Searching for Stochastic Background with LIGO: Results and Implications on Pre-Big-Bang Models

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Stochastic Background of Gravitational Waves

- Energy density:

$$\rho_{GW} = \frac{c^2}{32\pi G} < \dot{h}_{ab}\, \dot{h}^{ab} >$$

- Characterized by log-frequency spectrum:

$$\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}(f)}{d \ln f}$$

- Related to the strain spectrum:

$$S(f) = \frac{3H_0^2}{10\pi^2} \frac{\Omega_{GW}(f)}{f^3}$$

- Strain scale:

$$h(f) = 6.3 \times 10^{-22} \sqrt{\Omega_{GW}(f)} \left(\frac{100\ \mathrm{Hz}}{f}\right)^{3/2} \mathrm{Hz}^{-1/2}$$

Detection Strategy

- Cross-correlation estimator

$$Y = \int_{-T/2}^{+T/2} dt_1 \int_{-T/2}^{+T/2} dt_2 \, s_1(t_1) \, s_2(t_2) \, Q(t_2 - t_1)$$

$$Y = \int_{-\infty}^{+\infty} df \, \tilde{s}_1^*(f) \, \tilde{s}_2(f) \, \tilde{Q}(f)$$

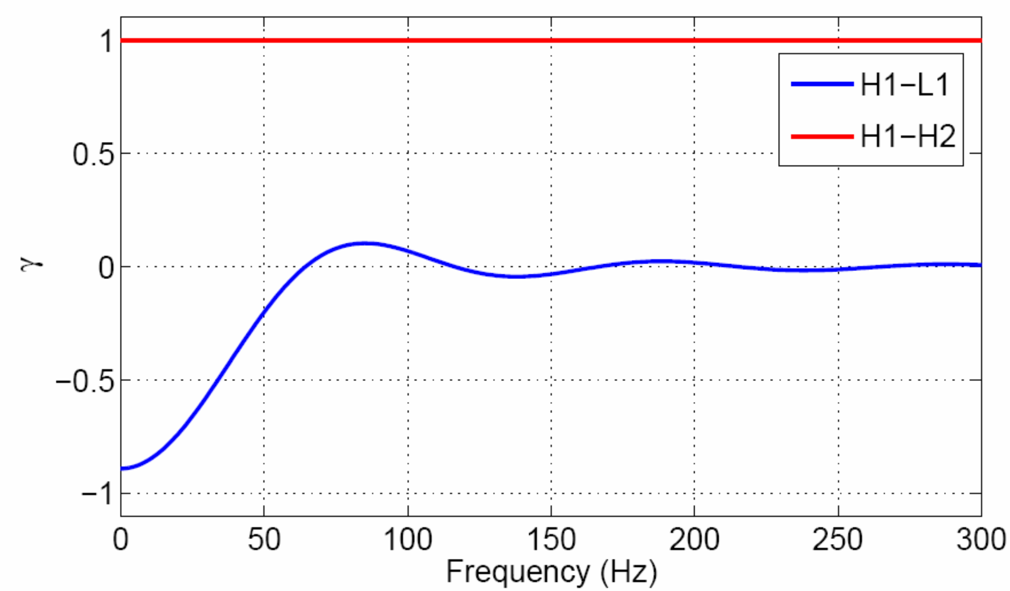
- Theoretical variance

$$\sigma_Y^2 \approx \frac{T}{2} \int_0^{+\infty} df \, P_1(f) \, P_2(f) \, | \, \tilde{Q}(f) \, |^2$$

- Optimal Filter

$$\tilde{Q}(f) = \frac{1}{N} \frac{\gamma(f) \, \Omega_t(f)}{f^3 \, P_1(f) \, P_2(f)}$$

Overlap Reduction Function



For template: $\Omega_t(f) = \Omega_\alpha (f/100 \text{ Hz})^\alpha$

Choose *N* such that: $< Y >= \Omega_\alpha T$

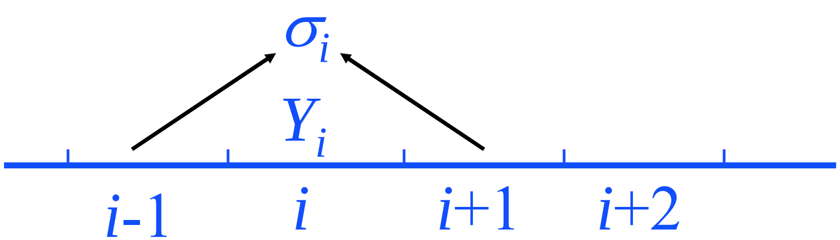
LIGO

Analysis Details

- Data divided into segments:
  - Y_i and σ_i calculated for each interval *i.*
  - Weighed average performed.
- Sliding Point Estimate:
  - Avoid bias in point estimate
  - Allows stationarity (Δσ) cut
- Data manipulation:
  - Down-sample to 1024 Hz
  - High-pass filter (40 Hz cutoff)
- 50% overlapping Hann windows:
  - Overlap in order to recover the SNR loss due to windowing.

$$Y_{\text{opt}} = \frac{\sum_i \sigma_i^{-2} Y_i}{\sum_i \sigma_i^{-2}}$$

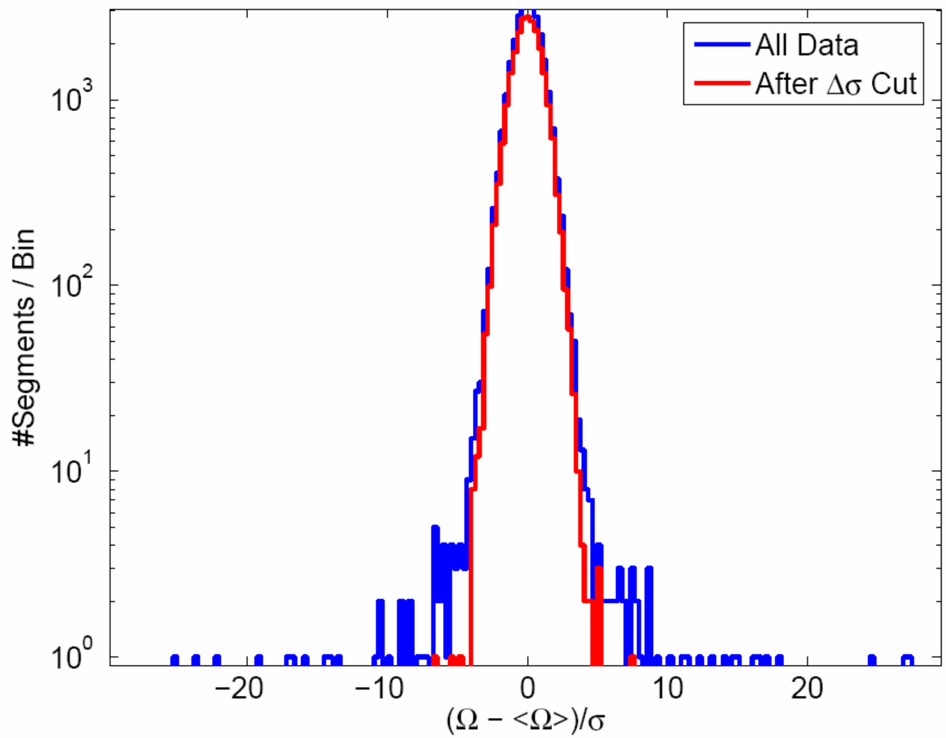
$$\sigma_{\text{opt}}^{-2} = \sum_i \sigma_i^{-2}$$



S3 Results (1)

- LIGO S3 run took place between 31 Oct 2003 and 9 Jan 2004.
- Used H1-L1 pair, 60-sec segments, with ¼ Hz resolution.
- Notched:
  - » 60 Hz harmonics
  - » 16 Hz harmonics
  - » Pulsar lines
- After Δσ cut, 218 hours of exposure.
- Procedure verified by successfully recovering hardware and software injections!

Δσ cut: $$\left| \frac{\sigma_i - \sigma_{i\pm 1}}{\sigma_i} \right| < 20\%$$



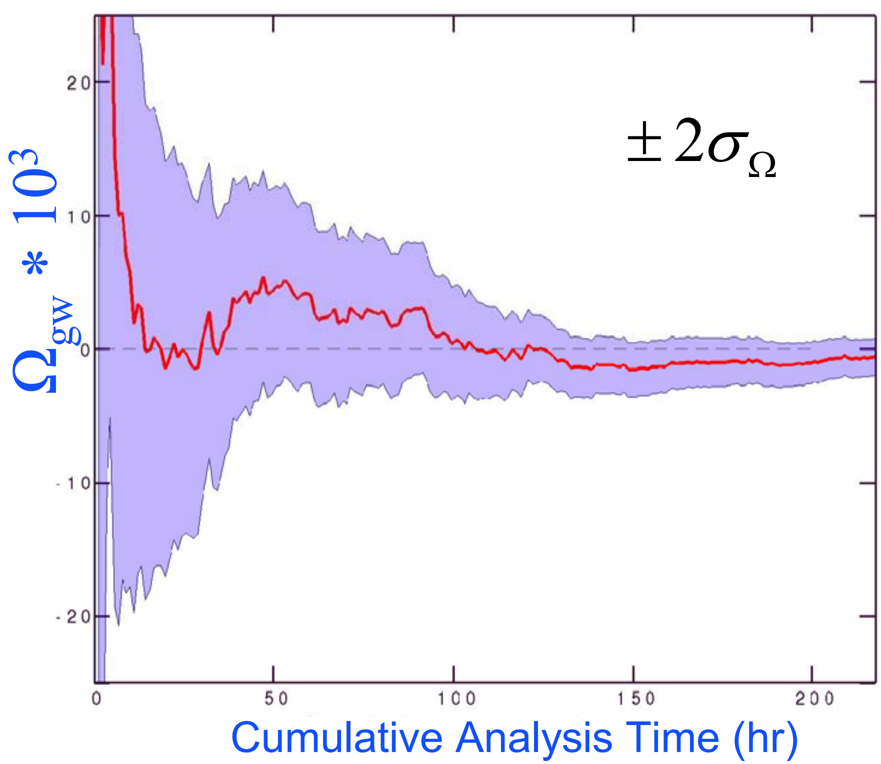
S3 Results (2)

