Bumps in the Big Bang

Simulating Fluctuations in a Gravitational Wave Background

Madeleine Udell[1]
Collaborator: Grant Meadors[2]
Mentors: Stefan Ballmer[3] and Vuk Mandic[3,4]

[1]Yale University [2]Reed College [3]California Institute of Technology
[4]University of Minnesota

Caltech Summer Seminar Day
07-08-14

Outline

1. Introduction
   - The SGWB
   - Sources
2. Motivation for Spherical Harmonics
   - An Example
   - Advantages
3. Searches
   - Types of Searches
   - Results
   - Ongoing Work
4. Conclusion

What is a Stochastic Background?

Definition

The Stochastic Background of Gravitational Waves *(SGWB)*

- **random signal**
- **weak, independent, and unresolved** gravitational wave sources
- characterized by statistical properties

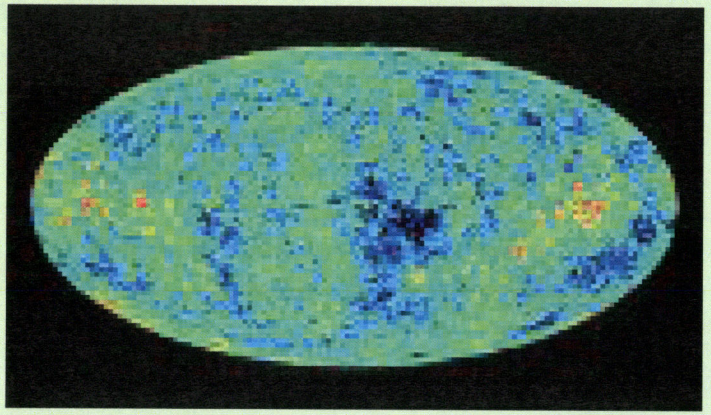
Expectation value depends on:

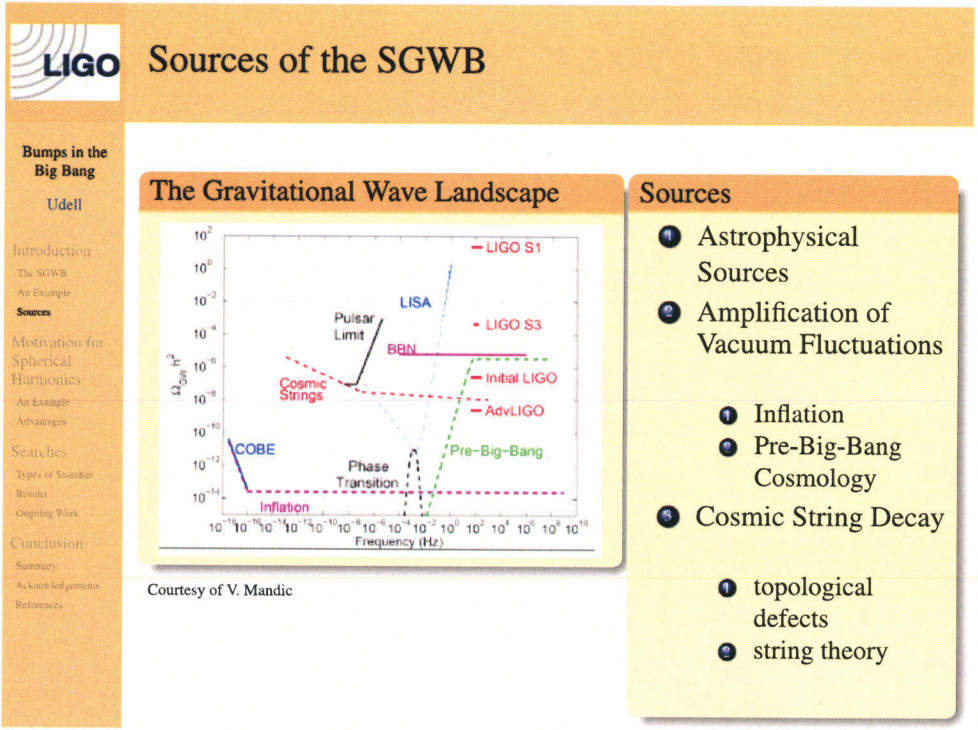
1. frequency $H(f)$
2. direction $P(\Omega)$

