Population statistics of gravitational-wave pulsars

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GWs from spinning neutron stars

- A spinning neutron star is expected to emit gravitational waves if it is not perfectly symmetric about its rotation axis
- For the purpose of modelling their GW emission, spinning neutron stars are often modelled as triaxially shaped rigid bodies rotating around one of their principal axes
- Measure of deformation: ellipticity $\epsilon = (I_2 - I_1)/I_3$ (I_1, I_2, I_3 denote the principal moments of inertia, I_3 corresponds to the axis of rotation)
- Frequency of emitted waves is twice the frequency of rotation of the neutron star
- GW amplitude is $h \approx \frac{16\pi^2 G}{c^4} \frac{f^2}{r} I_3 \epsilon$

GW detectability of spinning neutron stars

- Currently 120 pulsars known with $f_{\rm rot} > 20{\rm Hz}$ ($f_{\rm GW} > 40{\rm Hz}$)
- Theoretical models predict a maximum sustainable ellipticity of the order of 10^{-7}
- For $f_{\rm rot} = 100{\rm Hz}$, $d = 1{\rm kpc}$, $\epsilon = 10^{-7}$,

$$h \approx 4 \cdot 10^{-26}$$

- Spinning neutron stars emit weak but periodic signals
- Methods have been developed to analyse data from GW detectors for GWs from known pulsars; all-sky searches are harder because of high computational requirements

Ellipticity distribution

- The goal of this project: to develop a method to determine probability distributions of maximum and mean ellipticity of the pulsar population
- Data available: results of searches for individual pulsars, in the form of probability density functions $p(h_{0i}|P_i)$
- By $h \approx \frac{16\pi^2 G}{c^4} \frac{f^2}{r} I_3 \epsilon$, results for individual pulsars can be scaled into $p(Q_i|P_i)$, where $Q_i = I_3 \epsilon$
- Bayes' theorem:

$$p(\text{Hypothesis}|\text{Data}) \propto p(\text{Hypothesis}) p(\text{Data}|\text{Hypothesis})$$

Likelihoods

- Likelihoods $p(\{P_i\}|Q_{\rm max})$ and $p(\{P_i\}|Q_{\rm mean})$ computed as

$$p(\{P_i\}|Q_{\rm max}) = \prod_{i=1}^{N} \int_0^{+\infty} p(P_i|Q_i) p(Q_i|Q_{\rm max}) dQ_i$$

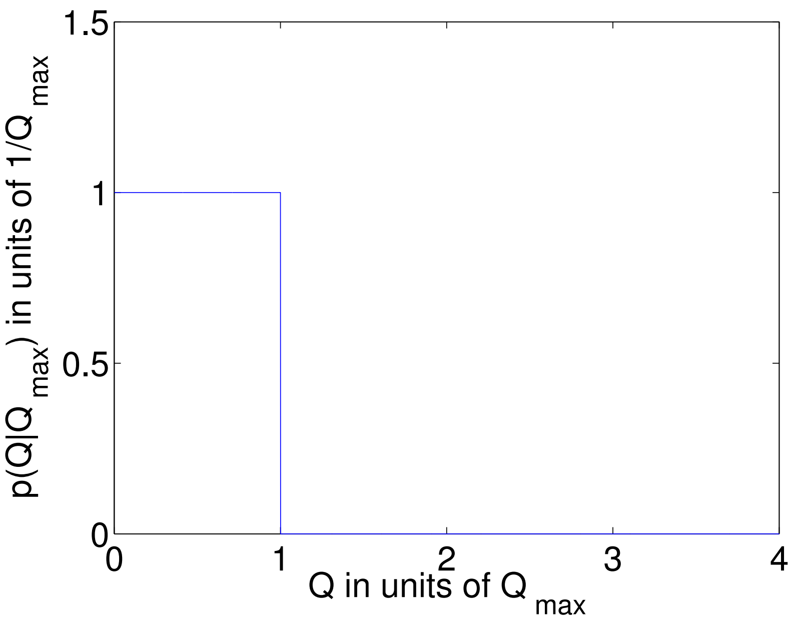
$$p(\{P_i\}|Q_{\rm mean}) = \prod_{i=1}^{N} \int_0^{+\infty} p(P_i|Q_i) p(Q_i|Q_{\rm mean}) dQ_i$$

