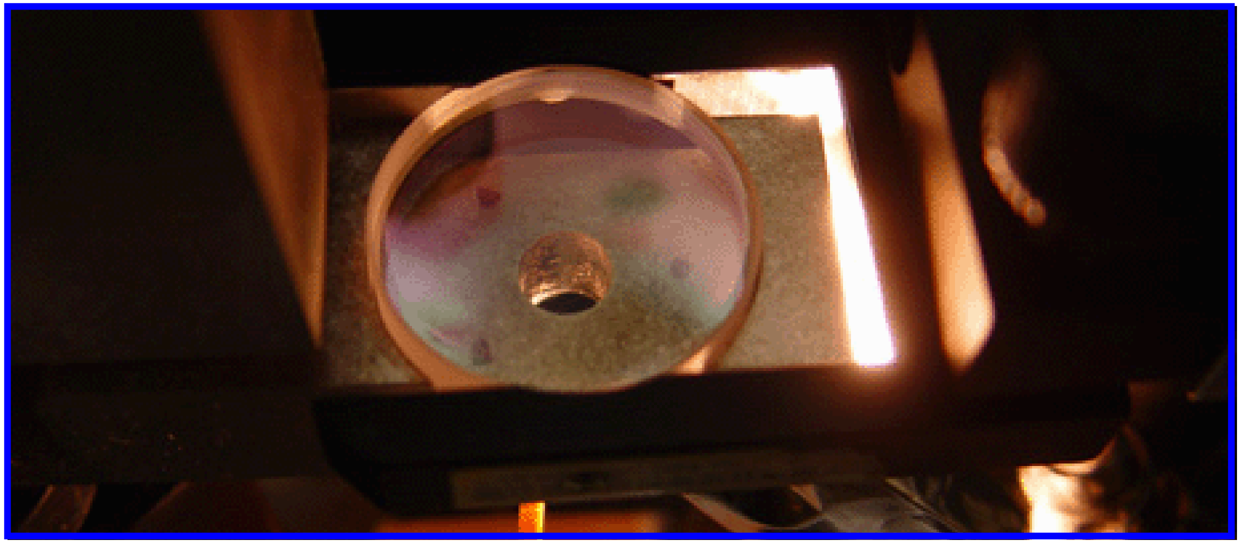
Parametric Effects in a Macroscopic Optical Cavity

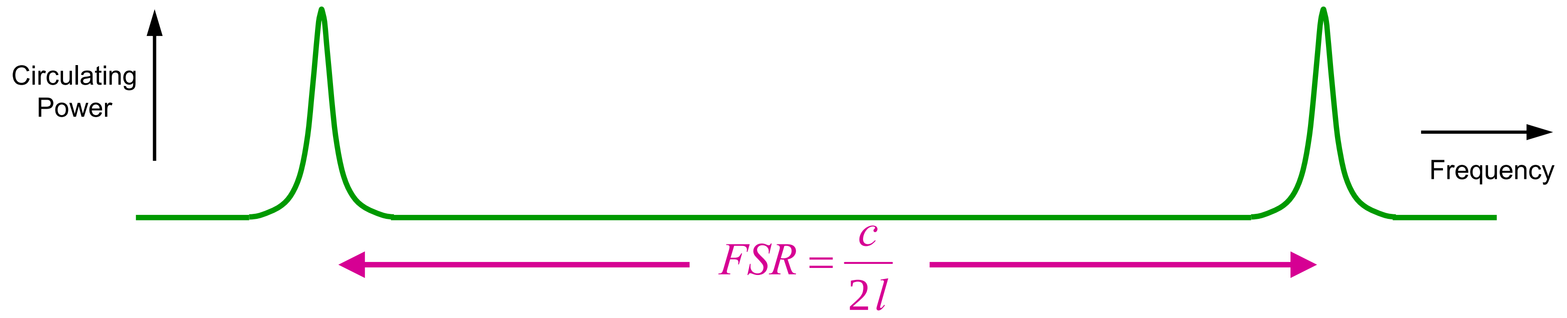
Sascha Schediwy
schediwy@physics.uwa.edu.au