A Stochastic Background: The CMB

Example

Figure: The Cosmic Microwave Background is a stochastic background of electromagnetic radiation





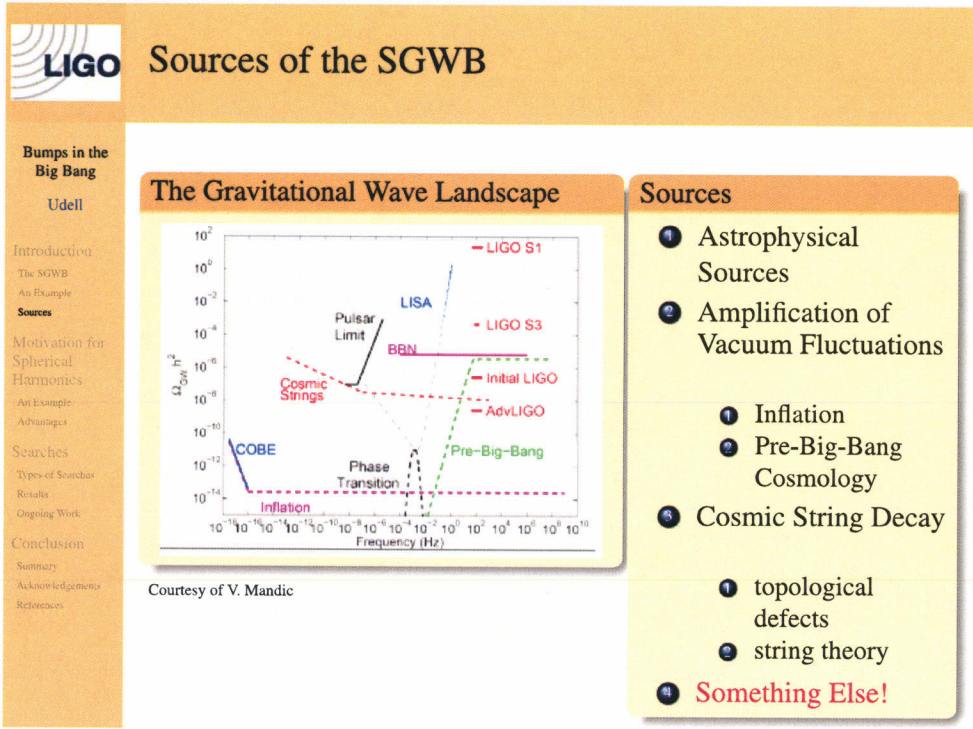
Sources of the SGWB

The Gravitational Wave Landscape

Courtesy of V. Mandic

Sources

1. Astrophysical Sources
2. Amplification of Vacuum Fluctuations
    1. Inflation
    2. Pre-Big-Bang Cosmology
3. Cosmic String Decay
    1. topological defects
    2. string theory



Sources of the SGWB

The Gravitational Wave Landscape

Courtesy of V. Mandic

Sources

1. Astrophysical Sources
2. Amplification of Vacuum Fluctuations
    1. Inflation
    2. Pre-Big-Bang Cosmology
3. Cosmic String Decay
    1. topological defects
    2. string theory
4. Something Else!

