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Estimating the Spatial Structure of a Stochastic Gravitational Wave Background

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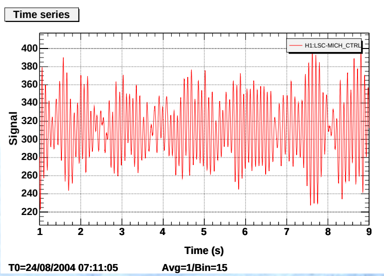
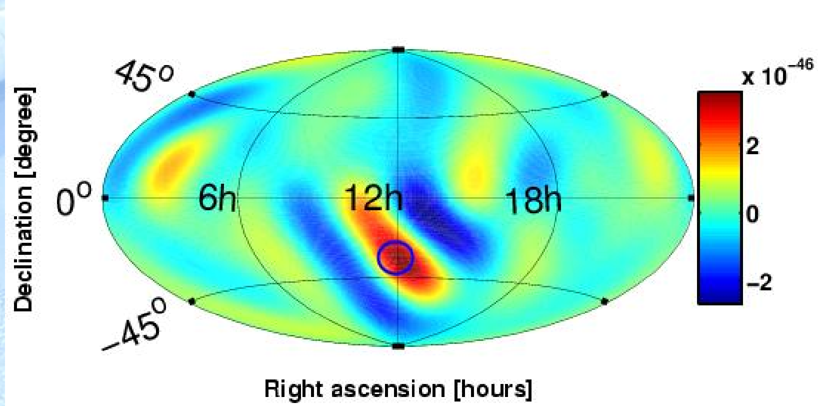
The generic Problem



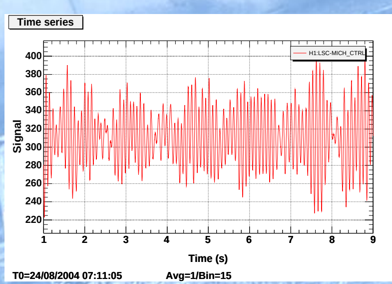
Detector 1

Spinning Earth

?



A spatially resolved map



Detector 2

Only restriction: stationary signal

Some Definitions (sorry)

- The Signal:

$$\langle h^*_{Af\hat{\Omega}} h_{A'f'\hat{\Omega}'} \rangle = \delta_{AA'} \delta(f - f') \delta^2(\hat{\Omega}, \hat{\Omega}') P(\hat{\Omega}) \frac{H(|f|)}{4}$$

- The Measurements:
  - Detector X-Power as function of time and frequency.

$$C_{ft} = s_1^*(f,t) s_2(f,t)$$

  - They are all independent, i.e.

$$N_{ft,f't'} := < (C_{ft} - < C_{ft} >)^* (C_{ft} - < C_{ft} >) > \approx \delta_{tt'} \delta_{ff'} P_1(f,t) P_2(f,t)$$

The Expectation Value

- Expectation value of Measurement: depends on time due to earth rotation

$$\left\langle C_{ft} \right\rangle = \int d\Omega \, \gamma_{ft\Omega} H_f P_\Omega$$

- This defines the geometry factor

$$\gamma_{t\Omega}(f) = \frac{1}{2} \sum_{A=+,\times} e^{i 2\pi f \hat{\Omega} \frac{\Delta\vec{x}(t)}{c}} F_{1,t}^{A}(\Omega) F_{2,t}^{A}(\Omega)$$

