Ground-based constraints on the stochastic gravitational wave background

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Outline

- Ground-Based SGWB Search Technique
- Statistical Aside: Frequentist & Bayesian ULs
- Current Upper Limits

Reminder: ground-based detectors sensitive at 10s–1000s of Hz

Stochastic GW Spectrum

- For isotropic backgrounds,
  define spectrum i.t.o. GW contribution to $\Omega = \frac{\rho}{\rho_{\text{crit}}}$:

$$\Omega_{\text{gw}}(f) = \frac{1}{\rho_{\text{crit}}} \frac{d\rho_{\text{gw}}}{d \ln f} = \frac{f}{\rho_{\text{crit}}} \frac{d\rho_{\text{gw}}}{df}$$

- Note $\rho_{\text{crit}} \propto H_0^2$, so $h_{100}^2 \Omega_{\text{gw}}(f)$ is independent of

$$h_{100} = \frac{H_0}{100\,\text{km/ s/ Mpc}}$$

  Most recent results assume $h_{100} = 0.72$

- Equivalent GW strain power (in interferometer w/$\perp$ arms)

$$S_{\text{gw}}(f) = \frac{3H_0^2}{10\pi^2} f^{-3}\, \Omega_{\text{gw}}(f)$$

How to Tell Stochastic Signal from Random Noise

- Ground-based detectors noise-dominated $S_{\text{gw}}(f) \ll P(f)$ & can't be pointed "off-source"
  $\rightarrow$ identifying a SGWB in a single detector impractical

- Need correlations among detectors
  - Detector 1: $s_1 = h_1 + n_1$, Detector 2: $s_2 = h_2 + n_2$
  - h=stoch GW signal, n=noise (usu. much larger)

- Assume noise uncorrelated with signal & between detectors

- Cross-correlation:

$$\langle s_1 s_2 \rangle = \langle n_1 n_2 \rangle + \langle n_1 h_2 \rangle + \langle h_1 n_2 \rangle + \langle h_1 h_2 \rangle$$

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  only surviving term is from stochastic GW signal

Sensitivity to Stochastic GW Backgrounds

- Optimally filtered CC statistic

$$Y = \int df\, \tilde{s}_1^*(f)\, \tilde{Q}(f)\, \tilde{s}_2(f)$$

- Optimal filter $\tilde{Q}(f) \propto \frac{f^{-3}\Omega_{\mathrm{gw}}(f)\gamma_{12}(f)}{P_1(f)P_2(f)}$
  (Initial analyses assume $\Omega_{\mathrm{gw}}(f) \propto f^{\alpha}$ across band)
- Optimally filtered cross-correlation method sensitive to

$$\Omega_{\mathrm{gw}} \propto \left(T \int \frac{df}{f^6} \frac{\gamma_{12}^2(f)}{P_1(f)P_2(f)} \right)^{-1/2}$$

  for $\alpha = 0$
- Significant contributions when
  - detector noise power spectra $P_1(f)$, $P_2(f)$ small
  - overlap reduction function $\gamma_{12}(f)$ (geom correction) near ± 1

