$\mathcal{F}$-Statistic Searches for White Dwarf Binaries in the Mock LISA Data Challenges

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Outline

1. LISA and Gravitational Wave Searches
   - Crash Course in Gravitational Wave Physics
   - The LISA Mission
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2. White Dwarf Binary Search
   - $\mathcal{F}$-Statistic Search for Periodic Gravitational Waves
   - MLDC1 Pipeline and Results
   - MLDC2 Pipeline and Results

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Motivation

- In Newtonian gravity, force dep on distance btwn objects
- If massive object suddenly moved, grav field at a distance would change instantaneously
- In relativity, no signal can travel faster than light
  $\longrightarrow$ time-dep grav fields must propagate like light waves

Gravity as Geometry

- Minkowski Spacetime:

$$ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

$$= \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\text{tr}} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = \eta_{\mu\nu} dx^\mu dx^\nu$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\text{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = g_{\mu\nu} dx^\mu dx^\nu$$

Gravitational Wave as Metric Perturbation

- For GW detection, spin-2 “graviton tensor” $h_{\mu\nu}$ is difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

  $$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$

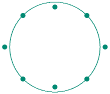
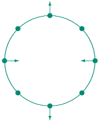
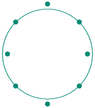
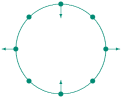
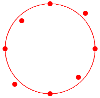
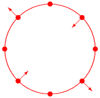
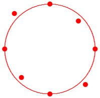
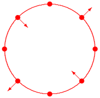
  ($h_{\mu\nu}$ “small” in weak-field regime, e.g. for GW detection)
- Gauge: transverse ($\eta^{\nu\lambda}\partial_\lambda h_{\mu\nu}=0=h_{0\mu}=h_{\mu 0}=0$) & traceless ($\eta^{\mu\nu}h_{\mu\nu}=0$)
- E.g. Plane wave propagating in z direction

  $$\{h_{\mu\nu}\}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i2\pi f(z-t)}$$

  h_+ and $h_\times$ are amplitudes of “plus” and “cross” pol states.

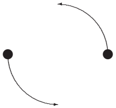
Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

Plus (+) Polarization				
Cross (×) Polarization				

Gravitational Wave Generation

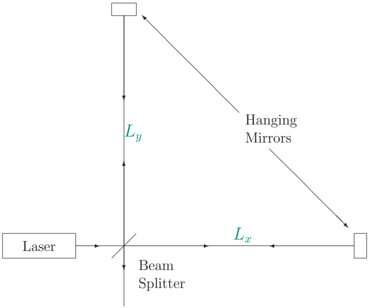
- Generated by moving/oscillating mass distribution
- Lowest multipole is quadrupole
- Classic example: orbiting binary system



(e.g., Binary Pulsar 1913+16
– Observed energy loss agrees w/GW prediction)

Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



- Measure small change in

$$L_x - L_y = \sqrt{g_{11}L_0^2} - \sqrt{g_{22}L_0^2} = \sqrt{(1+h_{11})L_0^2} - \sqrt{(1+h_{22})L_0^2} \approx L_0\frac{h_{11}-h_{22}}{2} \sim L_0 h_+$$

- More gen, $(L_1 - L_2)/L_0 = \overleftrightarrow{h} : \overleftrightarrow{d}$ with “response tensor”

$$\overleftrightarrow{d} = \frac{\hat{n}_1 \otimes \hat{n}_1 - \hat{n}_2 \otimes \hat{n}_2}{2}$$

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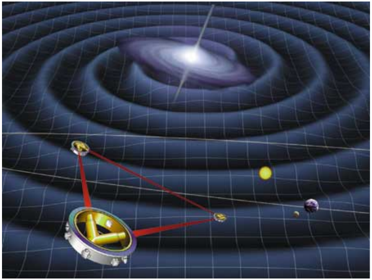
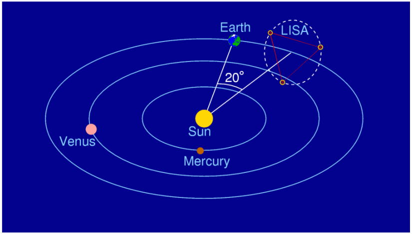
LISA: Interferometry in Space