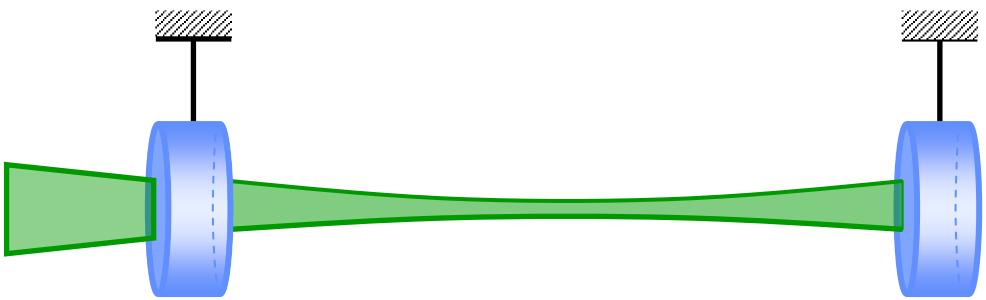


AIGO Workshop
Thursday 6th October

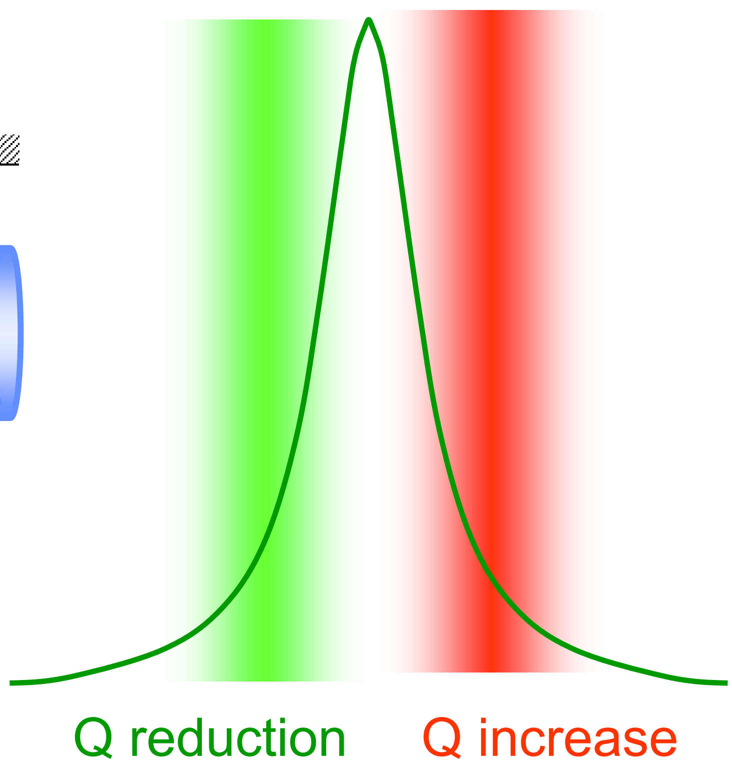
LIGO-G050532-00-Z

Optical Spring

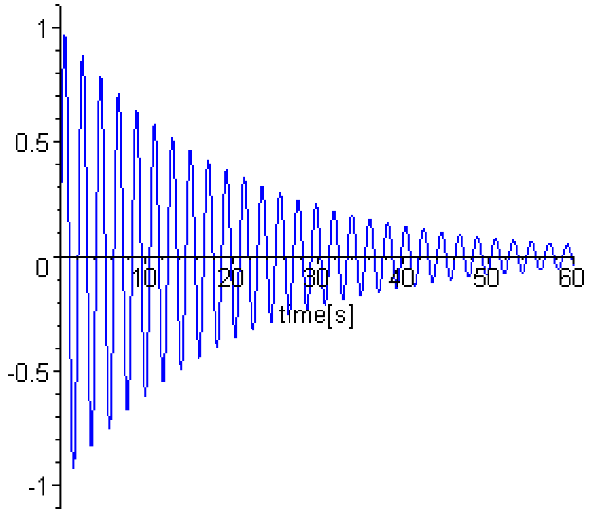


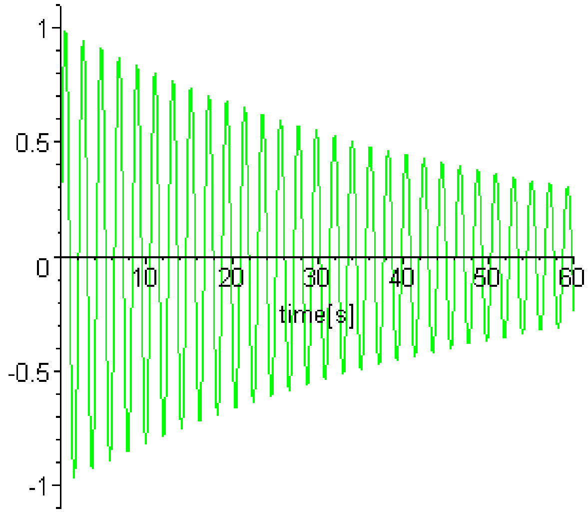


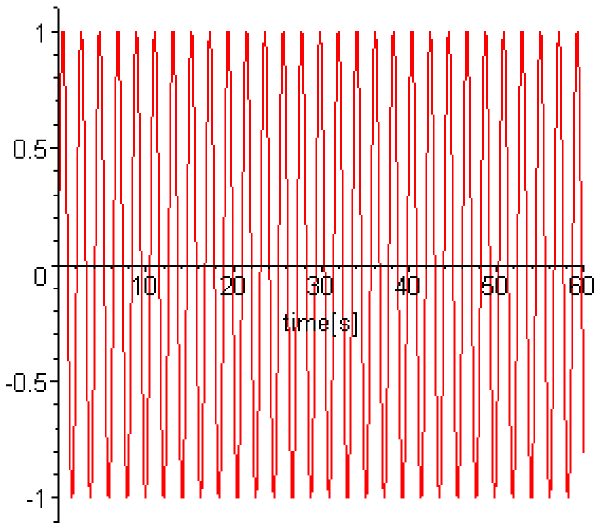
$$F_{rad} = \frac{P_{circ}}{c}(2R_2 + L_2)$$

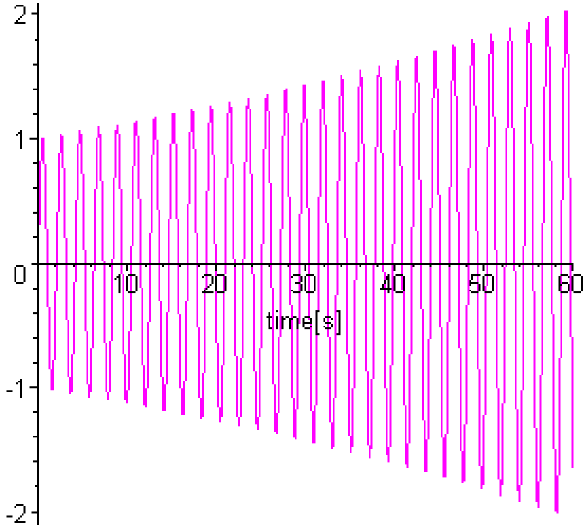


Q Increase - Parametric Gain

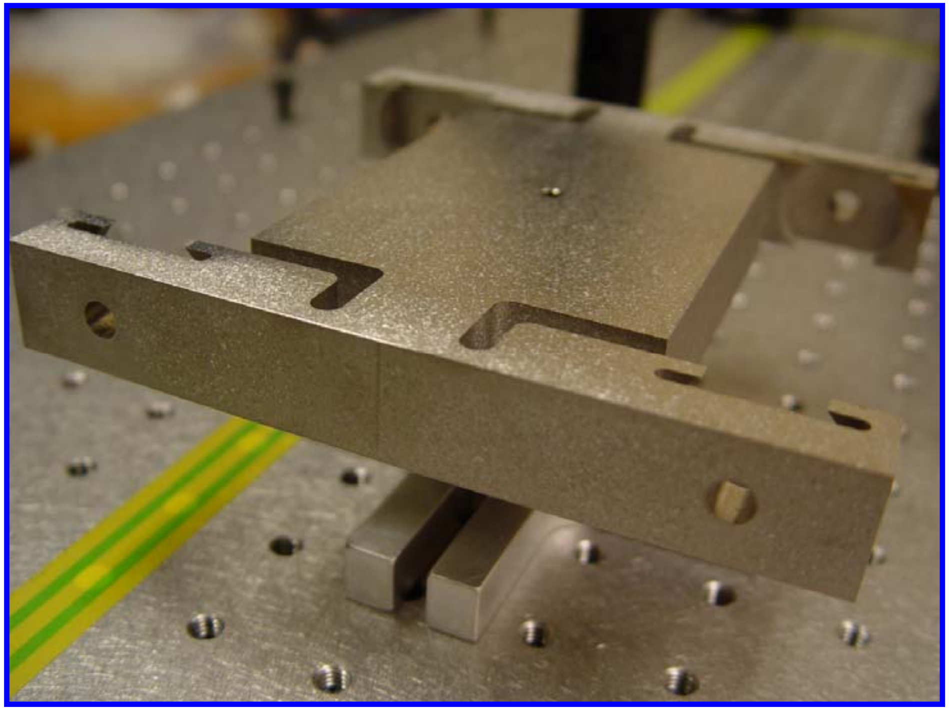




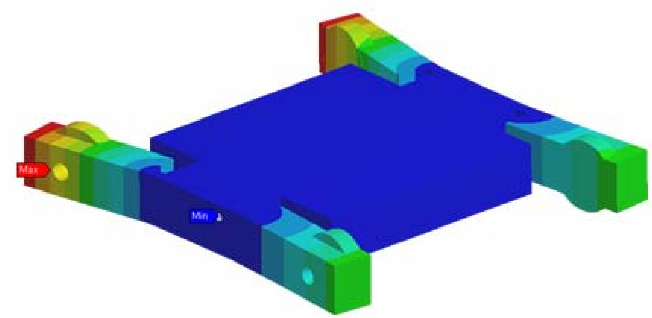




Niobium Resonator



- Cavity Properties
  - *Niobium*
  - $Q_m = 1.562\ (8) * 10^5$
  - $f_{mech} = 780$ Hz
  - $m_{eff} = 32.3$ g
  - $l = 0.10$ m