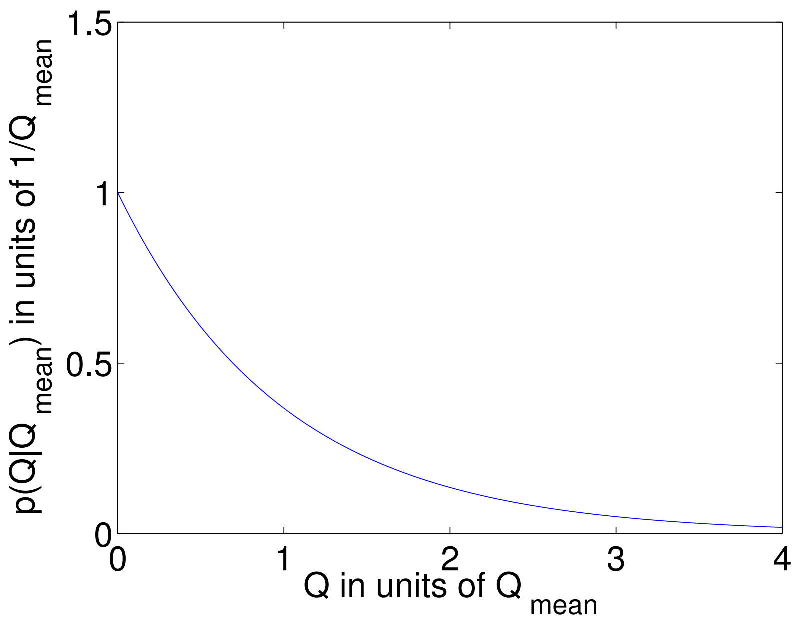
- $p(P_i|Q_i)$ - input data
- $p(Q_i|Q_{\rm max})$ and $p(Q_i|Q_{\rm mean})$ - priors

Priors

General principles of Bayesian analysis give us the forms of the priors

- Flat prior $p(Q|Q_{\rm max})$ on the interval $[0, Q_{\rm max}]$
- Exponential prior $p(Q|Q_{\rm mean}) = \frac{1}{Q_{\rm mean}} {\rm e}^{-Q/Q_{\rm mean}}$





Posteriors

- We assume flat priors $p(Q_{\mathrm{max}})$ and $p(Q_{\mathrm{mean}})$
- The posteriors:

$$p(Q_{\mathrm{max}}|\{P_i\}) \propto \prod_{i=1}^{N} \int_0^{+\infty} p(P_i|Q_i) p(Q_i|Q_{\mathrm{max}}) dQ_i$$

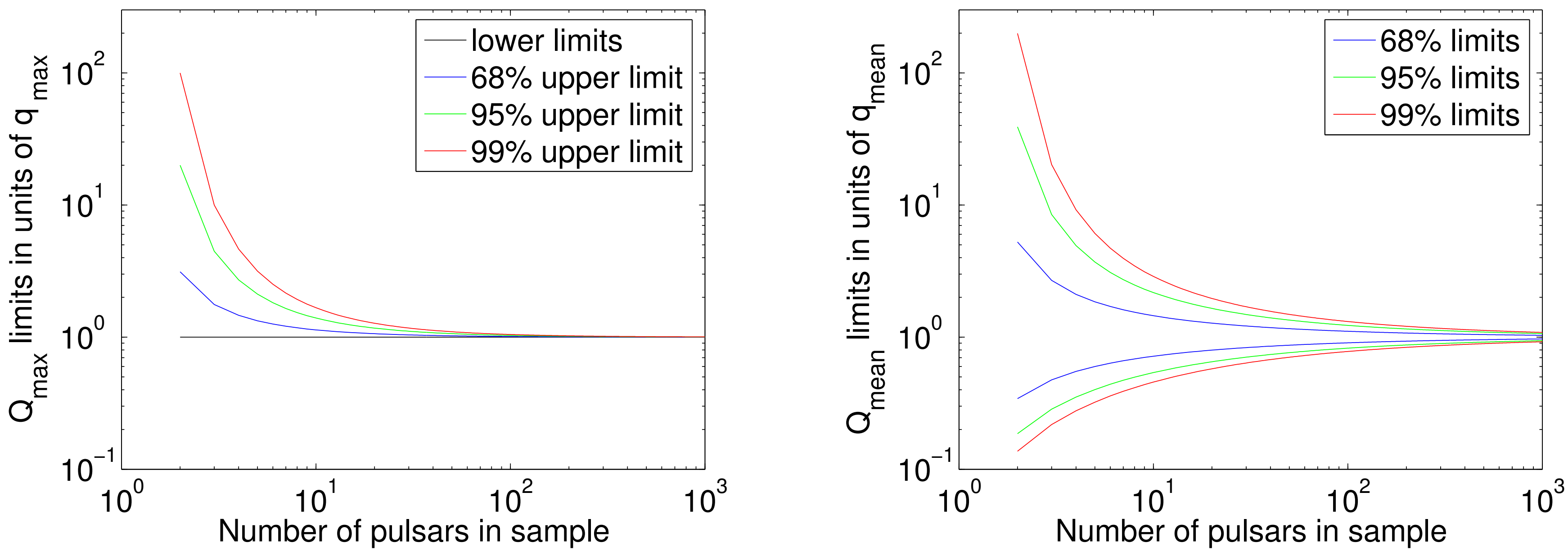
$$p(Q_{\mathrm{mean}}|\{P_i\}) \propto \prod_{i=1}^{N} \int_0^{+\infty} p(P_i|Q_i) p(Q_i|Q_{\mathrm{mean}}) dQ_i$$

Confidence intervals

- A p confidence interval is the shortest interval $[x_{\mathrm{l,p}}, x_{\mathrm{r,p}}]$ that contains the parameter x with probability p
- Confidence intervals can only be extracted from normalizable posteriors
- Posteriors in our method are normalizable for $N > 1$