- Planned Joint NASA-ESA Mission: to launch 2016 or later
- 3 spacecraft will orbit sun & track each other w/lasers
- Laser phase data combined to simulate IFO: “Time-Delay Interferometry” (TDI)

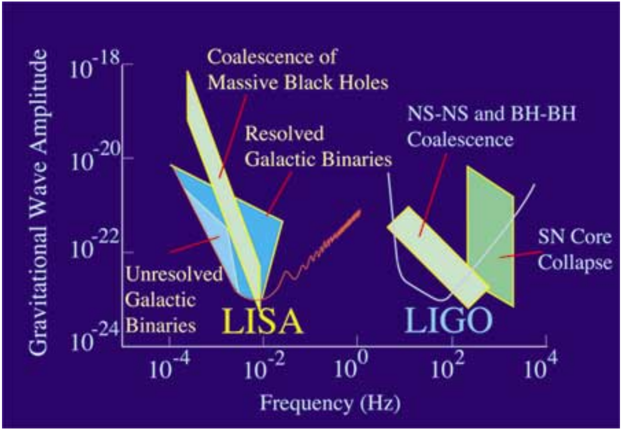




Credits: NASA/JPL; MPI for Gravitational Physics (AEI)/Einstein Online

Differences Between LISA and LIGO

- Diff noise sources & sizes mean diff frequency ranges



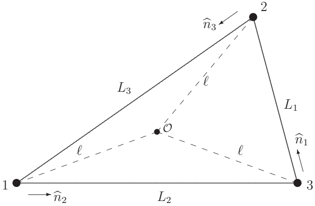
Credit: NASA/JPL

Differences Between LISA and LIGO

- Diff noise sources & sizes mean diff frequency ranges
- LIGO data noise-dominated; can seek one source at a time
  LISA data will contain many strong sources;
  ⟶ must worry about signal extraction
- LISA to observe GWs w/λ comparable to arm length
  ⟶ At higher frequencies, simple IFO picture breaks down
  & response depends on propagation direction

$$\widetilde{X}(f) = \frac{\widetilde{h}(f)}{R(f)} = \frac{\overleftrightarrow{h}(f) : \overleftrightarrow{d}(f, \widehat{k})}{R(f)}$$

LISA Response



- LISA spacecraft 1, 2, 3
- Arm lens L_1, L_2, L_3
  all $\approx L = 5\,\text{million km}$
  vary due to GW & orbit
- TDI vars X, Y, Z comb links
  btwn sc to cancel laser noise
- Convert into “strains”
  $\widetilde{h}^X(f) = R(f)\widetilde{X}(f) = \overleftrightarrow{h}(f) : \overleftrightarrow{d}^X(f, \widehat{k})$
  where in long-λ limit
  $\overleftrightarrow{d} \approx \frac{1}{2}(\hat{n}_2 \otimes \hat{n}_2 - \hat{n}_3 \otimes \hat{n}_3)$
  (etc. for Y & Z)

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Mock LISA Data Challenges

- LISA data analysis presents unusual challenges;
  Need to coördinate searches for different types of signals
  Need plan worked out before LISA flies
- LISA International Science Team (LIST) has organized MLDCs to build community expertise
  Extract simulated signals from simulated LISA noise
- MLDC1 ran from June-December 2006; results announced at GWDAW 11, Dec 2006, Potsdam, Germany
- MLDC2 ran from January-June 2007; results announced at GR 18 / Amaldi 7, Jul 2007, Sydney, Australia

First Mock LISA Data Challenge

- Data sets:
  - Challenge 1.1: White Dwarf Binaries: Periodic Sources
  - Challenge 1.2: Super-Massive Black Hole Inspirals
  - Challenge 1.3: Extreme Mass Ratio Inspirals (deadline postponed until MLDC2)
- Entries submitted by ten groups
- AEI group of Reinhard Prix & JTW searched for WD binaries w/$\mathcal{F}$-statistic method

