LIGO and the search for gravitational waves from spinning binaries

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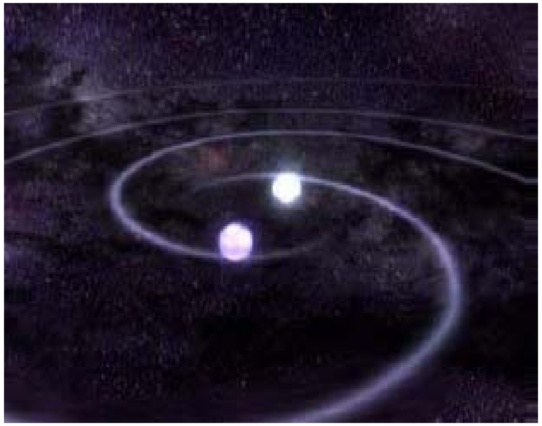
The theory behind gravitational waves

Einstein's equations admit wave-like solutions for weakly perturbed non-static space-times with metric $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$.

- In particular, spinning compact binaries such as neutron stars and black holes are expected to emit energy in the form of gravitational waves as they spiral toward each other until they finally merge.

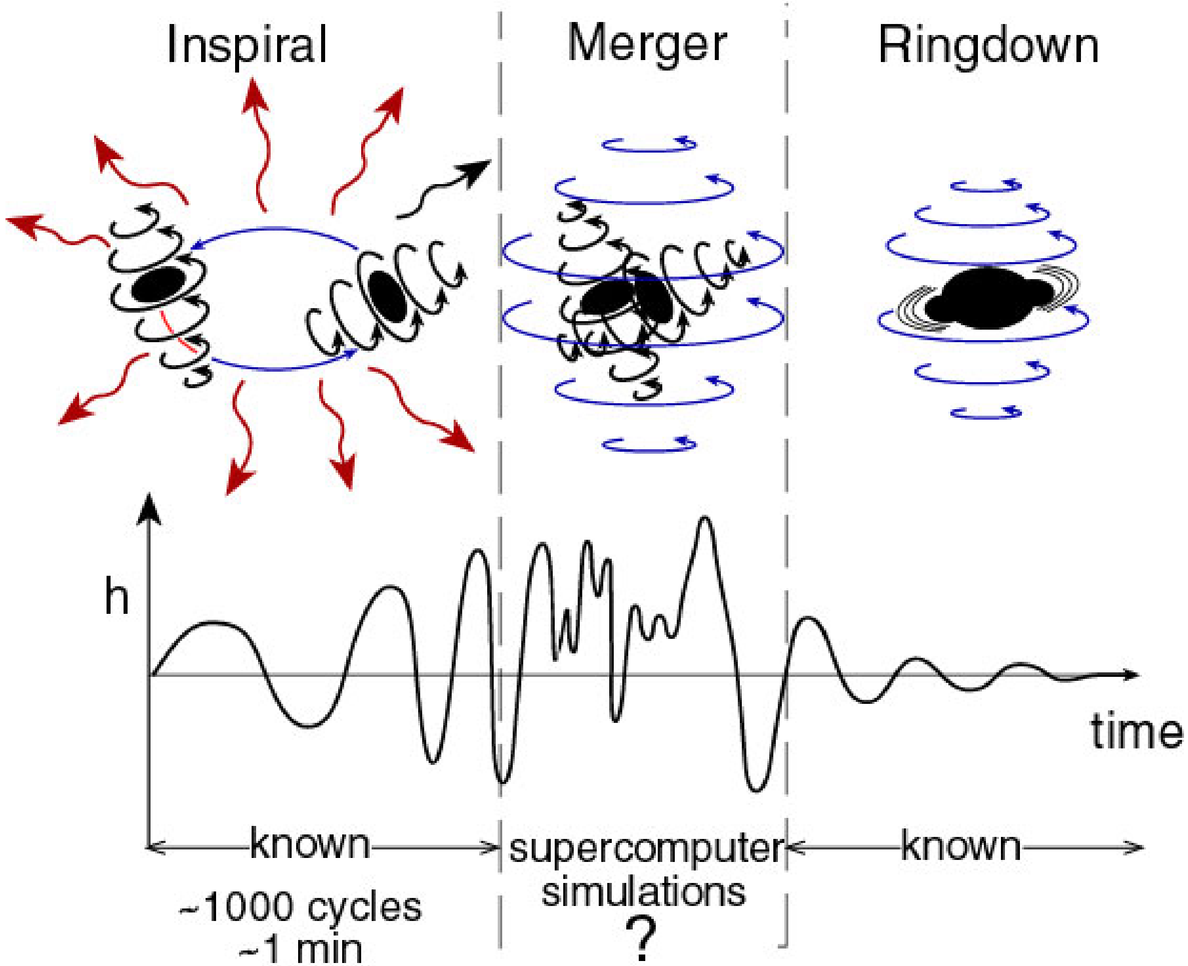


At lowest order the gravitational wave field is described by the quadrupole formula which, in the transverse-traceless gauge, reads

