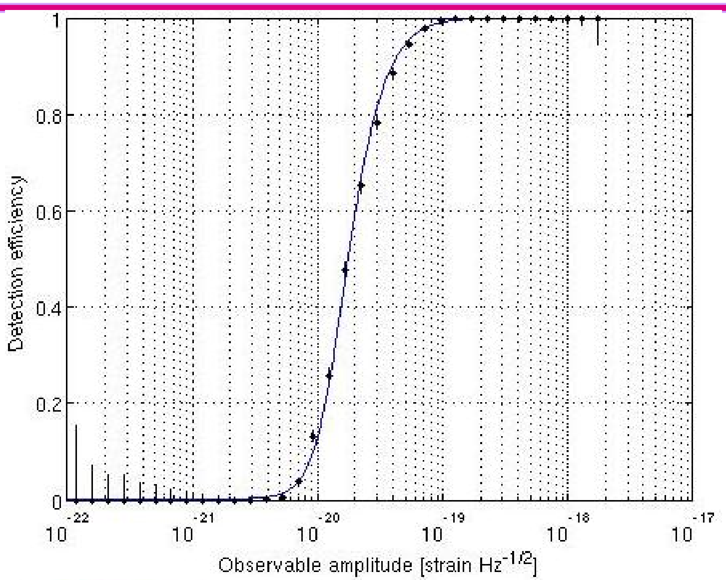
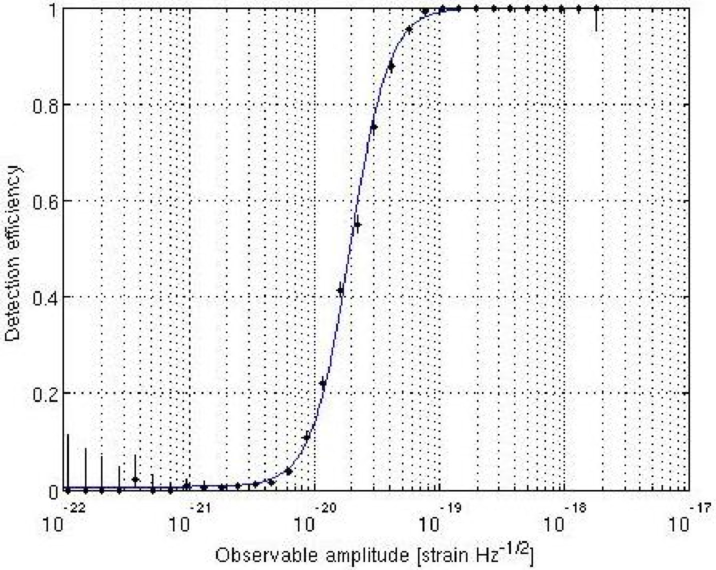
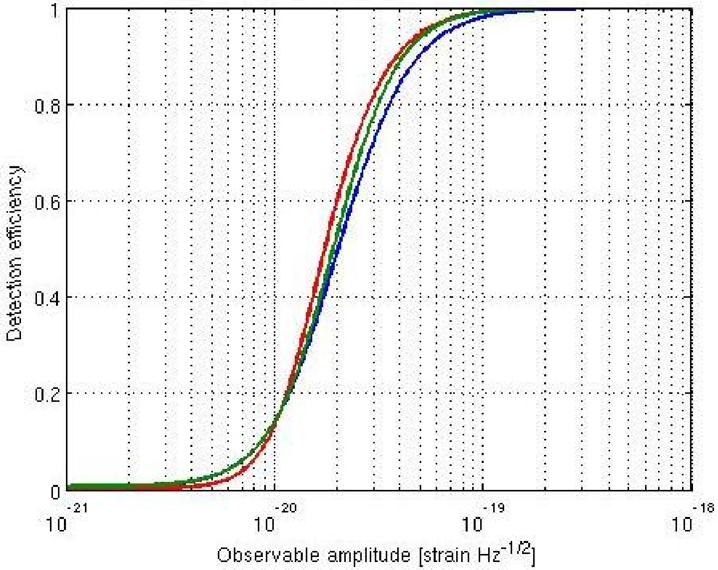
Single-Detector Efficiency

Optimally oriented efficiency curves for H2, L1

(ρ = 0)







Comparing H1, H2 and L1 efficiencies with no SNR threshold

Slide 10

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l40 here some results for H2 and L1 are shown with no SNR threshold