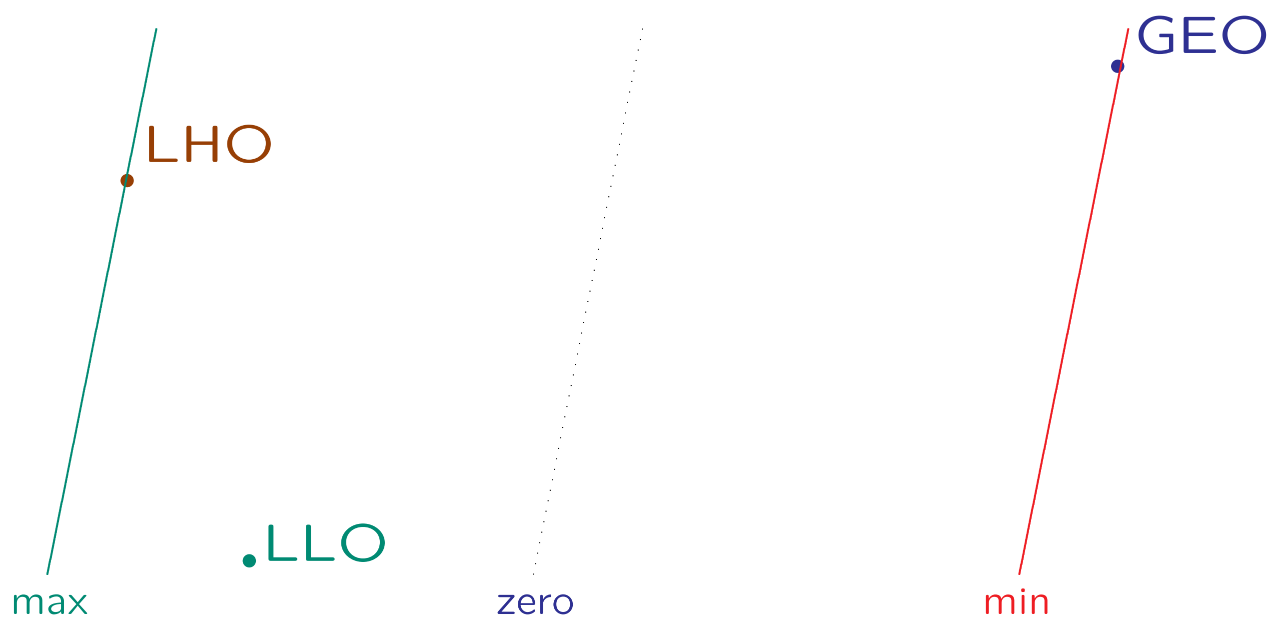
Overlap Reduction Function

$$\gamma_{12}(f) = d_{1ab} d_2^{cd} \frac{5}{4\pi} \iint_{S^2} d^2\Omega \; P^{\mathrm{TT}}{}^{ab}_{cd}(\widehat{\Omega}) e^{i2\pi f \widehat{\Omega}\cdot\Delta\vec{\mathbf{x}}/c}$$

Depends on alignment of detectors (polarization sensitivity)

