Glitch Rejection Properties of a Simple Coherent Network Algorithm for Detecting Unmodeled GW Bursts

The Waves Group Università degli Studi del Sannio

INFN Istituto Nazionale di Fisica Nucleare

Maria Principe, Innocenzo M. Pinto

TWG, University of Sannio at Benevento, Italy

principe@unisannio.it, pinto@sa.infn.it

LIGO G070834-00-Z

LSC

Introduction

In GWB detection we face a twofold difficulty. First the sought waveforms are unmodeled; second, transient instrumental disturbances (glitches) make the noise non-Gaussian and poorly statistically modeled. Coherent network algorithms exploit network redundancy to estimate the waveforms from the data; suitable versions of the likelihood ratio are accordingly obtained, usually based on the Gaussian background noise assumption, and ad-hoc arguments are added aimed at assessing the ability to discriminate spurious glitches from true GWBs. Here we propose a general promising model for the impulsive IFO noise component along the lines laid out by D. Middleton. We use this model to assess the performance of the cross-correlation based coherent detection algorithm proposed by Rakhmanov and Klimenko (RK), for the case of triggered search, highlighting the effect of glitches in terms of ROC degradation. The results are obtained in a readable analytic form.

Middleton's model for glitches

Experimental evidence suggests that instrumental noise glitches can be modelled as *atoms* (waveforms with almost-compact support) in the time-frequency (TF) plane, characterized in terms of energy content, occurrence time t_0, center frequency f_0, effective duration σ_t and bandwidth σ_f.

The impulsive noise component, $g(t)$, can be modelled as a random process consisting of a linear superposition of atoms, viz.

$$g(t)=\sum_{k=1}^{K[T]}\psi\left(t-t_0^{(k)};\underline{a}^{(k)}\right),\quad t\in\Theta$$

- $\psi(\cdot)$ chosen representation atom;
- $t_0^{(k)}$ glitch firing times (random);
- $\underline{a}^{(k)}$ atoms' shape vectors (random);
- $K[T]$ # of glitches in (T seconds wide) analysis window Θ (random).

The random parameters $\underline{a}^{(k)}$ are determined *independently* at each glitch instance (i.e. for each k); their distribution must be deduced from available data; $K[T]$ follows a Poisson distribution [Hurwitz and Kac, Ann. Math. Stat. **15** (1944) 173], i.e.

Expected # of glitches in Θ

$$\text{prob}\{K[T]=K\}=\frac{\bar{N}^K e^{-\bar{N}}}{K!}$$

The characteristic function of $g(t)$ can be computed exactly up to any order [D. Middleton, J. Appl. Phys. 22 (1951) 1143]. To first order

$$F_g(\xi,t)=\exp\left[\bar{N}\left(E\left[e^{i\xi\psi(t-t_0;\underline{a})}\right]-1\right)\right]$$

yielding for the first two moments

$$\mu_g^{(1)}=E[g(t)]=\bar{N}E\left[\psi(t-t_0;\underline{a})\right]_{t_0,\underline{a}}$$

$$\mu_g^{(2)}=E\left[g(t)^2\right]=\bar{N}^2E^2\left[\psi(t-t_0;\underline{a})\right]_{t_0,\underline{a}}+\bar{N}E\left[\psi^2(t-t_0;\underline{a})\right]_{t_0,\underline{a}}$$

Moments are time independent, such being by assumption the prior distributions of $t_0^{(k)}$, $\underline{a}^{(k)}$ in Θ.