Second Mock LISA Data Challenge

- Data sets:
  - Challenge 1.3: Extreme Mass Ratio Inspirals
  - Challenge 2.1: Galactic Binaries (30 Million)
  - Challenge 2.2: “Whole Enchilada”: Galaxy + EMRIs + BHB
- Entries submitted by thirteen groups
- AEI group of Reinhard Prix & JTW
  searched for WD binaries w/$\mathcal{F}$-statistic method
  (improved pipeline to distinguish sources)

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Periodic GW Sources

- Searching for sinusoidal signals is easy: Fourier transform $\widetilde{x}(f)$ & look for peaks
- But signal won't be sinusoidal:
  - Motion of detector doppler-shifts signal
  - Change in orientation changes projection $\overleftrightarrow{h} : \overleftrightarrow{d}$
- Signal parameters:
  - 4 Amplitude params: GW amp, initial phase, inclination & orientation of WD orbit (or NS spin); Combine into $\{\mathcal{A}^{\mu}\}$
  - 3+ Doppler params: intrinsic f, ecliptic lat & lon of source (also spindown if appropriate); represent by θ

Searching for Periodic GWs

- Measured strain (= noise + signal) is (implicit $\sum_{\mu=1}^{4}$)
  $x(t;\mathcal{A},\theta) = n(t) + \mathcal{A}^{\mu} h_{\mu}(t;\theta)$
  $n(t)$ & $h_{\mu}(t;\theta)$ depend on detector, $\mathcal{A}$ does not
- Given data x w/noise spectrum $S_n(f)$ can calculate $x_{\mu} = \int \frac{\widetilde{h}_{\mu}(f)^{*}\widetilde{x}(f)}{S_n(f)} df$ and $\mathcal{M}_{\mu\nu} = \int \frac{\widetilde{h}_{\mu}(f)^{*}\widetilde{h}_{\nu}(f)}{S_n(f)} df$ for each choice of doppler params θ
- Most likely signal (for a given θ) minimizes

$$\int \frac{|\widetilde{x}(f) - \mathcal{A}^{\mu}\widetilde{h}_{\mu}(f)|^2}{S_n(f)} df = \mathcal{A}^{\mu}\mathcal{M}_{\mu\nu}\mathcal{A}^{\nu} - 2\mathcal{A}^{\mu}x_{\mu} + \int \frac{|\widetilde{x}(f)|^2}{S_n(f)} df$$

The $\mathcal{F}$-Stat Method (JKS 1998)

Jaranowski, Królak, Schutz 1998:

- Most likely signal (for given θ) maximizes $-\mathcal{A}^{\mu}\mathcal{M}_{\mu\nu}\mathcal{A}^{\nu} + 2\mathcal{A}^{\mu}x_{\mu}$
- Maximized by amplitude parameters $\mathcal{A}^{\mu}_{\text{MLE}} = \mathcal{M}^{\mu\nu}x_{\nu}$ where $\mathcal{M}^{\mu\nu}$ is matrix inverse of $\mathcal{M}_{\mu\nu}$; max value is $2\mathcal{F} = x_{\mu}\mathcal{M}^{\mu\nu}x_{\nu}$
- $\mathcal{F}$-stat search technique:
  - Make a grid of doppler params θ (freq & sky pos)
  - For each choice of θ, calculate $2\mathcal{F}$ from data
  - High values are candidate sources w/amp params $\mathcal{A}_{\text{MLE}}$

Currently the basis of LIGO searches for spinning neutron stars including Einstein@Home

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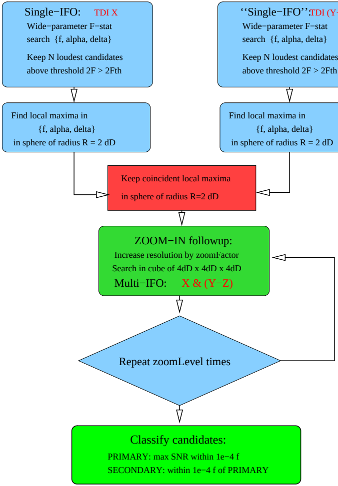
Pipeline for Prix/Whelan MLDC1 Search