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l41

Network Efficiency

- Through direct integration, i.e. solving numerically

$$E_{nw}(h_{rss}) = \int_0^{\pi} \partial\psi \int_0^{2\pi} \partial\varphi \int_0^{\pi} \partial\theta \sin\theta \prod_{i=1}^{N} E_i(|h_{obs}(\psi,\varphi,\theta)|)\, p(\varphi,\theta)\, p(\psi)$$

$$h_{obs}(\psi,\varphi,\theta) = F_+(\psi,\varphi,\theta) \cdot h_{rss}$$

- ψ-dimension is sampled uniformly
- θ and φ dimensions are sampled uniformly over the sky
- p(φ, θ) is the distribution of sources over the sky
- p(ψ) is the distribution of polarization angle
- Sigmoid fitting function turns out ok also for network efficiency curves

l41 Next step is computing network effeiciency basing on the known single-detector efficiencies. I did this through direct integration, i.e. approxmating shown integral with a discrete sum. it basically averages nw efficiency over source angle and polarization angle.

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I42

I15

Network Efficiency

$N_\theta = 22$

I16

$N_\psi = 20$

$p(\theta,\varphi)$ uniform

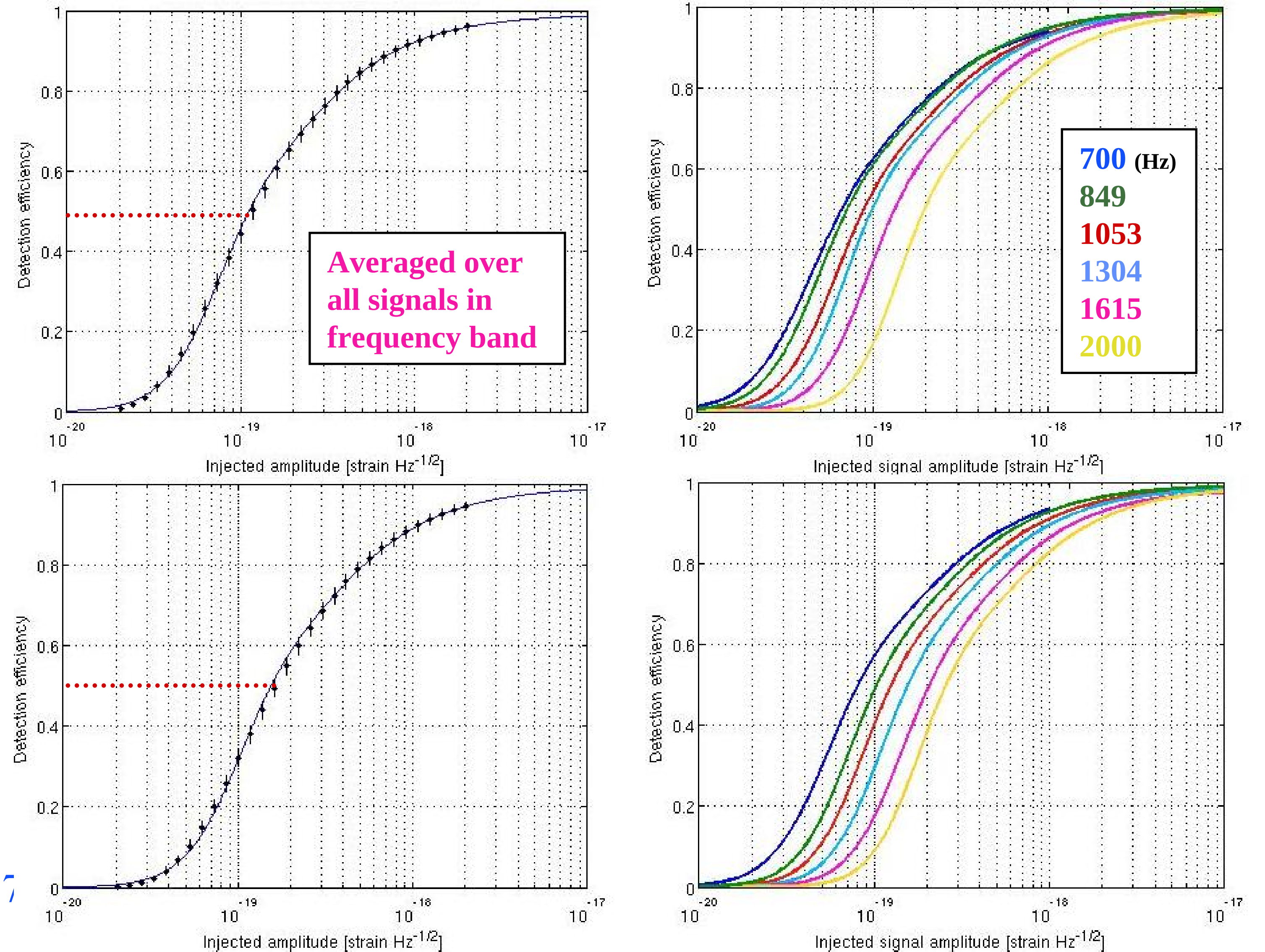
$p(\psi)$ uniform

H1-H2-L1

ρ-set: (0, 0, 0)

H1-H2-L1

ρ-set: (0, 0, 5)



