Robust Vetoes for GW Burst Triggers Using Known Instrumental Couplings

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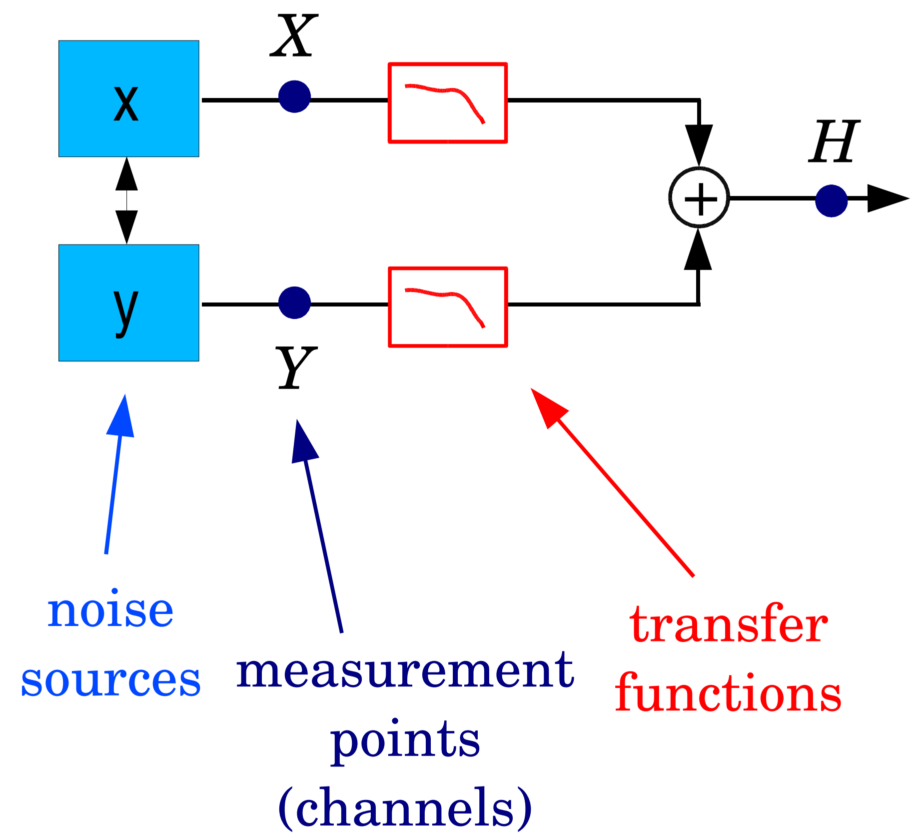
NOISE TRANSFER

- Transfer function from X to H can be measured by injecting noise at X and measuring $X(f)$ and $H(f)$ [1]

$$T_{\mathrm{XH}}(f) = \frac{\tilde{X}(f)^{*}\tilde{H}(f)}{\tilde{X}(f)^{*}\tilde{X}(f)}$$

- Assuming that the system is linear and time-invariant, the noise in X can be *transferred* (*mapped*) to H by

$$\tilde{X}'(f) = T_{\mathrm{XH}}(f)\,\tilde{X}(f)$$



[1] J R Smith et al, CQG 23, 527-537 (2006)

VECTOR SPACE PICTURE

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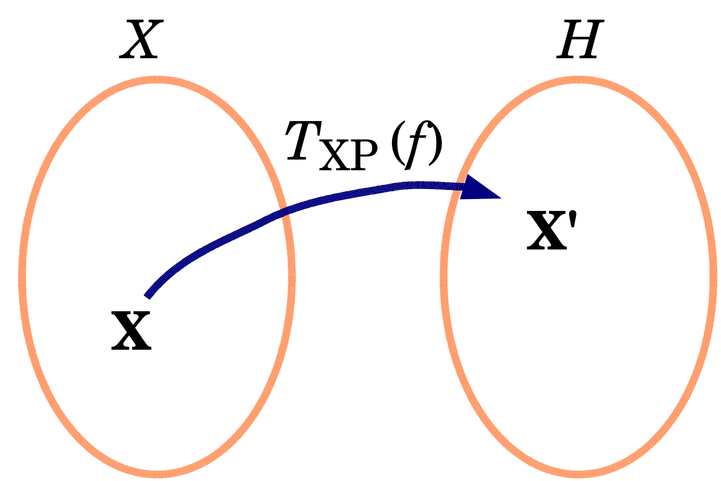