Frequency dependence from cancellations when $\lambda \lesssim$ distance

$\longrightarrow$ Widely separated detectors less sensitive at high frequencies



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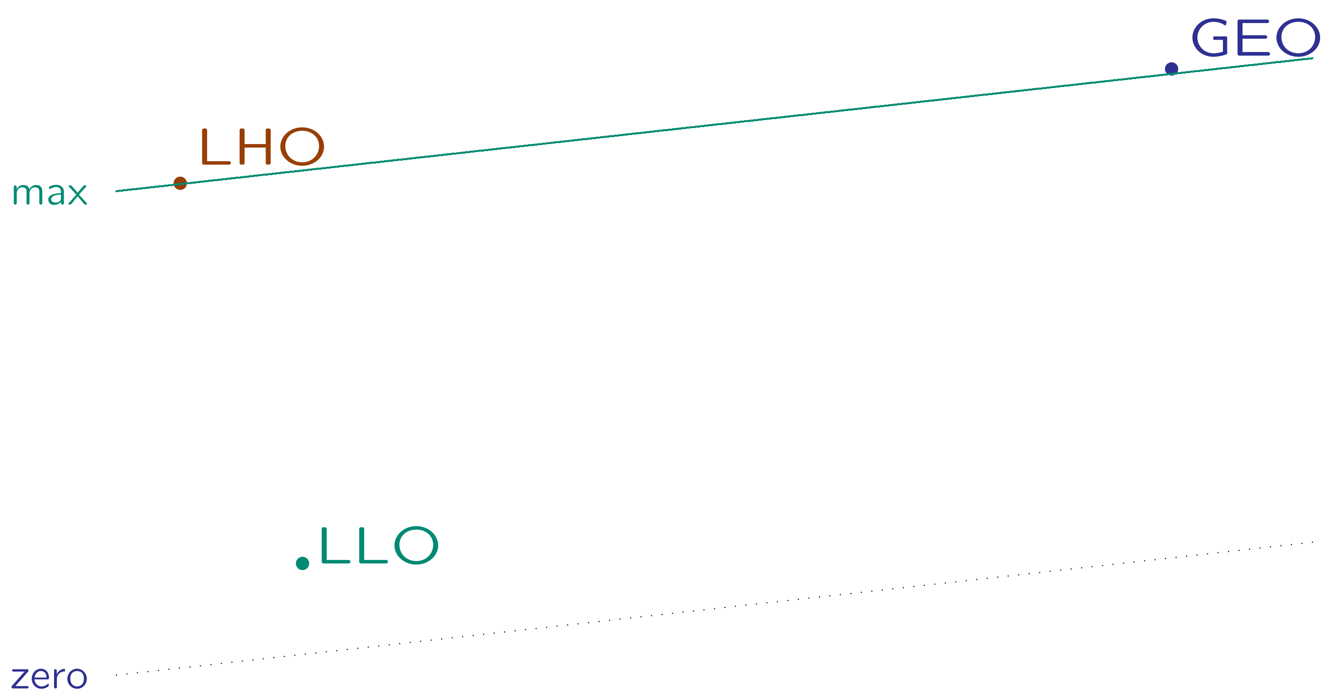
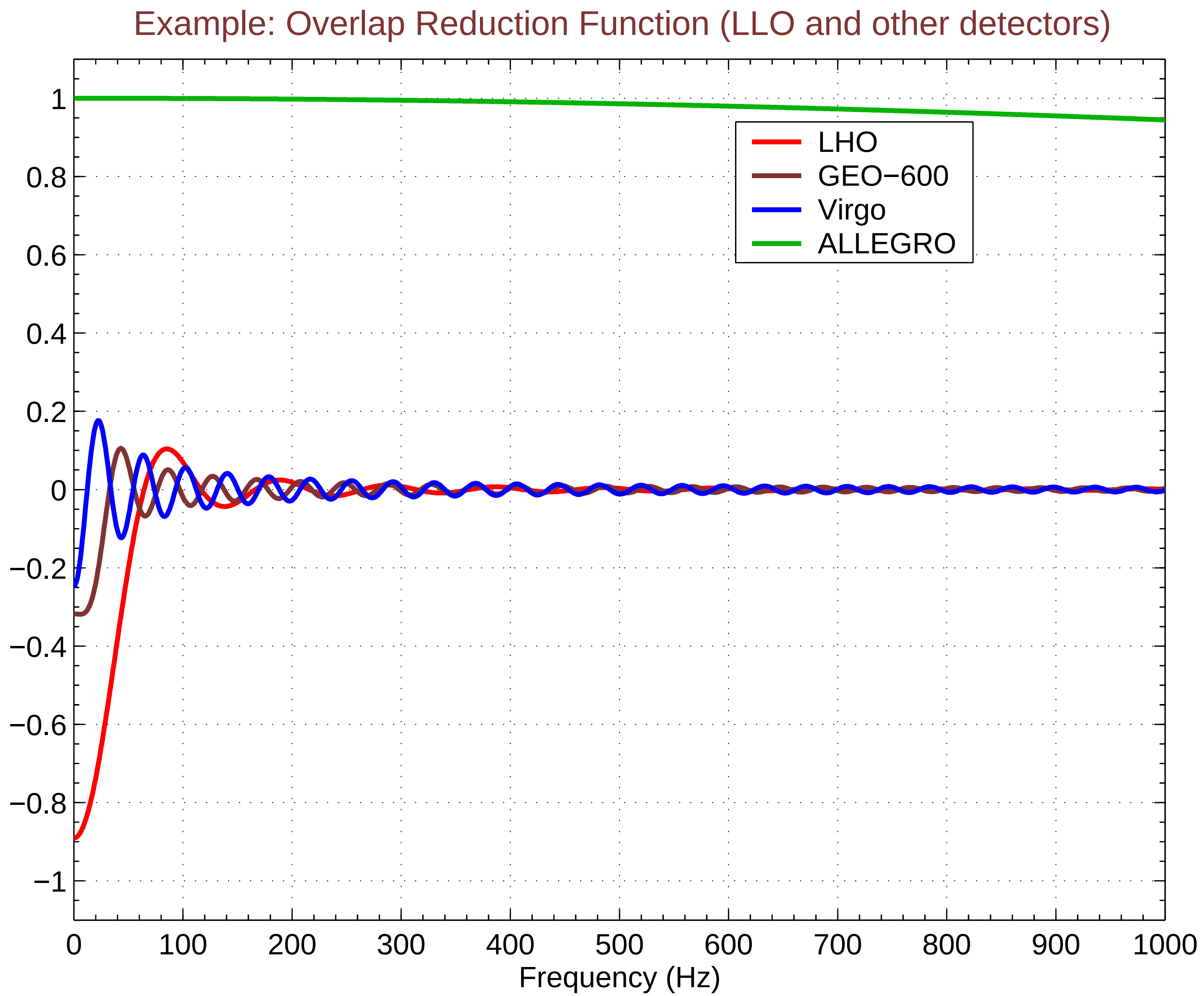