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l15 To see how rho-set effects nw efficiency, let's try to lower L1 threshold. L1 is the less sensitive, it has the highest level noise among LIGO detectors. So you can think to low L1 threshold to reduce its FAR. The result is that the nw sensitivity get worse, and we can see it from the center of sigmoid curve, which increase roughly by 10%

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l16 Ntheta equals to 22 means 617 point over the sphere, which multiplied by Npsi gives total number of integration points.

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l42 For H1-H2-L1 network and for rho-set 000, results are shown, both for all of the injections and for each kind of simulated signals at different frequencies. Efficiencies for each kind of simuated signlas have of course a greater uncertainty because of the smalle r number of simulations and they show network sensitivity as function of the frequency in the considered frequency range: at lower frequencies it's more senistive than higher frequencies

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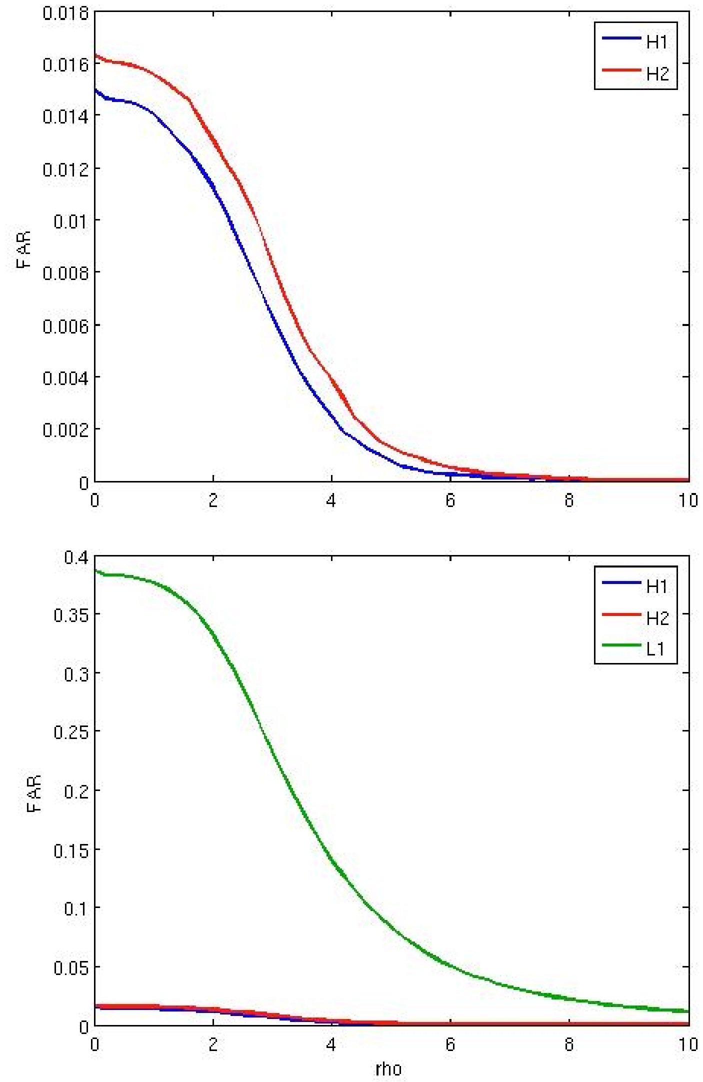
Detector False Alarm Rate

- *Time rate of background noise events occurring with SNR above fixed threshold*
- Estimate single-detector time FAR
  - » Based on trigger list and total observation time
  - » Background noise is modeled as a Poisson process
    - – Best estimator: $R_t = \frac{N_e}{T_{obs}}$