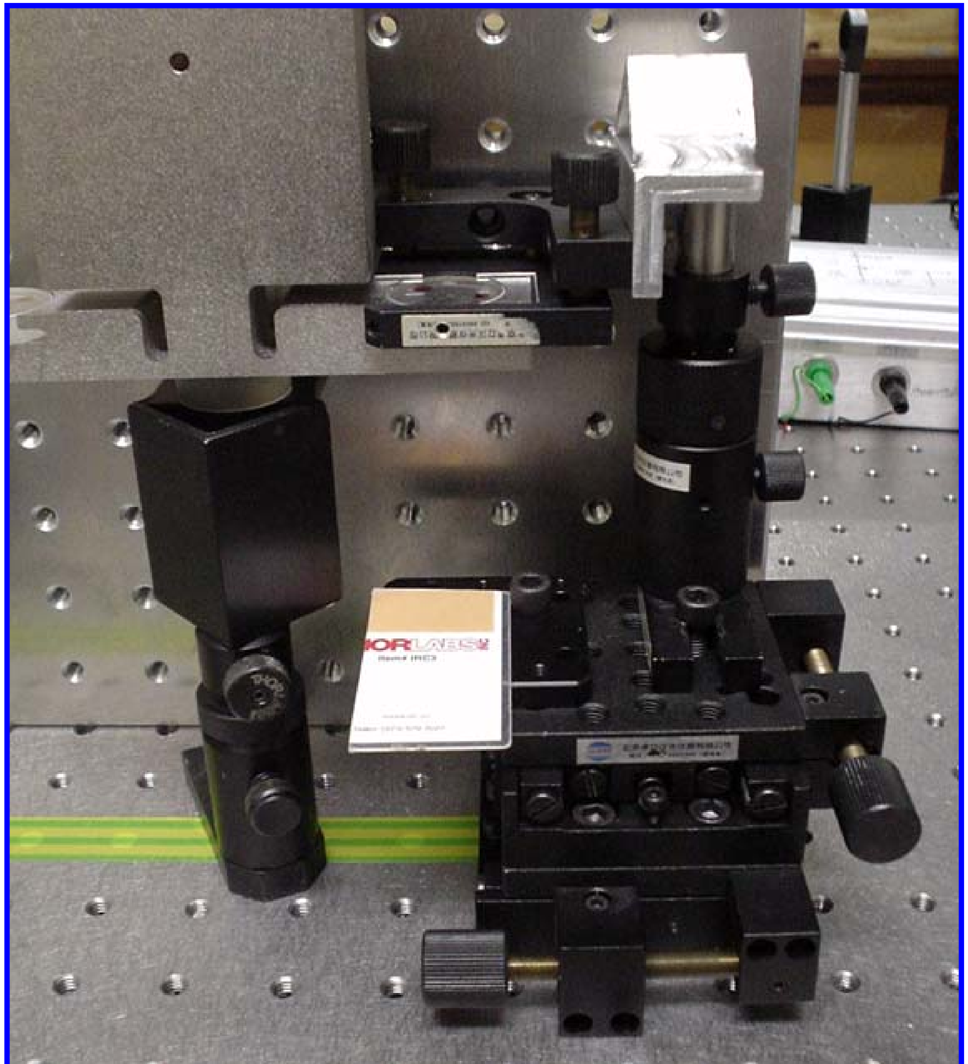
- Proxy Mirrors
  - $R = 0.98$
  - $\mathcal{F} = 155$
  - $roc = 10.0$ m
- Super Mirrors
  - $R \sim 0.99968$ (rated: 0.9994+)
  - $\mathcal{F} \sim 9800$ (rated: 5200+)
  - $roc = 1.0$ m

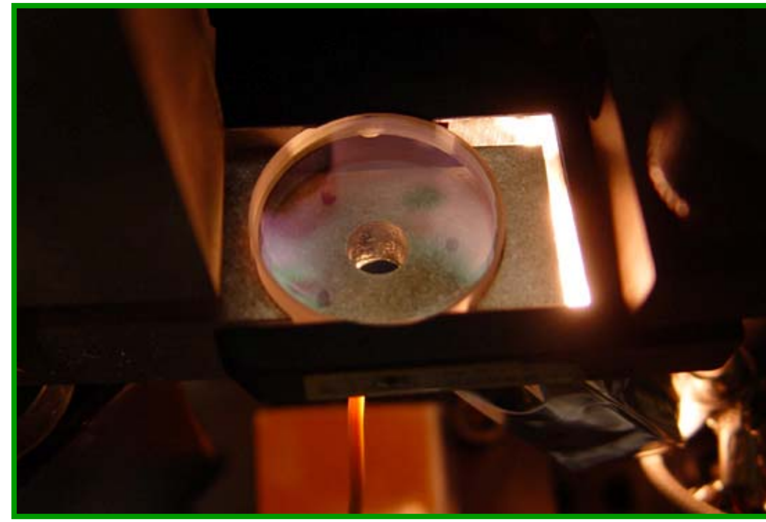
Yacca Gum Properties

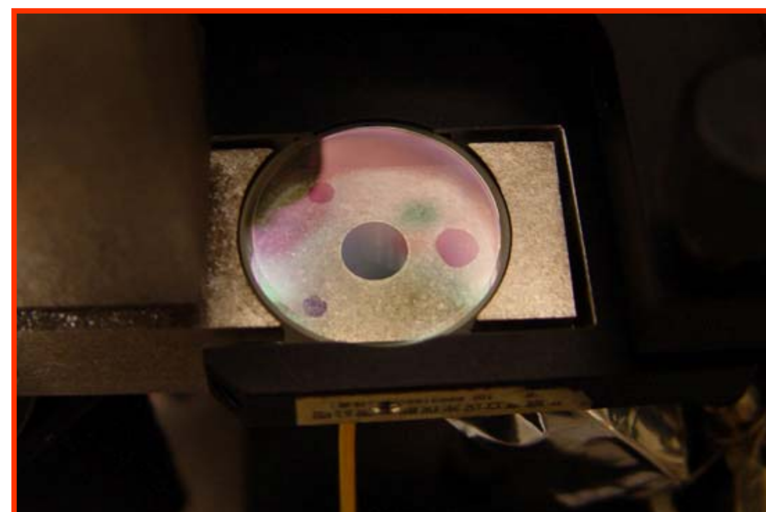
- viscous at 80°C+
- reversible bonds
- dissolves in alcohol
- relatively low loss
- Q ~ 100