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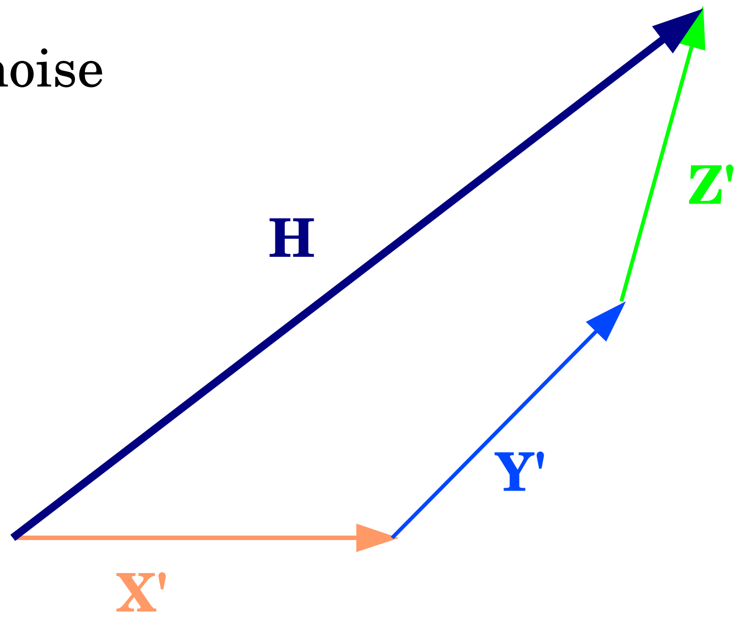
- **H** is made up of many such 'mapped' noise vectors.



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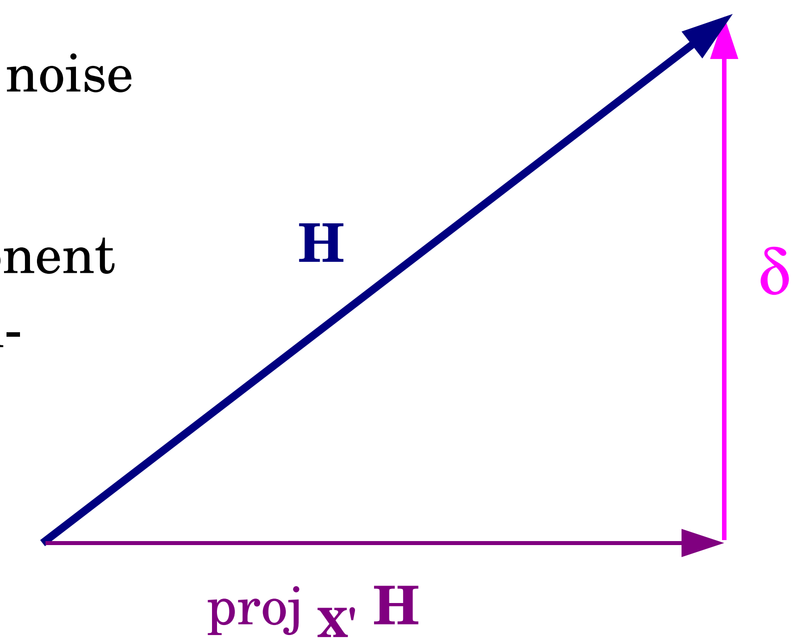
- Transfer function from X to H maps **X** to the space of **H**.
- **H** is made up of many such 'mapped' noise vectors.
- Given **X'**, one can calculate the component of **H** that is orthogonal to **X'** by a Gram-Schmidt orthogonalization

$$\tilde{\delta} = \tilde{\mathbf{H}} - \mathrm{proj}_{\tilde{\mathbf{X}}'} \tilde{\mathbf{H}}$$



VETO STRATEGY

- If a non-stationarity (glitch) originates from channel X, it changes the statistics of that segment of data in channel X, and hence, in channel H.
- But, the statistics of the component of $\mathbf{H}$ that is orthogonal to $\mathbf{X}'$ remain unchanged.
- This can be tested by a statistical hypothesis test.
- We take the set of coincident triggers in channels X and H and compute δ from the segment of the data containing the burst.
- If δ at the time of the burst is statistically the same as at other times ⇒ the non-stationarity is originated from channel X. Thus, we veto the trigger.

TEST STATISTIC

- Construct the 'excess-power' statistic [2] from $\boldsymbol{\delta}$

$$\epsilon = \sum_{k=m}^{m+M} P_k \;, \; P_k = \frac{|\tilde{\delta}_k|^2}{\sigma_k^2}.$$

$\boldsymbol{\delta}$ in kth frequency bin

Expected variance of $\boldsymbol{\delta}$ in kth frequency bin

[2] W G Anderson et al. PRD **63**, 042003 (2001).

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- If the non-stationarity originated in channel X, ϵ will follow a Gamma distribution [3] with some expected parameters.

[3] P E Johnson et al, IEEE Trans. on Signal Proc. **47**, 1255 (1999).