$$h_{jk}^{TT} = \frac{2G}{c^4 r} \frac{d^2 I_{jk}^{TT}(t-r)}{dt^2}$$

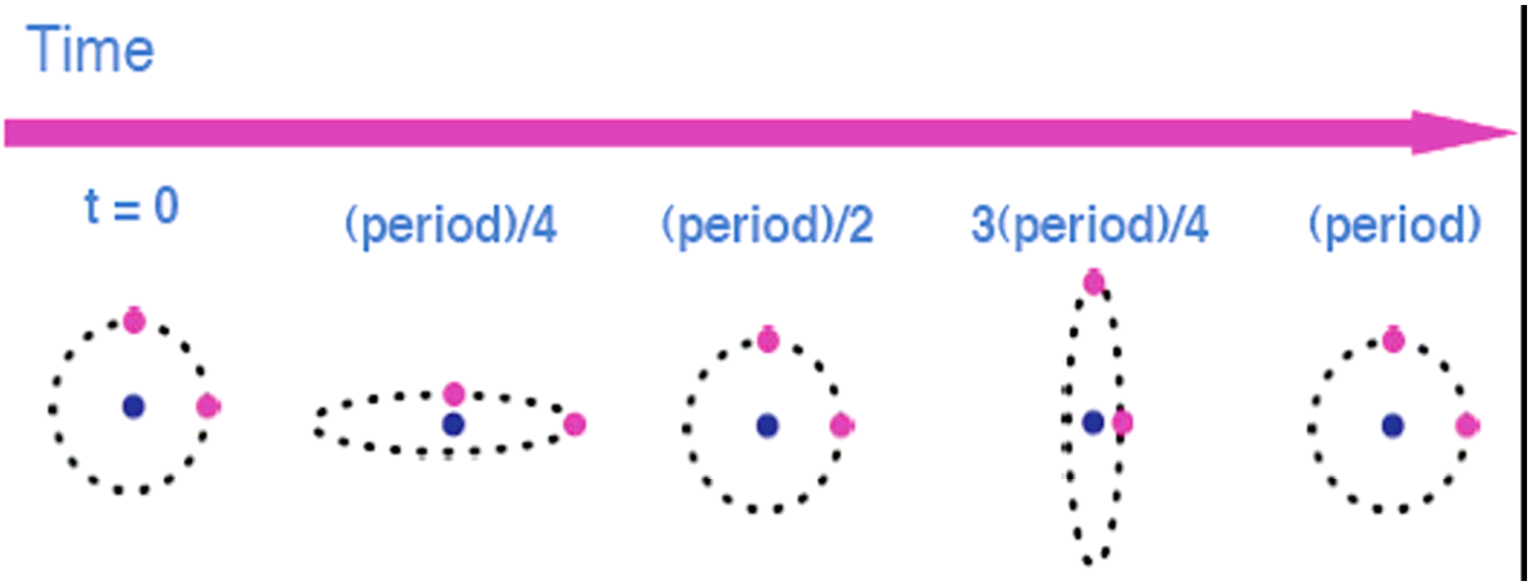
where I_{jk} is the binary quadrupole moment.