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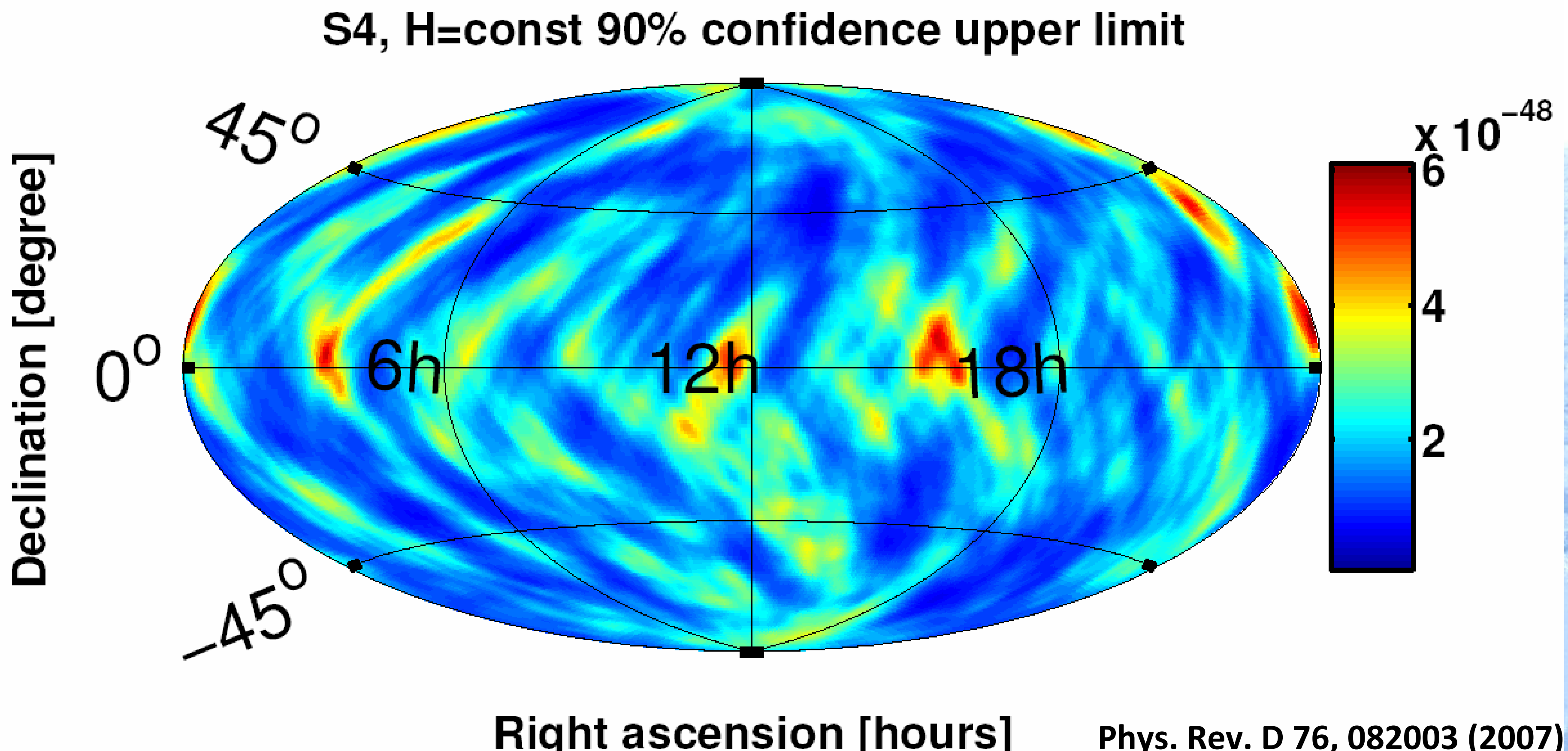
So far:
Looking for Point Sources

- For each pixel:

$$Y_\Omega \propto \sum_t \int df \, \gamma^*_{\Omega,ft} \frac{H_f}{P_{1,f} P_{2,f}} C_{ft}$$

- Optimal to find isolated Point Sources
- Computationally inexpensive
- Neighboring pixels correlated (“blurred map”)
- S4 result: Phys. Rev. D 76, 082003 (2007)

S4 Result: Limit on Point Sources



Phys. Rev. D 76, 082003 (2007)