$h_{100}=0.72$

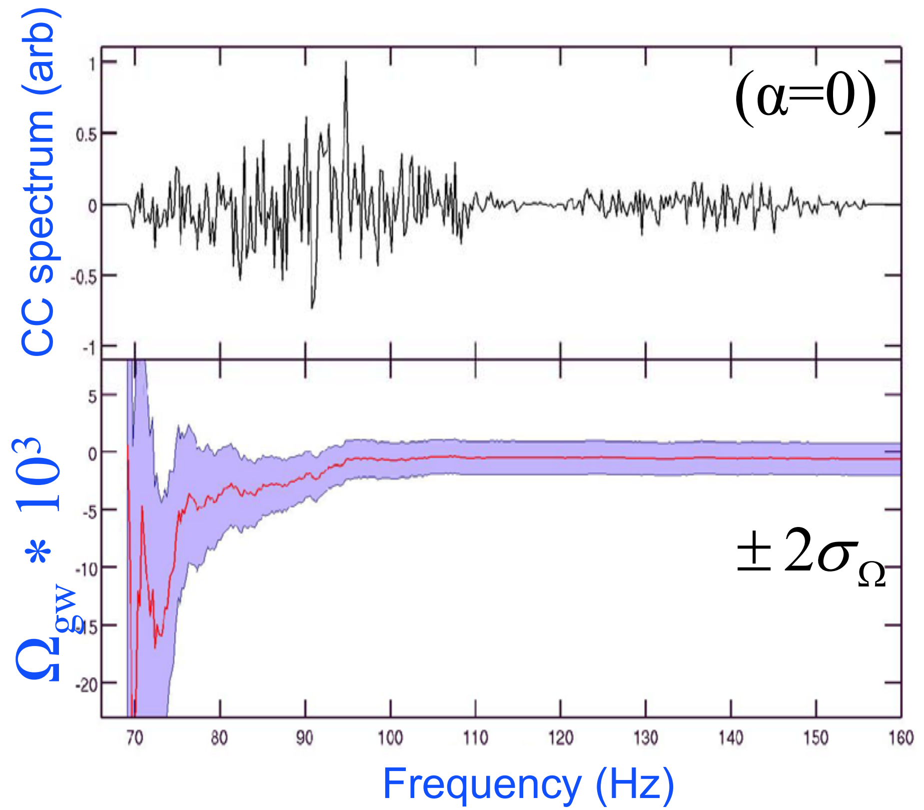
Power law	Freq. Range	$\left(\hat{\Omega}_{gw} \pm \hat{\sigma}\right) \times 10^{-4}$ at 100Hz	Upper Limit $\Omega_{gw} \times 10^{-4}$	Upper Limit $S_{gw}^{1/2} \times 10^{-23}\, Hz^{-1/2}$
α=0	69-156Hz	-6.0 ± 7.0	8.4	$1.2 \times (100Hz/f)^{3/2}$
α=2	73-244Hz	-4.7 ± 7.2	$9.4 \times (f/100Hz)^2$	$1.2 \times (100Hz/f)^{1/2}$
α=3	76-329Hz	-4.0 ± 6.2	$8.1 \times (f/100Hz)^3$	1.2

S3 Results (3)

Running Point Estimate

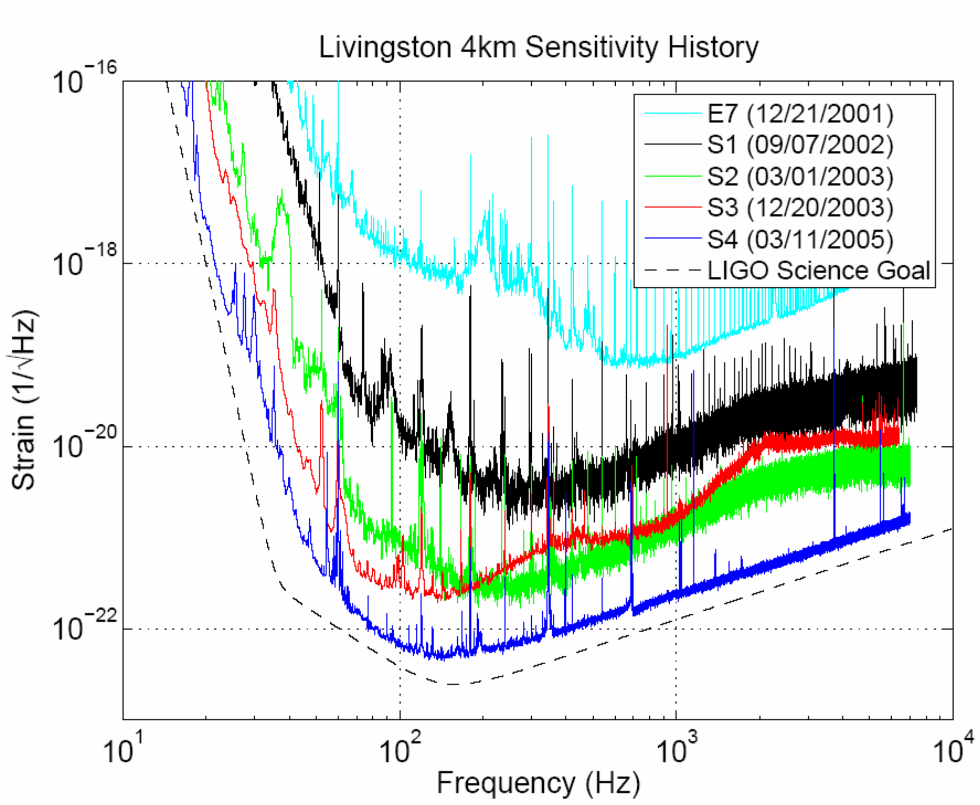


Cross-Correlation Spectrum



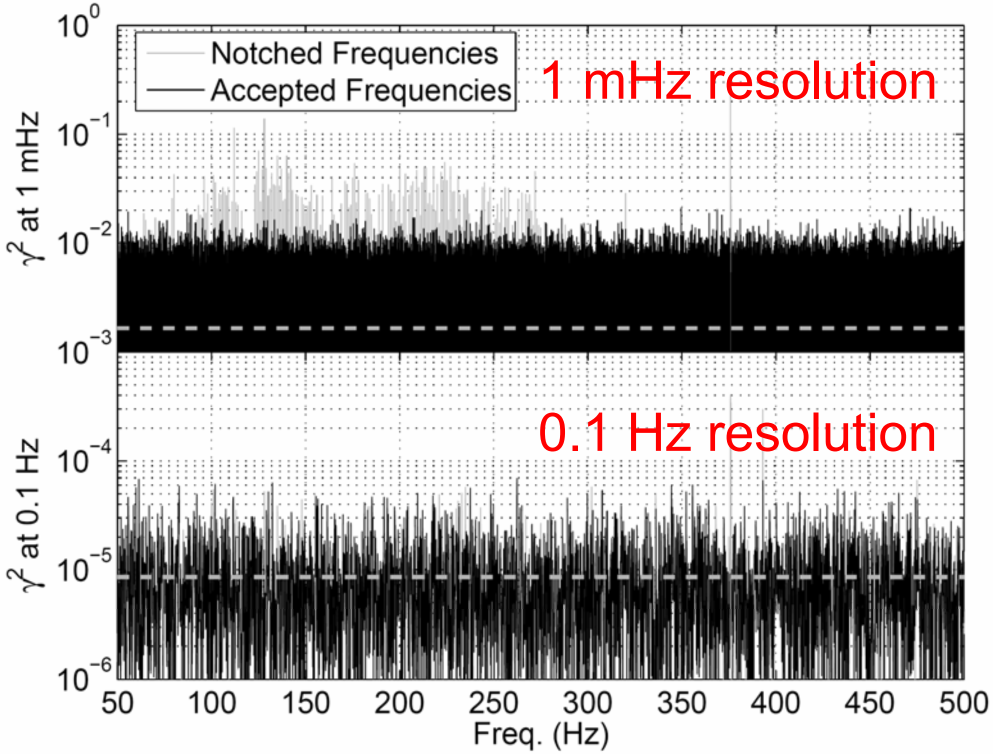
Since Then…

- Significant improvements in interferometer sensitivities:
  - » Laser power increase
  - » Active seismic isolation at LLO…
- Factor of ~10 improvements at some frequencies.
- LIGO S4 science run took place between 22 Feb 2005 and 23 Mar 2005.
- Since March, further improvements were made to all interferometers:
  - » Year-long science run (S5) at design sensitivity has started in November!



S4: H1L1 Coherence

- Calculated over all of S4.
  - » Using the same data as in stochastic analysis.
- At 1 mHz resolution, many 1 Hz harmonics are observed.
  - » Sharp features, not visible at 0.1 Hz resolution.
  - » One source was the GPS synchronization signal.
  - » Expect improvement for S5.
- Also see simulated pulsar lines.