The evolution of a binary system



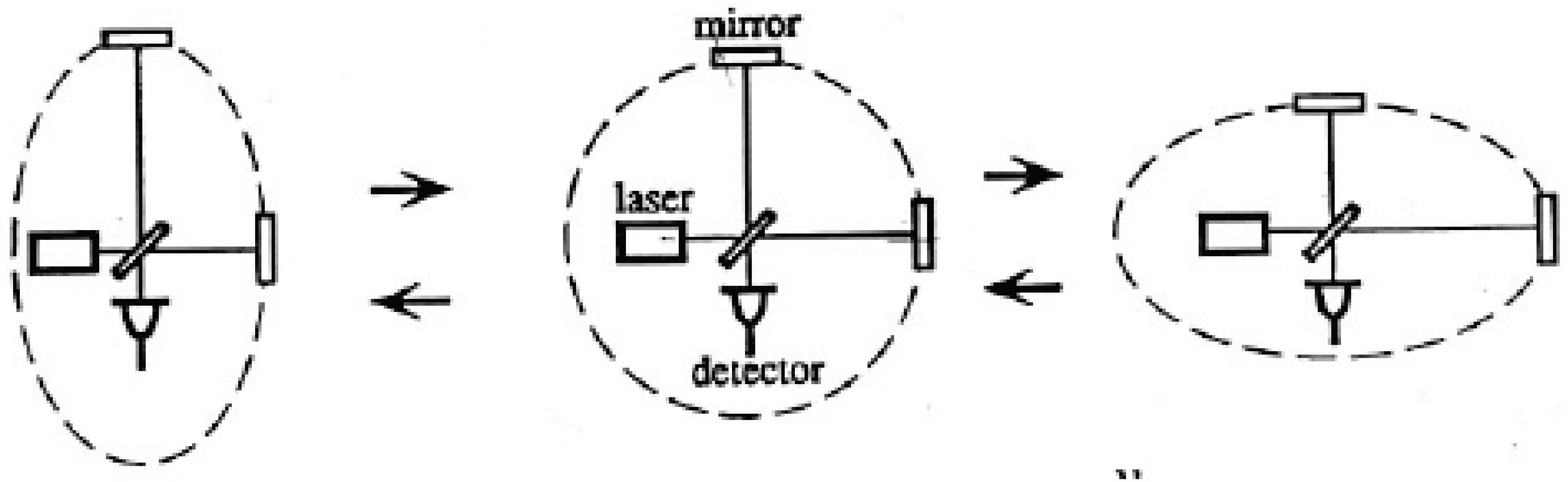
The effect of a gravitational wave

- When a gravitational wave passes through matter it causes the distance between freely falling particles to vary in time. If we consider a ring of freely falling particles, a gravitational wave will induce over a period the motions shown below



What kind of detector do we need?

- Michelson-like interferometers can measure with high accuracy the relative displacement of mirrors placed at the ends of the two orthogonal optical paths, by measuring the phase shift of the two recombining laser beams. The time evolution of this phase describes the gravitational wave!

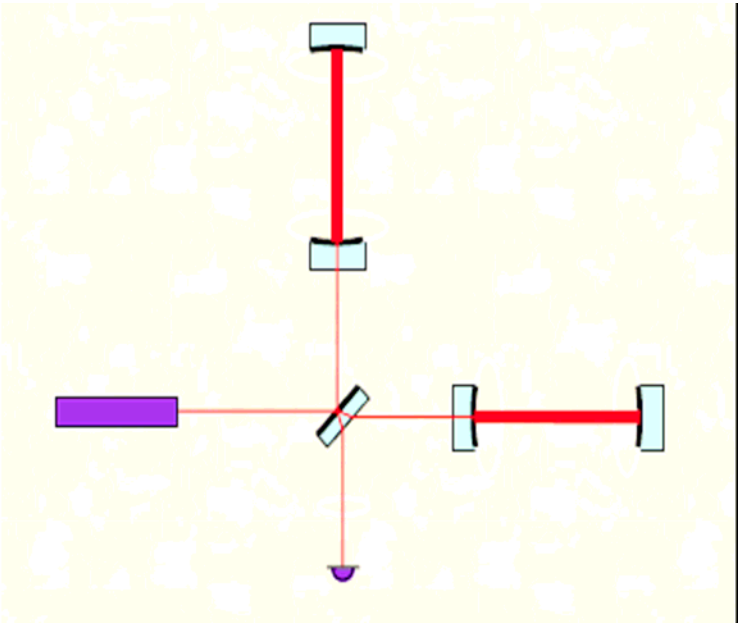


The Challenge!

- The strength of a gravitational wave is measured by the strain $h(t)=\Delta L(t)/L$ and for a neutron star binary with component masses of 1.4 M_{sun} in the VIRGO cluster $h \cong 10^{-21}$
- If we choose the interferometer's arms to be 4 Km long (like LIGO!), then the relative displacement beteween the mirrors will be of the order of $\Delta L = h \times L \cong 10^{-21} \times 4 \cdot 10^{3} m \approx 10^{-18} m$
- Therefore the challenge for LIGO is to measure lengths ~1000 times smaller than the atomic nucleus!!! In order to increase the interferometer sensitivity we need the phase shift to be as large as possible, therefore we need L to be as large as possible: should we build 100 Km interferometers? It's not necessary! Just fold the light path 100 times!!!

The LIGO detectors - I

- LIGO interferometers are Fabry-Perot resonant cavities; two more mirrors are added which allow to store the light inside the interferometer arms and obtain an effective optical path 200 times longer than a normal Michelson



The LIGO detectors - II

Two observatories 3002 Km apart:

- Hanford, WA
- Livingston, LA

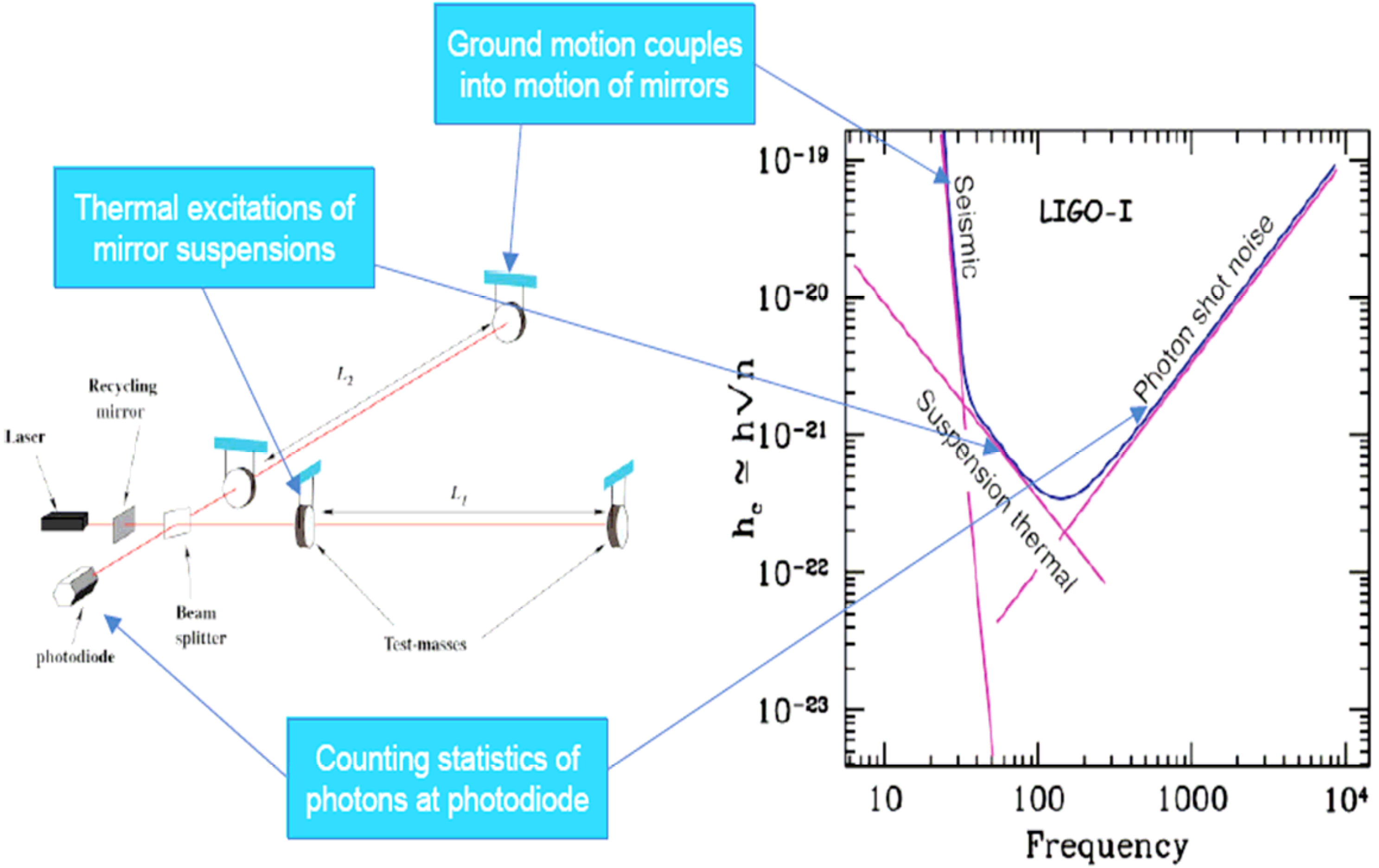
The LIGO Observatory Sites

Interferometers are aligned along the great circle connecting the sites

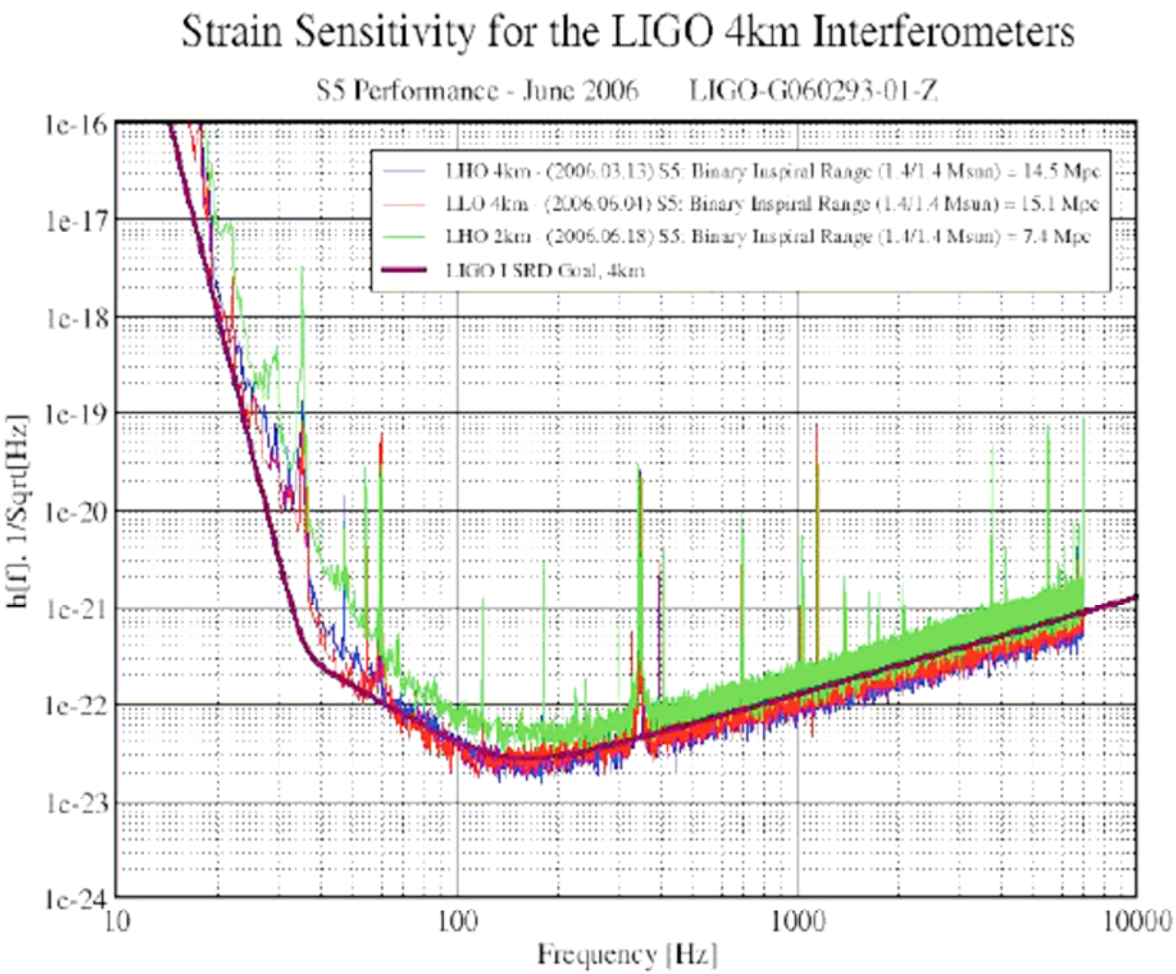
Noise sources in LIGO

- LIGO has formidable isolating and noise-suppressing systems, both passive (mechanical attenuators) and active (feedbacks), however noise still dominates the interferometer's output
- We have two main types of noise: stationary noise and transient noise
- Stationary noise is always present and is intrinsic to the interferometer mechanical system and electronics
- Conversely transient noise is due to random events which may happen in the interferometer's surroundings, such as earthquakes, eccess electronic noise in the control system, airplanes passing by…

Noise sources in LIGO



Current LIGO sensitivity



Matched filtering

How do we recover gravitational waves from the data stream?

- We construct theoretical models of the waveform called templates which depend on few parameters: masses and spins of the compact objects, orbital angular momentum, direction and orientation angles of the binary with respect to the detector, initial phase and time of arrival
- We evaluate the convolution of the the data stream s with the template h (called the match) and maximize over the parameters to obtain the signal-to-noise ratio (SNR) at every instant of time t

$$\rho(t) = \max \langle s, h \rangle (t) = \max \int_{f_{\min}}^{f_{\max}} \frac{\tilde{s}^*(f)\tilde{h}(f)}{S_n(f)} e^{2\pi i f t} df$$

Matched filtering

- We decide that we have a trigger at time t_0 if the SNR at that time is bigger than a chosen threshold $\rho(t_0) \geq \rho_0$