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- Threshold τ giving a rejection probability ψ can be calculated from

$$\psi = \int_0^{\tau} \Gamma(x; \alpha, \beta)\, \mathrm{d}x$$

Prob. density of Gamma dist. with scale parameter α and shape parameter β

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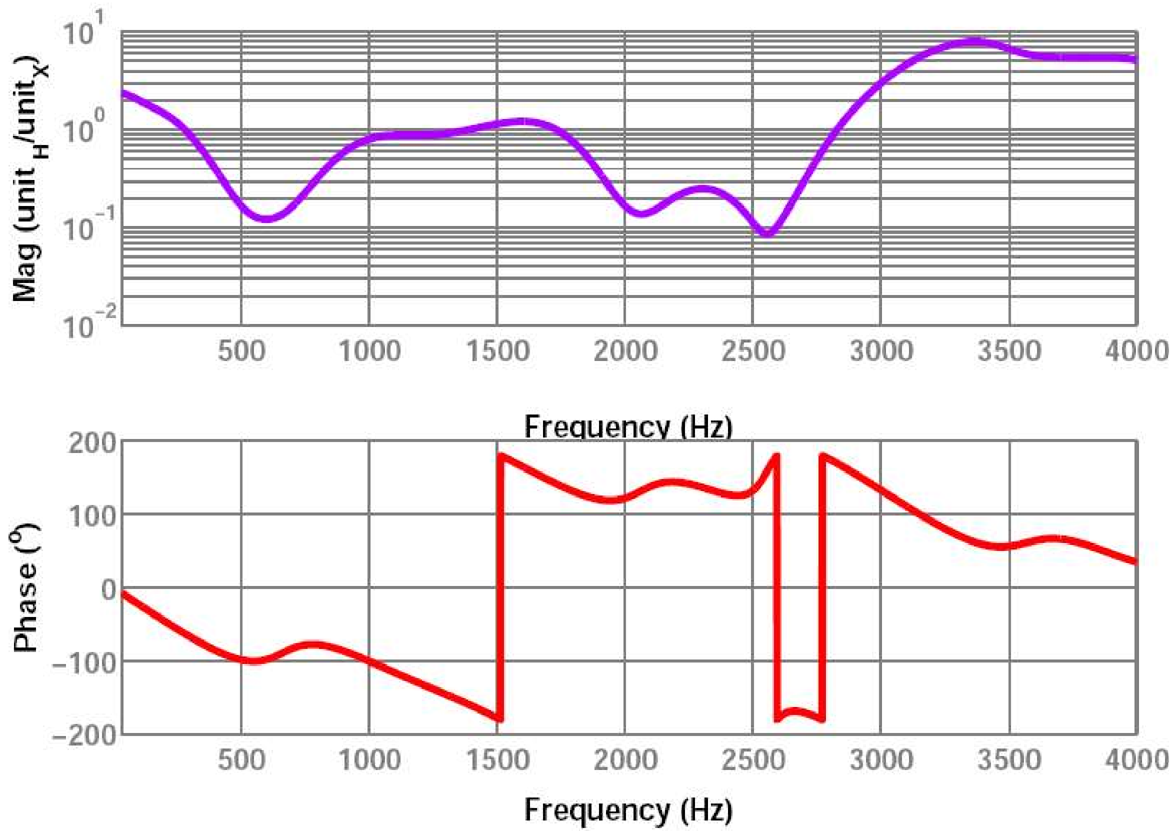
- α and β are estimated from stationary data around the glitch.

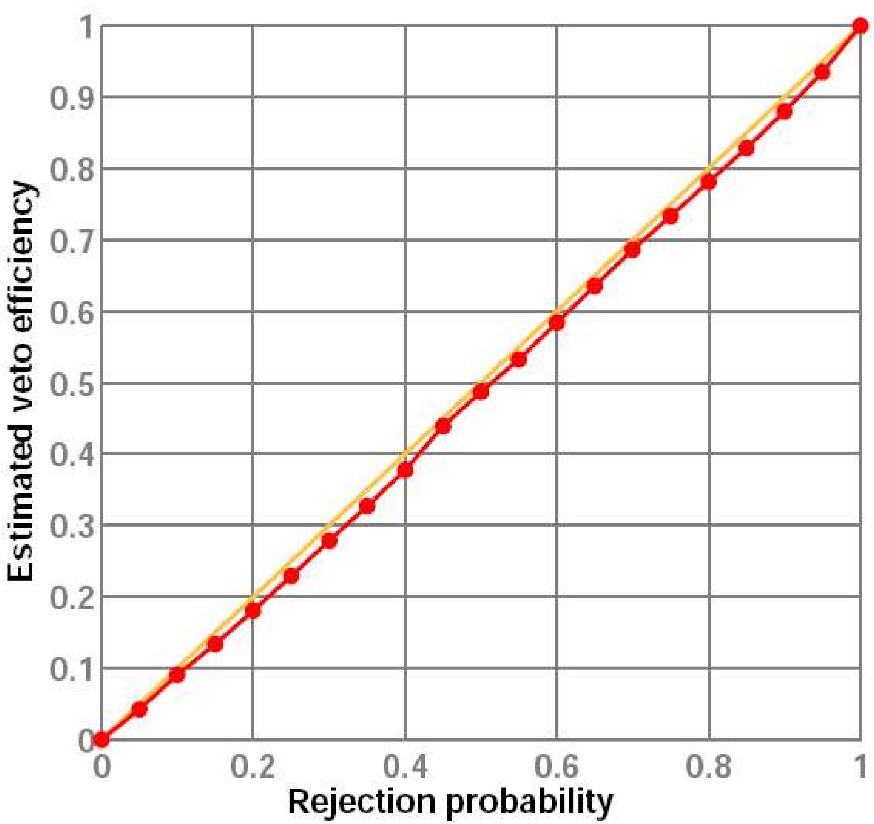
SOFTWARE INJECTIONS

- Q9 sine-Gaussians are injected to white-noise – channel *X*.
- To get channel *H*: Channel *X* data is filtered using a time-domain filter, then some extra noise is added to simulate other components of *H*.
- The response of the filter is the transfer function from *X* to *H*.



