Reaching for the spindown limit on the Crab Pulsar

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The Crab Pulsar

- Remnant of a supernova in 1054 AD
- ~7000 light years distant
- Surrounded by the Crab Nebula
- Rotating Neutron star
  - » Pulsations detected from radio to gamma rays

(Credits: X-ray: NASA/CXC/ASU/J. Hester et al.; Optical: NASA/HST/ASU/J. Hester et al.)

Vital Statistics

(as determined by electromagnetic observation)

- Also known as PSR 0534+2200
- Spin Frequency (f): 29.78 Hz
- 1st Spindown Rate (df/dt): -3.729e-10 Hz/s
- 2nd Spindown Rate (d^2f/dt^2) : 1.242e-20 Hz/s^2

Spin-down and breaking index

- The formula for expressing pulsar spin-down is

$$\dot{\Omega} = K\,\Omega^n$$

- K is the “torque function” which contains all the physics of the source of the torque and n is the breaking index
- We can solve for n in terms of observed frequency and its derivatives: $n = \dfrac{\Omega\,\ddot{\Omega}}{\dot{\Omega}^2}$

Magnetic Dipole Radiation and Spin-down

- For pure dipolar magnetic radiation $n_{em} = 3$
- In which case K_{em} is

$$K_{em} = \frac{-2}{3c^3}\frac{B^2}{I}R^6\sin^2(\alpha)$$

,where B is the strength of the magnetic field, R is the neutron star radius, α is the angle between the rotation and the magnetic axes, and I is the moment of inertia

Gravitational Quadrupole Radiation and Spin-down

- For Quadrupole gravitational radiation $n_{gw} = 5$
- Similarly we can express K_{gw}:

$$K_{gw} = \frac{-32}{5} \frac{G}{c^5} I \epsilon^2$$

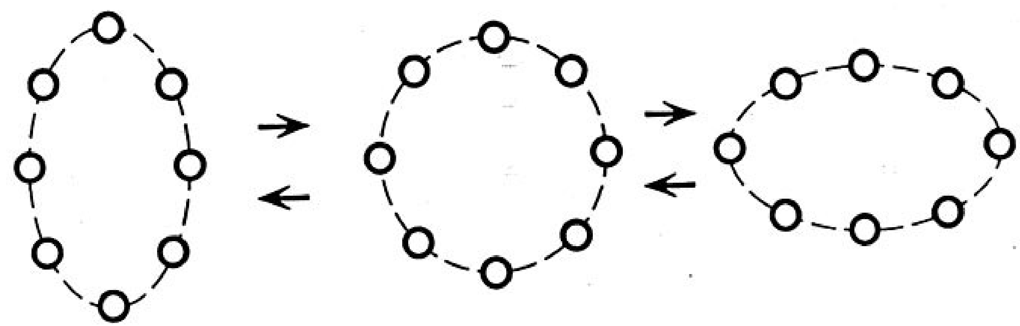
, where G is the gravitational constant, c is the speed of light, ϵ is the ellipticity of the star and I is the moment of inertia

The Classic GW Upper Limit Calculation

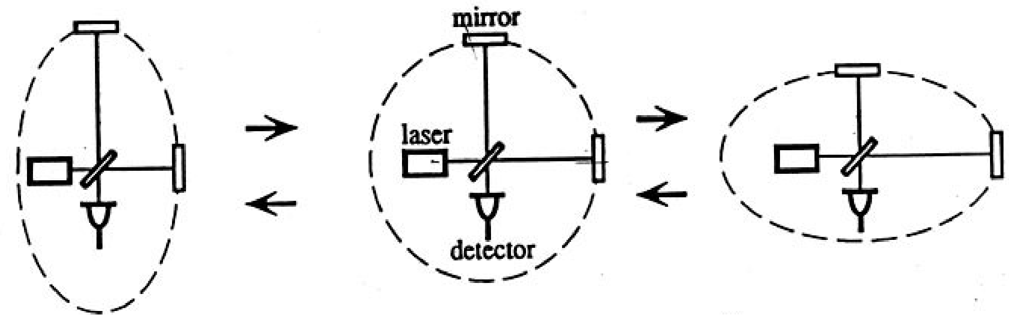
- To determine the upper limit on the amount of gravitational radiation coming from a Neutron star one can simply assume that all of the spin-down is due to gravitational radiation
- $E = \frac{1}{2} I \Omega^2$ and $\dot{E} = I \Omega \dot{\Omega}$ can be used with the previous K_{gw} torque function to find the energy leaving in gravitational radiation
- For a typical I = 10^{38} kg m^2 we have $\dot{E} = -4 \text{x} 10^{31} J$
- But how do we measure it?