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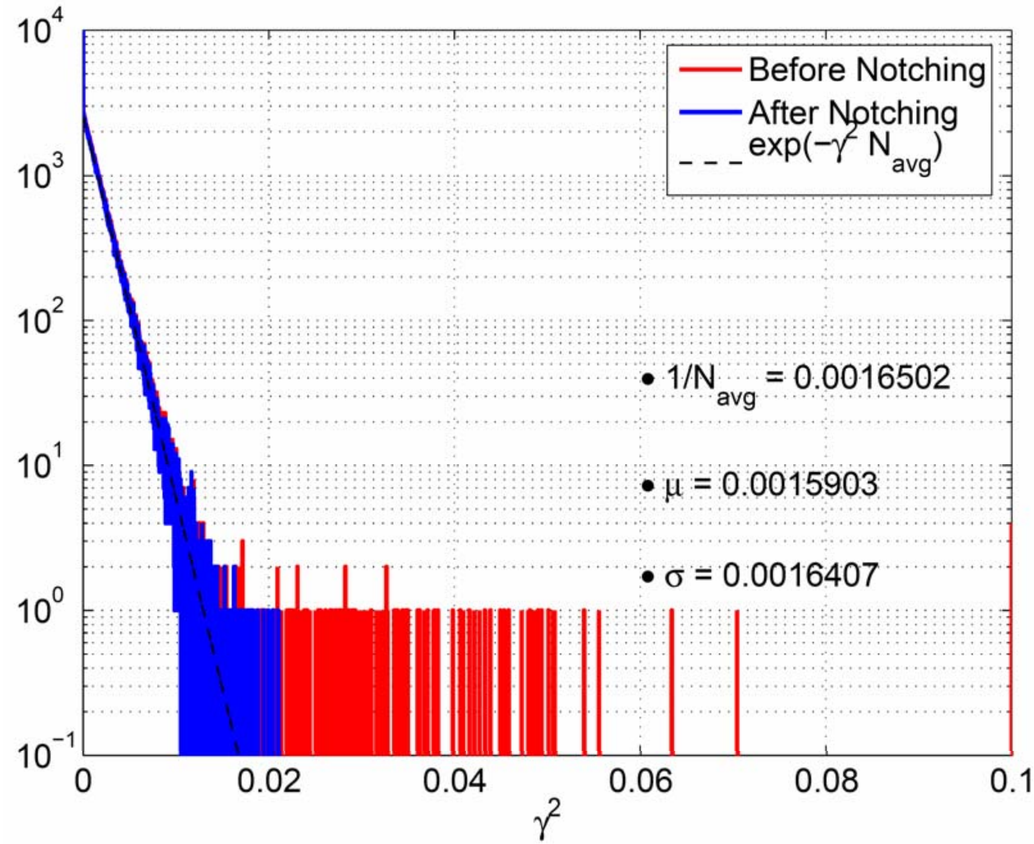
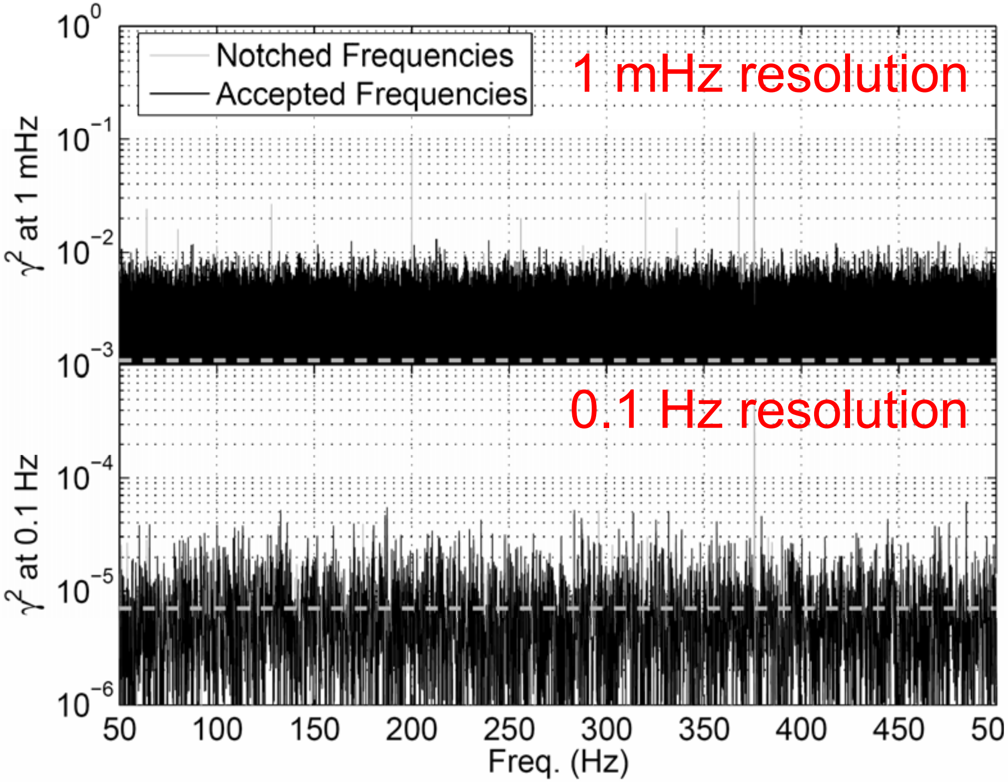
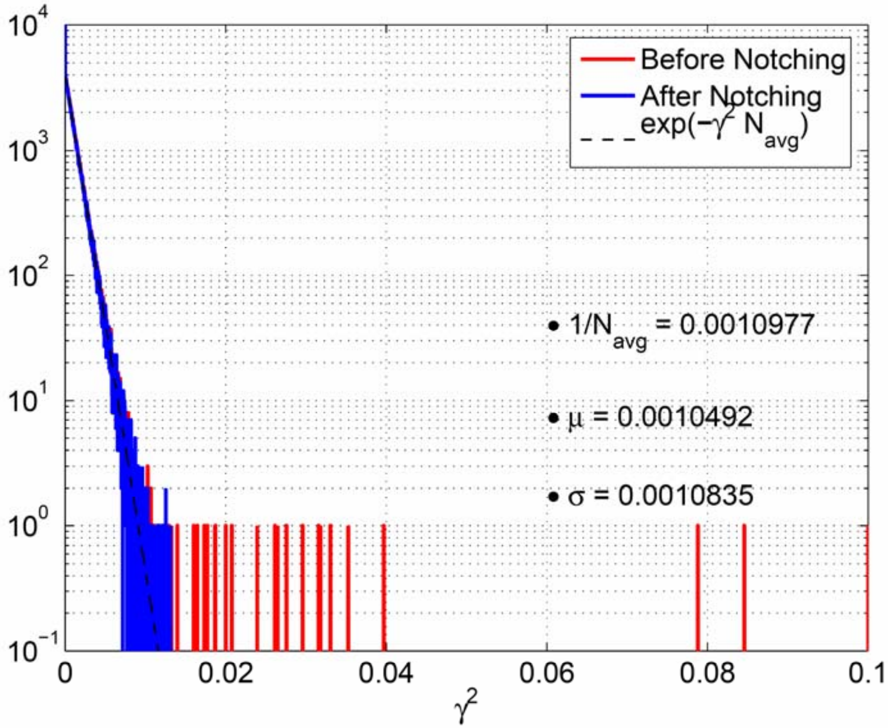
Coherence Histogram at 1 mHz



S4: H2L1 Coherence

H2L1: fewer 1 Hz harmonics.



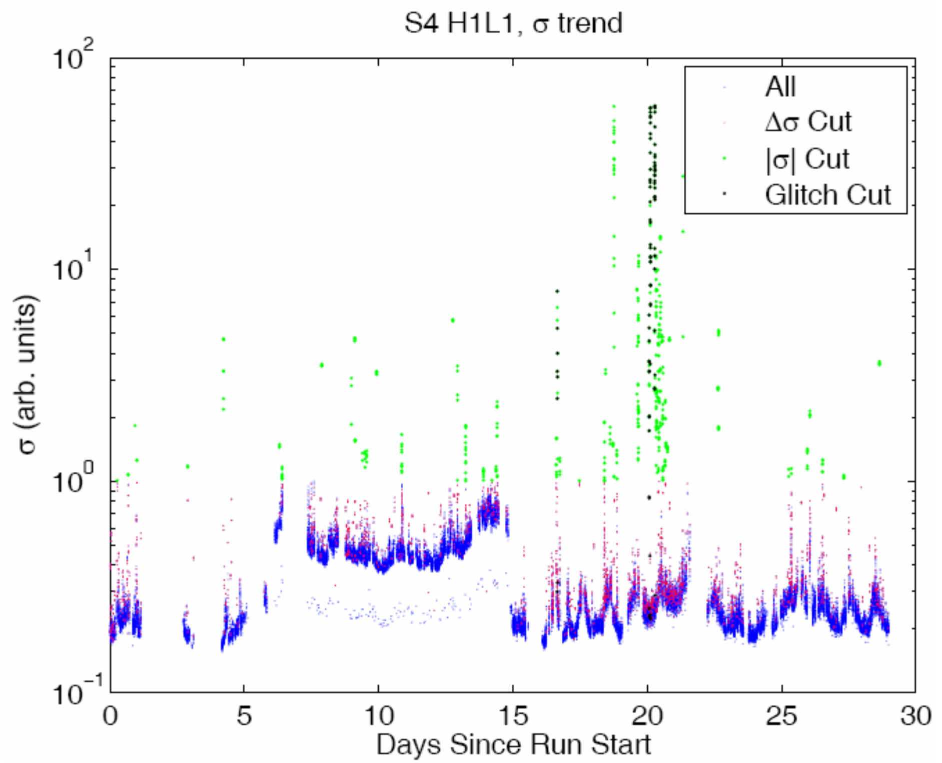


S4: Frequency Notching

- To avoid the 1 Hz harmonics:
  - » Use 1/32 Hz resolution instead of ¼ Hz.
  - » Use 192-sec segments instead of 60-sec segment.
- PSD's calculated by averaging 22 periodograms (50% overlapping).
  - » Bias increases from ~2.1% to ~5.6%.
- Notch 1 bin for:
  - » 1 Hz harmonics
  - » 60 Hz harmonics
  - » Simulated pulsar lines
- Lose ~3% of the total bandwidth.