Spatial Dependence of the CMB

Example

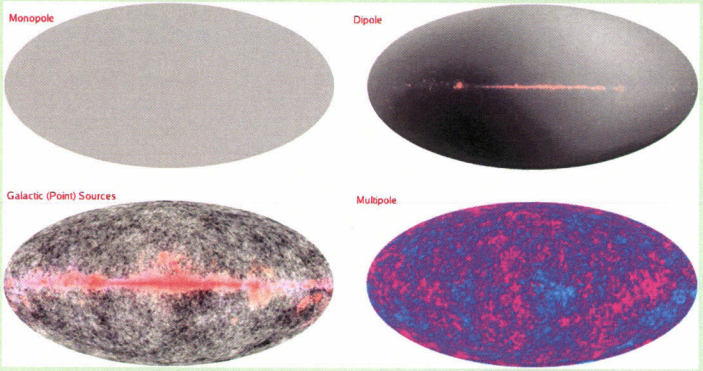


Figure: The contrast in the bottom images is enhanced 8,000 times

Courtesy of E. Wright http://www.astro.ucla.edu/ wright/CMB-DT.html

Spherical Harmonics Encode the Size of Inhomogeneities

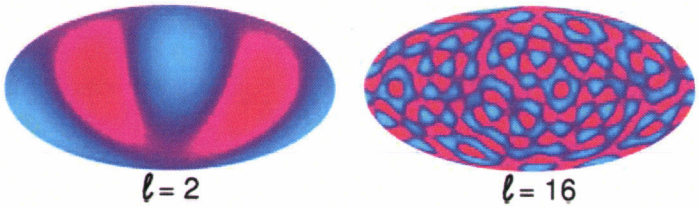


Figure: Spherical harmonics maps for l=2 and l=16

Courtesy of E. Wright http://www.astro.ucla.edu/ wright/CMB-DT.html

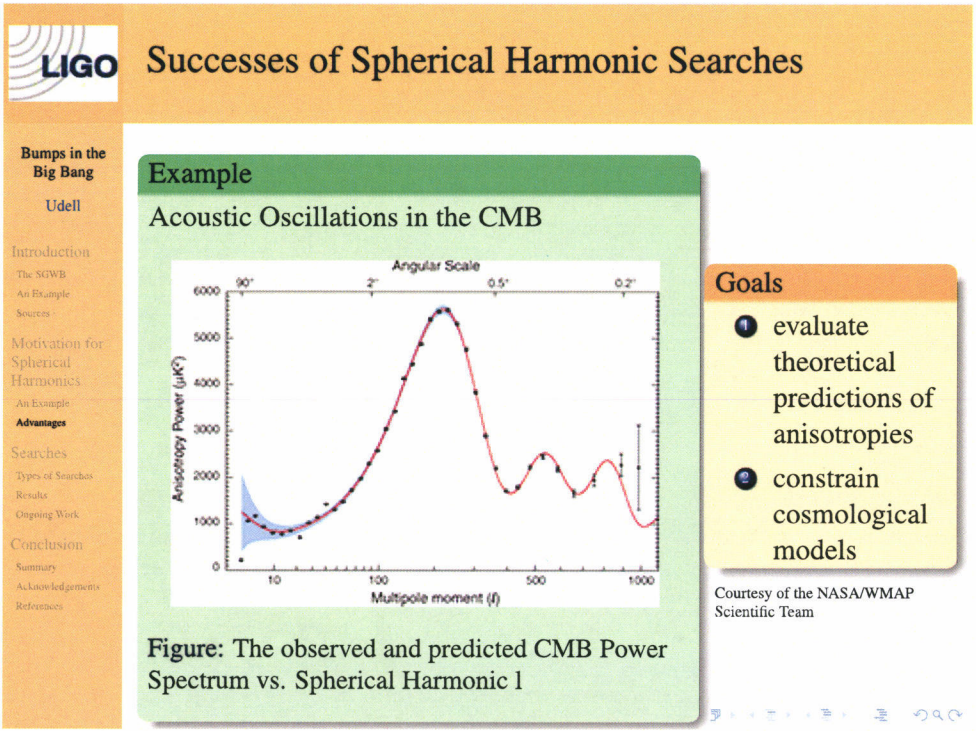
Choosing a Basis

Point-Source

1. Galactic Sources

Spherical Harmonics

1. Galactic Motion against Monopole Background
2. Cosmological Sources


LIGO
Successes of Spherical Harmonic Searches
Bumps in the Big Bang
Udell
Introduction
The SGWB
An Example
Sources
Motivation for Spherical Harmonics
An Example
Advantages
Searches
Types of Searches
Results
Ongoing Work
Conclusion
Summary
Acknowledgements
References
Example
Acoustic Oscillations in the CMB
Angular Scale
90°
2°
0.5°
0.2°
Anisotropy Power (μK²)
6000
5000
4000
3000
2000
1000
0
10
100
500
1000
Multipole moment (l)
Figure: The observed and predicted CMB Power Spectrum vs. Spherical Harmonic l
Goals
1 evaluate theoretical predictions of anisotropies
2 constrain cosmological models
Courtesy of the NASA/WMAP Scientific Team