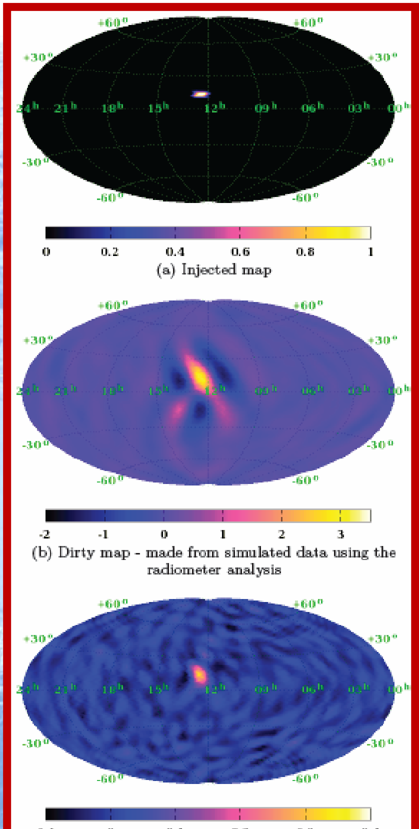
$$H_{90\%} = (0.85 - 6.1) \times 10^{-48}\, Hz^{-1}$$

What about that blur?

- "Blur" described by covariance (Fisher) matrix:

$$\Gamma_{\Omega'\Omega} = \sum_t \int df\ \gamma^*_{ft\Omega'} \frac{H_f^2}{P_{1,f} P_{2,f}} \gamma_{ft\Omega}$$

- In principle not hard to calculate,
  - But Γ needs to be inverted to calculate noise
  - Size = (# Pixel)^2
  - No exploitable symmetry for Pixel basis
- S. Mitra et. al., to be published in PRD
  - Fisher matrix not fully inverted



(a) Injected map

(b) Dirty map - made from simulated data using the radiometer analysis

(c) Clean map - obtained by deconvolution of the dirty map using 15 CG iterations (NMSE = 1.22)

A better basis

- Pixel basis not optimal for deconvolution
  - Does not respect rotational symmetry
  - No natural resolution cut-off

- The obvious candidate:
  - Spherical Harmonics
    - Have all of the above

Spherical Harmonics

- Maximum likelihood estimation gives:

$$P_{lm} = (\Gamma^{-1})_{ml,l'm'} X_{l'm'}$$

$$\Gamma_{lm,l'm'} = \gamma^*_{ft,lm} \frac{H^2(f)}{P_1(f,t)P_2(f,t)} \gamma_{ft,l'm'}$$

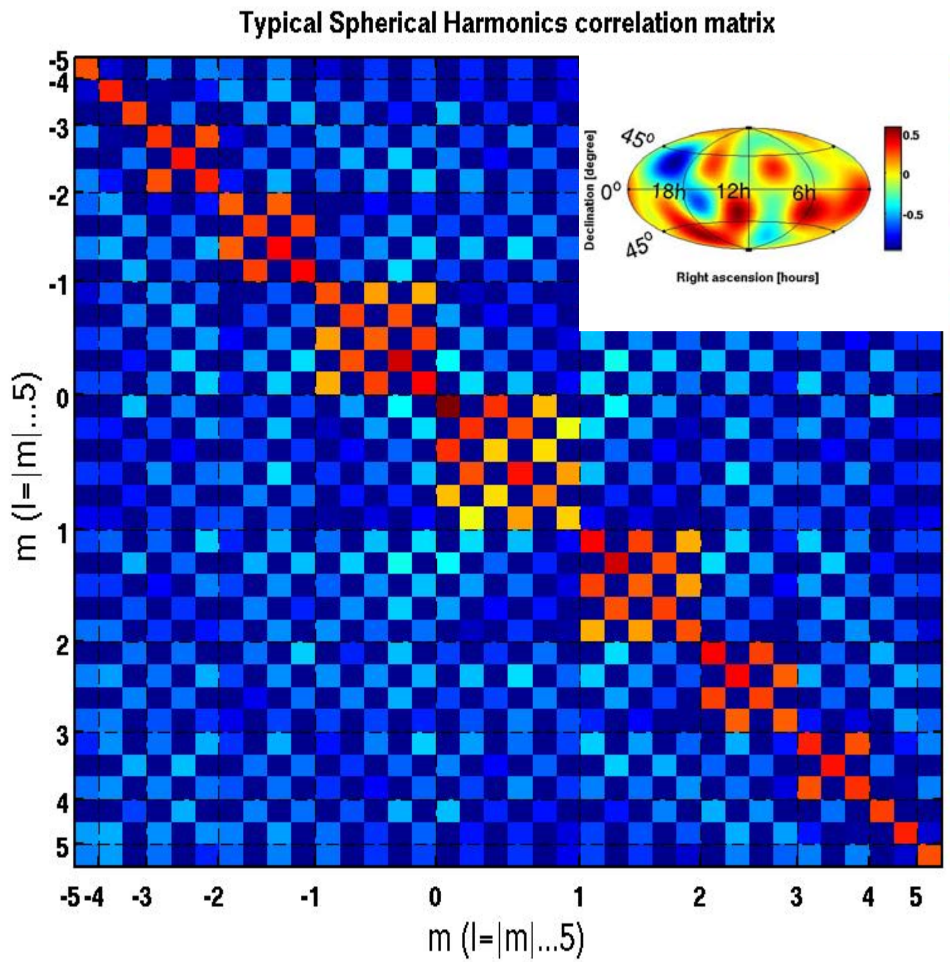
$$X_{lm} = \gamma^*_{ft,lm} \frac{H(f)}{P_1(f,t)P_2(f,t)} C_{ft}$$