This wave (same λ) drives LHO & GEO in phase


Example: Overlap Reduction Function (LLO and other detectors)
LHO
GEO−600
Virgo
ALLEGRO
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
100
200
300
400
500
600
700
800
900
1000
Frequency (Hz)

Statistics in SGWB Upper Limits

- CC stat provides estimate $\widehat{\Omega}$;
  From noise PSDs, calculate theoretical std dev σ

- If actual value is Ω, prob of measuring $\widehat{\Omega}$ is

$$P(\widehat{\Omega}|\Omega) \propto e^{-(\widehat{\Omega}-\Omega)^2/2\sigma^2}$$

- Frequentist UL (e.g., 90% CL):
  If $\Omega = \Omega_{\mathrm{UL}}$, odds of measuring a higher value than $\widehat{\Omega}$ are 90%

$$\Omega_{\mathrm{UL}} = \widehat{\Omega} + 1.28\,\sigma$$

- Problem: if $\widehat{\Omega} < 0$, Ω_{UL} can be unreasonably small, or negative!

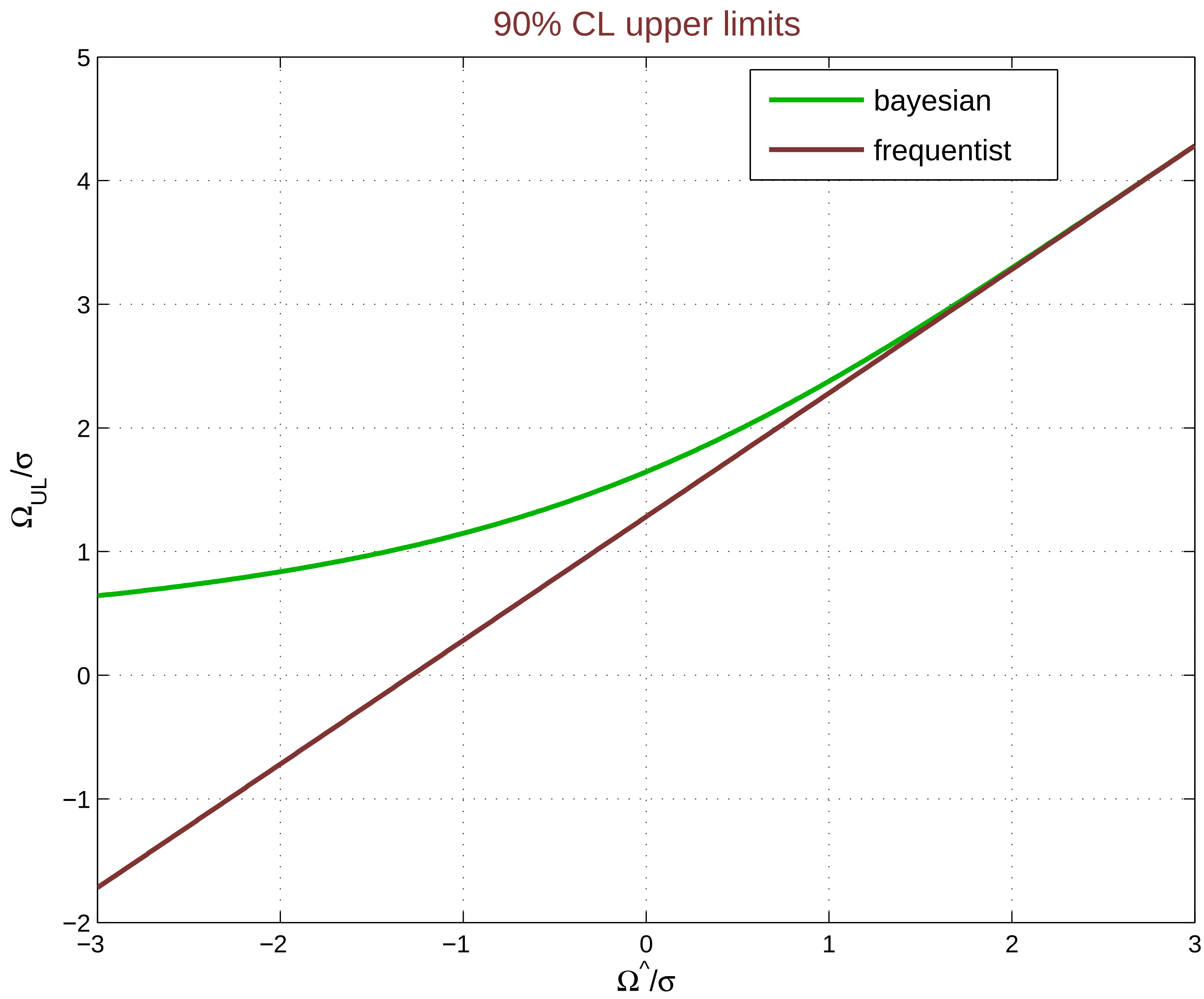
Bayesian UL in SGWB Search

- Alternative method: use Bayes's Theorem to find posterior

$$P(\Omega|\widehat{\Omega}) \propto P(\widehat{\Omega}|\Omega)P(\Omega) \propto e^{-(\widehat{\Omega}-\Omega)^2/2\sigma^2} P(\Omega)$$

- Use simple prior $P(\Omega) = \text{const}$ for $0 < \Omega < \Omega_{\text{max}}$
  More conservative than Jeffreys prior $\propto 1/\Omega$