SOFTWARE INJECTIONS

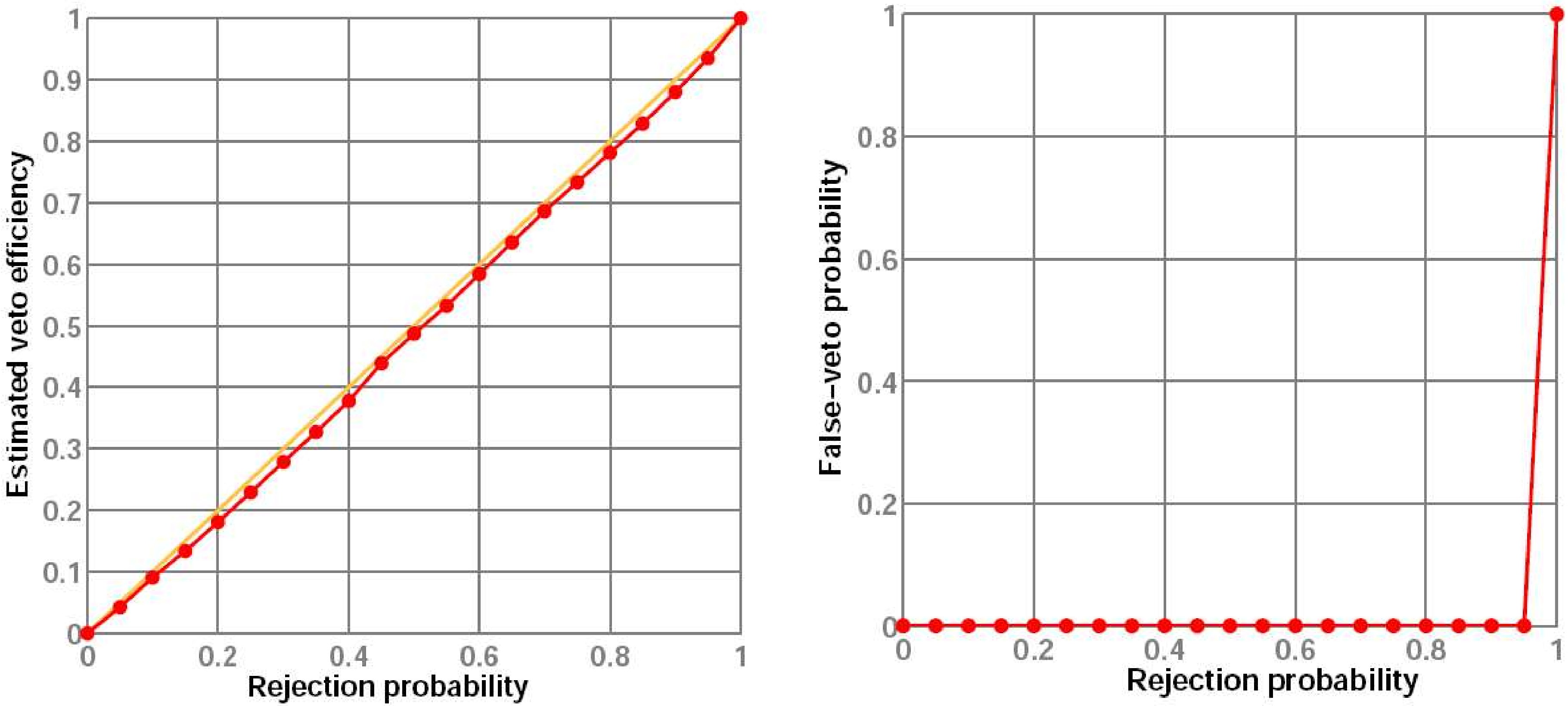
• Perform the veto analysis after choosing different thresholds. Compare the estimated veto fraction with the predicted rejection probability corresponding to each threshold.



SOFTWARE INJECTIONS

• Perform the veto analysis after choosing different thresholds. Compare the estimated veto fraction with the predicted rejection probability corresponding to each threshold.

• A plausible estimation of false-veto probability: Inject SG waveforms with random parameters to X and H. Perform the veto analysis.