LIGO Detector

- Laser Interferometers can very precisely measure the strain caused by passing gravitational waves



Effect of GW on 'test' masses



Interferometric measurement

Strain at the LIGO detector

- For an optimally oriented neutron star source (such that the angle between the rotation axis and the line of sight is zero) the strain, h_0, reaching Earth is

$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I \epsilon f^2}{r}$$

,where r is the distance to the source, f is twice the rotational frequency, I is the moment of inertia and ϵ is the ellipticity

Classic GW Upper Limit Calculation

- The ellipticity necessary (assuming a typical $I = 10^{38}$) for 100% of the energy loss to be due to gravitational radiation is $\epsilon = 7.5\text{x}10^{-4}$
- The corresponding h_0 strain (and upper limit) is $1.4\text{x}10^{-24}$
- Its interesting to note that there are some suggestions that the Crab Pulsar could have a moment of inertia between 1 and 3 times the classic $I = 10^{38}$, which could increase the upper limit by up to a factor of 3

But...

- However, the Crab's breaking index n can be determined from observed quantities

- $n = \dfrac{\Omega \ddot{\Omega}}{\dot{\Omega}^2} = 2.5$,which is distinctly not 5 or even 3

- Thus the energy loss cannot be due only to Gravitational radiation or even pure magnetic dipole radiation

How to get better Upper Limits

- In 2000 Palomba published a paper on how to do better than the classic upper limit
- Assume $\dot{\Omega} = \dot{\Omega}_{em} + \dot{\Omega}_{gw}$, where we let the EM term vary below 3 by assuming something more complex than simple magnetic dipole radiation is going on
- Define a ratio $Y(\Omega) = \frac{\dot{\Omega}_{gw}}{\dot{\Omega}_{em}} = \frac{K_{gw}}{K_{em}} \Omega^{5-n_{em}}$

The limit to reach (and beat)

- We can than write and numerically solve

$$\dot{\Omega} = K_{gw} \Omega^5 \left(\frac{1 + Y(\Omega)}{Y(\Omega)}\right)$$

- The solution will depend on the ellipticity (contained in K_{gw}), n_{em} and the initial angular velocity Ω_i
- Knowing the actual age of the Crab and choosing the solution with the smallest n_{em} consistent with the overall n = 2.5, Paloma arrived at the upper limit of $h_0 = 5.5\text{x}10^{-25}$ and $\epsilon = 3\text{x}10^{-4}$ with a ratio Y of 0.18

LIGO searches

- The LIGO interferometers have been taking data for their 5th science run since November 2005
- The Continuous Wave working group (or more accurately Matt Pitkin) has been analyzing this data with a Time Domain Search (TDS) method looking at known pulsars including the Crab

LIGO

Time Domain Search

- The TDS assumes the gravitational wave emission from a triaxial neutron star is tightly coupled and phase locked with the electromagnetic emission and thus uses one template
- It heterodynes time domain data with the known phase evolution of the pulsar
- It then uses Bayesian parameter estimation of the unknown pulsar parameters using data from all three LIGO interferometers