Run 15 ρ = 0	H1	H2	L1
R_t	0.0176	0.0153	0.3699
T_{obs}	$1.0065\ 10^6$	$1.0056\ 10^6$	$1.0008\ 10^6$



l32 Using data with injections is needed to subtract to total observation time (NumInj*dur_inj) to avoid underestimating FAR
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l45 shourov9/21/2005

Avoid confusing "false alarm rate" with "false alarm probability".

false_rate = false_probability * measurement_rate

measurement rate is the number of measurements per second, which is usually difficult to predict. For
TFClusters, it depends on the statistical independence of time-frequency tiles.
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l46 Detector FAR is estimated in order to estimate nw FAR. It is estimated using trigger list and knowing total obser time. Poisson, so the best estiamator for the parameter of the process is the number of false events divided by tot obs tm. In the table are shown estimated FAR for LIGO detectors using run 15 data with no injections and no threshold.
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I44

LIGO

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Network False Alarm Rate

- To minimize the probability of falsely claiming a GW detection, we require any candidate GWB to be observed simultaneously by all detectors I26
- If so, they are required to be in frequency coincidence
- Further they must be coincident in amplitude
  - Such comparison is made difficult by the differences in the alignment of the detectors
  - Simple comparison is possible to apply only for aligned detectors
- Expected network FAR is given by I28

$$R_{nw} = R_{t\_nw} \cdot R_{f\_nw} \cdot R_{h\_nw}$$

NTFAR: Probability for background noise events to occur simultaneously in all detectors

NFFAR: Probability for background noise events to occur in frequency coincidence in all detectors

NAFAR: Probability for background noise events to occur approximately with the same amplitude in all detectors

l26 To reduce nw FAR
ligo, 9/20/2005

l28 Nw FAR is given by the product of these three quantities, to which we refer as nw time FAR, nw frequency FAR and nw amplitude FAR
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l44 Succesive step was estimating NwFAR. To minimize the probability of falsely claiming a GW detection... Triggers passing time coincidence test are required to be in frequency coincidence and further outliving triggers are required to be in amplitude coincidence. Amplitudute comparison is made hard beacuse of differences in the alignements of the detcteors. Not aligned detectors are sensitive to different combinations of the 2 polarization component of the GW signal, so a trivial ampl comparison is possible only for aligned detectors.

NwFAR is then given by the product of these 3 quantities. the first one is... and I will refereto it with NTFAR,..
The product rule is valid in the hypothesis tha time, frequency and amplitude coincidences are indipendent from each other and this is supposed to be true
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I47

Network False Alarm Rate

NTFAR

- *Probability that a background noise event can occur in the all detectors in time coincidence*
- Coincidence test:
  - » Events from 2 detectors are defined to be in coincidence if

$$\left|t_i - t_j\right| < w_t + \frac{1}{2}\left(\Delta t_i - \Delta t_j\right)$$

t - peak time of the event

w_t - coincidence window

Δt - trigger duration

I21

  - » w_t takes into account for the light travel time between the detectors
    - – In practice 10-20 ms longer than the light travel time

I20

  - » Second term can be considered as an allowance for the uncertainty in the determination of the peak time

l20 because it allows for the estimated peak time of coincident triggers to be farther apart if the trigers are long compared to Wt
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l21 Ideally Wt should be as short as possible, to minimize the rate of accidental coincidences, while still being long enough that all simuated signals detected are in coinicdence
ligo, 9/20/2005

l47 estimate now the first quantity in that product. Its values depends on the test performed on triggers.
2 events are defined to be in tm coincidence if this condition is satisfied, where..

4 safety
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Network False Alarm Rate

NTFAR

- A set of event triggers is defined to be in coincidence if each pair is in coincidence
- The expected network background rate for a set of N detectors with rates R_i is

$$R_{t\_nw} = 2 w_t^{N-1} \prod_{i=1}^{N} R_i$$

  - assuming $R_i \, w_t << 1$
  - w_t is supposed to be the same for each pair of detectors
- Using previously computed detector rates
  - H1-H2-L1 network
  - $w_t = 0.02$ s

$$R_{t\_nw} = 1.5905 \; 10^{-7}$$

l25 Always in the hypothehsis that background noise is a Poisson process for each detcteor. This formula is obtaibed assuming Riw<<1 and supposing wt ro be the same for each pair of detectors. One of futre targets is to figure out the formula with different windows for each pair.