80°C

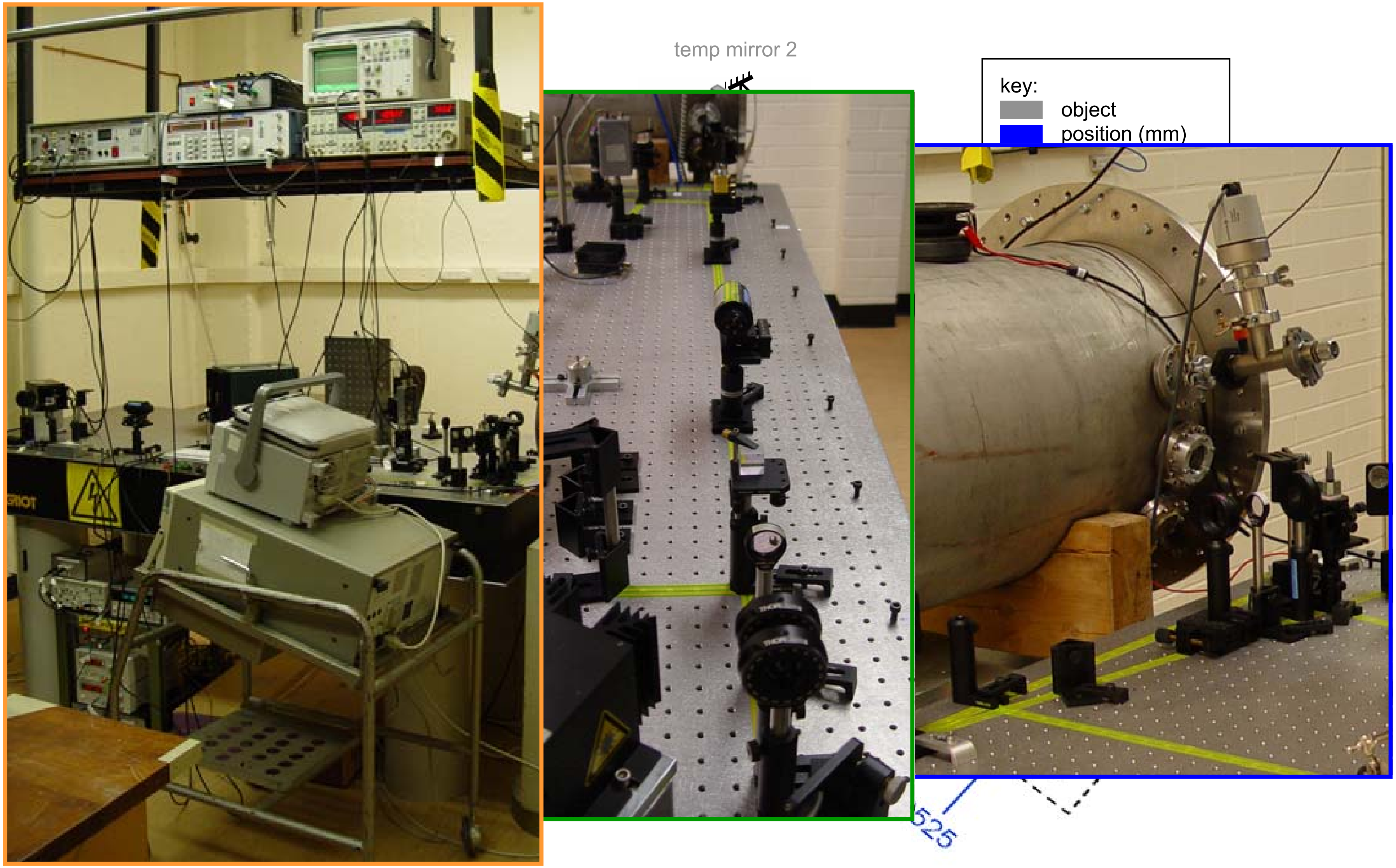
Yacca Gum Bonding





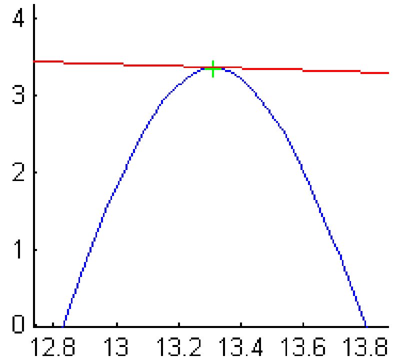


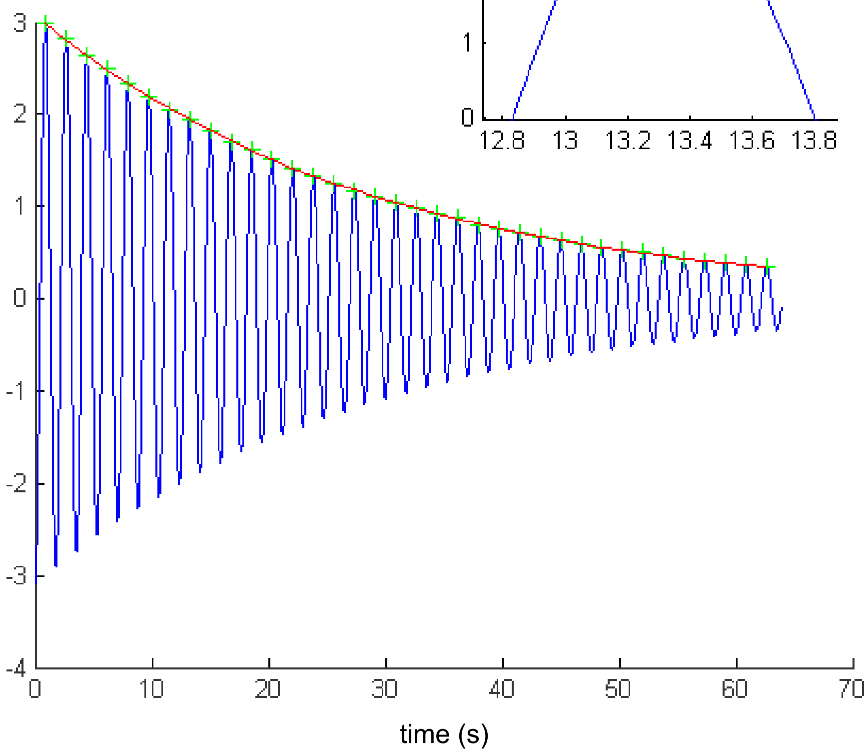
Experimental Design