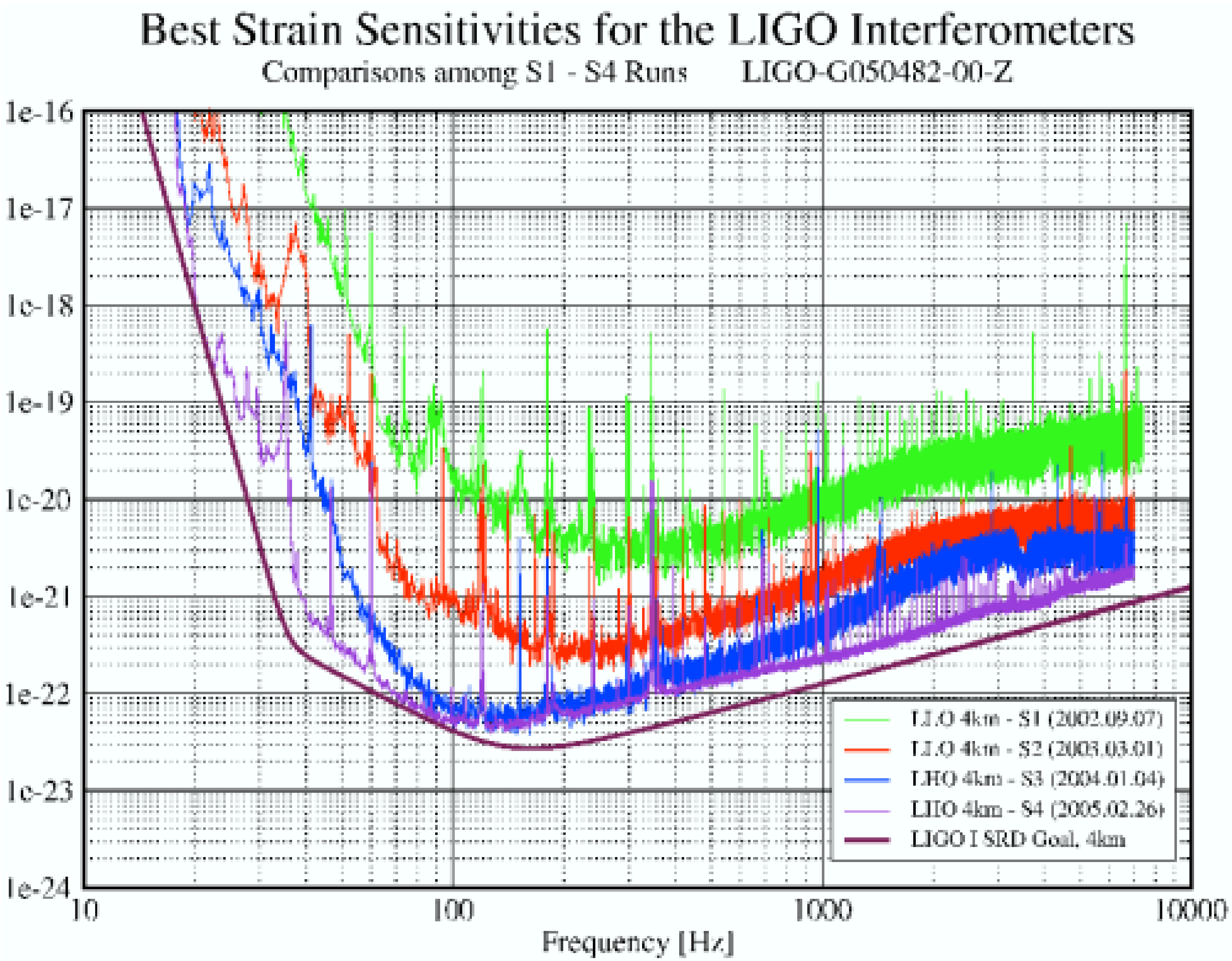
- Bayesian UL (e.g., 90% CL):
  90% of the area under the posterior PDF lies below Ω_{UL}



90% CL upper limits
bayesian
frequentist
Ω_{UL}/σ
$\hat{\Omega}/\sigma$
5
4
3
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

Cartoon courtesy of E. Coccia, ROG Group (Rome)

LIGO Approaching Design Sensitivity



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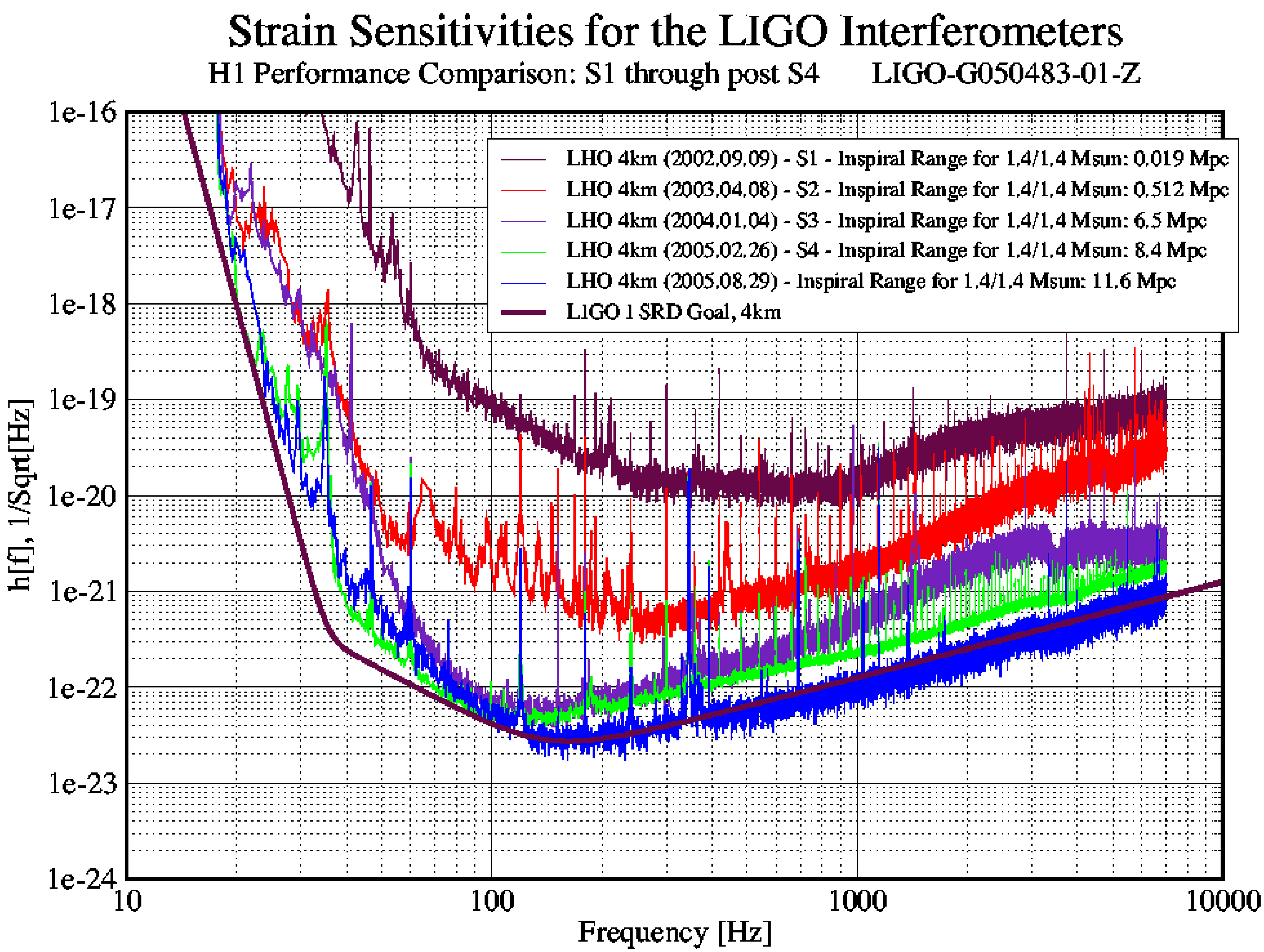