Is it all so easy???

- Of course not, because the data stream is dominated by noise and high SNR's could correspond to non-stationary noise transients (or "glitches")!

How do we discard noise artifacts?

- We remember we have 3 interferometers! If an event is a real gravitational wave it should show up in all the instruments, so we look for coincident triggers, i.e. triggers which appear in at least two instruments within the light (or gravitational wave) travel time

Matched Filtering

So, have we found a gravitational wave?

- No, we just increased our confidence that the trigger can be a real event; the matched filtering-coincidence scheme alone is not capable of ruling out glitches
- We therefore perform consistency checks (like χ^2 and r^2 vetoes) and if the trigger survives all these than we say that we have an event candidate
- Finally the candidate goes through the detection checklist (which is a “manual” follow-up) and only at the end of this we can state that we have a detection, i.e. a real gravitational wave

Dynamical evolution of spinning binaries

- Very complicated, due to Spin-Spin and Spin-Orbit couplings which cause **precession** of the orbital angular momentum $\hat{L}_N$ and therefore **modulations** in the phase and amplitude of the GWs emitted.
- Waveforms depend on a relatively large number of parameters to be maximized over in matched filtering:
- **extrinsic** parameters can be searched over quickly via analytical or numerical maximization
- **intrinsic** parameters influence the shape of the waveform and need to be searched over using a different template for every point in the parameter space (template bank)