Adding to the nw a detctero with rate Ri and window W, it causes a decrease of ne FAR by a factor of (2RiW)

Actually for H1-H2 could be used a smaller window

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l48 10^-7 is 10 raised to the negative seventh power

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Network False Alarm Rate

NFFAR

- Estimate single-detector background noise distribution over central frequency and frequency bandwidth
  - 2-dimensional histogram
- Coincidence test:
  - 2 events are defined to be coincident if

$$\left|f_i - f_j\right| < w_f + a\left(\Delta f_i - \Delta f_j\right)$$

f - central frequency of the event

w_f - coincidence window

Δf - frequency bandwidth

  - Multiple events are defined to be in coincidence if each pair is in coincidence
- Estimate NFFAR through Monte Carlo
  - H1-H2-L1
  - ρ-set: (0, 0, 0)
  - w_f = 0; a = 0.5;
  - 10^6 trials

$$R_{f\_net} = 0.008159$$

Slide 17

l49 Concerning the NFFAR, estimating background noise distribution over cf and bw is required first, in order to estimate NFFAR thorug Monte Cralo.
The frequency coincidence test is quite similar to that performed in time.
the rule to define multiple events to be in coincidence is the same. performing a Monte Carlo integration ovre frquency and bandwidth, the obtained result is..
That is the probability that background noise events occuring in all detectros satisfy coincidence condition.
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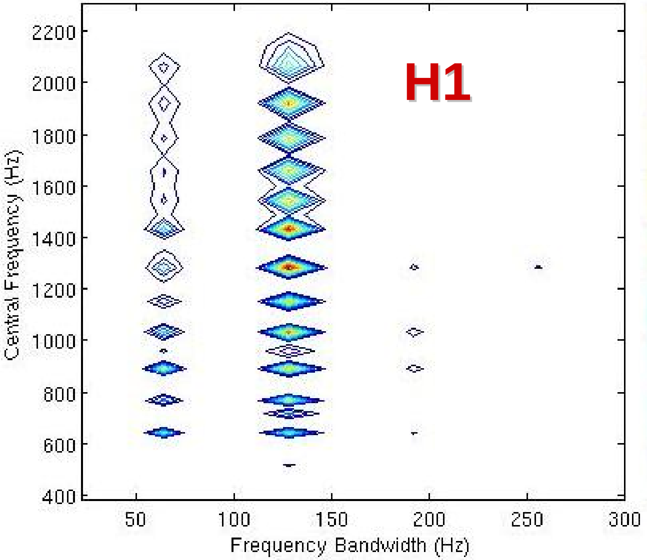
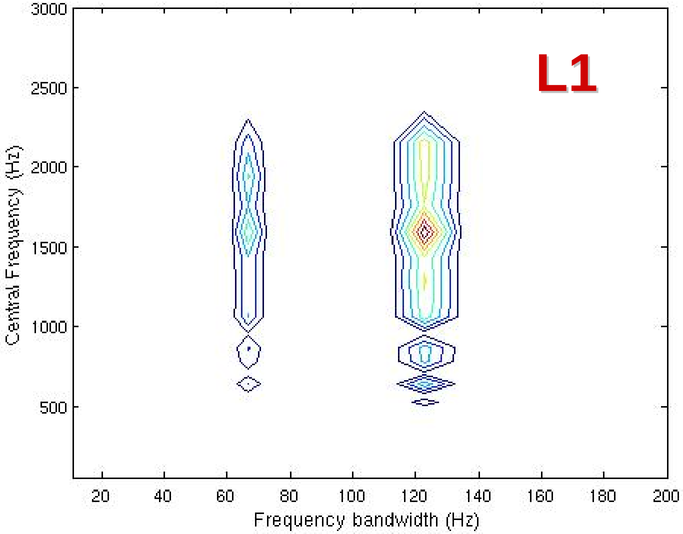
Network False Alarm Rate