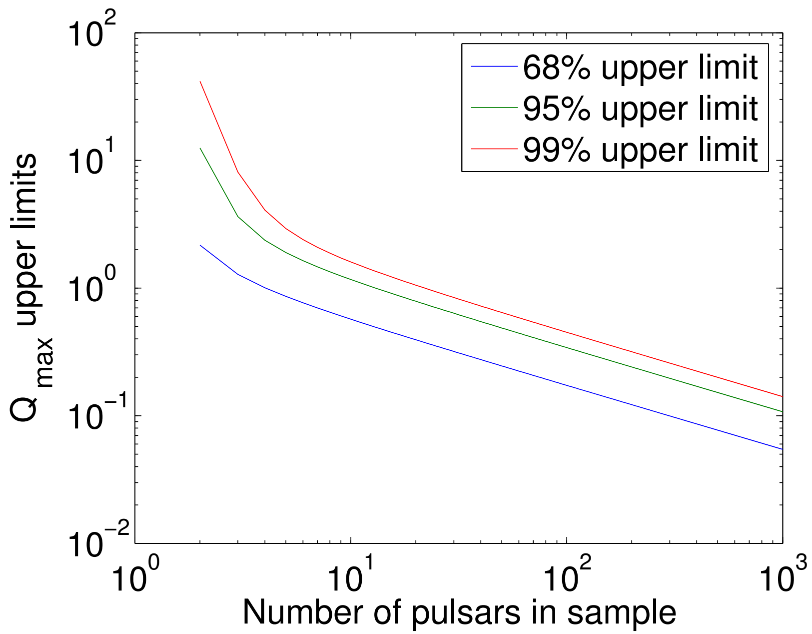
Ideal signals without noise

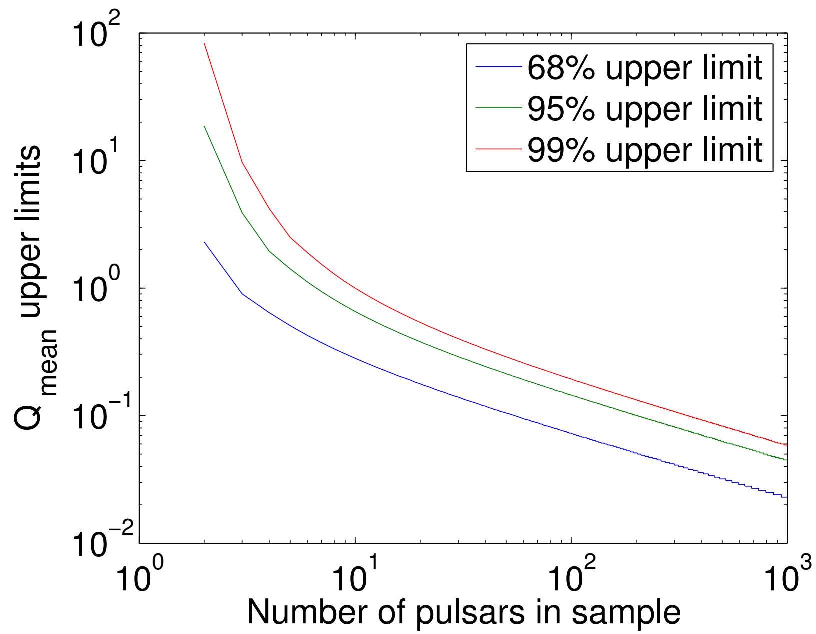
The method is applied to a set of N simulated pulsars with known quadrupole moments q_i



Noise without detectable signals

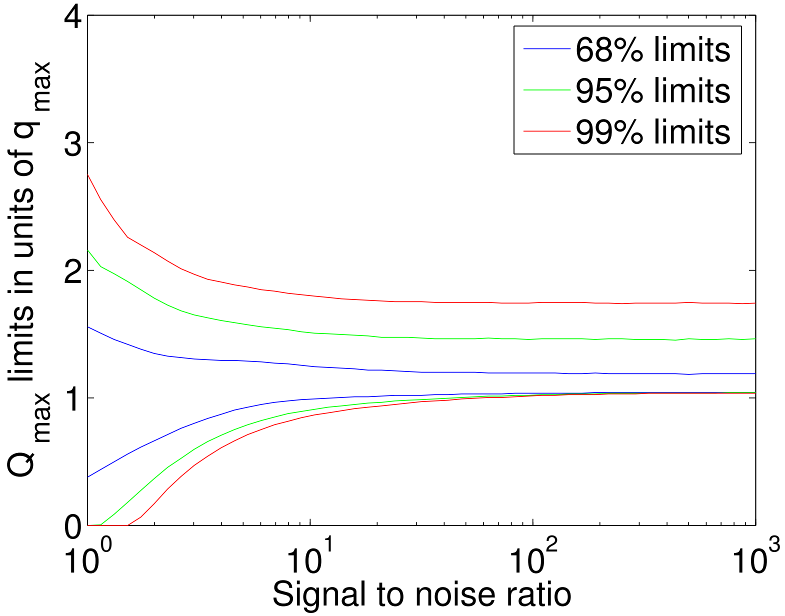
- The method is applied to a set of N pulsars with the same noise levels and no detectable signals
- For large N, the upper limits decrease proportionally to $\frac{1}{\sqrt{N}}$