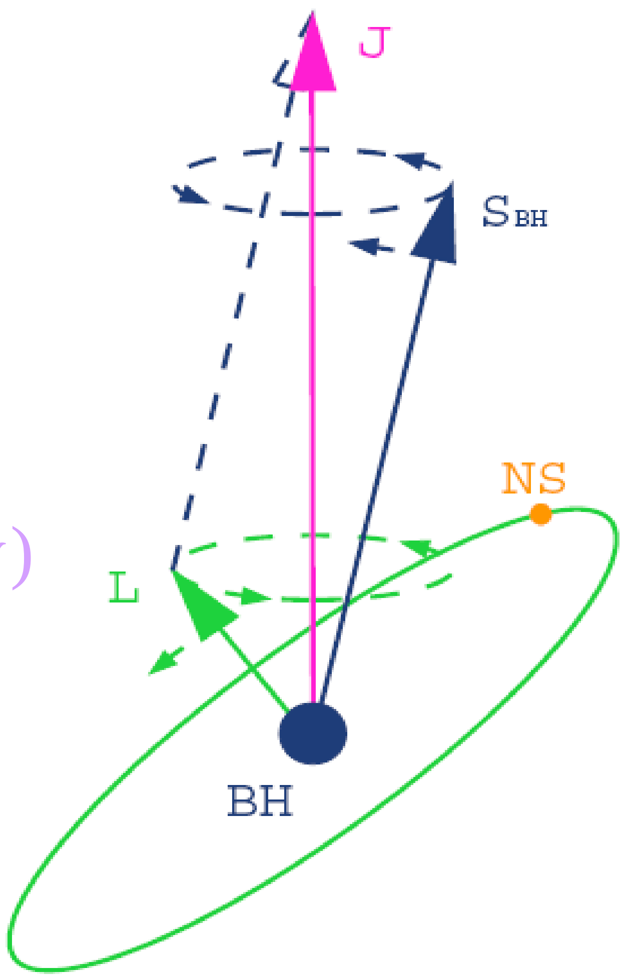
Single spin binaries

The detector response to GWs emitted by binaries (BBH or BH-NS) with a single spinning BH depends on 11 parameters. Upon defining $\chi_1 \equiv S_{BH} / m_{BH}^2 \,,\; \kappa_1 \equiv \hat{L}_N \cdot S_{BH}$ we have:

- 4 extrinsic parameters $(\Theta, \varphi, \Phi_0, t_0)$
- 7 intrinsic parameters $(M, \eta, \chi_1, \kappa_1, \phi, \theta, \psi)$

(Θ, φ) specify the direction to the detector in the source frame.

(ϕ, θ, ψ) specify the orientation of the detector with respect to the radiation frame.



Matched-filtering detection:DTF vs PTF

Searching over more than 3-4 intrinsic parameters in matched filtering is too computationally expensive ➡ **Detection Template Families** (DTF) depend on a smaller number of phenomenological **non-physical** parameters; good for the purpose of detection **but** not for parameters estimation. Relations with physical parameters not well defined ➡ a larger parameter space is searched over ➡ **high false alarm rate!** Reducing the number of parameters and converting some intrinsic parameters to extrinsic with a reparametrization we can still get exact waveforms ➡ **Physical Template Families** (PTF)

PTF - Parameter reduction

Pan, Buonanno, Chen, Vallisneri - Physical Review D 69, 104017 (2004)

We can write the interferometer response to the gravitational wave in particularly compact form as

$$h = -\frac{2\mu}{D}\frac{M}{r}\left([\mathbf{e}_+]^{ij}\cos 2(\Phi+\Phi_0) + [\mathbf{e}_\times]^{ij}\sin 2(\Phi+\Phi_0)\right)\left([\mathbf{T}_+]_{ij}F_+ + [\mathbf{T}_\times]_{ij}F_\times\right)$$

The dependence on the orientation angles (θ, ϕ, ψ) is only through F_+ and $F_\times$, and all 3 angles can be replaced by only one angle α, simply redefining

$$\begin{Bmatrix} F_+ \\ F_\times \end{Bmatrix} \equiv \sqrt{F_+^{\,2} + F_\times^{\,2}} \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} \equiv F \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} = \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix}$$

where template normalization allows to set F=1.

Reparametrization - 1

The template can then be rewritten as

$$h = Q^{ij}[M, \eta, \chi_1, \kappa_1; \Phi_0, t_0; t]\, P_{ij}[\Theta, \varphi; \alpha]$$

where

$$Q^{ij} = -\frac{2\mu}{D}\frac{M}{r}\left([\mathbf{e}_+]^{ij}\cos 2(\Phi + \Phi_0) + [\mathbf{e}_\times]^{ij}\sin 2(\Phi + \Phi_0)\right)$$

$$P_{ij} = [\mathbf{T}_+]_{ij}\cos\alpha + [\mathbf{T}_\times]_{ij}\sin\alpha$$

and now we are left with 5 extrinsic parameters and only **4 intrinsic parameters** $M, \eta, \chi_1, \kappa_1$, all contained in Q^{ij}.
Q^{ij} and P_{ij} are two 3-dimensional Symmetric Trace-Free (STF) tensors with 5 independent components each and they can be expanded on an orthonormal STF basis.