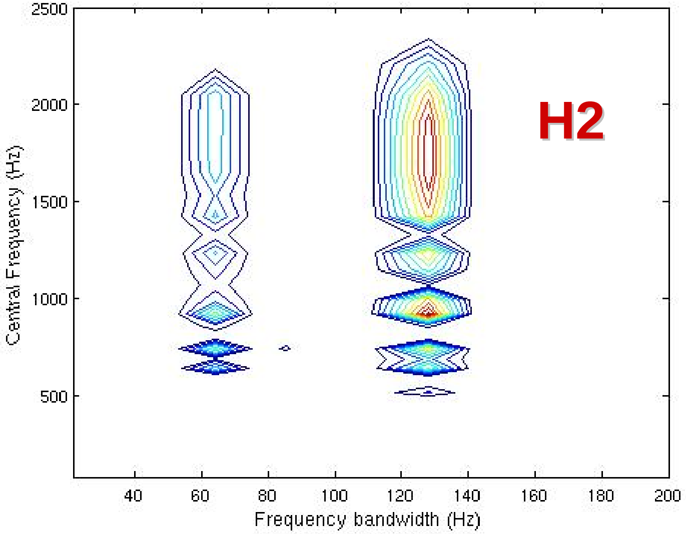
NFFAR

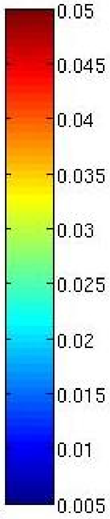
- 15 run
- no ρ threshold





Empirical probability density function of background noise over central frequency and bandwidth





l29

Here are shown epdf of background noise over f and bw
Since discrete nature of TFCluster algorithm we can detect only discrtete value of frquency, because of the TFCluster resolution.
Frequency values are spaced by 64.
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LIGO

Network False Alarm Rate

NAFAR

- Estimate single-detector background noise distribution over amplitude
- Coincidence test:
  - only for aligned detectors
  - 2 events are defined to be coincident if

$$|H_i - H_j| < w_h$$

$H = \log(h)$

$h =$ amplitude of the observed signal

w_h - coincidence window

  - Multiple events are defined to be in coincidence if each pair is in coincidence
- Estimate NAFAR through Monte Carlo
  - H1-H2
  - ρ-set: (0, 0)
  - $w_f = 0.3$
  - 10^6 trials

$$R_{h\_net} = 0.4161$$

l50 In a similar manner to NFFAR etsiamtion we can estimate NAFAR. So estimateing distriution of background noise over amplitude and efining a similar amplitude coincidence test, we can achieve through Monte Carlo an estiamtion of probability that background noise venets occur in amplitude coincidence in the detectros. Test..
only 4 aligned det.
Comparison between amplitude received by not aligned detectors cannot be done is a trivial manner.
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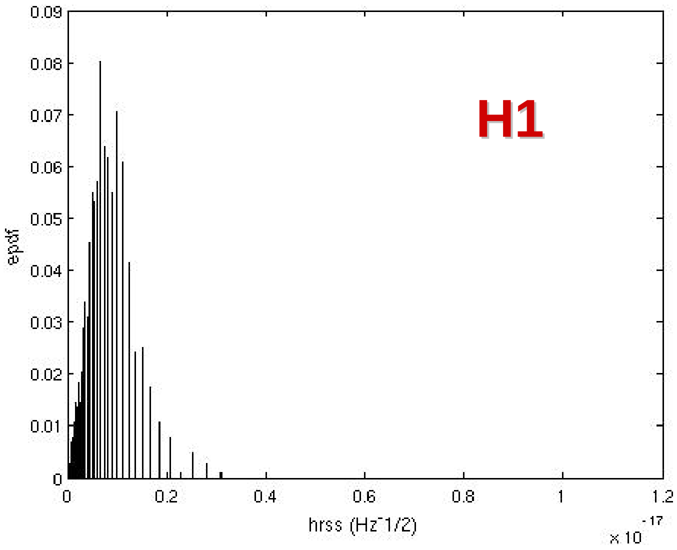
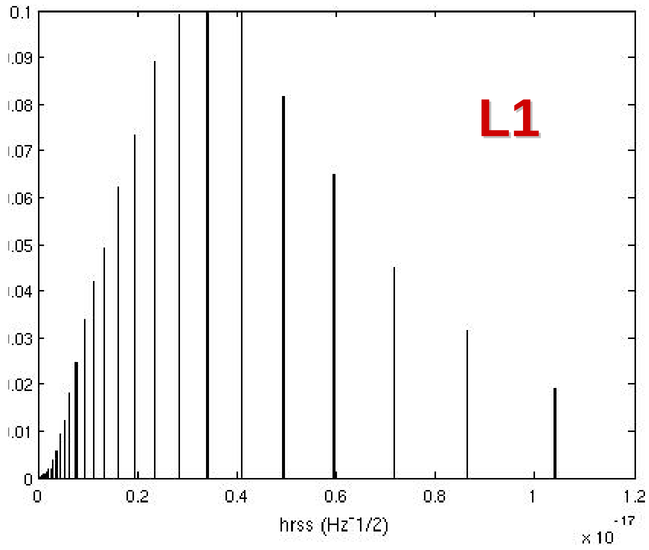
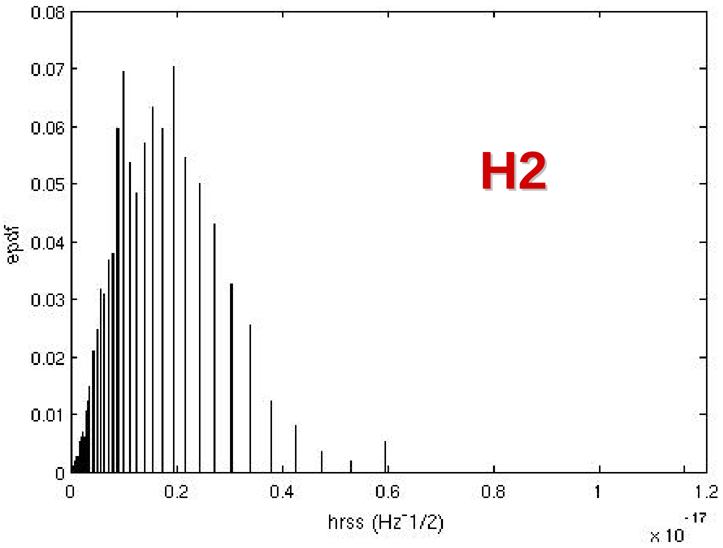
Network False Alarm Rate