Time Domain Results

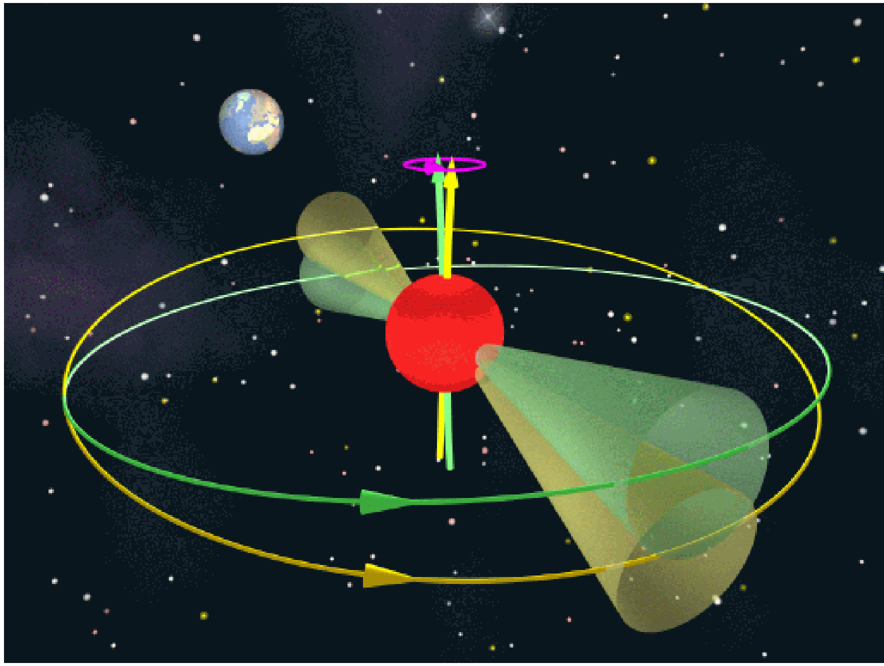
- The TDS then produces probability distribution functions for those unknown parameters and marginalizes over angles to set 95% confidence upper limits on h_0
- Matt applied the TDS method to 9 months of data on the Crab pulsar and found $h_0 < 4.3 \times 10^{-25}$ and $\epsilon < 2.3 \times 10^{-4}$ with 95% confidence
- These beat the classic GW upper limit of 1.4×10^{-24} and also Palomba's much better limit of 5.5×10^{-25}

Resolution of the Time Domain Search

- Consider the resolution in frequency space (df) of the TDS using only one template exactly matched to the electromagnetic ephemeris
- $df = \frac{1}{T_{obs}} = \frac{1}{9\,months} = 4\text{x}10^{-8}\,Hz$
- If for whatever reason the gravitational radiation doesn't match the electromagnetic to within 40 nanohertz we would be rapidly losing sensitivity to it

Free Precession

- If the rotation axis is not aligned with a principle axis, the Crab could precess like a top



Free Precession

- There's no large angle precession going on with the Crab pulsar, since that would show up clearly in the radio timing data
- However, very small angle precession could be occurring
- If the wobble angle θ is small, and a simple knife beam model of the pulsar is used, during the precession time the phase of the pulses could arrive early or late by at most $\theta/\tan(\lambda)$, where λ is the angle between the rotation axis and the spot on the pulsar generating the EM signal

Free Precession Period

- If we assume a simple axisymmetric model for the neutron star where $I_1 = I_2 \neq I_3$, we can write the precession angular frequency as

$$\Omega_p = \Omega \frac{(I_1 - I_3)}{I_3} \cos(\theta) = \Omega \epsilon \cos(\theta)$$

- The angular frequency of the gravitational radiation will occur at Ω and 2Ω, while the frequency of the electromagnetic radiation will be $\Omega + \Omega_p$
- An ellipticity of 10-4 and $\theta << 1$ could cause a shift up to $3\mathrm{x}10^{-3}$ Hz