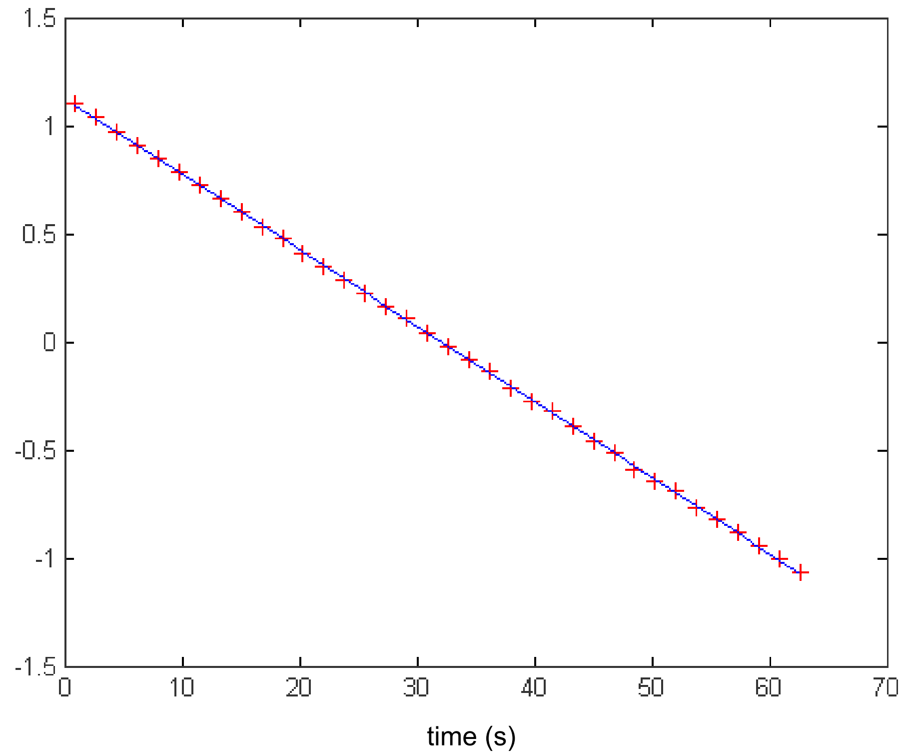
Optical Error Signal Ringdown

Ringdown



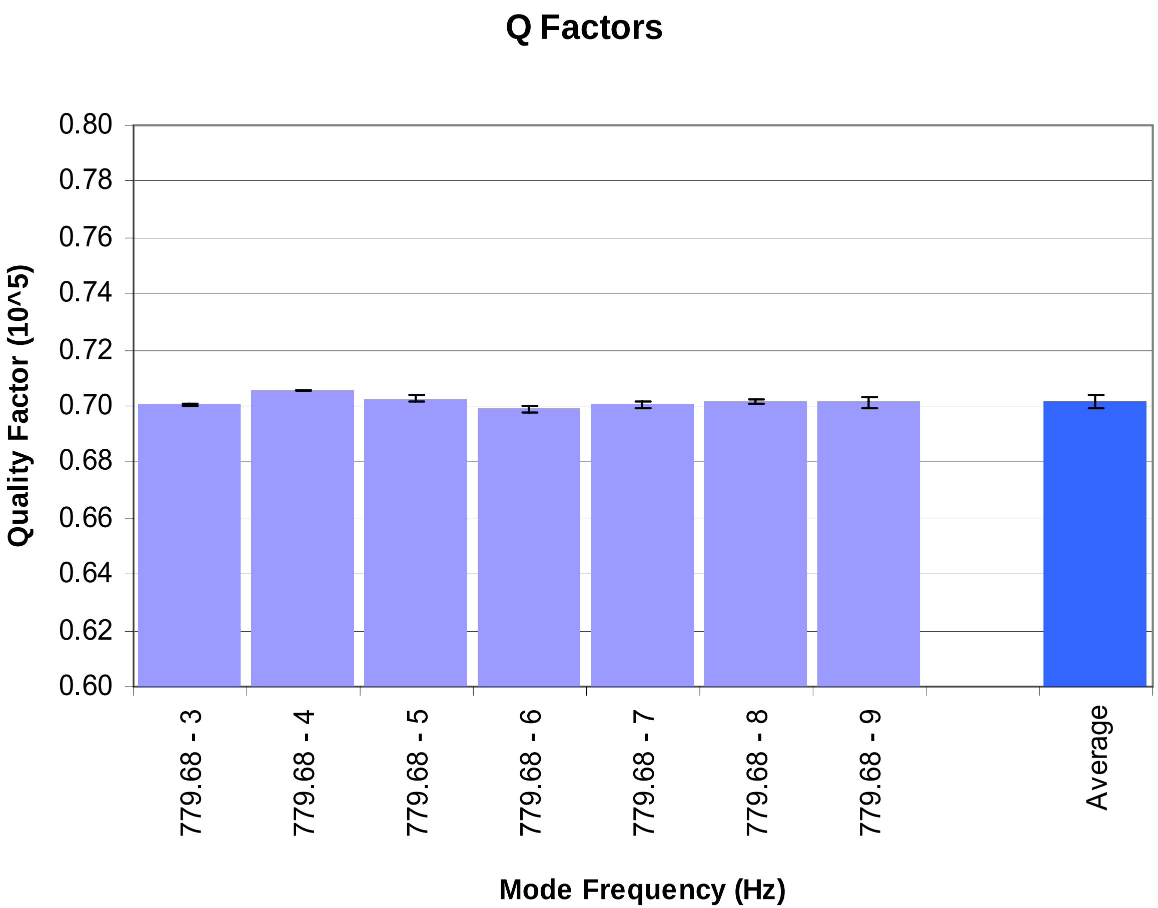


Linear Fit



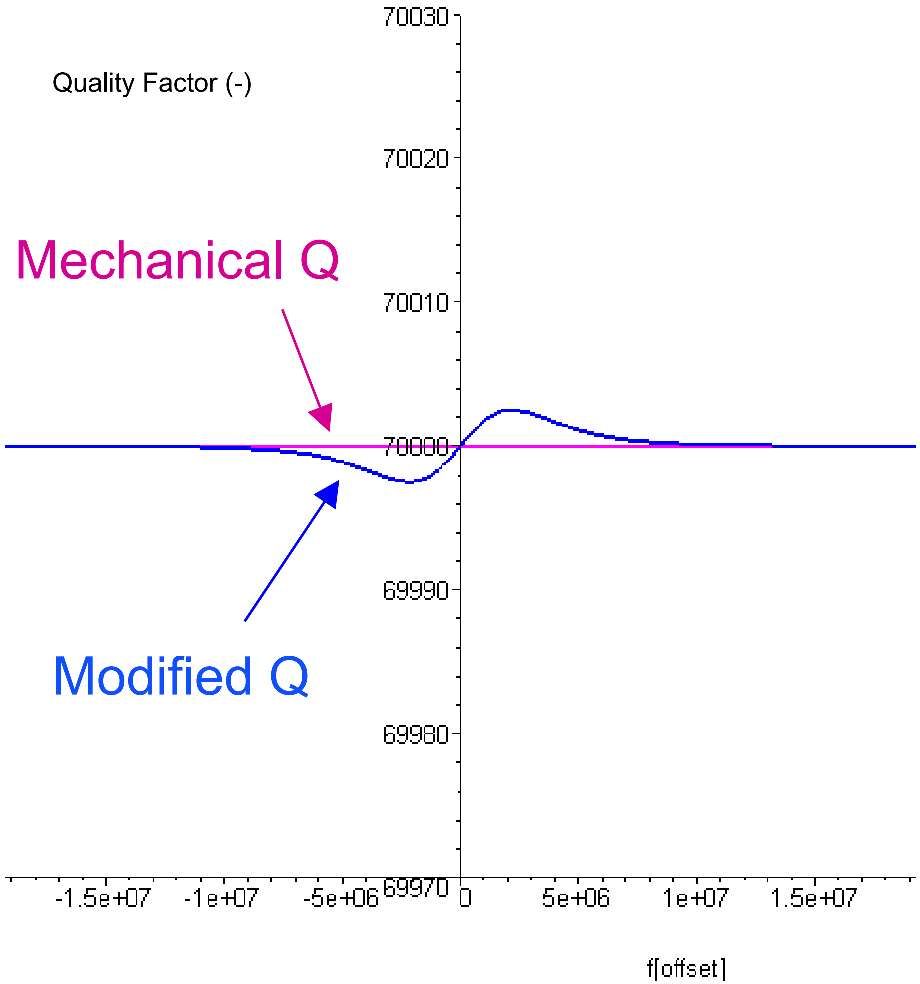
(experimental results)