NAFAR

- 15 run
- no ρ threshold





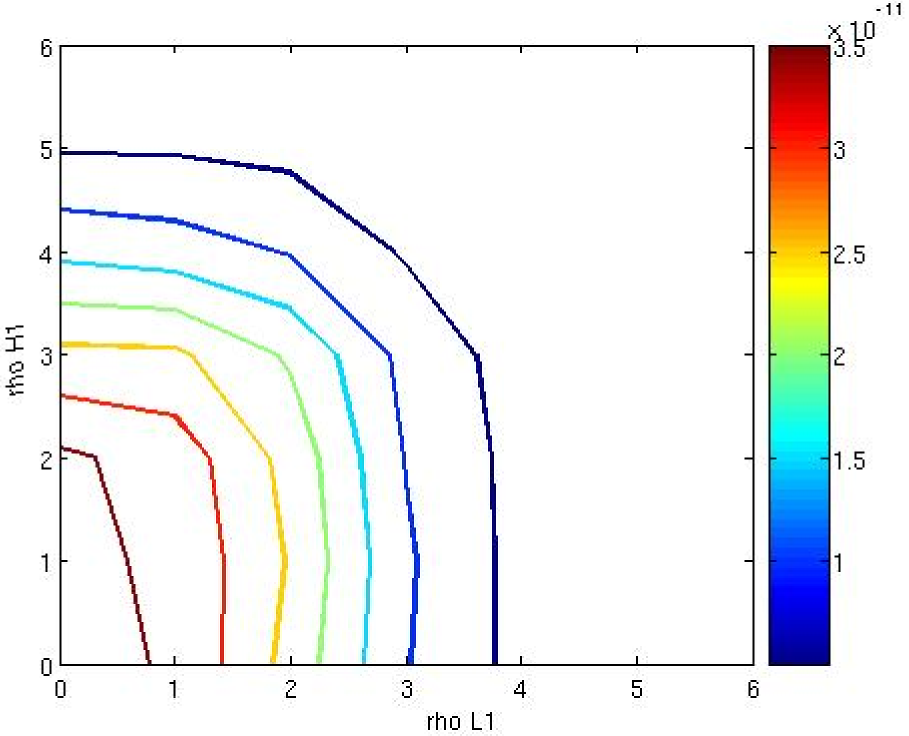


Empirical probability density function of background noise over detected amplitude

Network False Alarm Rate

- Last step is computing network false alarm rate by the product of previously obtained quantities
    - » ρ-set : (0, 0, 0)
    - » H1-H2-L1
    - » w_t = 0.02 s
    - » w_f = 0; a = 0.5
    - » w_h = 0.3

$$R_{net} = 5.4\ 10^{-10}$$



l51 Last step in NFAR estimation is multiply ...