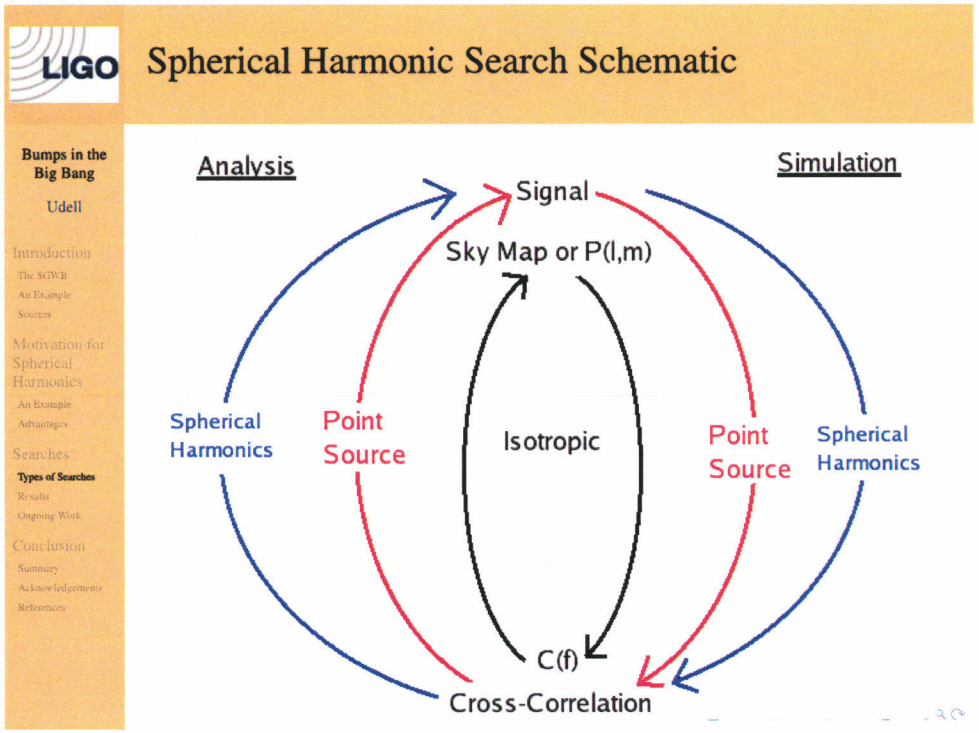
Cross-Correlation of Detectors

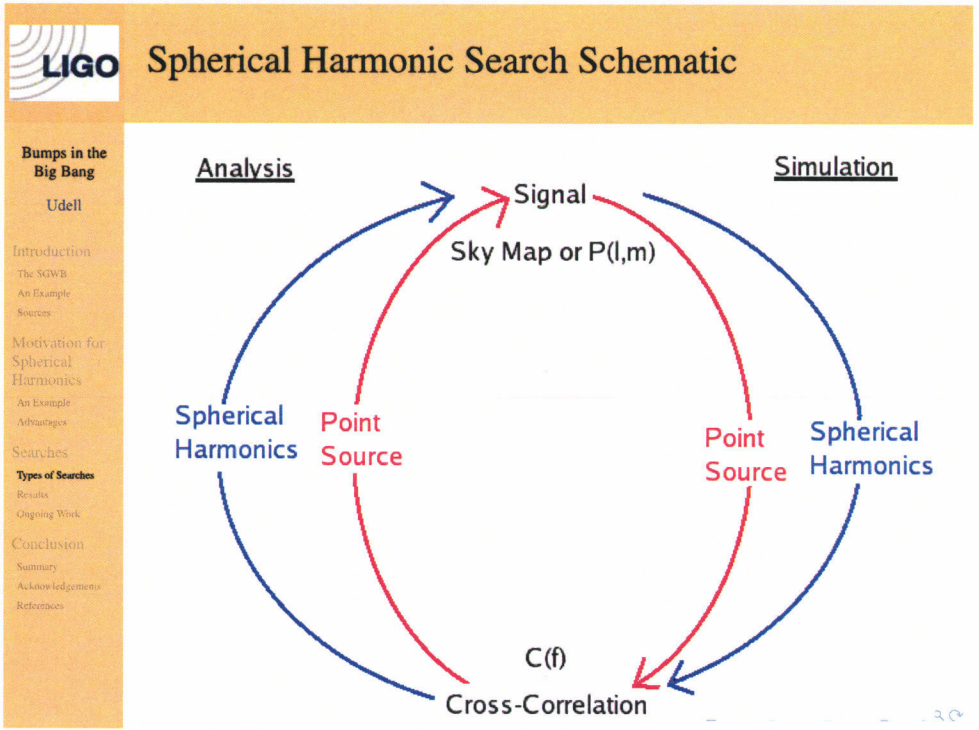
Definition

The Cross-Correlation between two detectors

$$C(f) := < s_1^*(f) s_2(f) >$$

- eliminates noise
- preserves signal


LIGO
Spherical Harmonic Search Schematic
Bumps in the Big Bang
Udell
Introduction
The SGWB
An Example
Sources
Motivation for Spherical Harmonics
An Example
Advantages
Searches
Types of Searches
Results
Ongoing Work
Conclusion
Summary
Acknowledgements
References
Analysis
Simulation
Signal
Sky Map or P(l,m)
Spherical Harmonics
Point Source
Isotropic
Point Source
Spherical Harmonics
C(f)
Cross-Correlation