AN ALTERNATIVE METHOD: 'TRIGGER MAPPING'

- An ETG is run over channels X and H and two sets of triggers are generated.
- Parameters of the burst triggers in channel X can be mapped to channel H, making use of the transfer function from X to H.
- If a trigger in X, mapped to H, is consistent (in time, frequency and amplitude) with a trigger in channel H, veto it.
- A less-rigorous, but computationally inexpensive method.

TRIGGER MAPPING

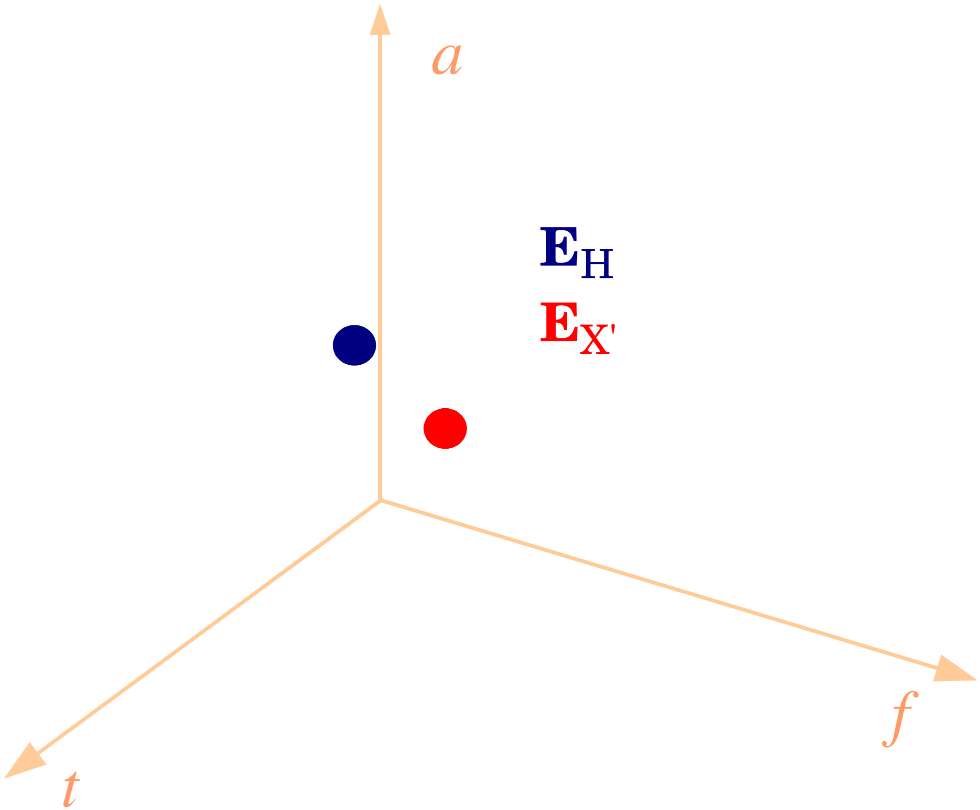
- A certain number of parameters are associated with each event $\mathbf{E}_X$ in channel X, like time, central freq, amplitude → $\mathbf{E}_X$ as point (E_X^a, E_X^f, E_X^t) in a 3-parameter space.

- If we make an assumption about the power spectrum of the underlying burst (like, it is a Gaussian with amplitude E_x^a, central frequency E_x^f etc.), we can map the power spectrum to channel H using the transfer function, and then can re-estimate the parameters from the 'mapped' power spectrum.

- Mapping the point $\mathbf{E}_X$ (in the space of X-triggers) to $\mathbf{E}_{X'}$ (in the space of H-triggers).

IDENTIFYING CONSISTENT EVENTS

If $\mathbf{E}_{X'}$ is 'sufficiently close' to $\mathbf{E}_H$, veto the trigger.