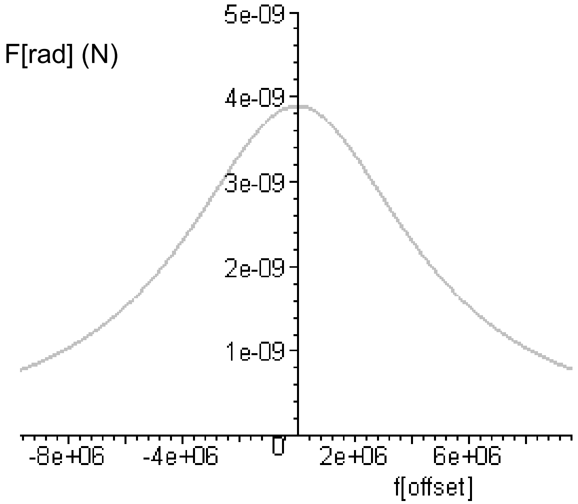
Quality Factor

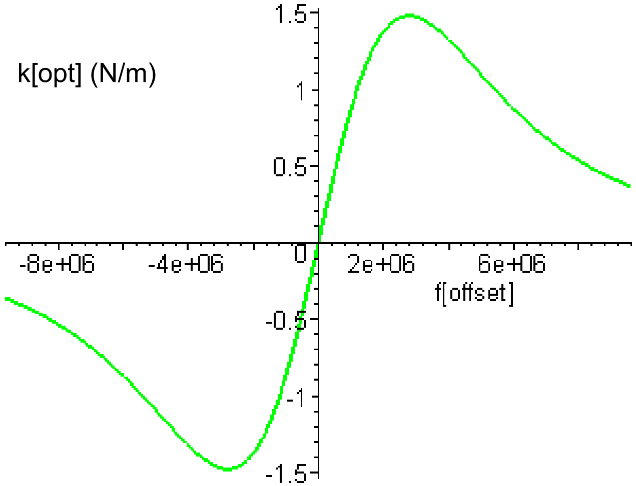


$Q = 0.700 \pm 0.002 \times 10^5$

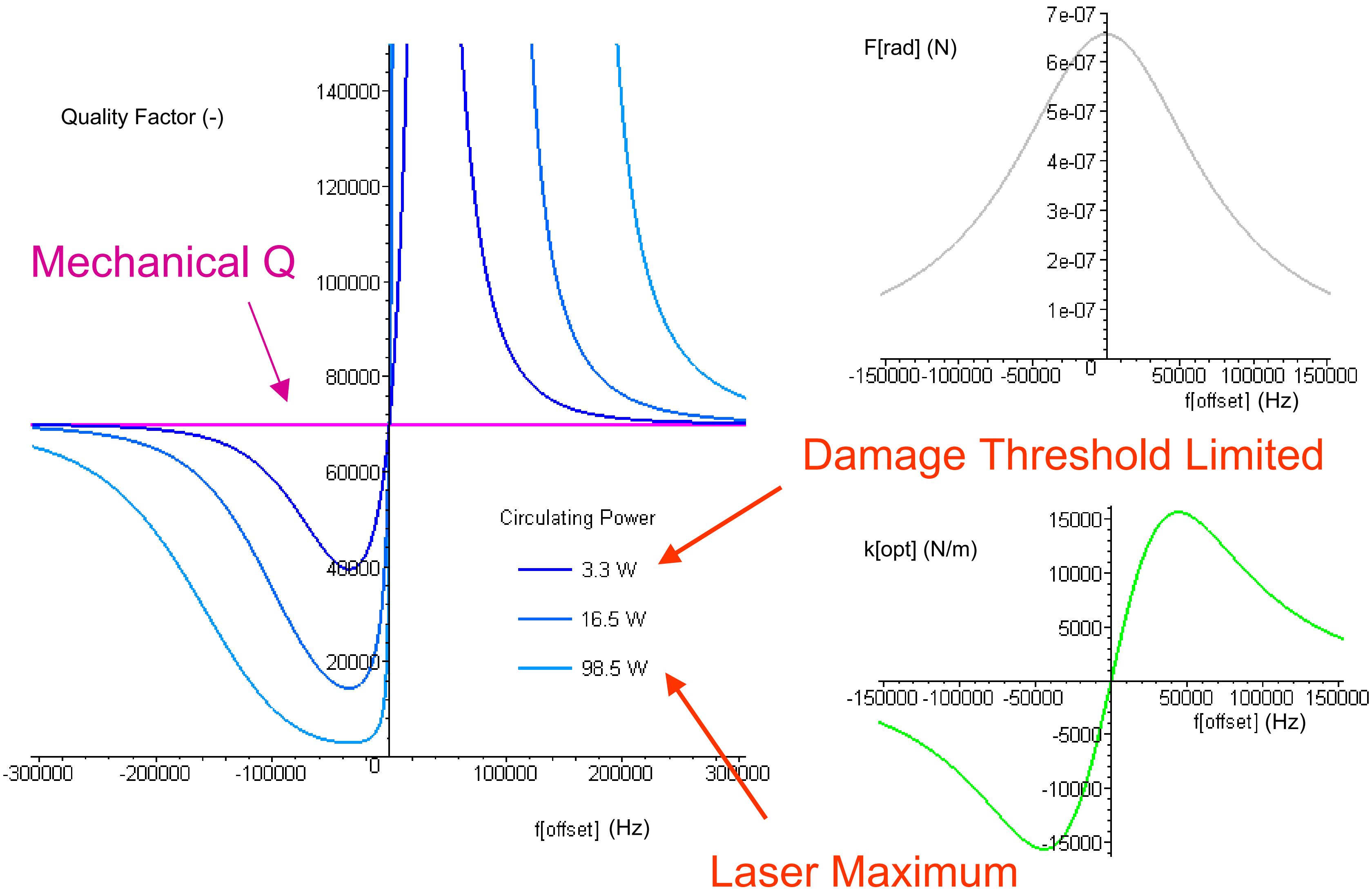
Proxy Mirror Q Modification



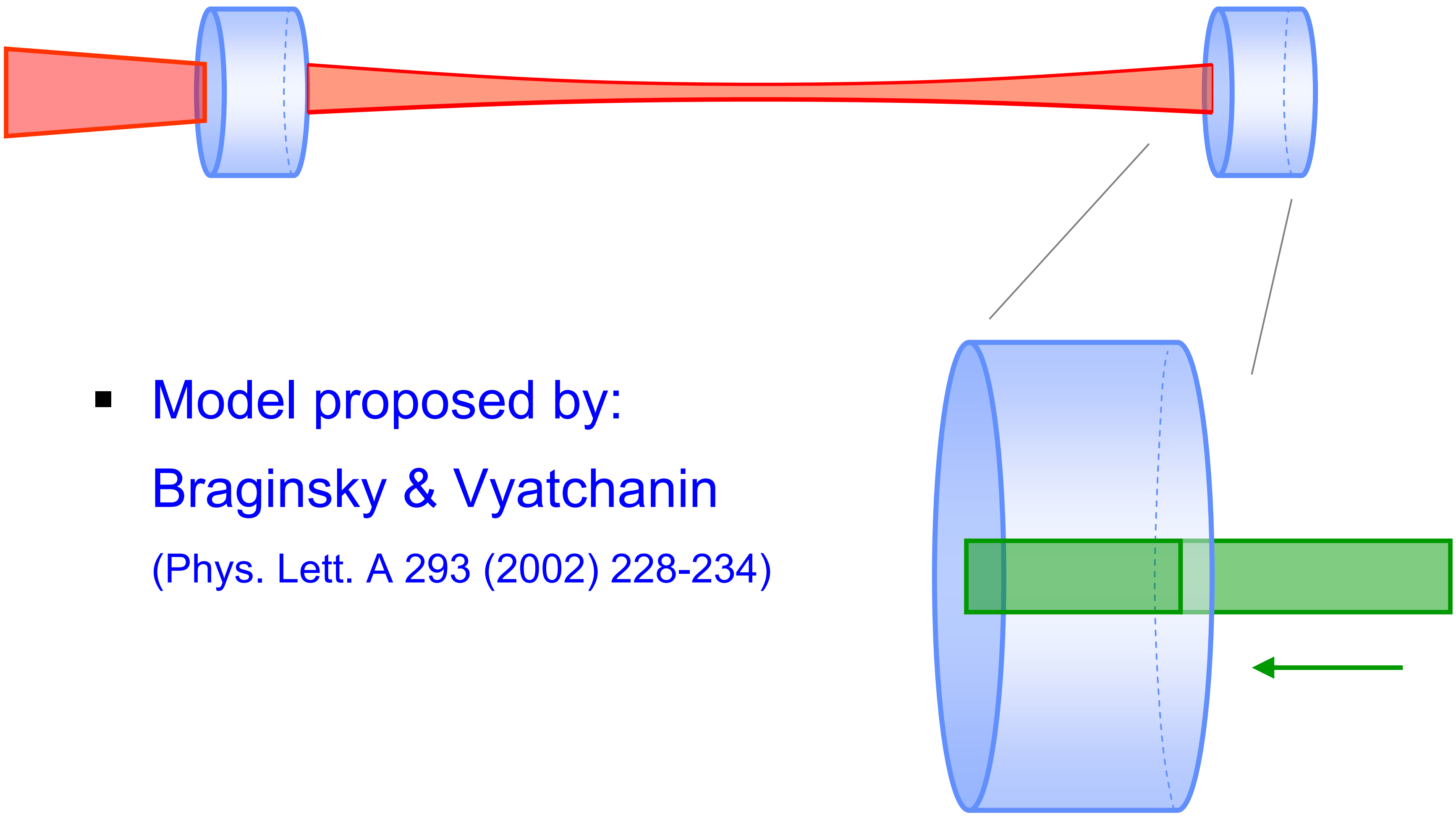




Super Mirror Q Modification



Parametric Instability Tranquilisation



- Model proposed by:

  Braginsky & Vyatchanin

  (Phys. Lett. A 293 (2002) 228-234)

Any Question?

Q Modification / Optical Spring Const.

$$k_{opt} = \frac{1024\pi\, l\, P_{input}\, f_{offset}}{\lambda\, c^2}\, \frac{1}{\sqrt{R}\,(1-R)}\, \frac{\sqrt{\eta}}{\left(2-\sqrt{\eta}\right)^3}\, \frac{1}{\left(1+\left(\frac{8\pi\, l\,(1-R)\, f_{offset}}{\pi\sqrt{R}\, c}\right)^2\right)^2}$$

$$Q_{alt} = \left(\frac{1}{Q_{mech}} - \frac{4096\,\pi\, P_{input}\, f_{offset}\, l^2\sqrt{\eta}}{m_{eff}\, c^3 f_{mech}\,\lambda\,(1-R)^4\left(2-\sqrt{\eta}\right)^5\left(1+\left(\frac{8\pi\, l\, f_{offset}}{c\,(1-R)\left(2-\sqrt{\eta}\right)}\right)^2\right)^3}\right)^{-1}$$

Circulating Power

- Cavity Properties
  - Rayleigh Range / Beam Waist

$$R_1 = +z_1 + \frac{z_0^2}{z_1} \qquad z_1 = +\frac{R_2}{2} \pm \frac{1}{2}\sqrt{R_1^2 - 4z_0^2}$$

$$R_2 = +z_2 + \frac{z_0^2}{z_2} \qquad z_2 = +\frac{R_2}{2} \pm \frac{1}{2}\sqrt{R_2^2 - 4z_0^2}$$

$$\boxed{z_0 = \sqrt{\frac{(2R-L)L}{4}}}$$

  - $R_1 = R_2 = 1.0\ m$
  - $L = 0.1\ m$

$$\boxed{z_0 = 0.2179m} \qquad w_0 = \sqrt{\frac{\lambda\, z_0}{\pi}} = 2.716\times10^{-4}m$$

  - Spot Size at Mirror ($z = 0.05\ m$)

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_0}\right)^2} = 2.787\times10^{-4}m$$