LIGO
Spherical Harmonic Search Schematic
Bumps in the Big Bang
Udell
Introduction
The SGWB
An Example
Sources
Motivation for Spherical Harmonics
An Example
Advantages
Searches
Types of Searches
Results
Ongoing Work
Conclusion
Summary
Acknowledgements
References
Analysis
Simulation
Signal
Sky Map or P(l,m)
Spherical Harmonics
Point Source
Point Source
Spherical Harmonics
C(f)
Cross-Correlation

The Simulation Algorithm

Isotropic

$$C(f) = PH(f)\gamma(f)$$

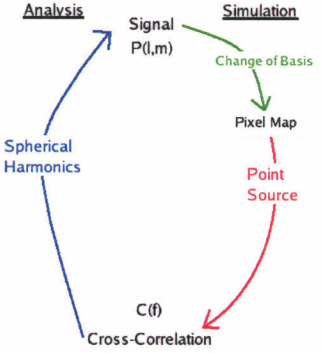
Point Source

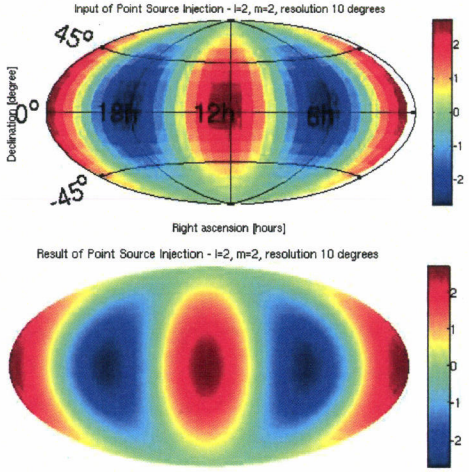
$$C(f) = \int_{all\ directions} d\Omega P(\Omega) H(f) \gamma(\Omega, f)$$