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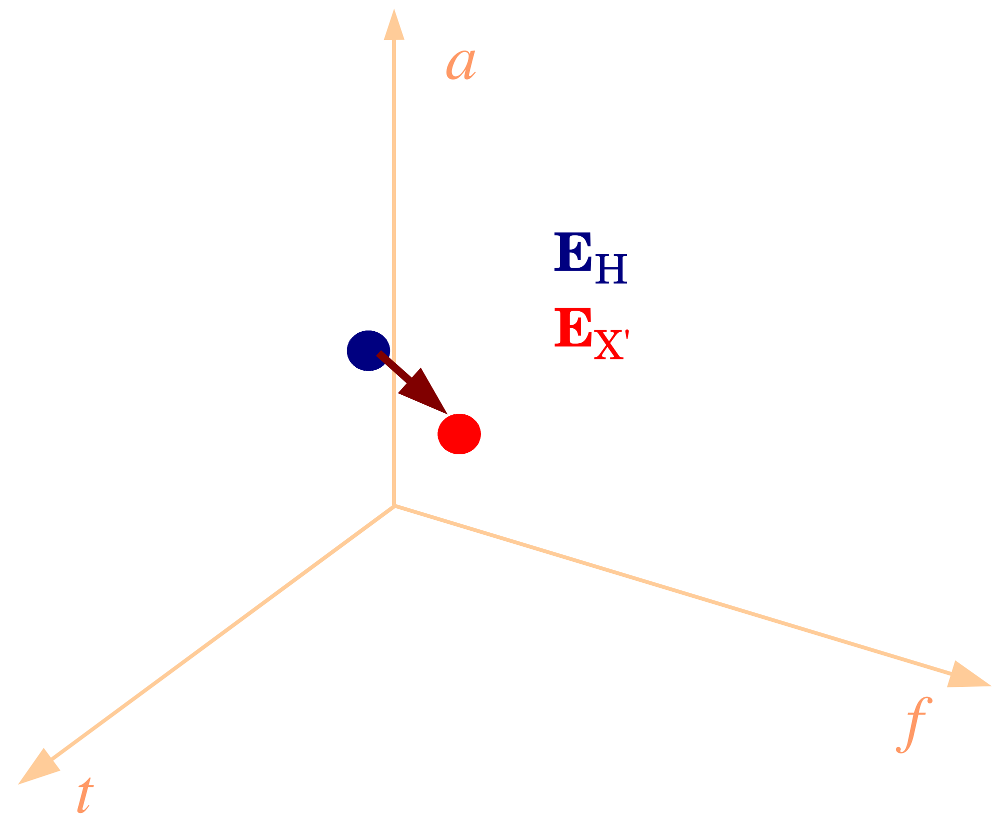
If $\mathbf{E}_{X'}$ is 'sufficiently close' to $\mathbf{E}_H$, veto the trigger.

- Compute the 'distance vector' between $\mathbf{E}_{X'}$ and $\mathbf{E}_H$:

$$\mathbf{w} \equiv \mathbf{E}_H - \mathbf{E}'_X$$



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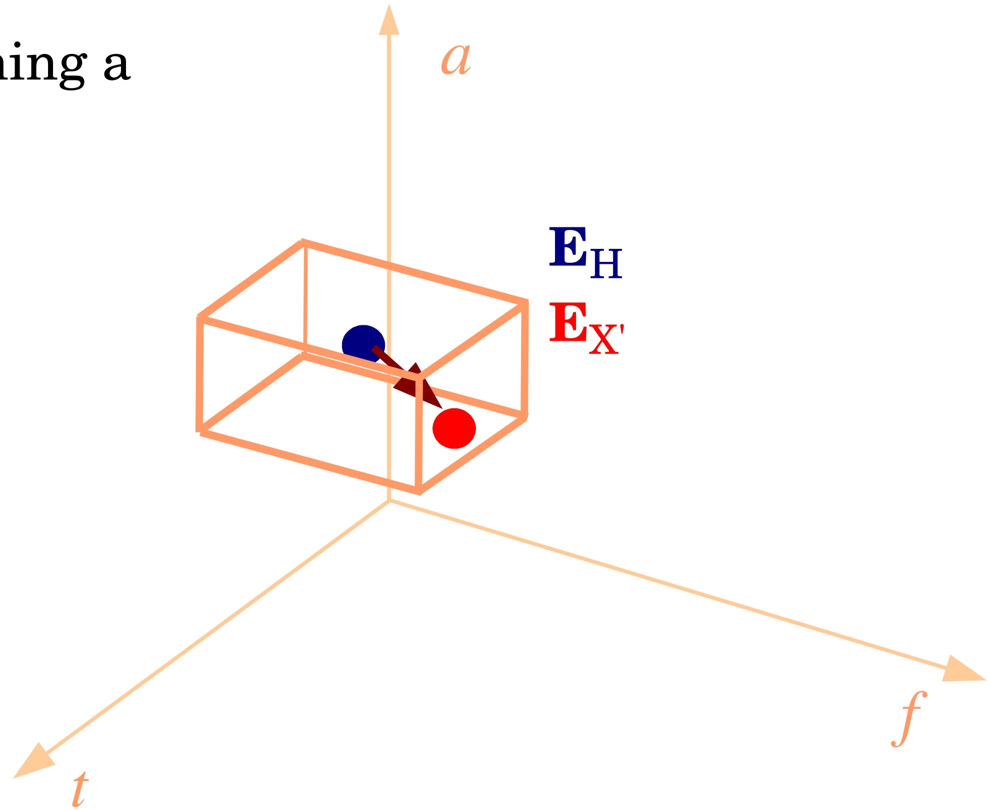
- If $\mathbf{w} \leq \boldsymbol{\tau}$, veto the trigger; thus defining a 'consistency volume'.