Reparametrization - 2

Introducing an STF orthonormal basis M^I_{ij} we get

$$Q^{ij} = Q^I (M^I)^{ij}, \quad P_{ij} = P^I (M^I)_{ij}$$

Now introducing $Q^I_0 \equiv Q^I(\Phi_0 = 0), \quad Q^I_{\pi/2} \equiv Q^I(\Phi_0 = \pi/4)$

we can factor out the initial phase to get

$$h = P_I [Q^I_0 \cos(2\Phi_0) + Q^I_{\pi/2} \sin(2\Phi_0)]$$

From the components $Q^I_0, \; Q^I_{\pi/2}$ we can construct the matrices

$$A^{IJ} \equiv \langle s, Q^I_0 \rangle_{t_0} \langle s, Q^J_0 \rangle_{t_0} + \langle s, Q^I_{\pi/2} \rangle_{t_0} \langle s, Q^J_{\pi/2} \rangle_{t_0}$$

$$B^{IJ} \equiv \langle Q^I_0, Q^J_0 \rangle$$

Maximization of the overlap ρ over the extrinsic parameters

The maximization of the overlap over Φ_0 is algebraic

$$\langle s, h_{norm} \rangle = \frac{\langle s,h \rangle}{\sqrt{\langle h,h \rangle}} \xrightarrow{\max \Phi_0} \sqrt{\frac{\sum_{I,J} P^I P^J A^{IJ}}{\sum_{I,J} P^I P^J B^{IJ}}}$$

and it now assumes a very simple form in terms of the new parameters P^I; we can therefore think about maximizing over the P^I however they are not free parameters but they must satisfy two constraints : $P_I P_J B^{IJ} = 1, \ \det P_{ij} = 0$ so the maximization of the overlap is actually constrained to a 3-dimensional physical submanifold $P^I(\Theta, \varphi, \alpha)$.

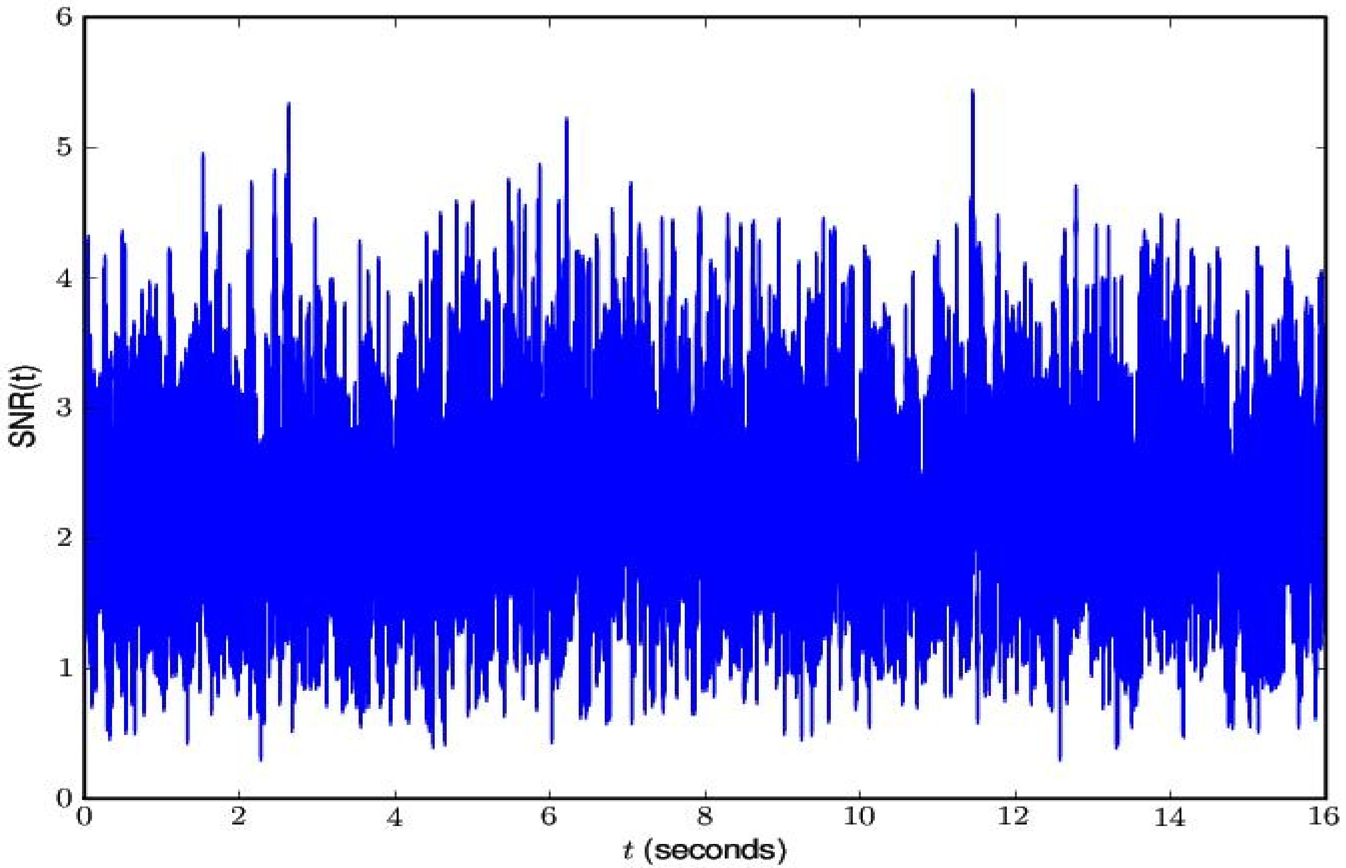
Two-stage search scheme

The constrained maximization over $(\Theta, \varphi, \alpha)$ is still too computationally expensive, so we build a two-stage search; an **unconstrained** analytical maximum ρ' over the new extrinsic parameters P^I is first evaluated and then the full constrained maximization procedure is performed only for the values of t_0 for which ρ' rises above a certain threshold ρ'^*.