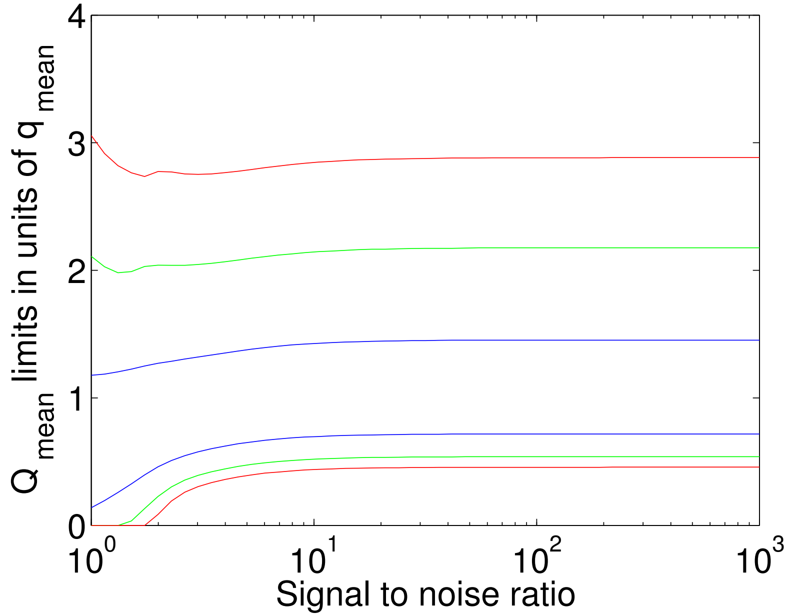




Results for varying SNR

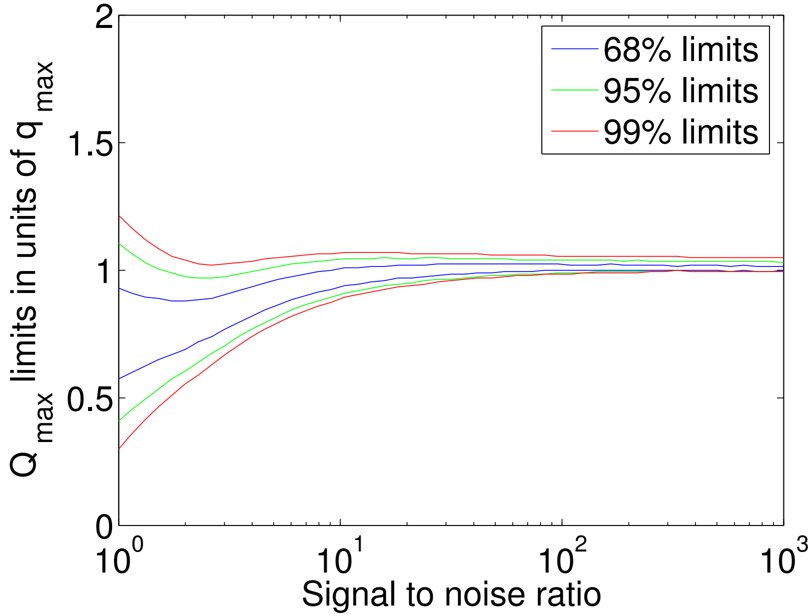
Results for a set of $N = 10$ simulated pulsars for a varying noise level