$$w(z)^2 = 7.767\times10^{-8}m^2$$
$$= 7.767\times10^{-4}cm^2$$

Circulating Power

- Newport Supermirrors
  - Damage Threshold:

$$DT = 1000\, W / cm^2$$

$$w(z)^2 = 7.767 \times 10^{-4}\, cm^2$$

  - Maximum Circulating Power:

$$\boxed{P_{circ(Max)} = 0.7767\, W}$$

- Maximum Input Power
  - $R_1 = R_2 > 99.94\%$
  - $T_1 > 0.06\%$

$$P_{in(Max)} = \frac{\left(1 - \sqrt{R_1 R_2}\right)^2}{T_1} P_{circ(Max)}$$

$$= 0.0006\, P_{circ(Max)}$$

$$= 0.466\, mW$$

Radiation Pressure Force

- Radiation pressure force for FP Cavity

$$F_{rad} = \frac{P_{circ}}{c}\underbrace{(2R_2 + L_2)}_{\approx \text{ unity}}$$

$$P_{circ} = \frac{T_1}{\left|1 - \sqrt{R_1 R_2}\, e^{-i\frac{4\pi x}{\lambda}}\right|^2} P_{in}$$

$$k_{opt} = -\frac{d\,F_{rad}(x)}{dx}$$

$$F_{rad} = \frac{(2R_2 + L_2)T_1}{c\left|1 - \sqrt{R_1 R_2}\, e^{-i\frac{4\pi x}{\lambda}}\right|^2} P_{in}$$

$$k_{opt} = -\frac{d}{dx}\left(\frac{(2R_2 + L_2)T_1}{\left|1 - \sqrt{R_1 R_2}\, e^{-i\frac{4\pi x}{\lambda}}\right|^2 c} P_{in}\right)$$

$$k_{opt} = -\frac{d}{dx}\left(\frac{A}{\left|1 - B\, e^{-iCx}\right|^2}\right)$$

- let: $A = \frac{(2R_2 + L_2)T_1 P_{in}}{c}$

  $B = \sqrt{R_1 R_2}$

  $C = \frac{4\pi}{\lambda}$

Optical Spring Constant

$$k_{opt} = -\frac{d}{dx}\left(\frac{A}{\left|1 - B\,e^{-iCx}\right|^2}\right)$$

$$P_{in(Max)} = 0.466\,mW$$

$$k_{opt} = -\frac{d}{dx}\left[\frac{A}{1 - B\left(e^{+iCx} + e^{-iCx}\right) + B^2}\right]$$

$$k_{opt} = -\frac{d}{dx}\left[\frac{A}{1 - 2B\,Cos\left(Cx\right) + B^2}\right]$$

$$k_{opt} = \frac{2ABC\sin\left(Cx\right)}{\left(1 - 2B\,Cos\left(Cx\right) + B^2\right)^2}$$