Single−IFO: TDI X
Wide−parameter F−stat search {f, alpha, delta}
Keep N loudest candidates above threshold 2F > 2Fth

‘‘Single−IFO’’: TDI (Y−Z)
Wide−parameter F−stat search {f, alpha, delta}
Keep N loudest candidates above threshold 2F > 2Fth

Find local maxima in {f, alpha, delta} in sphere of radius R = 2 dD

Find local maxima in {f, alpha, delta} in sphere of radius R = 2 dD

Keep coincident local maxima in sphere of radius R=2 dD

ZOOM−IN followup:
Increase resolution by zoomFactor
Search in cube of 4dD x 4dD x 4dD
Multi−IFO: X & (Y−Z)

Repeat zoomLevel times

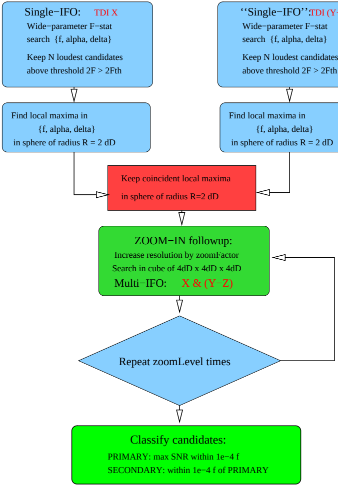
Classify candidates:
PRIMARY: max SNR within 1e−4 f
SECONDARY: within 1e−4 f of PRIMARY

We used open source
LAL & LALAPPS (LIGO) code,
with slight LISA mods

Note only took loudest cand
in freq window
to avoid secondary maxima

Results to appear in CQG;
arXiv:0707.0128

Pipeline for Prix/Whelan MLDC1 Search



Single−IFO: TDI X
Wide−parameter F−stat
search {f, alpha, delta}
Keep N loudest candidates
above threshold 2F > 2Fth

''Single−IFO'':TDI (Y−Z)
Wide−parameter F−stat
search {f, alpha, delta}
Keep N loudest candidates
above threshold 2F > 2Fth

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{f, alpha, delta}
in sphere of radius R = 2 dD

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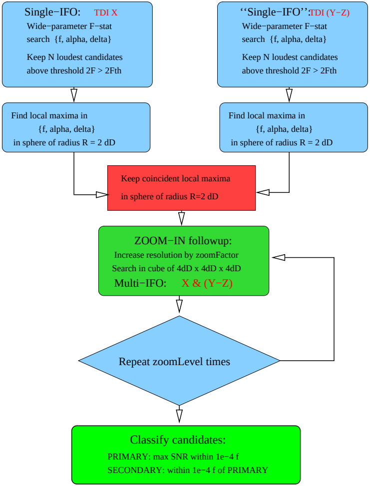

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Results to appear in CQG;
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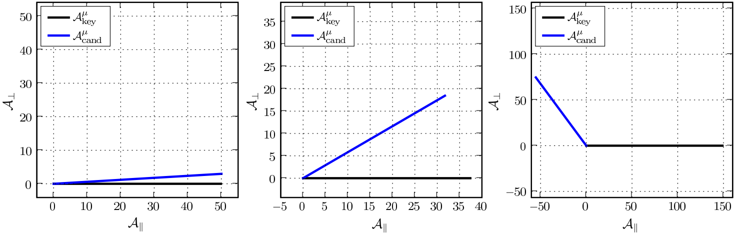
Challenge 1.1.1: Isolated Binaries