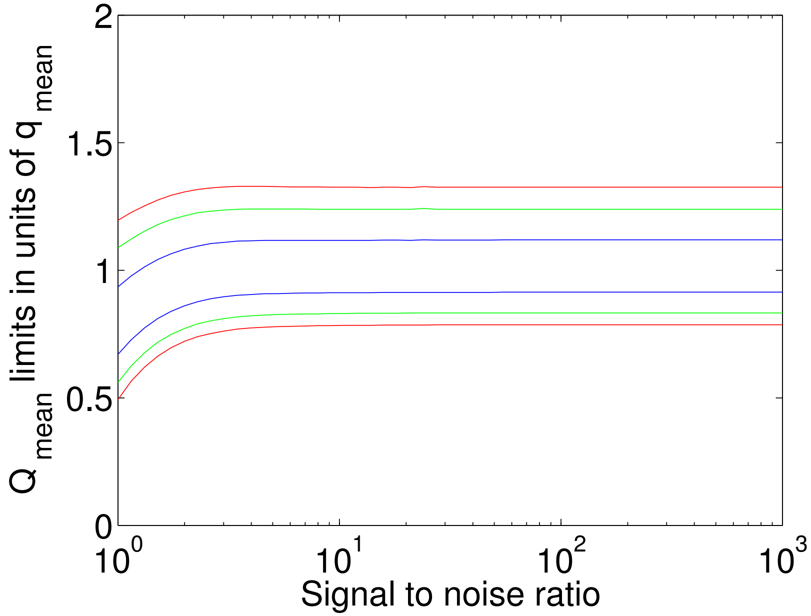




Results for varying SNR

Results for a set of $N = 100$ simulated pulsars for a varying noise level

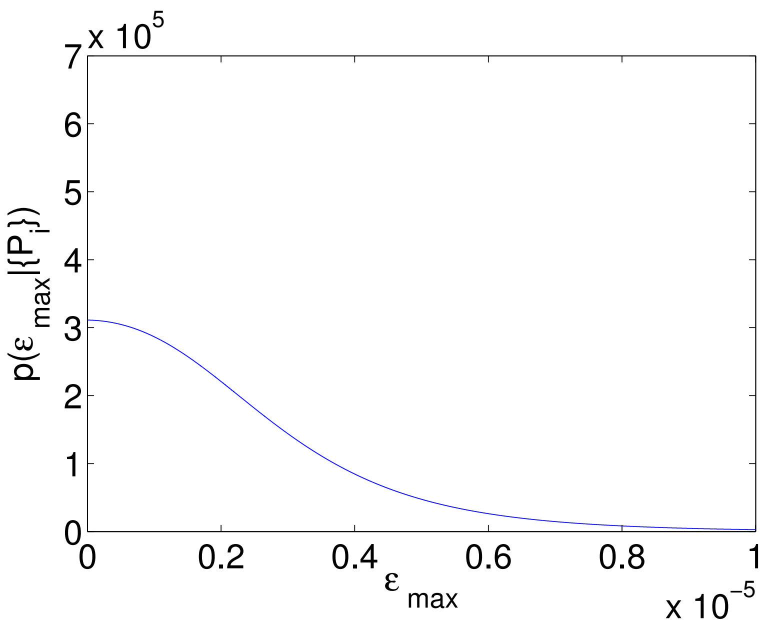


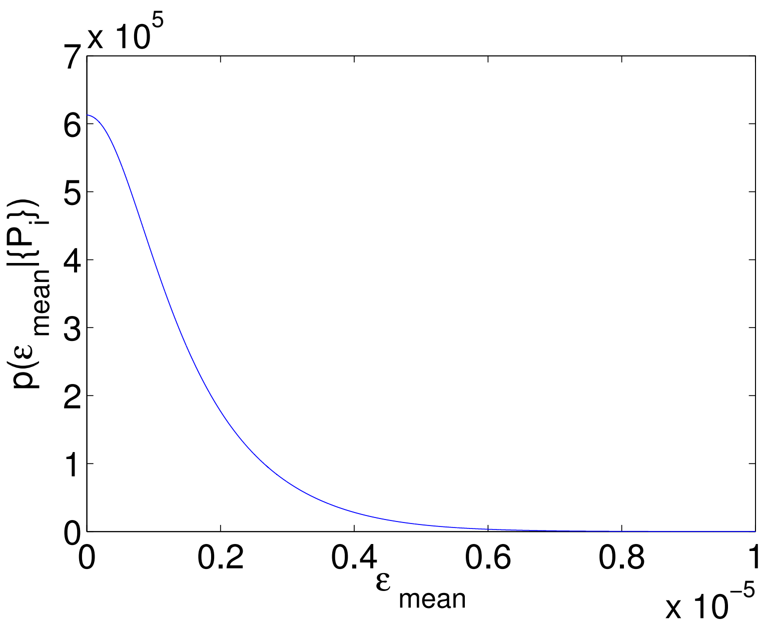


S2 results

S2 data has been searched for signals from 28 pulsars, with the tightest individual 95% upper limit $\epsilon_{95\%} = 4.8 \cdot 10^{-6}$. From the results of those searches we obtain