- substitute ABC back

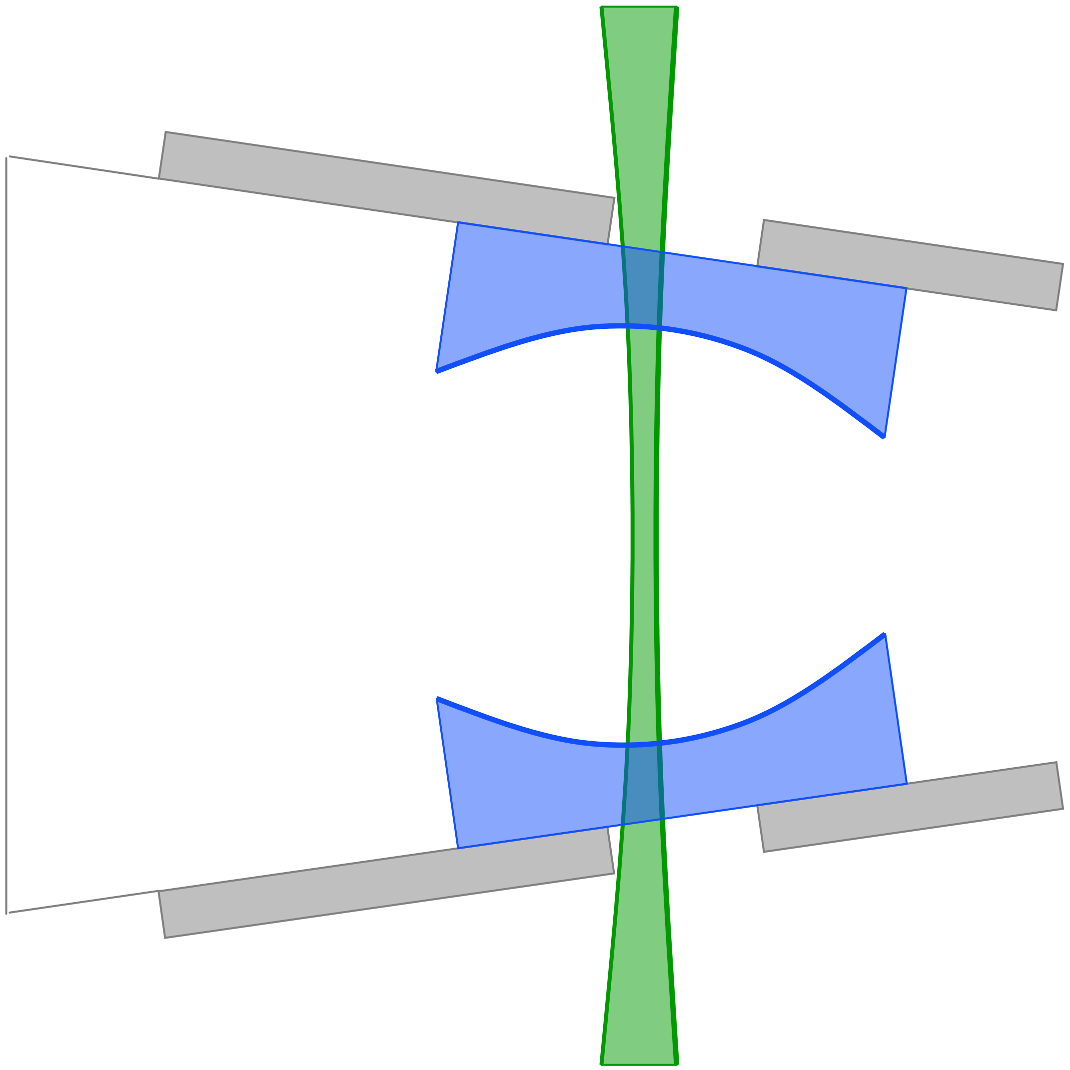
$$k_{opt} = \frac{8\pi P_{in}}{c\lambda}\,\frac{T_1\left(2R_2 + L_2\right)\sqrt{R_1R_2}\,\sin\left(4\pi x/\lambda\right)}{\left(1 + R_1R_2 - 2\sqrt{R_1R_2}\cos\left(4\pi x/\lambda\right)\right)^2}$$

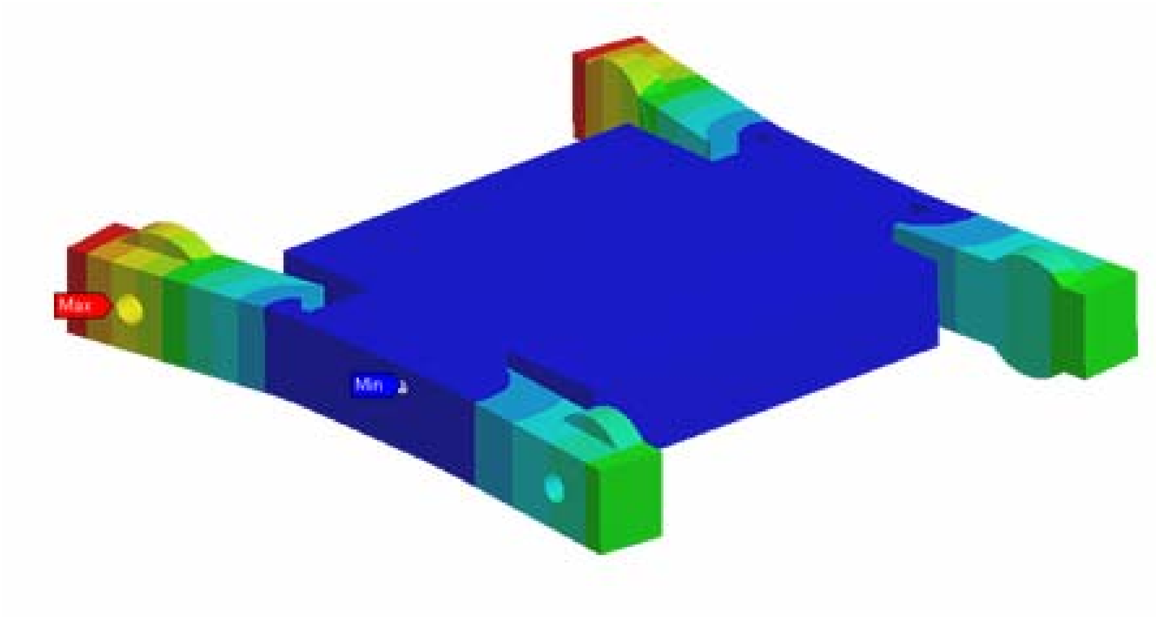
in x space

$$k_{opt} = \frac{8\pi P_{in}}{c\lambda}\,\frac{T_1\left(2R_2 + L_2\right)\sqrt{R_1R_2}\,\sin\left(4\pi l f_{offset}/c\right)}{\left(1 + R_1R_2 - 2\sqrt{R_1R_2}\cos\left(4\pi l f_{offset}/c\right)\right)^2}$$

in f space

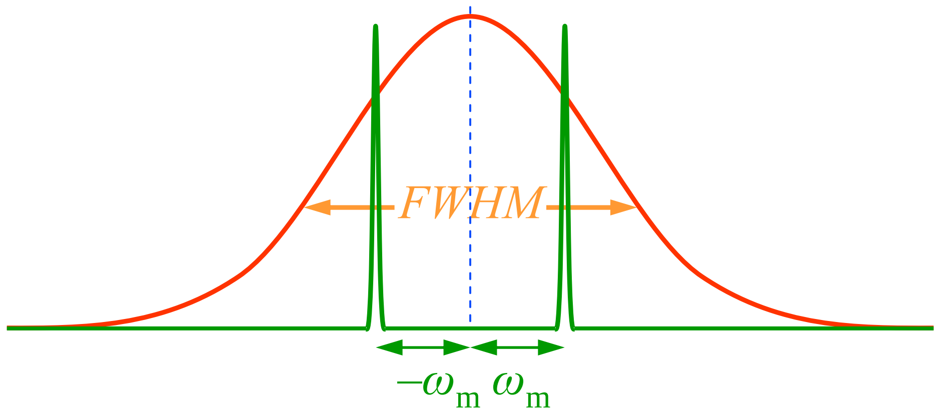
Cavity Alignment




Max
Min

"Low Q" / "High Q"

- Low Q – when the bandwidth is larger than ω_{m}



- High Q – when the bandwidth is smaller than ω_{m}