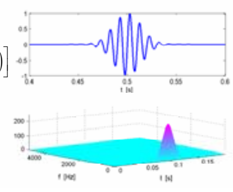
Here we adopt the (real) Sine-Gaussian (SG) atom, in view of its structural simplicity and minimum TF spread property ($\sigma_t\sigma_f=1/4\pi$)

$$\psi(t-t_0;g_0,f_0,\sigma_t)=g_0\sin\left[2\pi f_0(t-t_0)\right]e^{-(t-t_0)^2/\sigma_t^2}$$

for which

$$\mu_g^{(1)}\Big|_{\psi=SG}=0$$

$$\mu_g^{(2)}\Big|_{\psi=SG}=\frac{\bar{N}\sqrt{\pi}}{2}E\left[g_0^2\frac{\sigma_t}{T}\left(1-e^{-4\pi^2f_0^2\sigma_t^2}\right)\right]$$



Sine-Gaussian atom (top), and its time-frequency (Wigner – Ville) image (bottom)

The RK Correlation Algorithm

As a preparation to the more general (and complicated) case where one works in terms of the network likelihood ratio, we consider the simplest detection algo for *unmodelled* bursts devised by Rakhmanov and Klimenko [CQG **22** (2005) S1311], exploiting network redundancy to first construct *noisy* templates for detector i

$$W_i(t)=-A_jV_j(t)-A_\ell V_\ell(t)$$

for/from properly time-shifted (τ_i) IFO outputs

$$V_i(t)=S_i(t)+n_i(t)$$

where $A_i=F_j^+F_\ell^\times-F_j^\times F_\ell^+,\quad i\neq j\neq\ell$ and then to compute the *overlaps* (correlators):

$$C_i=\langle V_i,W_i\rangle\approx f_s^{-1}\sum_{m=1}^{N_s}V_{im}W_{im}$$

These are the best measure of overlap between V_i and W_i for Gaussian background noise, and still *asymptotically optimal* in the weak signal limit, for non-Gaussian noise, up to a suitable non-linear pre-processing of data (V_i), to be discussed elsewhere.

Pattern factors $F_i^{+,\times}$ and propagation delays τ_i depend on the direction of arrival (DOA) Ω_s. DOA and time of occurrence assumed known (*triggered search*).

Irrespective of the underlying noise properties, the RK correlators are Gaussian distributed (CLT) and thus completely characterized by their 1st and 2nd order moments.

Moments under H_1 hypothesis …

N_s : # of samples in Θ
f_s : sampling rate

$$\mu_1^{(i)}=E\left[C_i|H_1\right]\approx f_s^{-1}A_i\mathbf{S}_i\cdot\mathbf{S}_i^T$$

$$\left(\sigma_1^{(i)}\right)^2=Var\left[C_i|H_1\right]\approx f_s^{-2}A_i^2\left[\left(\sigma_i^2+\tilde{\sigma}_i^2\right)\mathbf{S}_i\cdot\mathbf{S}_i^T+\sigma_i^2\tilde{\sigma}_i^2N_s\right]$$

where σ is the noise variance in the detectors, and $\tilde{\sigma}_i^2=\left(A_j^2+A_\ell^2\right)A_i^{-2}\sigma_i^2$ is the noise variance in the template.

δ_S : SNR of V_i
δ_h : SNR of GW signal
$|F_i|$: antenna response at Ω_s

… corresponding deflection

$$d^{(i)}=\delta_S^{(i)}\frac{A_i}{\left[A_i^2+A_j^2+A_\ell^2+N_s\left(A_j^2+A_\ell^2\right)\left(|F_i|\delta_h\right)^{-2}\right]^{1/2}},\quad \delta_S^{(i)}=|F_i|\delta_h$$

GWB and glitches occurrence in Θ ruled out by hyp.

Moments under H_0 hypothesis

$$\Xi(\Omega_s)=\frac{2A_jA_\ell}{A_j^2+A_\ell^2}$$

$$\mu_0^{(i)}=E\left[C_i|H_0\right]=-f_s^{-1}N_s\left(A_j+A_\ell\right)\left(\mu_g^{(1)}\right)^2$$

$$\left(\sigma_0^{(i)}\right)^2=Var\left[C_i|H_0\right]=\underbrace{\left(\sigma_{AWGN}^{(i)}\right)^2}_{\text{AWGN term}}\left\{1+2\underbrace{\left[\mu_g^{(2)}+\Xi(\Omega_s)\frac{\left(\mu_g^{(1)}\right)^2}{\sigma^2}\right]}_{\text{Glitch correction}}\right\}$$