i.e. same as before

- But symmetries imply structure:
  - Parity: Δl odd → Γ = 0 (exact)
  - Rotational symmetry: m ≠ m’ → Γ ≈ 0 (approx.)

An example correlation matrix

- | Fisher matrix| for a random map ($l_{max} = 5$)
- Dark blue = 0
- Bright blue ≈ 0
  - ➔Inversion easier
- Some Y_{lm} hard to separate



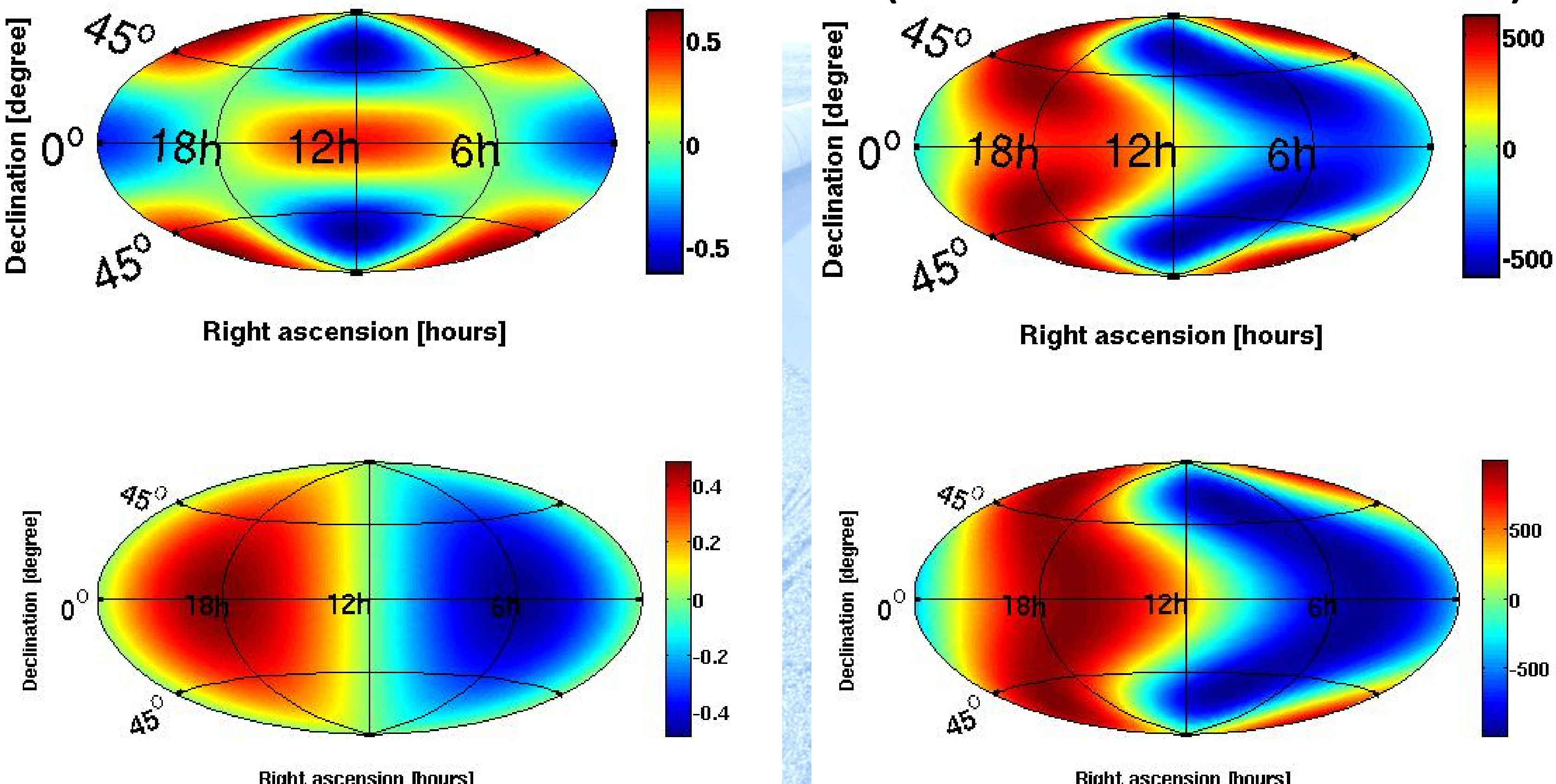
Y_{31} vs. Y_{11} as an example

Actual background

Seen by GW antenna pair (before deconvolution)

Y_{31}

Y_{11}



Conclusion

- Deconvolution allows estimation of spatial structure.
- Spherical Harmonics is appropriate basis
  - But some Y hard to separate
- Implementation Status
  - Analysis core exists in MATLAB
  - Further testing still required

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What about that blur?
Maximum likelihood formalism (1)

- Model:
  - Distribution:

$$P(\Omega) = \sum_{lm} P_{lm} Y_{lm}$$