Spherical Harmonic

$$C(f) = \sum_{l,m} P_{l,m} H(f) \gamma(l, m, f)$$

Point Source Simulation of Spherical Harmonics



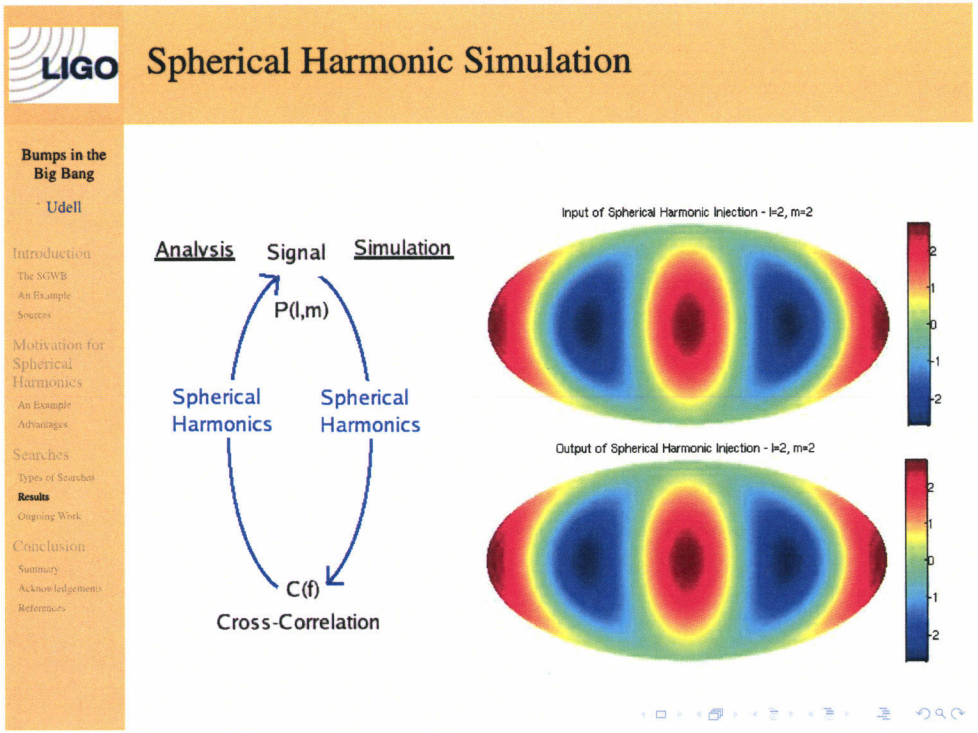


Signal Recovery: Point Source Simulation of Spherical Harmonics

Results

A Full Circle!

1. Point Source simulation works
2. Spherical Harmonic analysis works
3. Time $\propto$ # pixels included
4. LIGO is sensitive to 1,000 pixels


LIGO
Spherical Harmonic Simulation
Bumps in the Big Bang
Udell
Introduction
The SGWB
An Example
Sources
Motivation for Spherical Harmonics
An Example
Advantages
Searches
Types of Searches
Results
Ongoing Work
Conclusion
Summary
Acknowledgements
References
Analysis
Signal
Simulation
P(l,m)
Spherical Harmonics
Spherical Harmonics
C(f)
Cross-Correlation
Input of Spherical Harmonic Injection - l=2, m=2
2
1
0
-1
-2
Output of Spherical Harmonic Injection - l=2, m=2
2
1
0
-1
-2

Signal Recovery: Spherical Harmonic Simulation

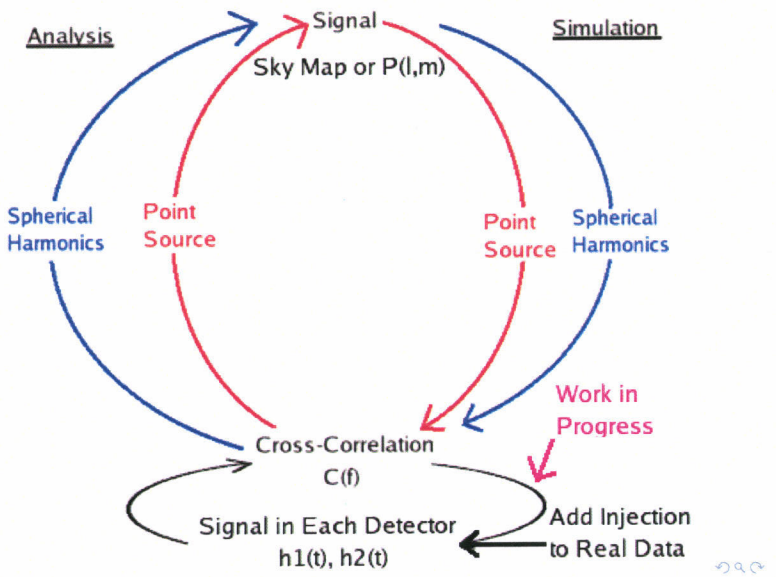
Results

1. Spherical Harmonic simulation works to Matlab precision

Advantages

1. Time $\propto$ # Spherical Harmonics included
2. LIGO is sensitive to about 9 Spherical Harmonics
3. For full sky maps, much faster than pixelated version

Ongoing Work



Summary

Spherical Harmonic Decomposition of the SGWB

1. Looks Beyond Isotropic Case
2. Basis used by Theories
3. Match Measurement and Theory
4. Code Simulates and Recovers

Acknowledgements

- **Mentors:** Stefan Ballmer and Vuk Mandic
- **Collaborator:** Grant Meadors
- LIGO Science Collaboration
- SURF / REU
- California Institute of Technology
- National Science Foundation

Bibliography

B. Allen and J. Romano, Phys. Rev. D59 (1999) 102001.

S. Ballmer, Class. Quantum Grav. 23 (2006) S179–S185

M. Maggiore, Gravitational wave experiments and early universe cosmology, gr-qc/990901v4 (2000)

Ballmer, S. and Romano, J. and Mandic, V. and Mitra, S., Formulae for stochastic multipole search, LIGO T070045-00-U (2007)