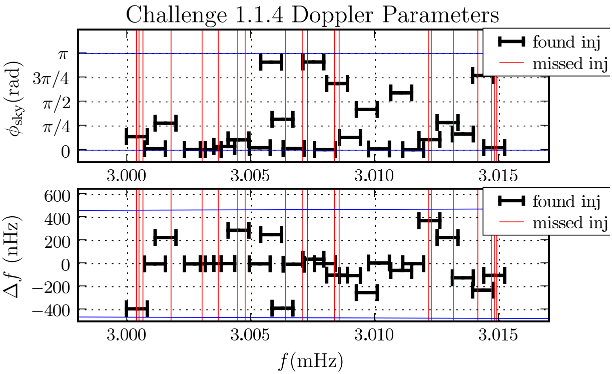


Challenge	f	lat	lon	Δf	$\phi_{\rm sky}$
1.1.1a	1.1 mHz	0.30π	1.62π	1.7 nHz	34.8 mrad
1.1.1b	3.0 mHz	-0.03π	1.47π	0.8 nHz	7.1 mrad
1.1.1c	10.6 mHz	-0.04π	1.48π	0.2 nHz	4.4 mrad

Good sky pos; amp params get worse as LWL breaks down

Challenge 1.1.4: Source Confusion

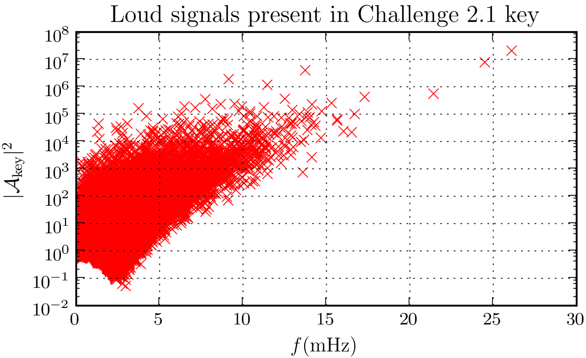
Stuff many signals into small freq range



Recover 21/45; sky position determination becoming unreliable as “freq bins” start to overlap

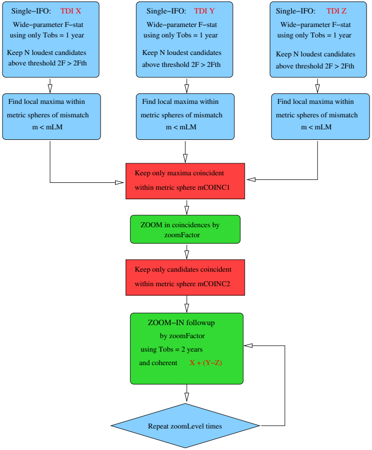
Outline

Galactic Binaries Injected in MLDC2



Challenge 2.1 has 26 million galactic WD binaries,
of which 59401 designated as “bright” sources

Pipeline for Prix/Whelan MLDC2 Search



Single–IFO: TDI X
Wide–parameter F–stat using only Tobs = 1 year
Keep N loudest candidates above threshold 2F > 2Fth

Single–IFO: TDI Y
Wide–parameter F–stat using only Tobs = 1 year
Keep N loudest candidates above threshold 2F > 2Fth

Single–IFO: TDI Z
Wide–parameter F–stat using only Tobs = 1 year
Keep N loudest candidates above threshold 2F > 2Fth

Find local maxima within metric spheres of mismatch m < mLM

Find local maxima within metric spheres of mismatch m < mLM

Find local maxima within metric spheres of mismatch m < mLM

Keep only maxima coincident within metric sphere mCOINC1

ZOOM in coincidences by zoomFactor

Keep only candidates coincident within metric sphere mCOINC2

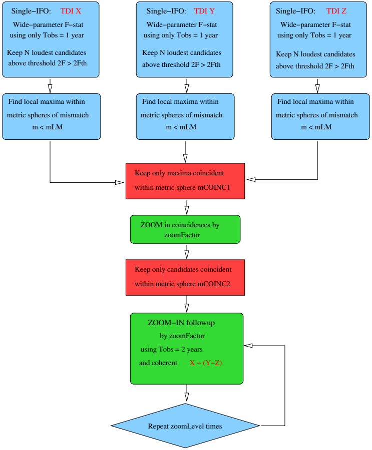
ZOOM–IN followup by zoomFactor using Tobs = 2 years and coherent X + (Y–Z)