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Find ρ-set satisfying Neymann-Pearson criterion

- Choose a grid of SNR threshold set
- Fix a network FAR threshold
- Check for sets allowing FAR to be below specified threshold
- For each of them compute optimally oriented efficiency curve for the network
- Look for the best efficiency
- The SNR threshold set corresponding to that curve is the wanted set

Slide 22

l52 Once we can compute nw efficiency and NFAR, a master function can be built to find best threshold set to satisfy NP cr. principal steps are

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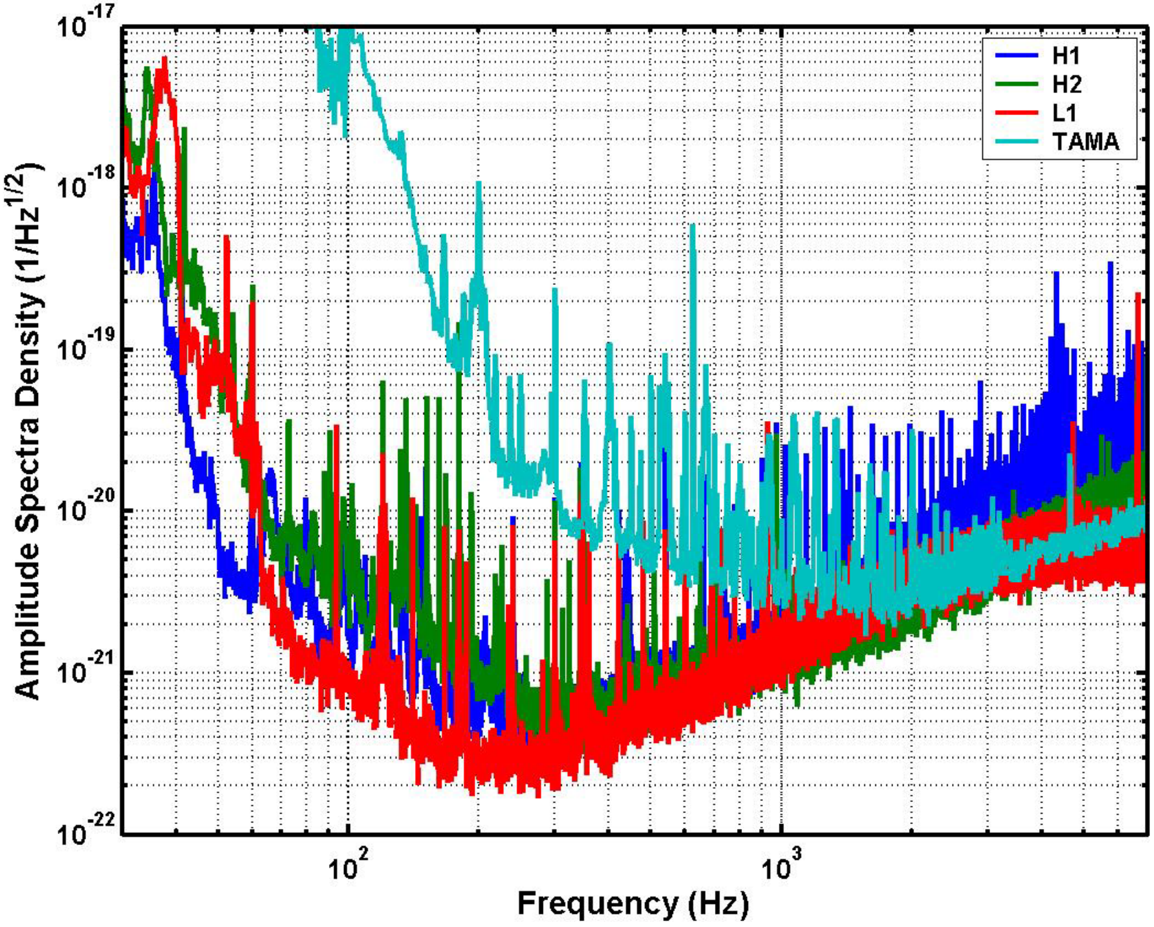
Future plan

- Further testing of the simulation tool
- Including the possibility to choose different coincidence windows for each pair of detectors
- Including the possibility to choose different SNR threshold ranges for different detectors
- Do simulations including TAMA detector
- Validate results

Acknowledgments

- Patrick Sutton
- Shourov Chatterji
- Caltech & SURF program staff
- Innocenzo M. Pinto
- Riccardo De Salvo
- Everything that let me enjoy this summer

LIGO-TAMA sensitivities



LIGO and TAMA look with best sensitivity at different frequencies:

- Tune for signals near minimum of envelope, [700-2000]Hz.
- Frequency, amplitude comparisons difficult.