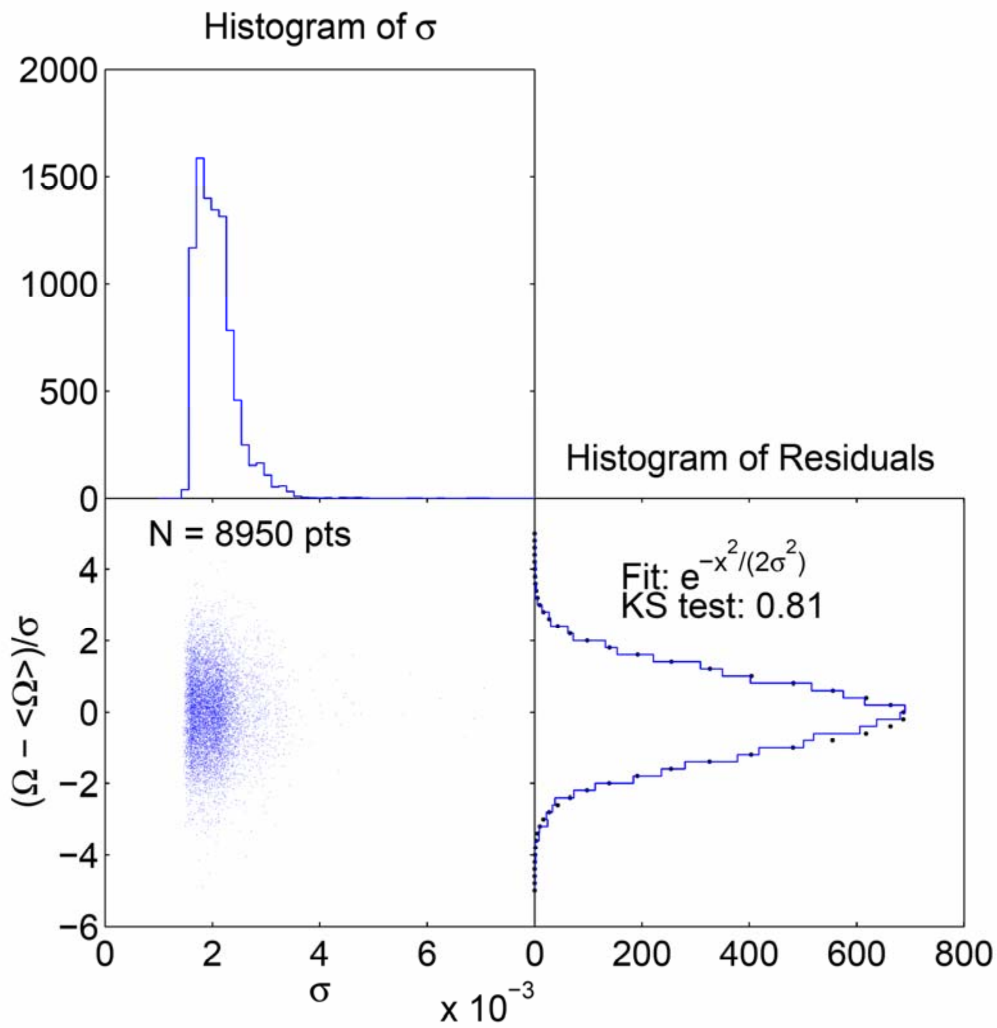
S4: Data Cleaning

- Using 60-sec analysis:
  - » Require $|\sigma_{i\pm1} - \sigma_i| / \sigma_i < 20\%$.
  - » Reject segments with large variance.
  - » Reject a handful of segments identified to contain glitches in coherence studies.
- Use the above bad 60-sec segments to identify bad 192-sec segments.
- Repeat independently for H1L1 and H2L1.
- Reject about 20% of the data.



S4: Gaussianity Checks

- The residuals follow Gaussian distribution.
- Kolmogorov-Smirnov statistic: 81%

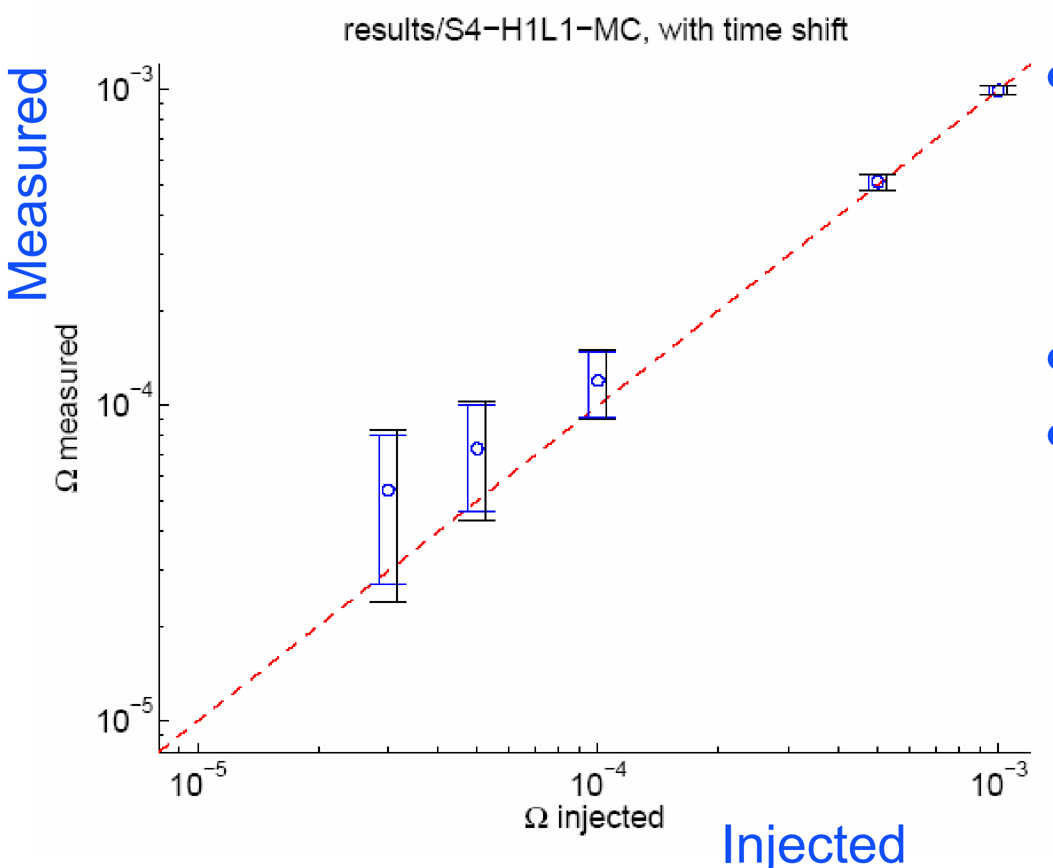


S4 Hardware Injections

Intended Ω	Actual Injected Ω	H1L1 Recovered Ω	H2L1 Recovered Ω
Pre-calculated at 0.04	3.6×10^{-2}	Calibration missing	$(3.6 \pm 0.5) \times 10^{-2}$
"On-the-fly" at 0.04	4.0×10^{-2}	$(3.4 \pm 0.1) \times 10^{-2}$ with 2-sec shift	$(3.5 \pm 0.2) \times 10^{-2}$ with 2-sec shift
Pre-calculated at 0.01	9.4×10^{-3}	$(8.5 \pm 0.9) \times 10^{-3}$ with no shift	$(1.1 \pm 0.2) \times 10^{-2}$ with no shift
"On-the-fly" at 0.005	Not checked yet	$(3.9 \pm 0.2) \times 10^{-3}$ with 22-sec shift	H2 corrupted

- "On-the-fly" injection code bug introduced LHO-LLO relative time-shift.
- Expect up to 10% change in σ due to calibration updates.