Glitches

- The Crab pulsar has been observed to have glitches
- A glitch is when the observed frequency changes abruptly, and in the case of the crab increases, and then slowly relaxes back towards the original frequency
- One occurred in August 2006, which was a natural breaking point for the TDS since its unknown what effect this has on the gravitational radiation frequency

Glitches

- The Crab undergoes glitches with a somewhat irregular frequency.
  - » Between 1969 and 1994 there were 6 glitches
  - » However since 1995 there have been more than 10
- These glitches are of order $\Delta\nu/\nu \sim 2\text{x}10^{-9}$ to $6\text{x}10^{-8}$
- These frequency jumps show a recovery towards the original frequency with timescale of ~3 days (for the smallest) to ~10 days (for the largest)

Two Component Model of glitches

- Since the crab pulsar always spins up during glitches, it possible that there are two components of the neutron star that are rotating at different rates
- When a glitch occurs some of the angular momentum from the faster part (which must not be part of the EM emission and thus under the surface) is transfered to the slower, resulting in a spin up
- Its possible that this second faster component would be generating the bulk of gravitational radiation rather than the surface component locked to the electromagnetic radiation

Glitches and Frequency

- The recovery time scale, $\tau_{coupling}$ of 3-10 days implies a certain strength torque coupling the two components
- The EM and GW emission mechanisms also have a determinable time scale, $\tau_{spin-down}$, which is of the order of the age of the Crab pulsar (if it originally had a spin period of ~10 ms)
- A relation of $\Omega_{gw} \approx 2\,\Omega_{em}\left(1+\frac{\tau_{coupling}}{\tau_{spin-down}}\right)$ would hold in such a case

1st Spindown and 2nd Spindown

- In both of the examined scenarios the gravitational radiation we're looking for could be within a band of $\Delta f = 6\text{x}10^{-3}$ Hz centered on twice the electromagnetic frequency.
- However, we also need to consider the parameter space in terms of the 1st Spindown and 2nd Spindown parameters

1^{st} Spindown

- One way to estimate this is to explicitly write our relation between f_{gw} and f_{em} $f_{gw} = 2f_{em}(1+\delta)$, where δ is our small deviation away from the exact EM frequency
- We can then note that $\dot{f}_{gw} = 2\dot{f}_{em}(1+\delta) + 2f_{em}\dot{\delta}$
- If we are willing to assume that the ratio of the current deviation δ over the rate of change of the deviation is either smaller or at most the same order as the current frequency over the rate of change of the frequency we can write $\Delta\dot{f}_{gw} \sim 3\dot{f}_{ew}\delta$

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Final Parameter space

- A similar argument can be made for the 2nd spindown as for the 1st spindown yielding $\Delta \ddot{f}_{gw} \sim 8 \dot{f}_{ew} \delta$
- Thus the size of the final parameter space with regards to the frequency and its derivatives is:

$$\Delta f_{gw} = 6\text{x}10^{-3}\, Hz$$

$$\Delta \dot{f}_{gw} = 1.2\text{x}10^{-13}\, Hz/s$$

$$\Delta \ddot{f}_{gw} = 2\text{x}10^{-23}\, Hz/s^2$$

Multi-IFO Compute F Statistic

- The search is being carried out by a code developed by the Continuous Wave working group called the Mutli-IFO Compute F Statistic
- The code uses a method known as maximum likelihood detection which looks for a signal and estimates its parameters (the maximized likelihood estimators)
- Uses data from multiple IFOs to calculate the total log likelihood function (a sum of log likelihood functions for each individual interferometer)