IDENTIFYING CONSISTENT EVENTS

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$$\mathbf{w} \equiv \mathbf{E}_H - \mathbf{E}'_X$$

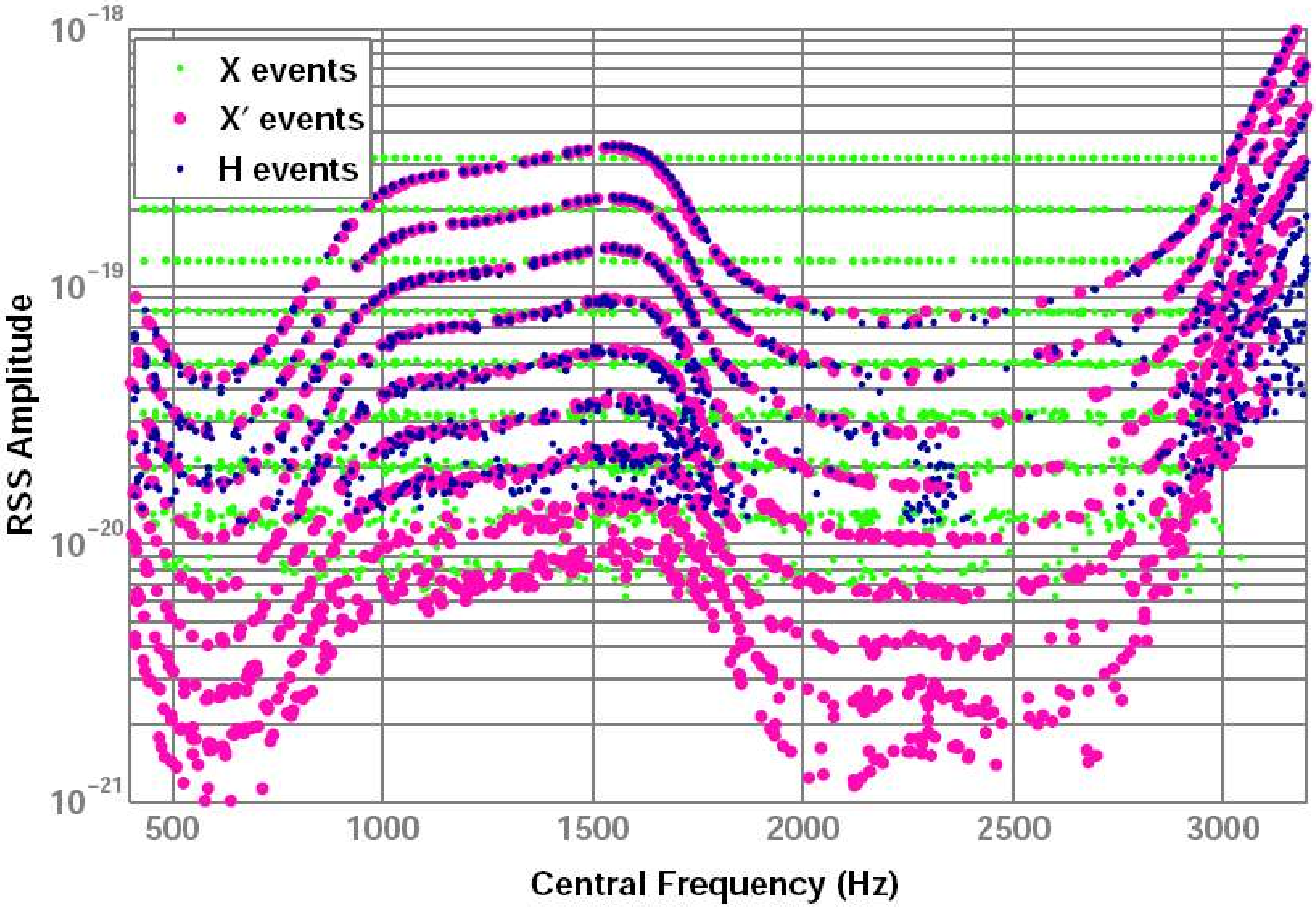
- If $\mathbf{w} \leq \tau$, veto the trigger; thus defining a 'consistency volume'.
- If the ETG errors are Gaussian distributed (with zero mean), $\mathbf{w}$ will be distributed according to Gaussian distributions of mean 0 and variance $\sigma^2(\mathbf{w})$.
- Threshold $\boldsymbol{\tau}$ corresponding go a given rejection probability can be calculated from the prob. densities of Gaussian distributions with mean zero and variance $\sigma^2(\mathbf{w})$.

SOFTARE INJECTIONS

- Data in channels X and H are generated as described before.
- The variance of errors in the parameter-estimation of the ETG is estimated by comparing the trigger-parameters with injected parameters.
- Power spectrum of the bursts in channel X is approximated by a Gaussian. (This is good for SG waveforms. But how good is it in real-life?).
- Veto analysis is performed by choosing different thresholds.