Upper Limits

- Best direct upper limit: LIGO Hanford (WA) aka LHO & Livingston (LA) aka LLO, S3 Run (LSC, Abbott et al, astro-ph/0507254): $\Omega_{\text{gw}}(f) \leq 8.4 \times 10^{-4}$ at $69\,\text{Hz} < f < 156\,\text{Hz}$

- Projected sens for 1 yr @ initial LIGO design: 10^{-6} (note LHO 2km-4km $\sim 5\times$ better than LHO 4km-LLO 4km)

- Projected sens for 1 yr @ advanced LIGO design: 10^{-9}

- Relevant indirect limit in ground-based freq band: Success of nucleosynthesis models means

$$\int_{10^{-8}\,\text{Hz}}^{\infty} \Omega_{\text{gw}}(f) \frac{df}{f} \leq 10^{-5}$$

The Power of Cross-Correlation

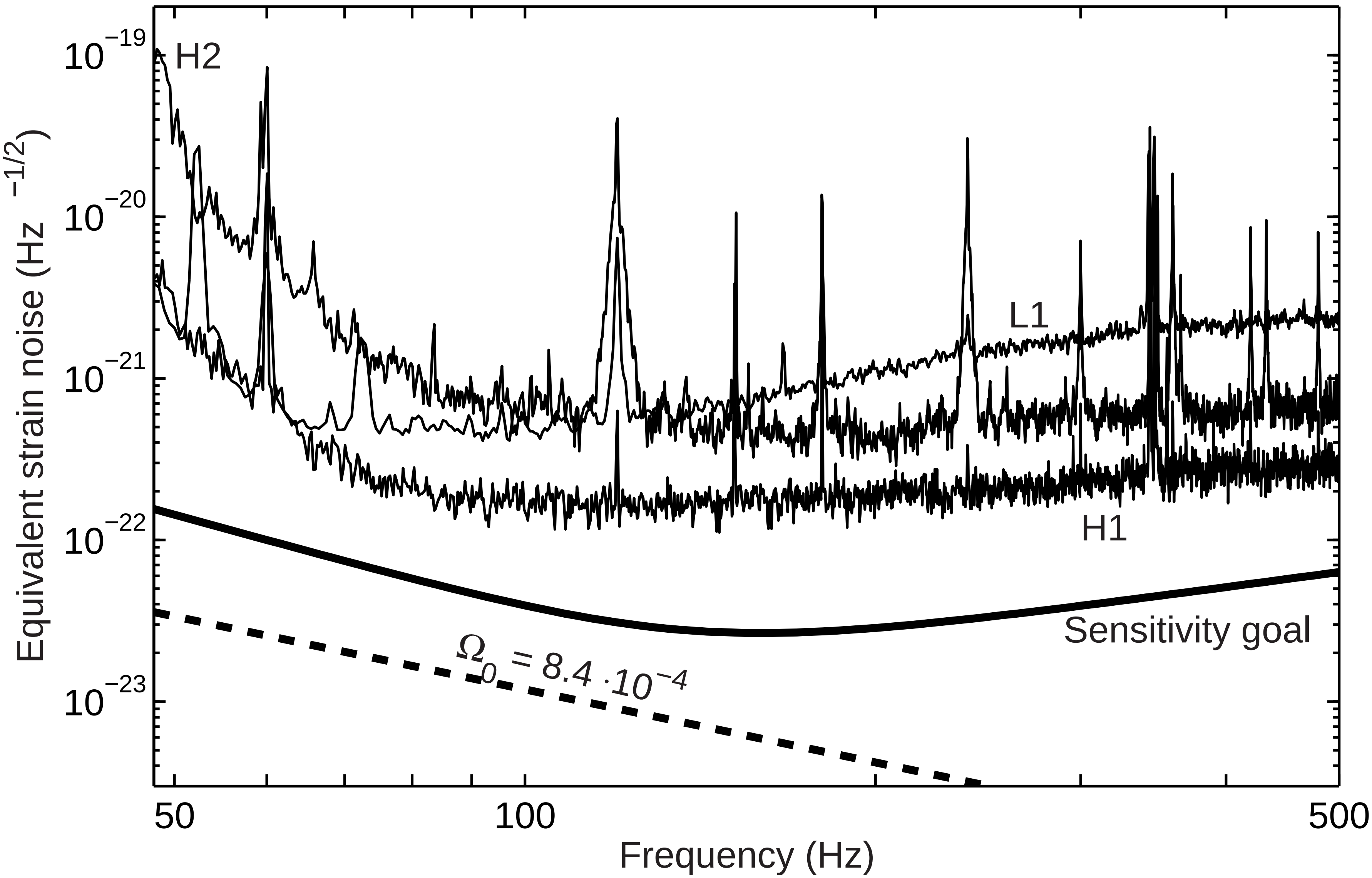


Figure from astro-ph/0507254

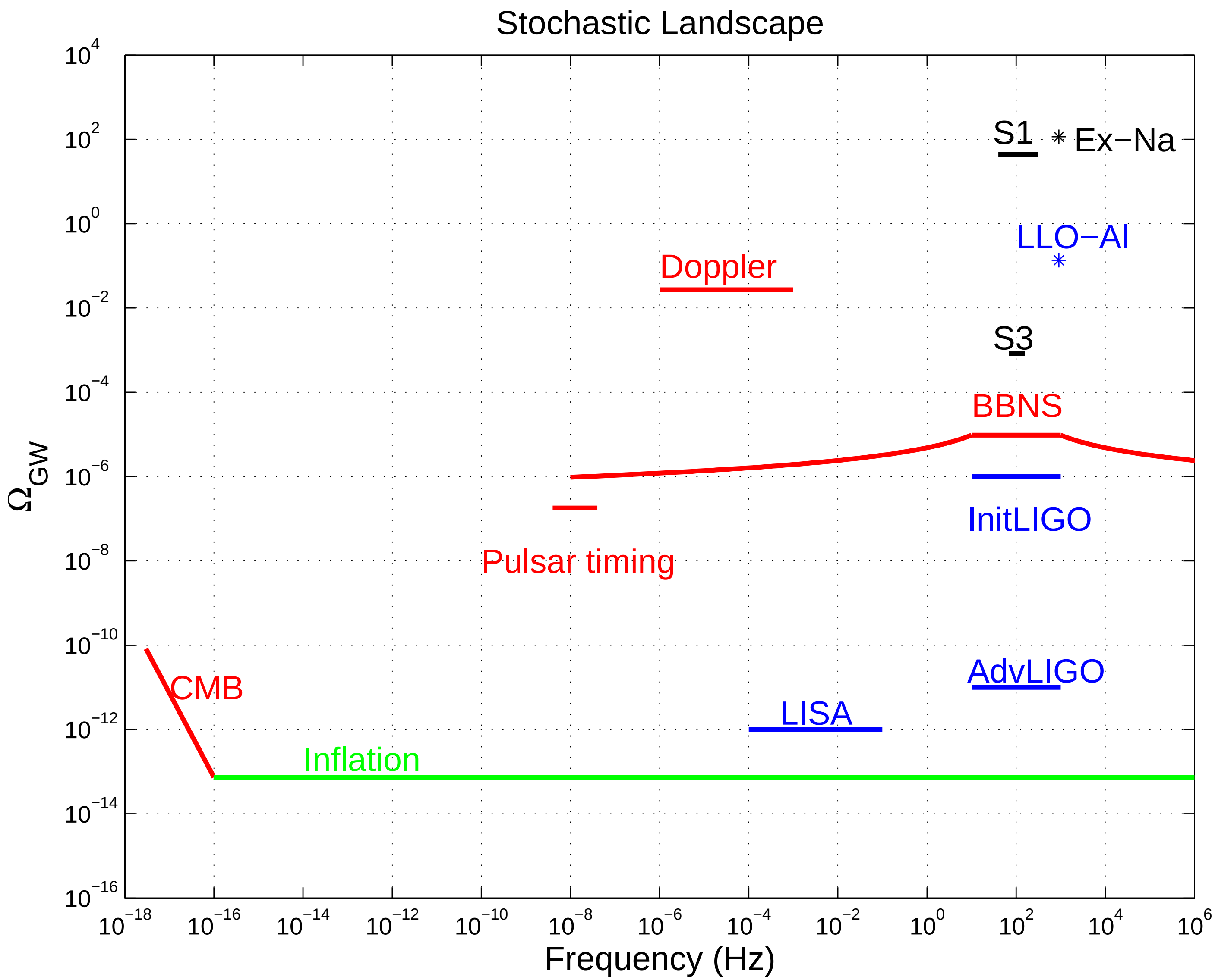
Other Ground-Based Measurements

- Correlation between Garching & Glasgow prototype IFOs [Compton et al, MG7 proceedings, 1994]: $h_{100}^2 \Omega_{\rm gw}(f) \lesssim 3 \times 10^5$