F Statistic without Signal

- In just the presence of Gaussian stationary white noise, twice the F Statistic (2F) is distributed according to a central χ^2 distribution with 4 degrees of freedom (due to the two gravitational wave polarizations times two for sine and cosine components)
- The probability density function is $p_0(F) = \frac{F}{2} e^{-F}$
- The false alarm probability of F is then just

$$P_0(F) = \int_F^\infty p_0(F')\, dF'$$

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F Statistic with Signal

- In the presence of signal, 2F is a non-central χ^2 distribution with 4 degress of freedom and a non-centrality parameter d^2 where d is the “optimal signal to noise”
- In this case the expected value of 2F is $4 + d^2$ (as opposed to an expected value of 4 when there is no signal)

Templates

- The multi-IFO Compute F Statistic code breaks the parameter space up into templates which define Right Ascension, Declination, Frequency, 1^{st} Spindown and 2^{nd} Spindown
- The other parameters such as h_0, inclination angle, and so forth are found by a least squares fit during the execution of the code

One Template

- For a GW pulsar with all the template parameters known (and assuming an average sky position, inclination angle, and polarization, and setting the false alarm rate at 1% and the false dismissal rate at 10%, the amplitude of the signal that could be detected in Gaussian stationary noise is

$$\langle h_0(f) \rangle = 11.4 \sqrt{\frac{S_h(f)}{T_{obs}}}$$

Multiple Templates

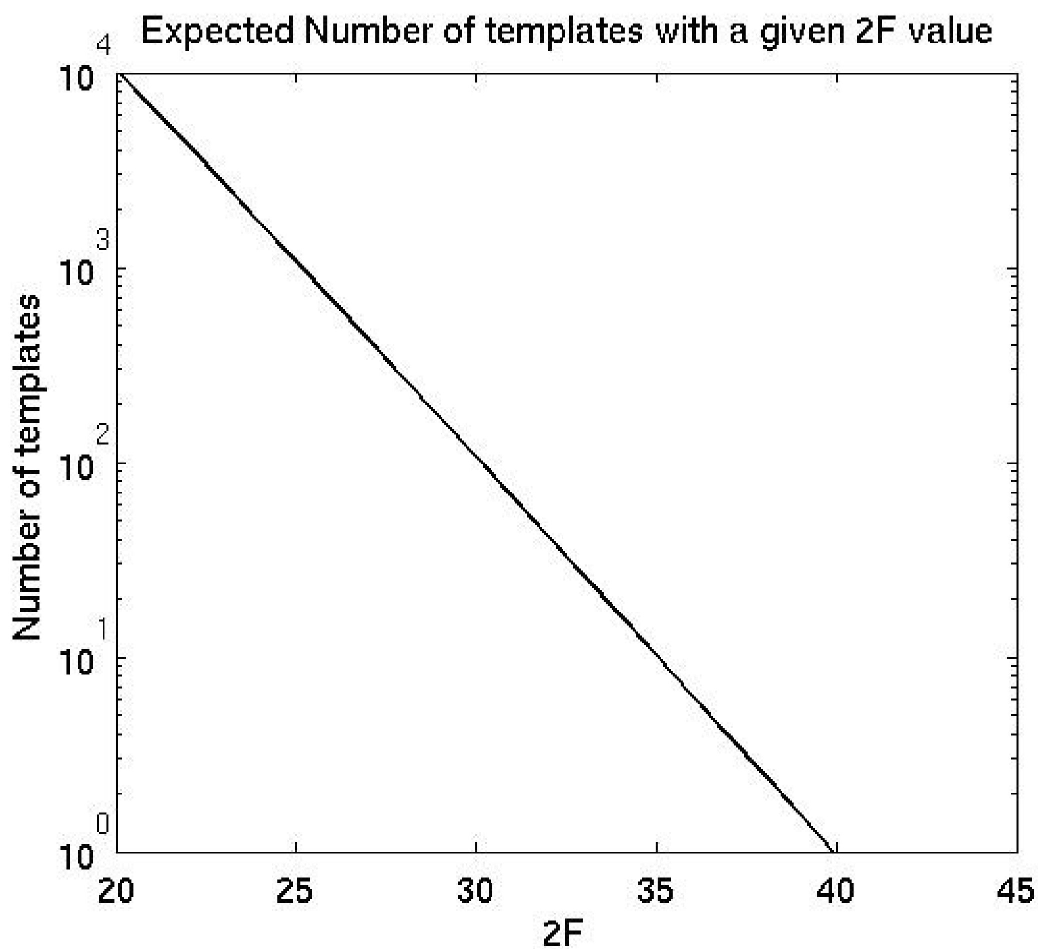
- As the parameter space we consider expands we need to use more templates
- The spacing between templates is determined by how much loss in power we're willing to accept between templates
- In this particular search I'm using a mismatch parameter of 0.3, or in other words if the signal were exactly between two templates, I'd be recovering 70% of the power in the signal