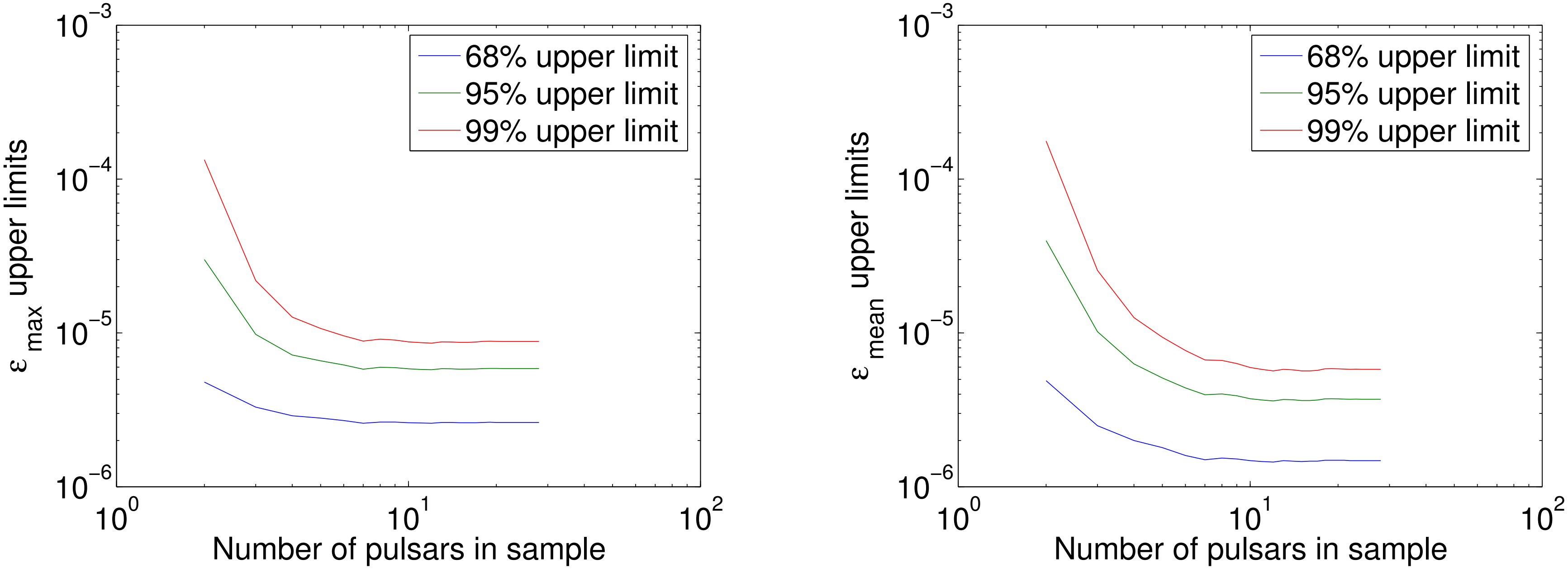
$$\epsilon_{\mathrm{max},95\%} = 5.9 \cdot 10^{-6} \qquad \epsilon_{\mathrm{mean},95\%} = 3.4 \cdot 10^{-6}$$





S2 results

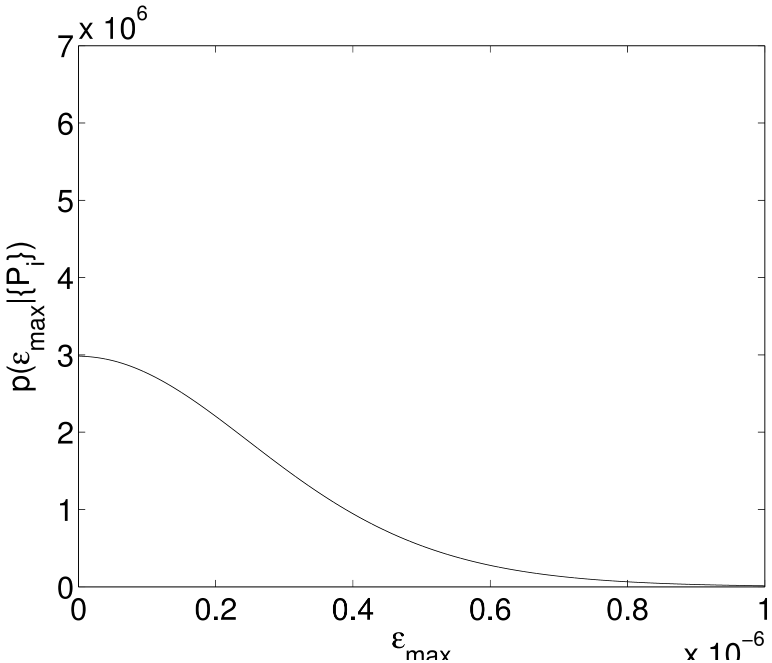
Results obtained by only including in the analysis the N pulsars with the tightest S2 upper limits, as a function of N

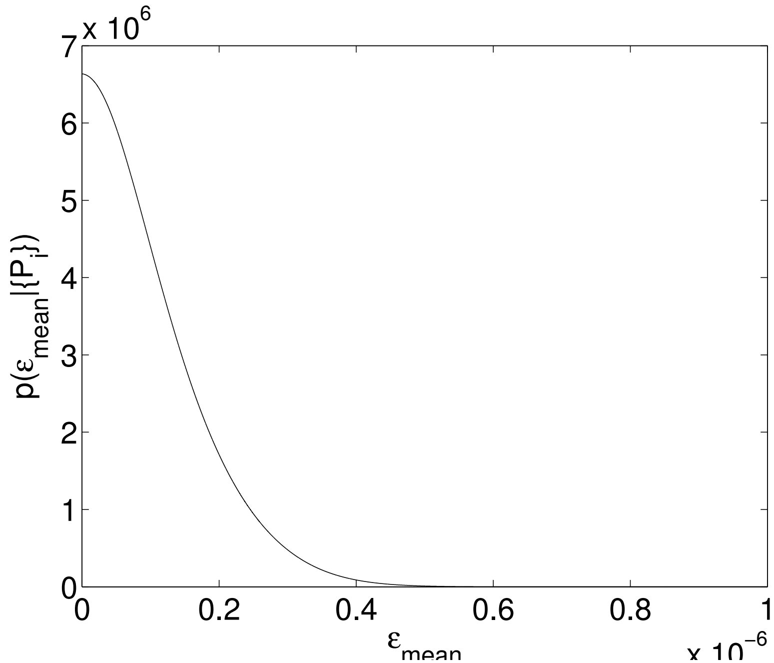


S3/S4 results

S3/S4 data has been searched for signals from 93 pulsars, with the tightest individual 95% upper limit $\epsilon_{95\%} = 8.4 \cdot 10^{-7}$. From the results of those searches we obtain