Repeat zoomLevel times

Use more careful coïncidence
in all doppler params
to eliminate secondary maxima

Still used long-λ limit
for submitted results

Pipeline for Prix/Whelan MLDC2 Search



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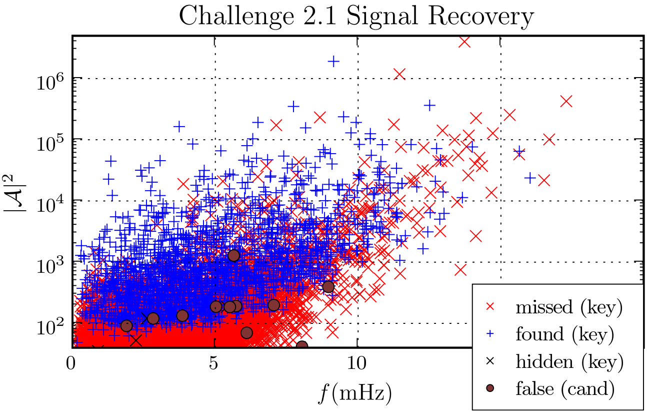
Overview of Galactic Signals Recovered



Found many signals, but still missed some bright ones (especially at higher *f*)

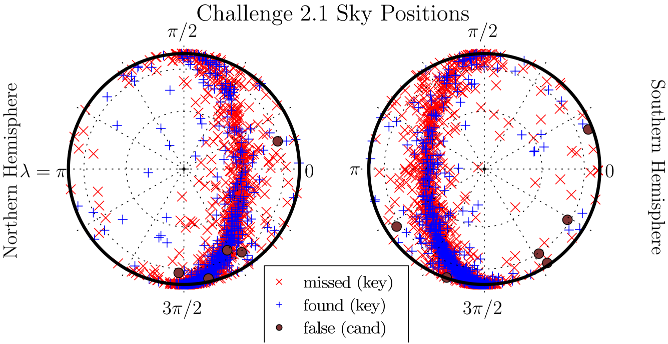
Statistics of Galactic Signals Recovered

Focus on sources w/expected $2\mathcal{F} > 40$

Freqs	Found	Missed	Hidden	False
0–5 mHz	1012	3642	2	3
5–10 mHz	679	1363	1	8
10–15 mHz	73	90	0	0
15–20 mHz	2	5	0	0
20–27 mHz	0	3	0	0

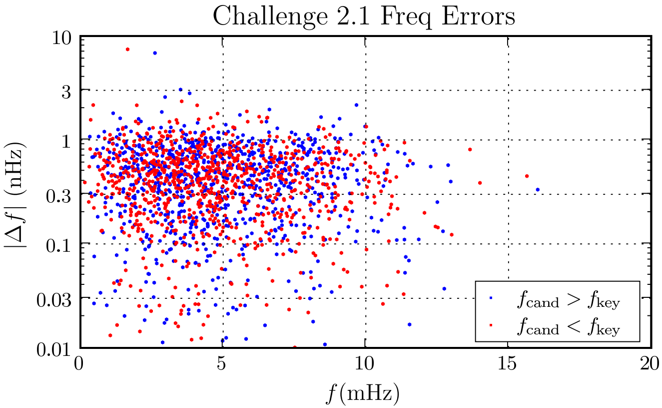
“Hidden” sources were obscured by brighter sources within doppler param window