Templates needed for the crab

- The total number of templates needed for the previously mentioned parameter space is roughly $3x10^7$
- There are $\sim 2x10^5$ frequency points (each template is spaced by $\sim 1/T_{obs}$ so $6x10^3$ / $3x10^{-8}$ = $2x10^5$)
- There are ~120 1st Spindown points (each template is spaced by $\sim 1/(T_{obs})^2$)
- Only a single 2nd Spindown is needed

Expected 2F template distribution

- Since we know the probability density function we can can estimate the distribution of 2F values for the $3x10^7$ templates
- Loudest 2F value due to noise should be ~40



Upper Limit Injections

- After the search has been run and we have the largest 2F value we need to turn that into an Upper Limit comparable those listed earlier
- This is done via Monte Carlo injections

Monte Carlo Injections

- We generate randomized GW signals within the searched parameter space but with a fixed h_0 and inject them into the data
- We then search for these fake GW signals with the same search algorithm (although restricting ourselves to the templates very close to the injection to save computation time)

Monte Carlo Injections

- We declare an injection found if the 2F is equal to or greater than the largest 2F value found in the actual search
- If we find less than about 95% of the injections at a given h0 out of sufficient number of injections (100-1000s) we raise the h_0 and continue with more injections
- Similarly if we find more than about 95% of the injections we decrease the h_0

Monte Carlo Injections

- Once we have the h_0 at which 95% of the injections are found and have a sufficient number of injections (~5000) we can stop and state our 95% confidence upper limit
- This is the h_0 at which we would have found 95% of possible gravitational wave signals if they were truly present in the data

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Expected Upper Limits for many templates

- The combination of Gaussian noise producing 2F values of up to 40 and the mismatch value of 0.3 means that the h_0 we can detect is worse (higher) than for the single template case, since we need better signal to noise, and some signals are going to be up to 30% smaller than when perfectly matched
- From past experience this generally results in an Upper limit 2-3 times worse than the value quoted earlier for a single template

Expected Upper Limits

- Looking at the LIGO Sensitivity curve for S5 around 59-60 Hz, taking into account the 9 month observation time, and past experience with similar searches of similar size leads to the following upper limit estimate: $h_0 = 3$ to 6×10^{-24}

Comparisons

- The estimated upper limit for this search is a factor of 6 worse than the time domain search and about a factor of 2 worse than the canonical upper limit (for a $I = 10^{38}$)
- However thats at the single template at the center of the search
- As one moves away from the center point the search does much better relative to the TDS search – at 6 template spacings they're roughly comparable and at the edges of the search this does orders of magnitude better ($\sim 10^5$)

LIGO

Conclusions

- A wide band search around the EM frequency of the Crab pulsar is needed to complement the TDS since several simple physical arguments show that the gravitational wave frequency can differ significantly, at which point the TDS sensitivity drops drastically
- This search is expected to have an upper limit of $3\text{-}6 \times 10^{-24}$, within a factor of ~2 of the canonical upper limit and a factor of ~6 away from the Palomba limit (again assuming $I = 10^{38}$)
- As LIGO and similar laser interferometers improve their performance, future searches like this one will continue to push further into astrophysically interesting territory