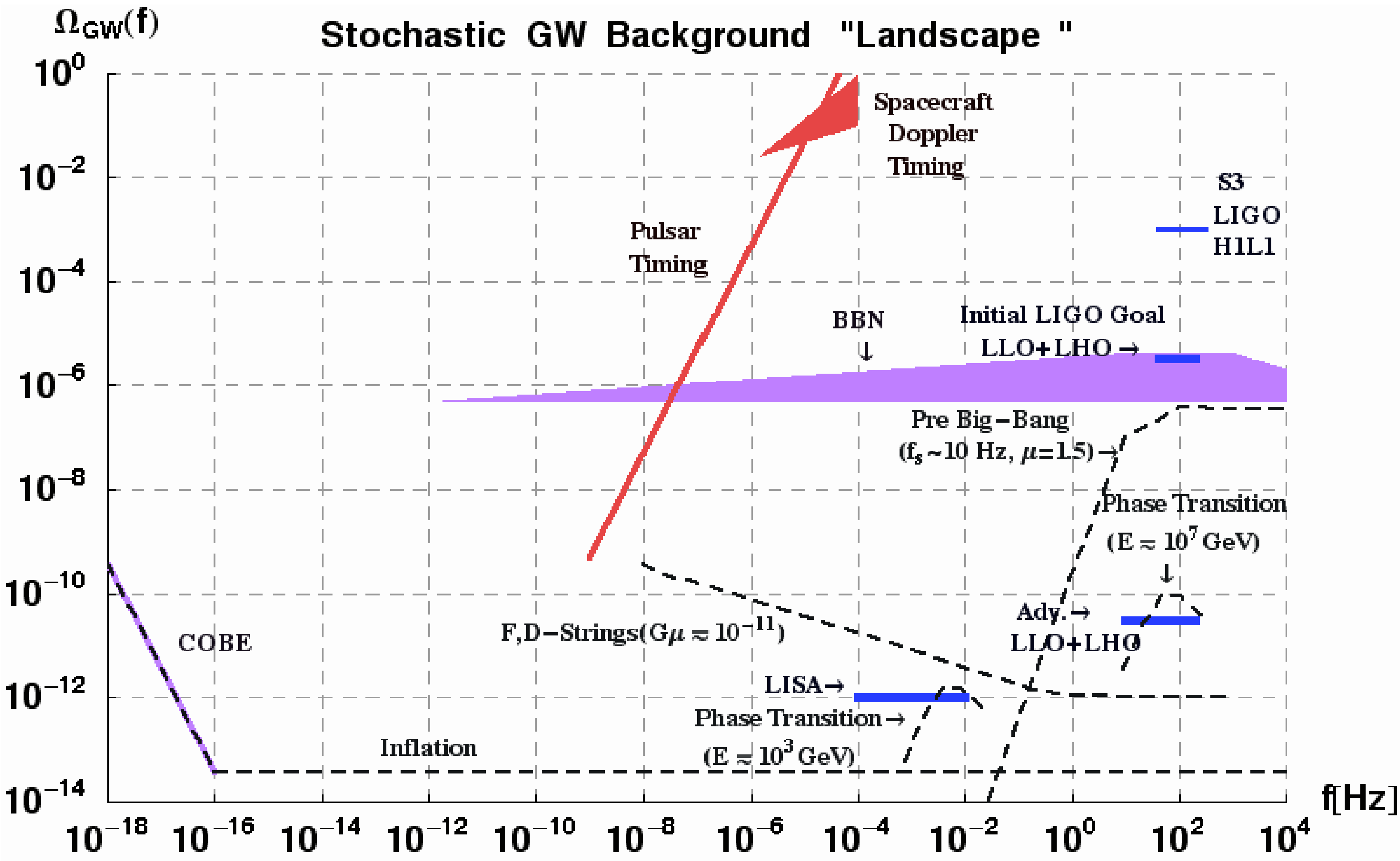
- Correlation between EXPLORER & NAUTILUS bars [Astone et al, A&A **351**, 811 (1999)]: $h_{100}^2 \Omega_{\rm gw}(907\,{\rm Hz}) \le 60$

- Correlation between LIGO Hanford & Livingston S1 data [LSC, Abbott et al, PRD **69**, 122004 (2004)]: $h_{100}^2 \Omega_{\rm gw}(f) \le 23$ at $64\,{\rm Hz} < f < 265\,{\rm Hz}$

- Correlation between LIGO Livingston and ALLEGRO bar:
  In progress; expect sens to $\Omega_{\rm gw}(f) \lesssim 10^0$ at $850\,{\rm Hz} < f < 950\,{\rm Hz}$



Plot adapted from one courtesy Joe Romano



Plot courtesy Albert Lazzarini