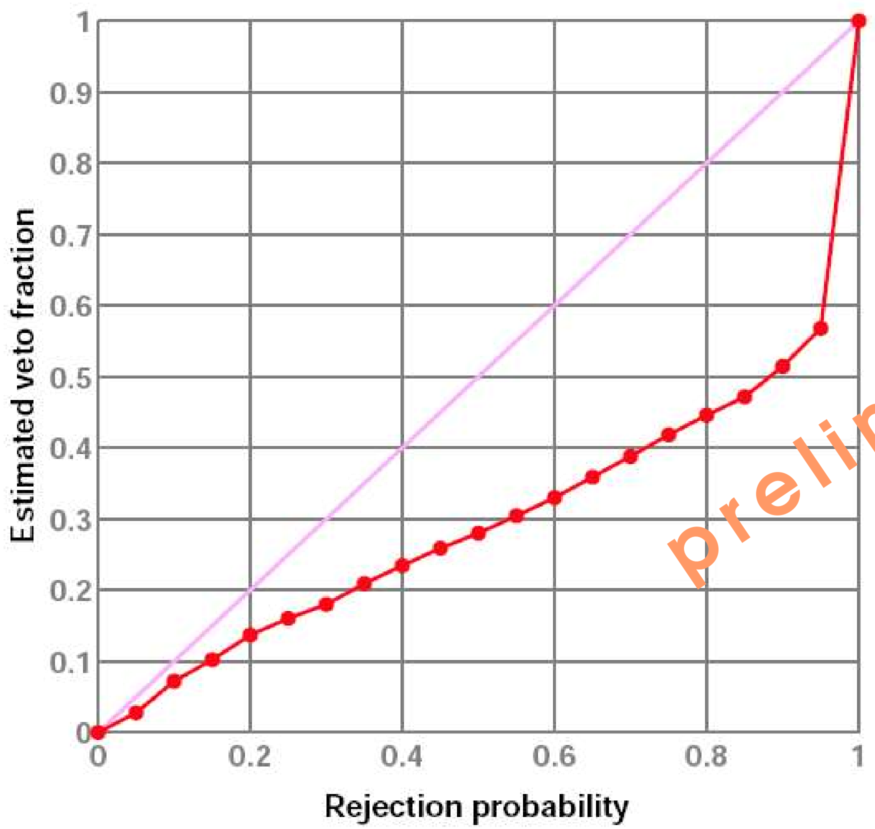
SOFTARE INJECTIONS

- An example from SG injections:

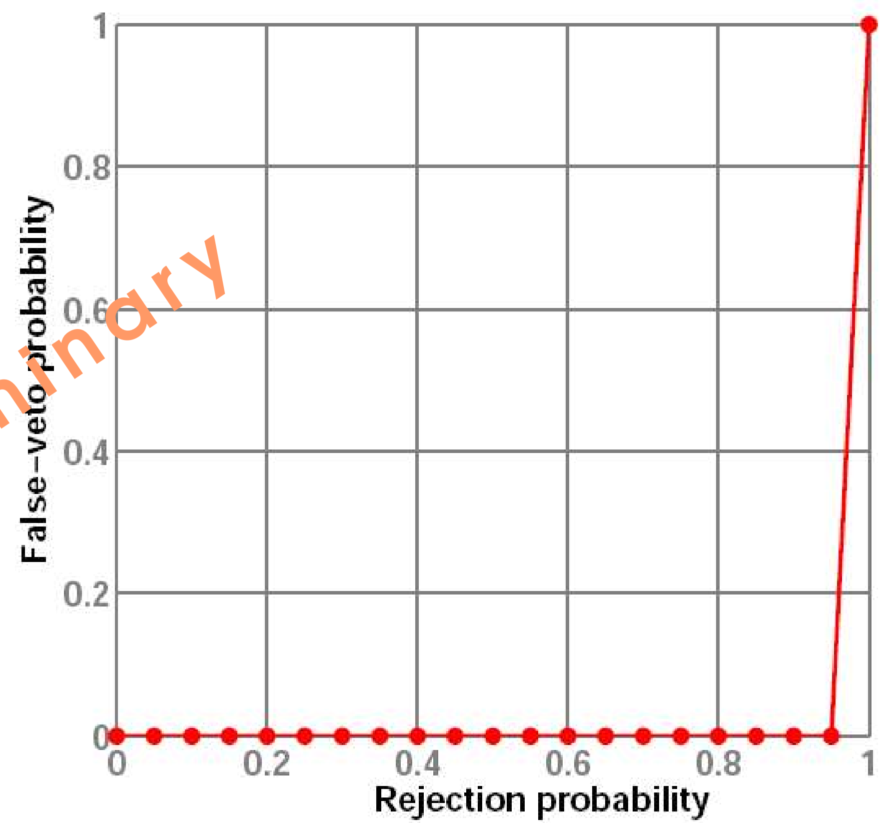


SOFTARE INJECTIONS

Estimating veto efficiency



Estimating false-veto probability



SUMMARY

- A robust veto strategy is formulated making use of the known instrumental couplings.
- Based on projecting the noise at the detector output into two orthogonal directions, making use of the measured transfer function.
- An alternative method – less rigorous, but computationally inexpensive – is proposed, making use of the trigger parameters estimated by the ETG.
- Work in progress to apply in to GEO600 data.