Sky Map of Found & Missed Binaries



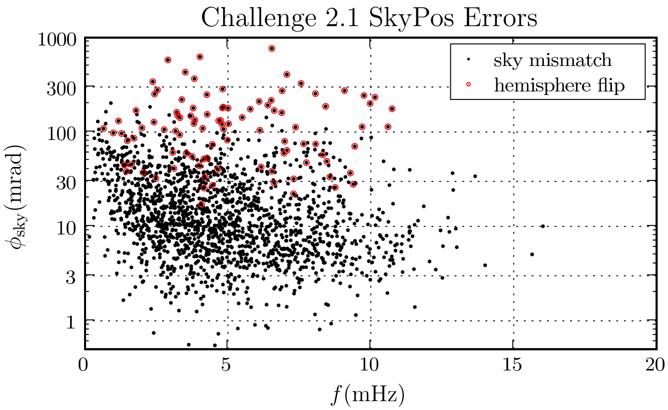
Galaxy clearly visible

Doppler Errors I: Frequency



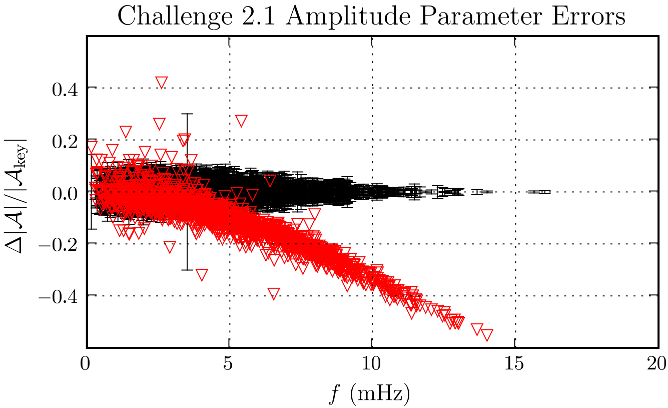
No noticeable bias; accuracy comparable to MLDC1

Doppler Errors II: Sky Position



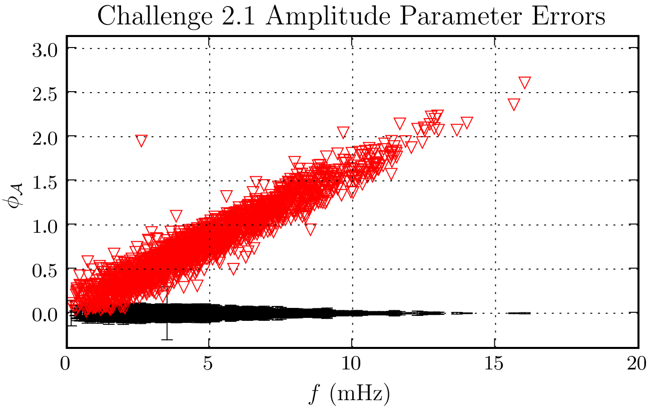
Biggest errors due to “hemisphere flip”:
wrong sign of ecliptic latitude

Amplitude Errors I: Magnitude



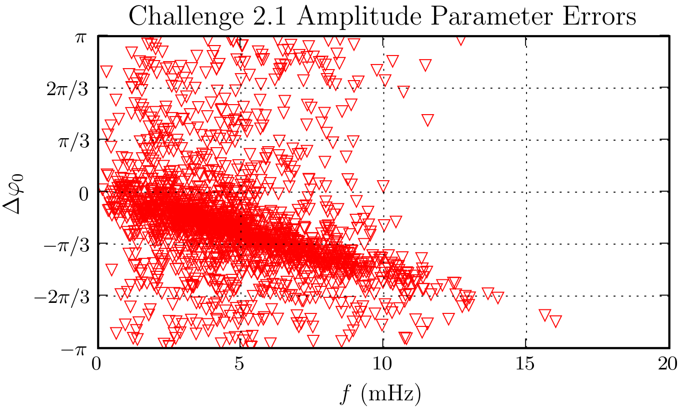
Systematic underestimate due to LWL response

Amplitude Errors II: Phase



Systematic phase error due to LWL response

Amplitude Errors III: Initial Phase



LWL errors lead to φ_0 error linear in f: time shift!

Moving Beyond the LWL (Deepak Khurana)

- More precise modelling of LISA interferometry gives (“Rigid Adiabatic”) expressions like