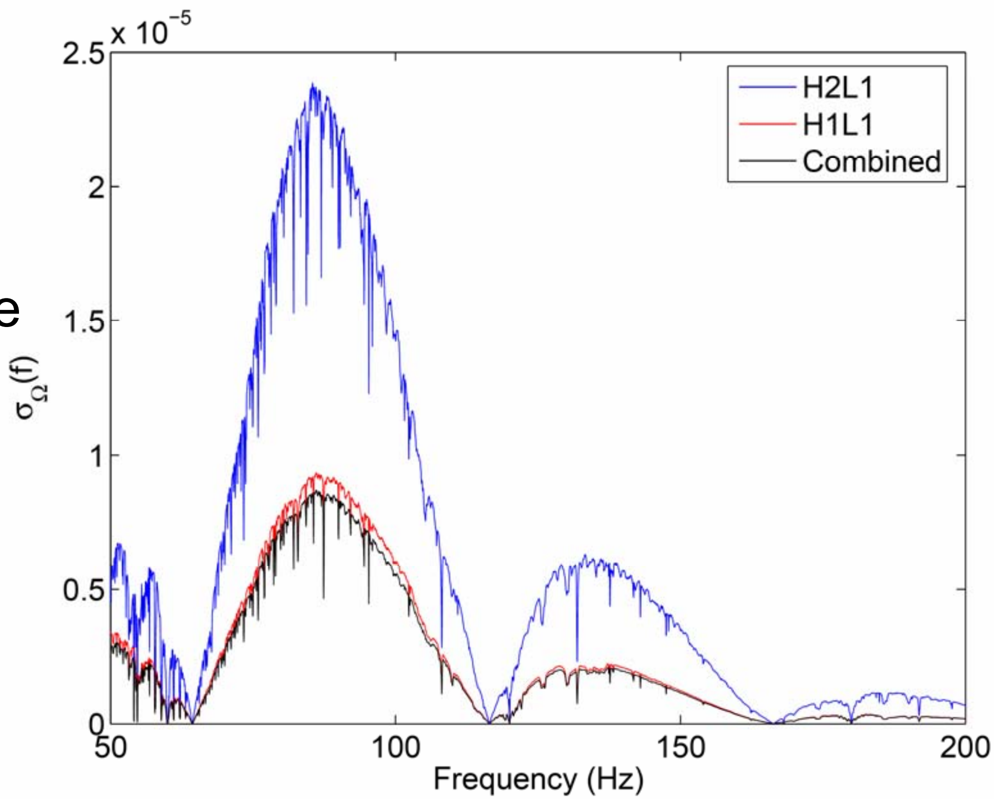
S4 Software Injections



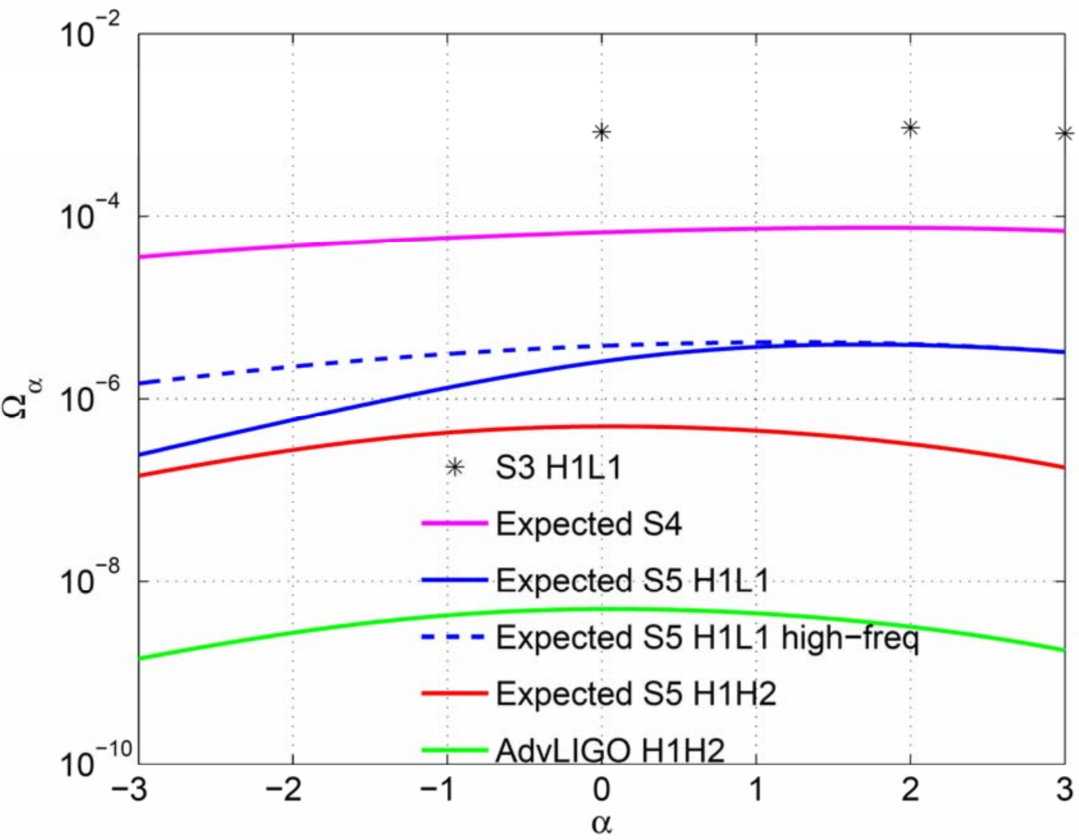
- 10 trials at each amplitude.
  - » Random relative shift in each trial, to properly sample distribution.
- Using a subset of S4 data.
- Theoretical variance agrees well with empirical standard deviation.

Expected Sensitivities

- H1L1: $\sigma_\Omega = 4.3 \times 10^{-5}$
- H2L1: $\sigma_\Omega = 1.1 \times 10^{-4}$
- Weighed average of H1L1 and H2L1 results:
  - » Separately for EACH frequency bin.
  - » Weights: 1/variance(f).
- Optimize frequency range to include 99% of inverse variance.
- Combined: $\sigma_\Omega = 4.1 \times 10^{-5}$
  - » h = 0.72
  - » Bias factor: 1.0556
- The theoretical errors may change by up to 10% due to calibration updates.



Reach as a Function of Spectral Slope



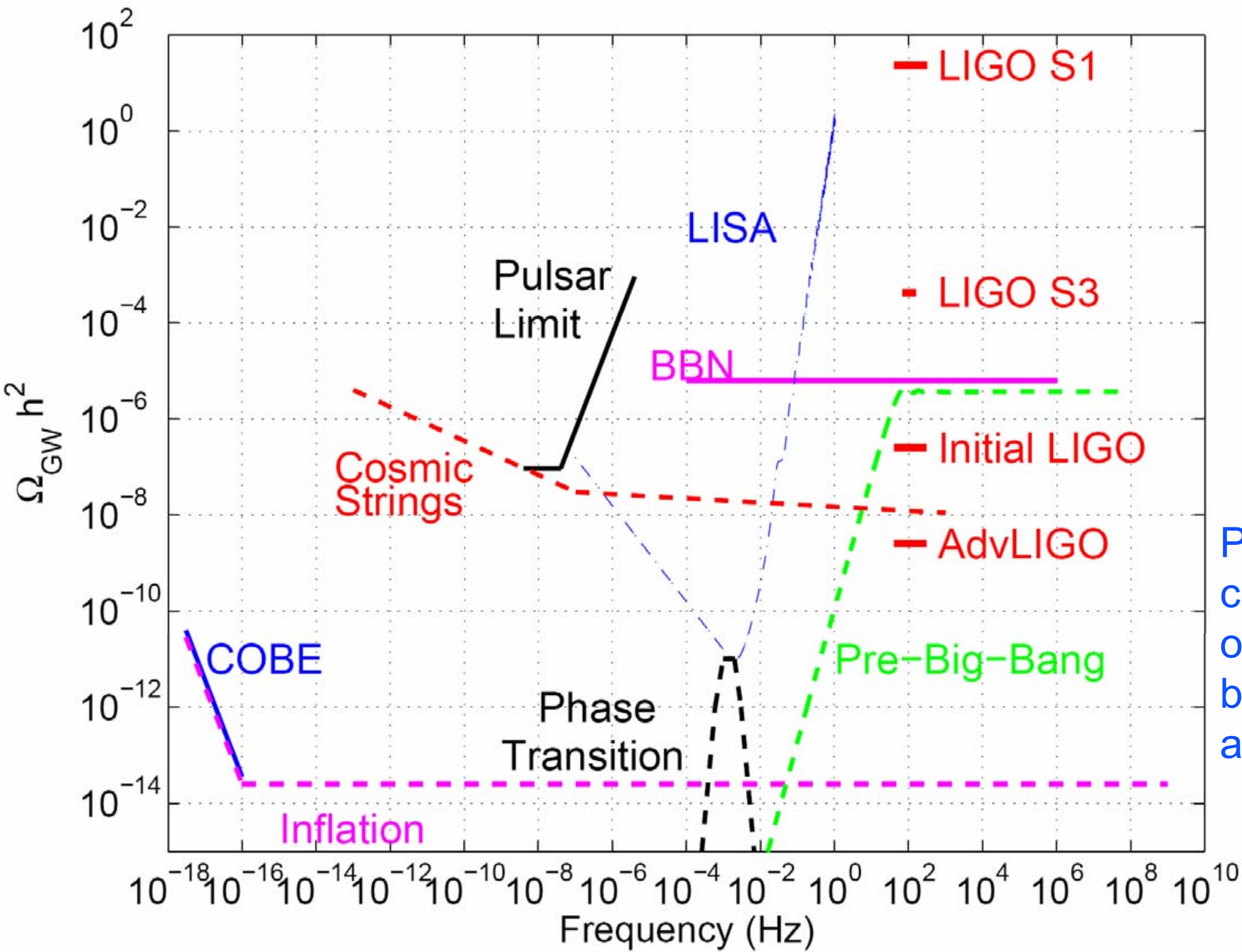
- S3 H1L1: Bayesian 90% UL on Ω_α for three values of α.

- Expected S4: using measured combined H1L1+H2L1 sensitivity.

- Expected S5: design strain sensitivity and 1 year exposure.
    - For H1L1, sensitivity depends on frequency band.

- Expected AdvLIGO: 10x better strain sensitivity than Initial LIGO design, and 1 year exposure.

Landscape

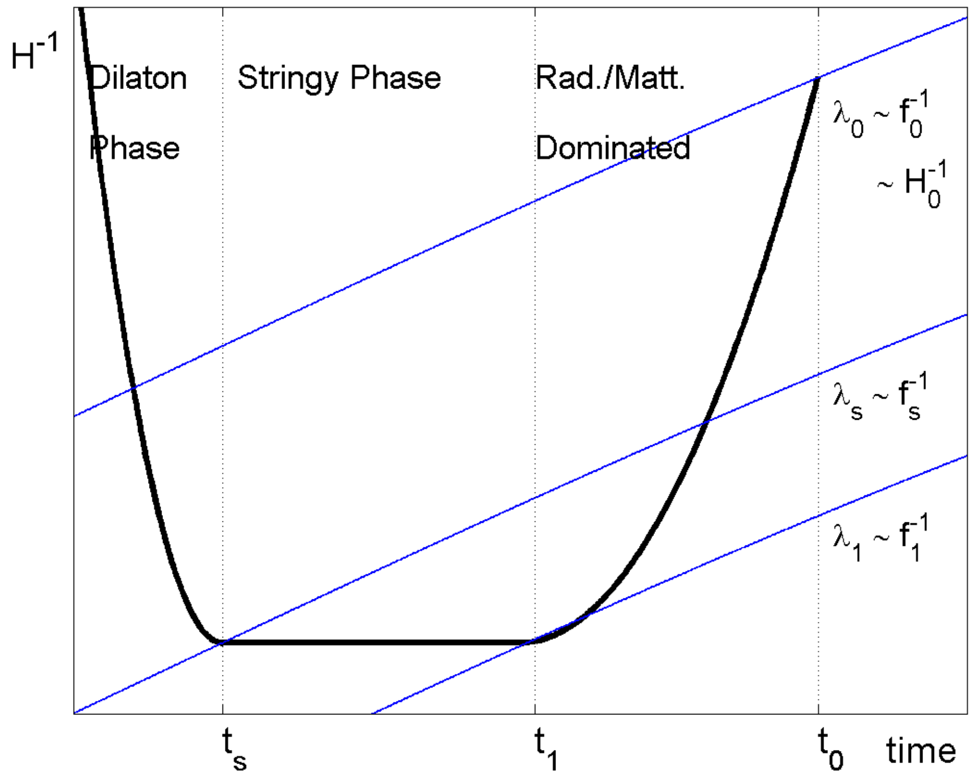


Pre-Big-Bang Models can easily escape other experimental bounds and be accessible to LIGO.