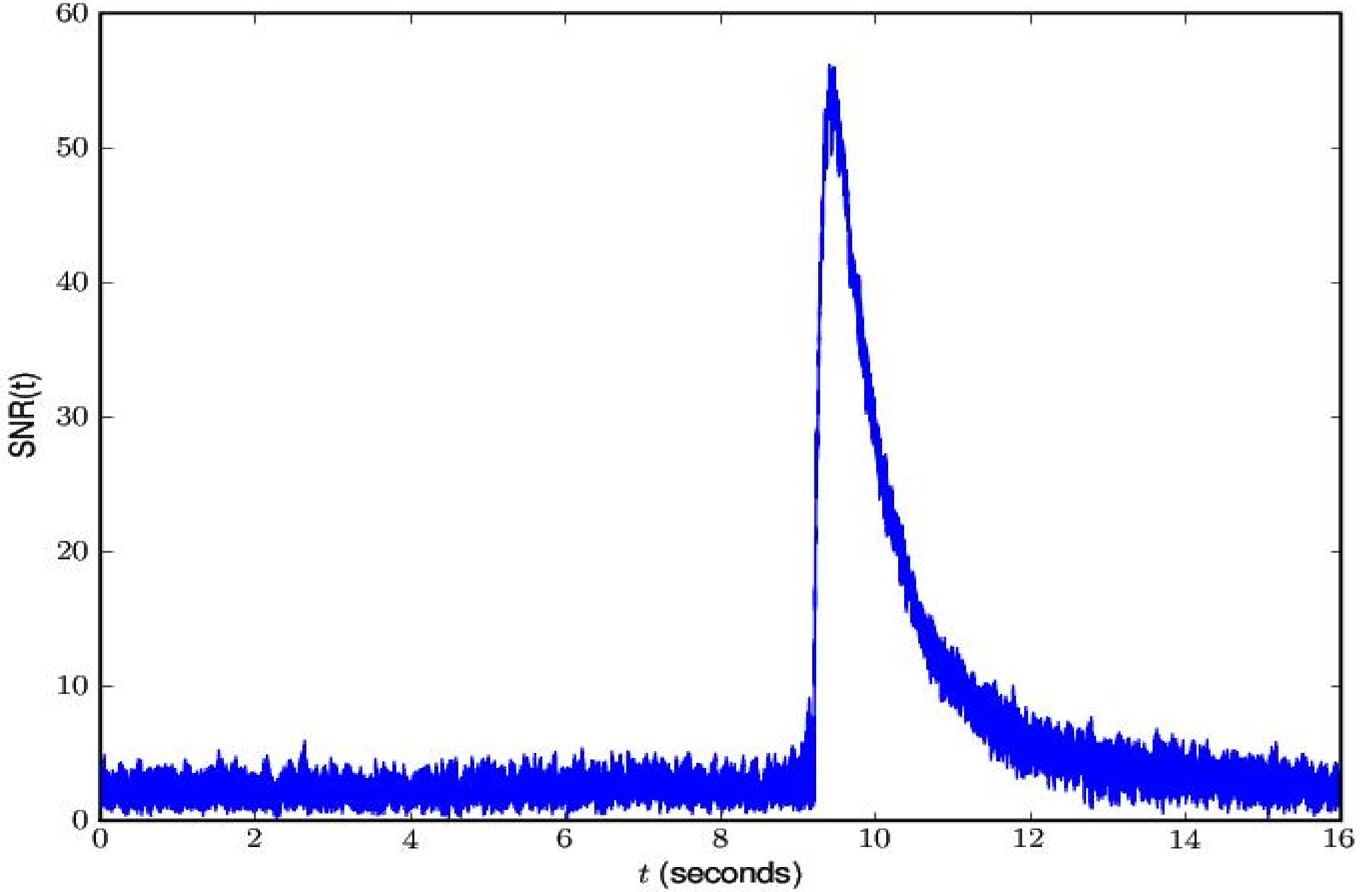
$$\rho' = \max_{P_I} \sqrt{\frac{P_I P_J A^{IJ}}{P_I P_J B^{IJ}}} = \sqrt{\max eigv\,[\mathbf{AB}^{-1}]}$$

The location of the approximated maximum provides also good initial guesses for Θ and φ.

Detection statistic for pure stationary Gaussian noise



Detection statistic with a simulated signal present



LIGO status and current developments

- LIGO is currently at design sensitivity and the fifth LIGO Science Run S5 started in November 2005 in under way; the goal is to take 1 year of coincident data!
- The code implementing the new PTF family has been written and testing is in progress!
- The new template bank will be used to analyze data from S5 in combination with the other ongoing searches to improve the reliability of the estimation of the sources physical parameters
- Hopefully we will make a detection!