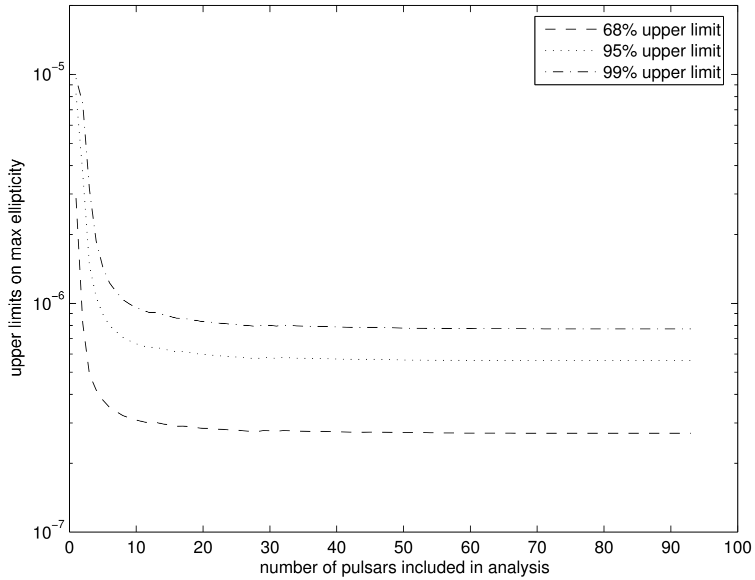
$$\epsilon_{\mathrm{max},95\%} = 5.6 \cdot 10^{-7} \qquad \epsilon_{\mathrm{mean},95\%} = 2.6 \cdot 10^{-7}$$

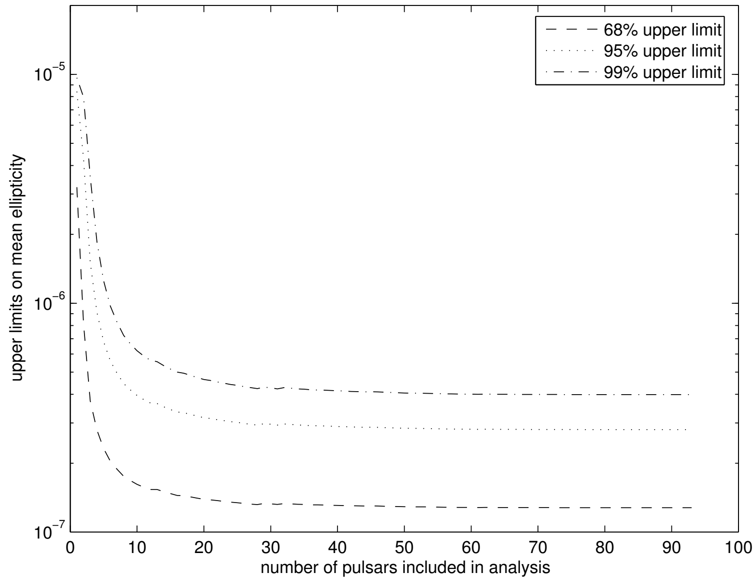




S3/S4 results

Results obtained by only including in the analysis the N pulsars with the tightest S3/S4 upper limits, as a function of N