Pre-Big-Bang Models

- Amplification of vacuum fluctuations
  - » Transition from one regime to another in the Universe (eg inflation to radiation dominated) on time-scale ΔT
    - – For cosmological setting, $\Delta T \sim H^{-1}$.
  - » Vacuum fluctuations are amplified only if transition is fast:
    - – $f \ll (2\pi\,\Delta T)^{-1}$ or $\lambda \gg 2\pi\, H^{-1}$ - i.e. super-horizon modes!
- Inflation:
  - » De Sitter inflation phase
  - » Radiation-dominated phase
  - » Matter-dominated phase.
- Pre-Big-Bang Models:
  - » Dilaton-dominated phase
  - » Stringy phase
  - » Radiation, followed by matter phase.

Pre-Big-Bang Models

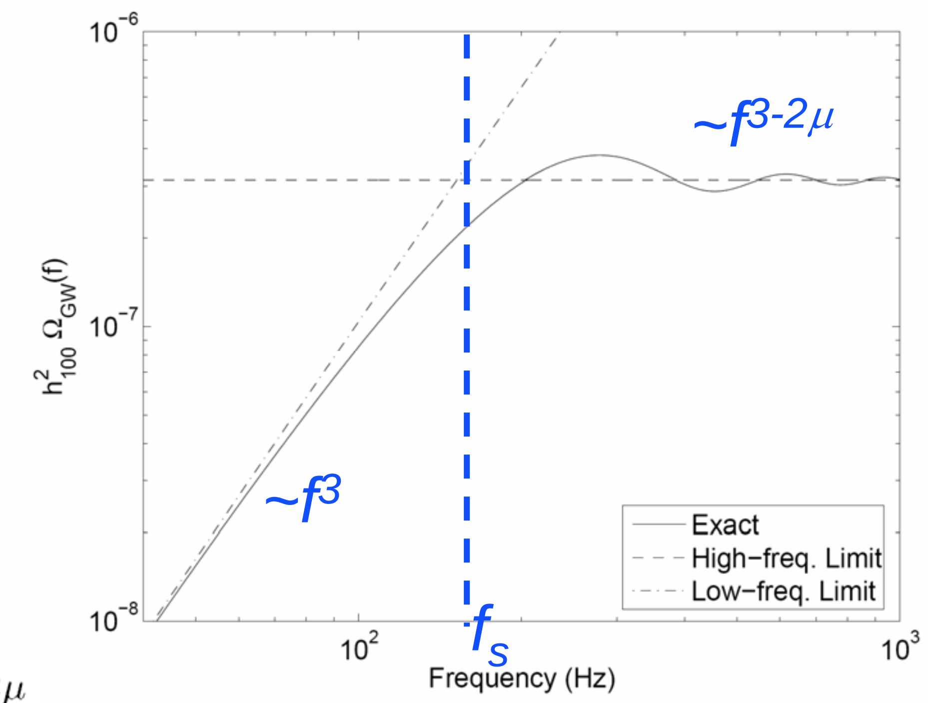


- Dilaton phase described by low-energy string effective action – good theoretical control.
  - » Universe "shrinks" until it reaches string scale.
- Transition to stringy phase at t_s – not well understood.
  - » Assume linearly growing dilaton, constant H – simplest guess.
  - » t_s not known.
- Transition to radiation dominated phase at t_1.
  - » "Big-Bang"
  - » Usual evolution of the Universe follows.

PBB Gravitational Wave Spectrum

- Low-frequency limit: ~f^3

$$h_{100}^2\Omega_{\rm GW}(f) \simeq \frac{(2\mu-1)^2}{192\mu^2\alpha}\frac{(2\pi f_s)^4}{H_{100}^2 M_{Pl}^2}\left(\frac{f_1}{f_s}\right)^{2\mu+1}\left(\frac{f}{f_s}\right)^3 \times\Big\{(2\mu\alpha-1+\alpha)^2 + \frac{4}{\pi^2}\Big[(2\mu\alpha-1+\alpha)\Big(\ln\frac{\alpha f}{2f_s}+\gamma_E\Big)-2\Big]^2\Big\}$$



- High-frequency limit: ~$f^{3-2\mu}$
  - » Independent of f_s
  - » Amplitude ~ f_1^4

$$h_{100}^2\Omega_{\rm GW}(f) \simeq \frac{4b(\mu)}{\pi^2\alpha}\frac{(2\pi f_1)^4}{H_{100}^2 M_{Pl}^2}\left(\frac{f}{f_1}\right)^{3-2\mu}$$

PBB Model Parameters

In collaboration with Alessandra Buonanno: **astro-ph/0510341**

- Typically, think of 2 free parameters:
  - » μ - determines the high-frequency slope
    - – $\mu < 1.5$ from theory.
    - – $\mu < 1$ not really accessible to LIGO.
    - – We consider $1 < \mu < 1.5$;
  - » f_s – the "turn-over" frequency
    - – From theory: $0 - f_1$
    - – $f_s < 30$ Hz preferred by LIGO due to f^3 at low frequency.
    - – If $f_s < 30$ Hz, then it does not impact LIGO accessibility much.
- But: High-frequency limit goes as $f_1^{\,4}$.
  - » It is estimated better by theory:

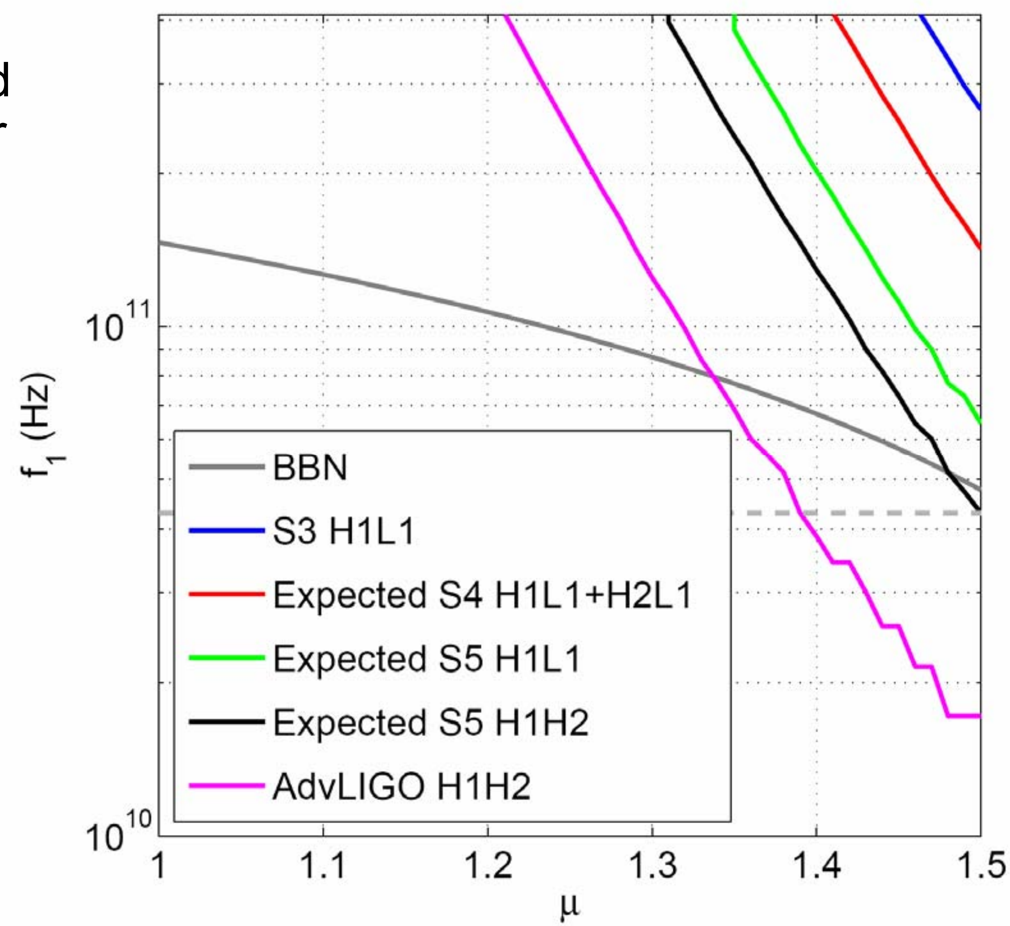
$$f_1 \simeq 4.3 \times 10^{10}\ \mathrm{Hz} \left(\frac{H_s}{0.15 M_{Pl}}\right) \left(\frac{t_1}{\lambda_s}\right)^{1/2}$$

  - » But it depends on string related parameters, which are not well known.
  - » So, treat it as another free parameter.
    - – Vary by factor of 10 around the most "natural" value.

f_1 - μ Plane

- Scan f_1 - μ plane for f_s=30 Hz.
- For each model, calculate $\Omega_{GW}(f)$ and check if it is within reach of current or future expected LIGO results.
- Beginning to probe the allowed parameter space.
- Currently sensitive only to large values of f_1.
- Sensitive only to spectra close to flat at high-frequency.
- But, not yet as sensitive as the BBN bound:

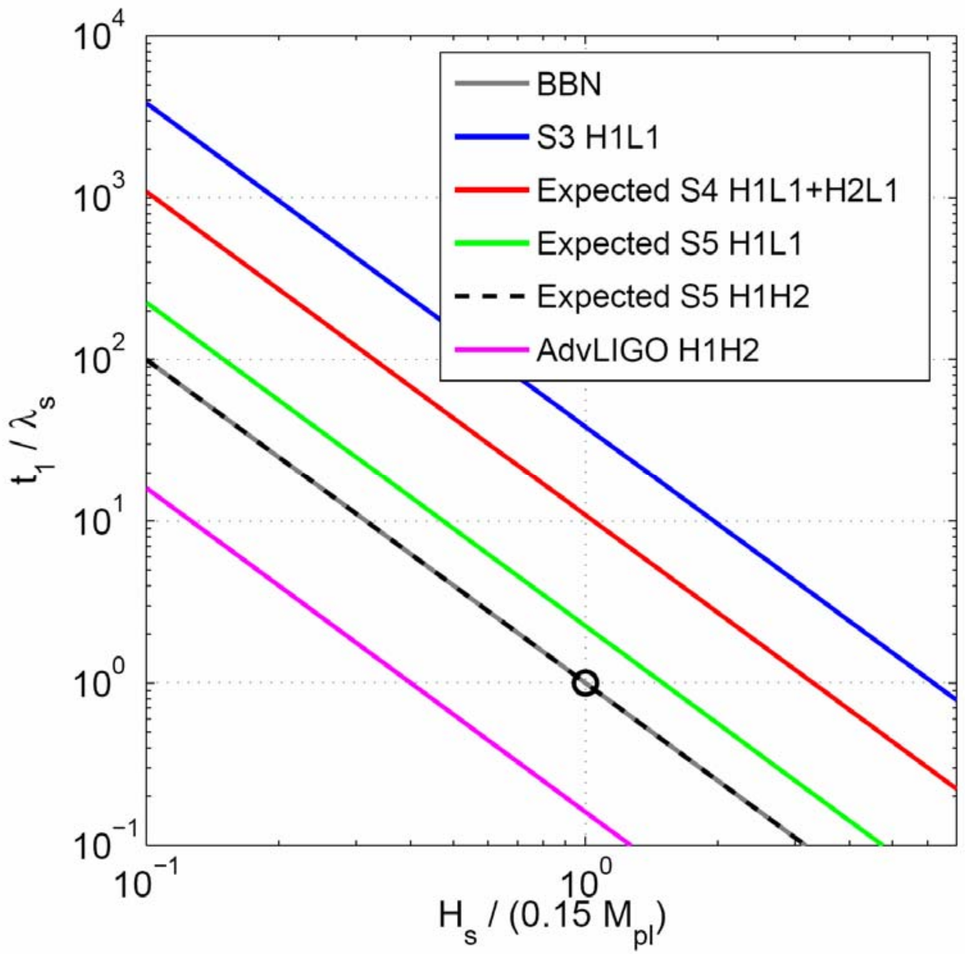
$$\int \Omega_{\rm GW}(f) h_{100}^2 d(\ln f) < 6.3 \times 10^{-6}$$



Limiting f_1

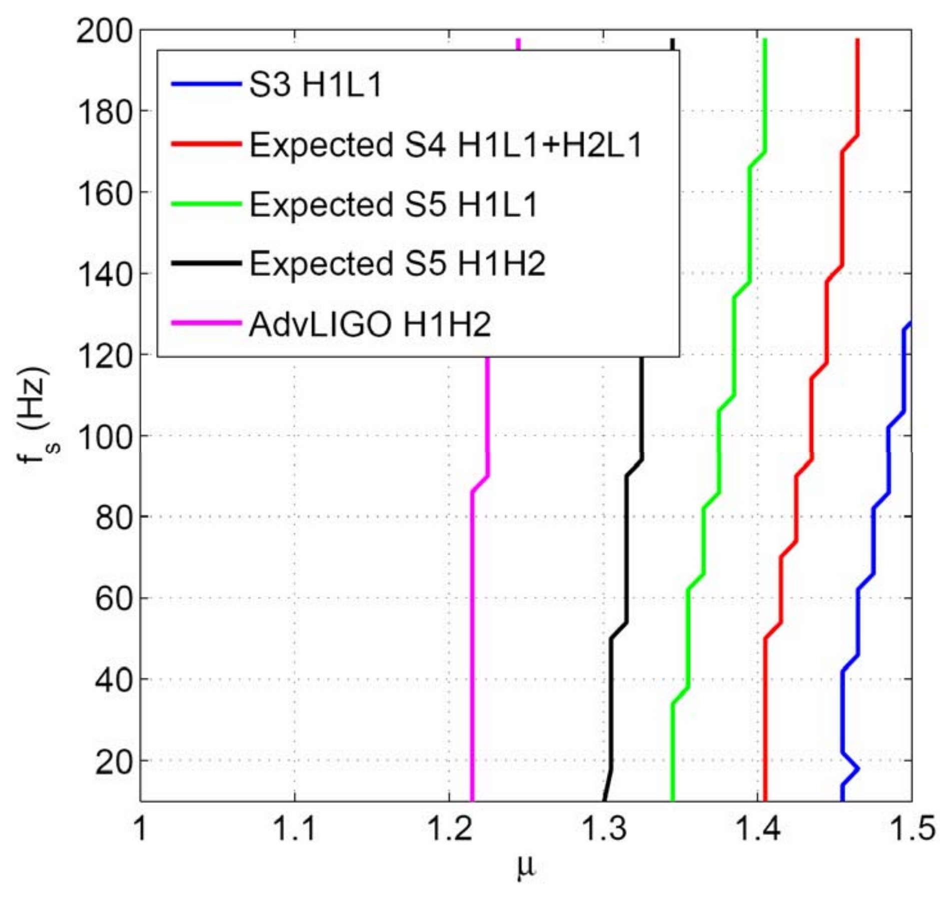
- For given μ and f_s, extract limit on f_1.
- This leads to a limit on more fundamental, string-related parameters.
  - » But only within the PBB model framework!
- Eventually should be able to probe the most “natural” values.

$$f_1 \simeq 4.3 \times 10^{10}\ \mathrm{Hz} \left(\frac{H_s}{0.15 M_{Pl}}\right)\left(\frac{t_1}{\lambda_s}\right)^{1/2}$$



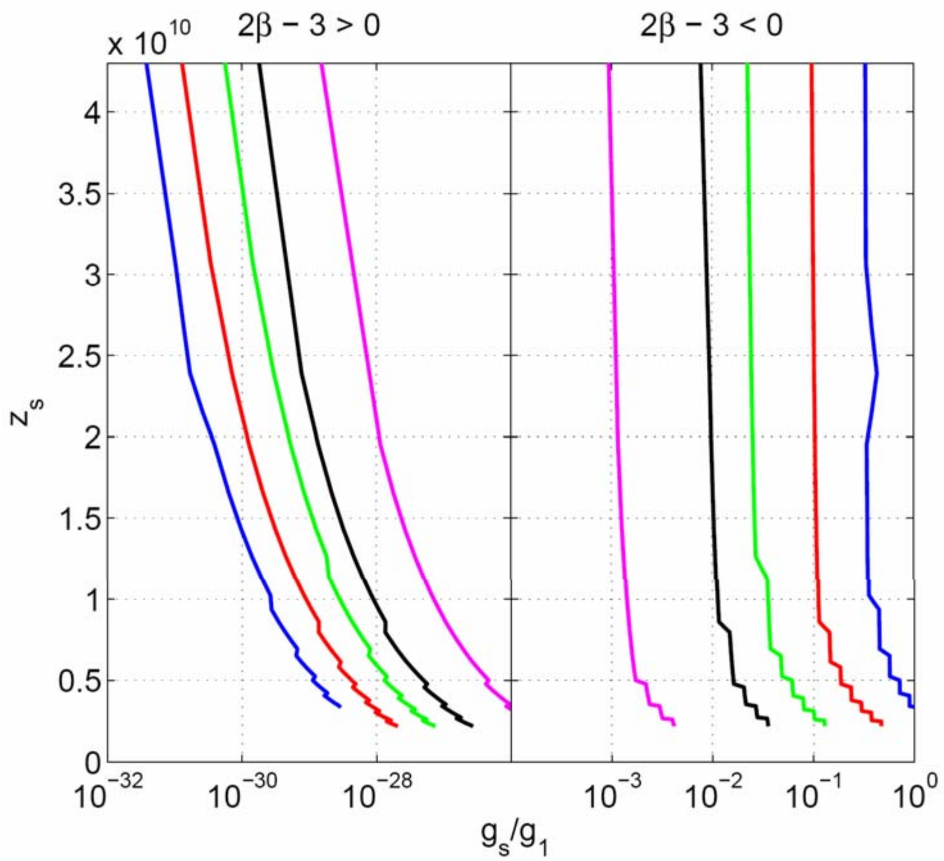
f_s - μ plane

- For the LIGO-favored, large value of $f_1 = 4.3 \times 10^{11}$ Hz, scan the f_s-μ plane.
- Again, for each model, calculate $\Omega_{GW}(f)$ and check if it is within reach of current or future expected results.
- S3 probing f_s<120 Hz, for close to flat spectra, but expect significant improvements for future runs.



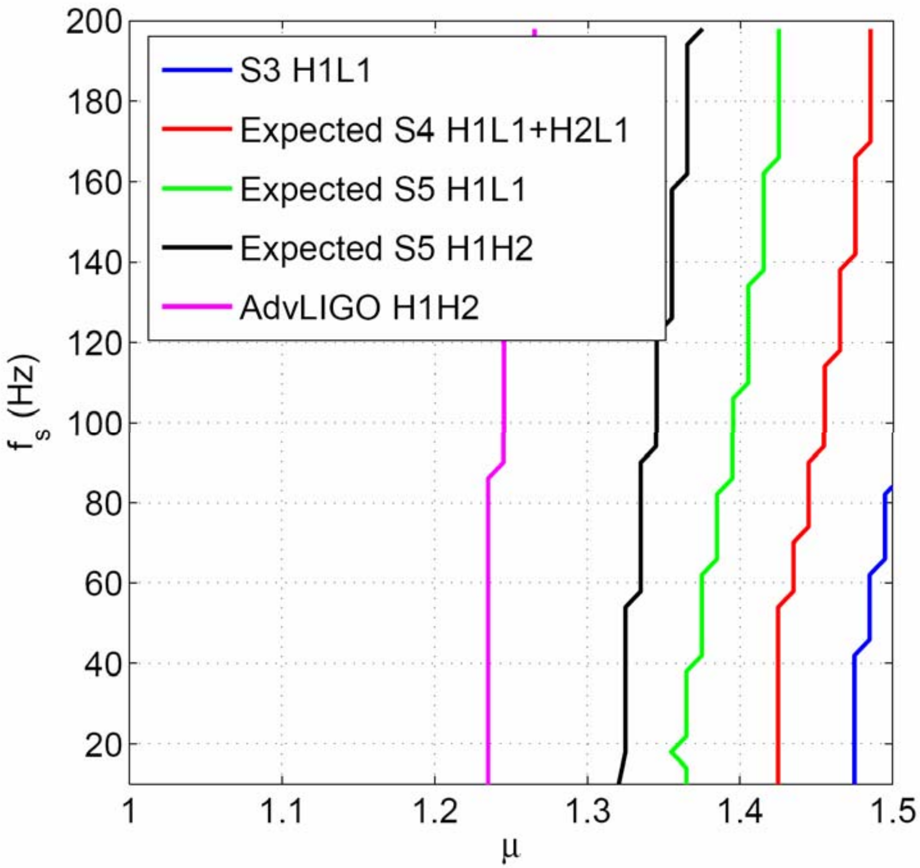
Parameter Redefinition

- Some papers prefer the following redefinition of parameters:
  - » $z_s = f_1/f_s$ is the total redshift in the stringy phase.
  - » $g_s/g_1 = (f_s/f_1)^\beta$, where $2\mu = |2\beta - 3|$
    - – g_s (g_1) are string couplings at the beginning (end) of the stringy phase
- Another way to probe fundamental, string-related parameters, in the framework of PBB models.
- For $f_1 = 4.3 \times 10^{11}$ Hz, transform the f_s-μ plane into the z_s vs g_s/g_1 plane.