Expectation value:

$$< C_{ft} >= H(f)\gamma_{ft,lm}P_{lm}$$

- Probability distribution: ...just Gaussian...

$$p(C_{ft}|P_{lm}) \propto \exp\left[-\frac{1}{2}\left(C^*_{ft} - H(f)\gamma^*_{ft,lm}P^*_{lm}\right)\left(N^{-1}\right)_{ft,f't'}\left(C_{f't'} - H(f')\gamma_{f't',l'm'}P_{l'm'}\right)\right]$$

What about that blur? Maximum likelihood formalism (2)

- Probability distribution:

$$p(C_{ft}|P_{lm}) \propto \exp\left[-\frac{1}{2}\left(C^*_{ft} - H(f)\gamma^*_{ft,lm}P^*_{lm}\right)(N^{-1})_{ft,f't'}\left(C_{f't'} - H(f')\gamma_{f't',l'm'}P_{l'm'}\right)\right]$$

- Is maximal at

$$P_{lm} = (\Gamma^{-1})_{ml,l'm'}X_{l'm'}$$

$$\Gamma_{lm,l'm'} = \gamma^*_{ft,lm}\frac{H^2(f)}{P_1(f,t)P_2(f,t)}\gamma_{ft,l'm'}$$

$$X_{lm} = \gamma^*_{ft,lm}\frac{H(f)}{P_1(f,t)P_2(f,t)}C_{ft}$$

- H(f): Spectral shape
- $\gamma_{ft,lm}$: Geometry factor