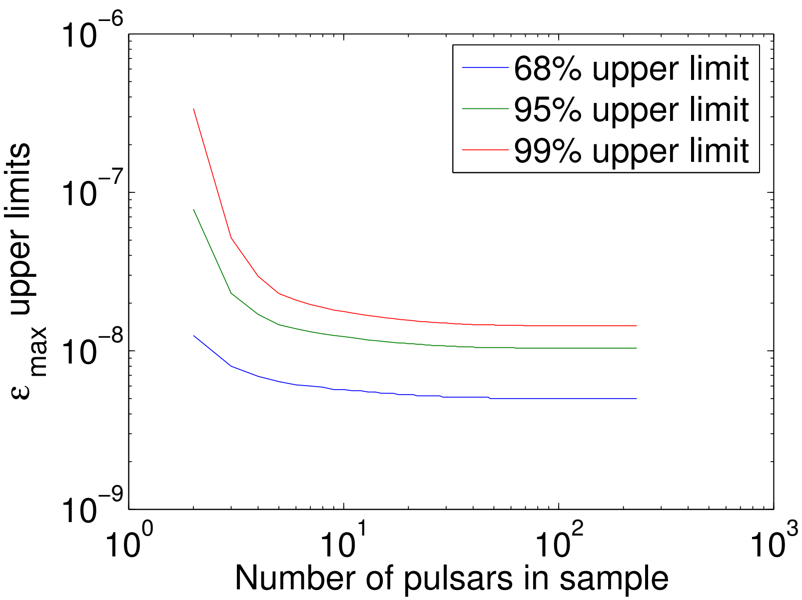


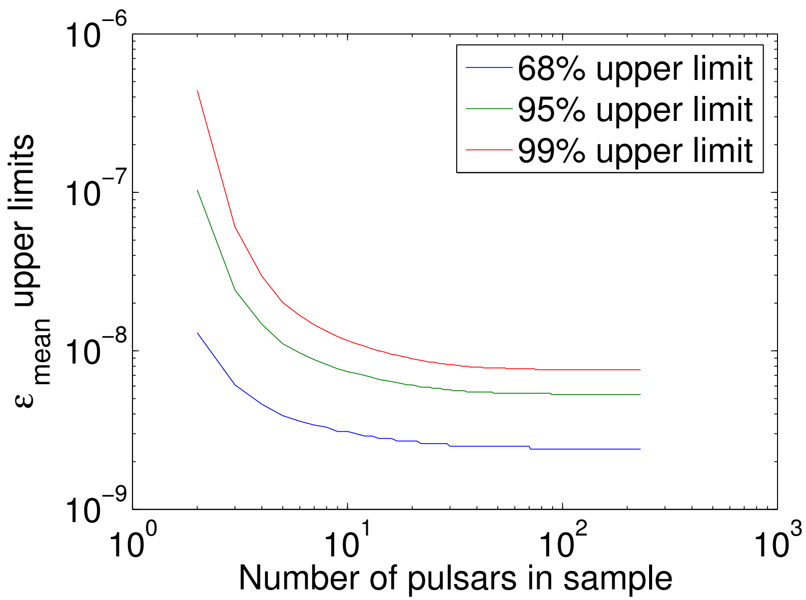


Advanced LIGO sensitivity

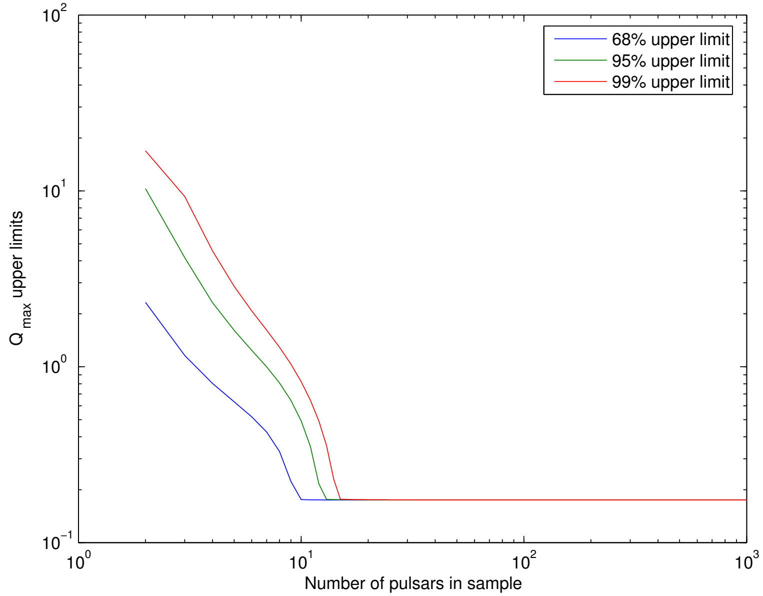
Results for the set of currently observed pulsars, with the design sensitivity of Advanced LIGO at 1 year of observations, under the assumption of no detection made

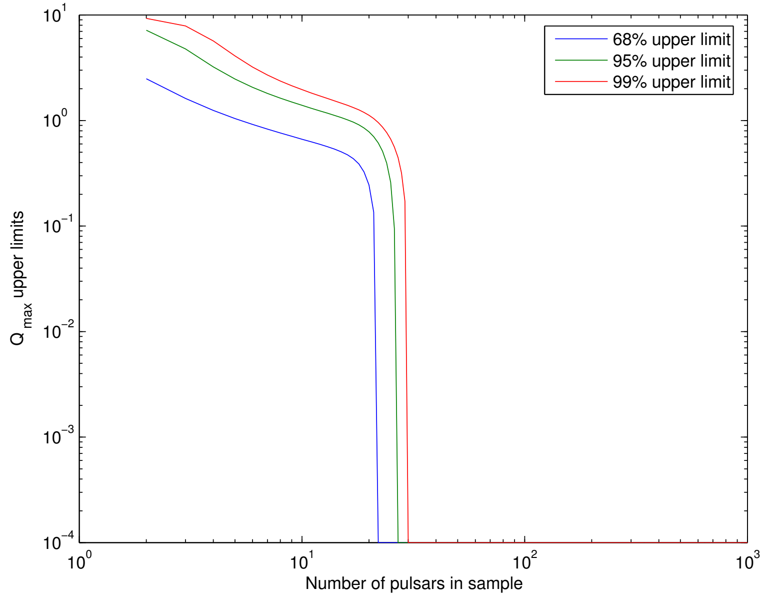
$$\epsilon_{\mathrm{max},95\%} = 1.04 \cdot 10^{-8} \qquad \epsilon_{\mathrm{mean},95\%} = 5.3 \cdot 10^{-9}$$





Beware of numerical errors





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