Detector Performance

We consider the network: {LHO (H), LLO (L), Virgo (V)}. From the Gaussianity of C_i, the false dismissal probability β is related to the false alarm probability α as follows (Receiver Operating Characteristics, ROC):

$$\beta^{(i)}=1-\text{erfc}\left[\frac{\sigma_0^{(i)}}{\sigma_1^{(i)}}\text{erfc}^{-1}(\alpha)+\frac{\mu_0^{(i)}}{\sigma_1^{(i)}}-\frac{\mu_1^{(i)}}{\sigma_1^{(i)}}\right]$$

Simulations

Figs 1,2 illustrate the performance of the RK detector compared to the ideal matched filter (MF). A degradation by a factor ~2 in terms of deflection at the optimum DOA and by a factor ~3 averaged over all sky is observed. Glitches are neglected in Figs 1,2. The effects of glitches are illustrated in Figs 3,4, for different firing rates (Fig 3) and SNR_{veto} levels (Fig 4).

The prior distributions of f_0 and σ_t for SG atoms, used in drawing Figs 3,4 were obtained from 1 week of instrumental triggers [http://ldas-jobs.ligo-wa.caltech.edu/qonline]. The glitch SNR is assumed as uniformly distributed between zero and SNR_{veto}. Performance deteriorates as either the glitch firing rate or/and the SNR_{veto} level increases.

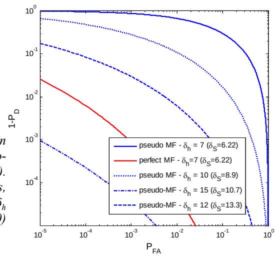


***Figure 1.** Performance in terms of ROCs of RK pseudo-correlator (H detector). Optimal DOA. T = 100 ms, f_s = 4096 Hz, several δ_h values. No glitches ($\bar{N}$ =0)*

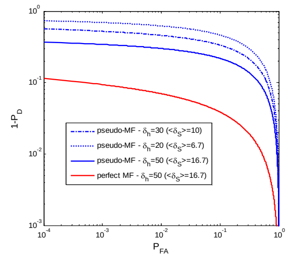


***Figure 2.** All-sky averaged performance in terms of ROCs of RK pseudo-correlator (H detector). T = 100 ms; fs = 4096 Hz. No glitches ($\bar{N}$ =0)*

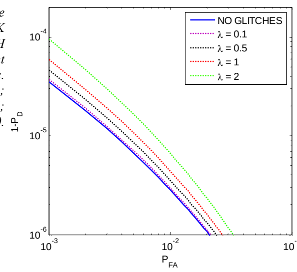


***Figure 3.** - Performance in terms of ROCs of RK pseudo-correlator (H detector) for different glitch firing rates. Optimal DOA. T=100ms; fs = 4096 Hz; δ_h=15; SNR_{veto} = 100.*

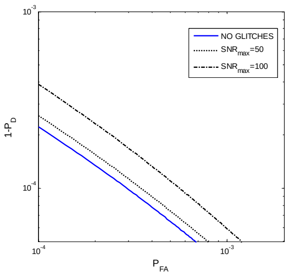


***Figure 4.** - Performance in terms of ROCs of the RK pseudo-correlator (H detector) for different SNR_{veto} levels. Optimal DOA. T= 100 ms; fs = 4096 Hz; δ_h=15; $\bar{N}$= 0.1.*

Conclusions/hints for future work

Main results:

- New characterization of the impulsive IFO noise component based on atomic representation for glitches and Middleton's model;
- Fully analytical model for RK detector's performance under CFAR operation (triggered search), including glitches obtained;
- RK detector robustness against instrumental glitches illustrated in terms of ROCs.

Directions for future work:

- Better glitch modeling (better atom dictionary and more accurate characterization of glitch parameters priors);
- Better detection statistics (e.g., direction-optimized combinations of the C_i);
- Implementation of "quasi-optimal" detector tailored to non-Gaussianity by nonlinear data conditioning before correlation (exploiting fully Middleton's model).