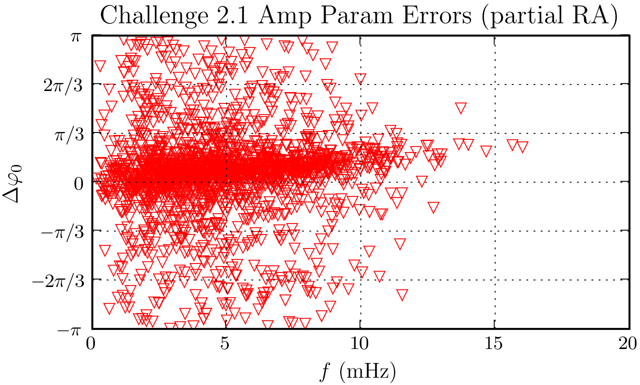
$$\frac{\overleftrightarrow{d}^X(f,\widehat{k})}{R(f)} = \left(-i\frac{4\pi fL}{c}\right) e^{-i4\pi fL/c}\,\mathrm{sinc}\left(\frac{2\pi fL}{c}\right) \times \left\{\mathfrak{T}_{\widehat{n}_2}(f,\widehat{k})\frac{\widehat{n}_2\otimes\widehat{n}_2}{2} - \mathfrak{T}_{-\widehat{n}_3}(f,\widehat{k})\frac{\widehat{n}_3\otimes\widehat{n}_3}{2}\right\}$$

- MLDC entries used $R(f)^{-1} = \left(-i\frac{4\pi fL}{c}\right)$ & $\overleftrightarrow{d} = \frac{\widehat{n}_2\otimes\widehat{n}_2}{2} - \frac{\widehat{n}_3\otimes\widehat{n}_3}{2}$
- Work on implementating full $\overleftrightarrow{d}(f,\widehat{k})$ (code restructuring) In the meantime, can use LWL $\overleftrightarrow{d}$ &

$$R(f)^{-1} = \left(-i\frac{4\pi fL}{c}\right) e^{-i4\pi fL/c}\,\mathrm{sinc}\left(\frac{2\pi fL}{c}\right)$$

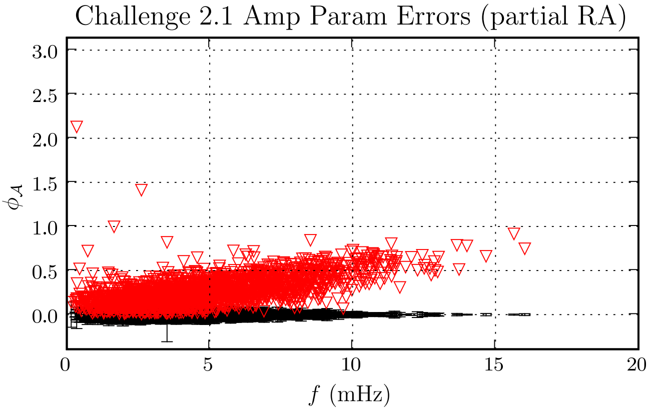
for partial Rigid Adiabatic Response

Amplitude Errors w/partial RAR: Initial Phase



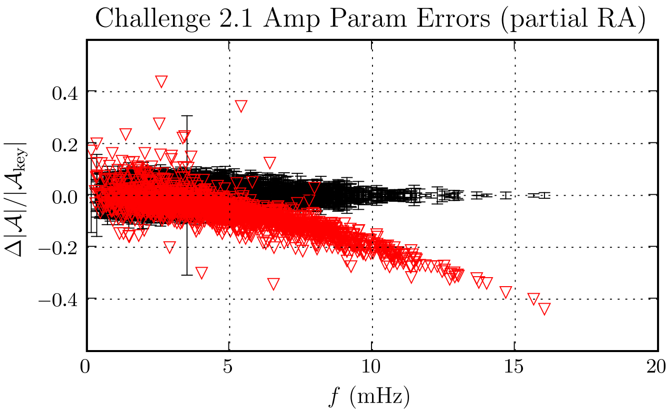
Most of systematic drift removed;
residual may be due to anisotropy of galaxy

Amplitude Errors w/partial RAR: Phase



Phase errors much smaller

Amplitude Errors w/partial RAR: Magnitude



Underestimate reduced by sinc fcn in $R(f)$

Conclusions

- $\mathcal{F}$-statistic method to find doppler-shifted periodic signals applied to mock LISA data
- Weaker signals can be mistaken for secondary maxima
  MLDC1 $\longrightarrow$ MLDC2 pipeline improvements mitigate
  (Still, LISA experts can find $10\times$ as many sources)
- Long-λ limit hampers determination of amp params
  doppler params still good at high freq
- Still working on full response,
  but partial RAR removes egregious phase errors