PBB Model Extensions (1)

- Radiation production below the string scale.
  - » Possibly needed to dilute the relics produced at the end of PBB phase.
- The amount, timing, and duration of this entropy production can affect the amplitude AND the shape of the PBB GW spectrum.
- Difficult to model, as not much is known!
- If 50% of current entropy is produced (exactly) at the end of stringy phase, all bounds weaken.
  - » $f_1 = 4.3 \times 10^{11}$ Hz

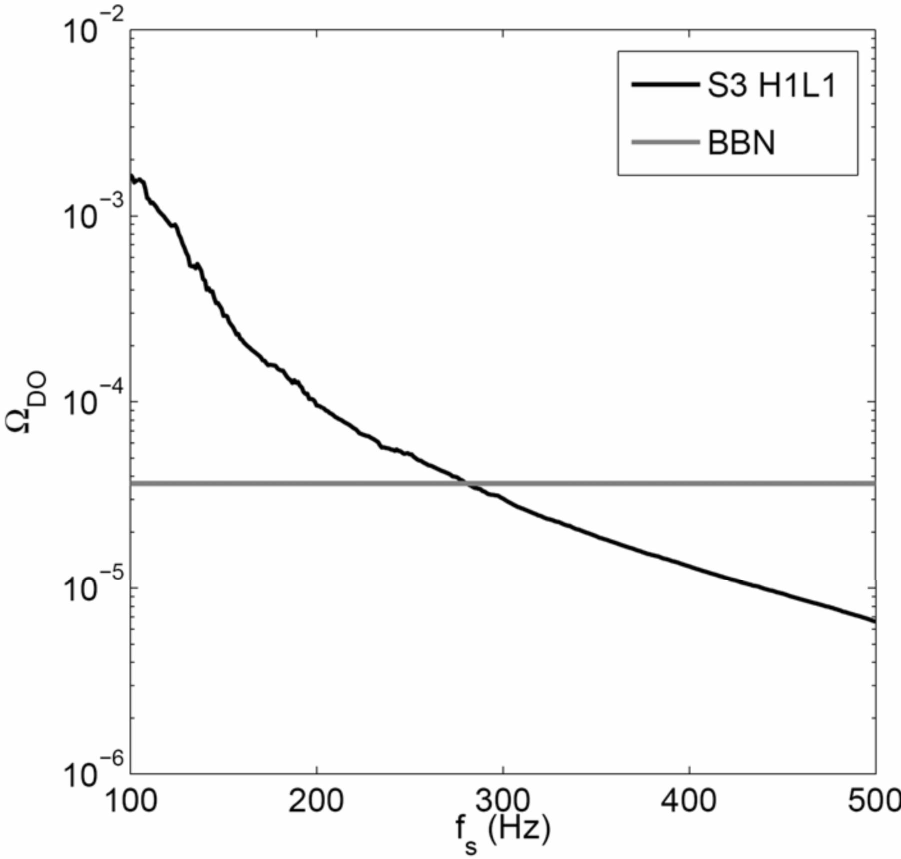


PBB Model Extensions (2)

- Allen and Brustein: no GW were produced in the stringy phase.

$$\Omega_{\rm GW}(f) = \begin{cases} \Omega_{\rm DO}\left(\frac{f}{f_s}\right)^3 & f < f_s \\ 0 & f > f_s \end{cases}$$

- Attractive for LIGO, as it represents the class of spectra that could peak in the LIGO band.
- S3 already better than the BBN bound above 300 Hz.
- Gasperini: Many phases are also possible!
  - » Significantly complicates the shape and amplitude of the spectrum.
  - » Correspondingly more difficult to constrain such extensions with LIGO.



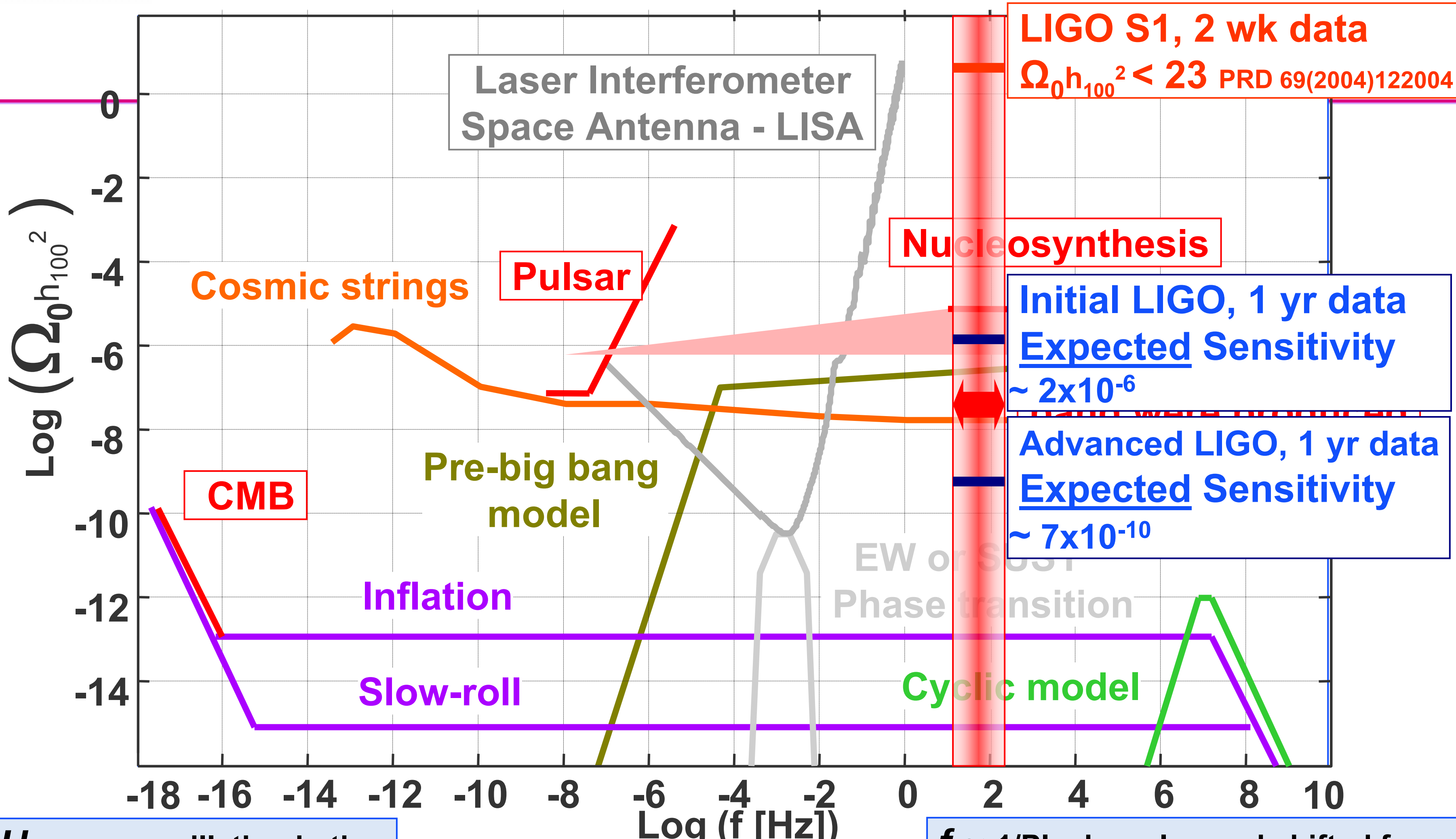
Possible Other Implications

- Low-Mass X-ray Binaries (LMXBs)
  - » Accretion onto neutron star leads to density non-uniformity.
  - » Neutron star spins at ~300 Hz $\Rightarrow$ GW at ~600 Hz.
  - » Integrating over the whole Universe could lead to observable signal.
  - » Preliminary calculations indicate strain ~10^{-27} – difficult to reach even with Advanced LIGO.
- Cosmic Strings
  - » Could lead to significant signal in LIGO range.
  - » Constrained by the pulsar limit at 10^{-8} Hz.
  - » Should be interesting to see how LIGO compares to the pulsar limit:
    - – The two limits could be complementary.

Conclusion

- LIGO S3: Upper limit at 8.4×10^{-4} for flat spectrum.
- LIGO Expectations – factors of 10:
  - » S4 H1L1 + H2L1: 10x more sensitive than S3 $\Rightarrow \sim10^{-4}$.
  - » S5 H1L1: 10x more sensitive than S4 $\Rightarrow \sim10^{-5}$.
  - » S5 H1H2: Up to 10x more sensitive than S5 H1L1 $\Rightarrow \sim10^{-6}$.
  - » Advanced LIGO: 100-1000x more sensitive than Initial LIGO $\Rightarrow \sim10^{-9}$.
- LIGO S3 result beginning to explore the parameter space of the PBB models.
- Currently, only large f_1 values are accessible.
- Future runs of LIGO and Advanced LIGO should be able to probe even the most "natural" parameter values and surpass the BBN bound.
- LIGO can be used to constrain the fundamental, string-related parameters in the PBB model.

Landscape



f ~ H_0 - one oscillation in the lifetime of the universe

f ~ 1/Plank scale – red shifted from the Plank era to the present time