
Search for gravitational waves from spinning binaries in LIGO data using a new family of Physical Templates

Diego Fazi

California Institute of Technology

and

LIGO Scientific Collaboration

LIGO-G080009-00-Z

Università di Bologna – January 15 2008

The theory behind gravitational waves

Einstein's equations admit wave-like solutions for non-static space-times for which the metric can be written as $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$

- In particular, **spinning compact binaries** such as neutron-star (BNS) and black-hole (BBH) binaries are expected to emit energy in the form of GW's during their coalescence phase.

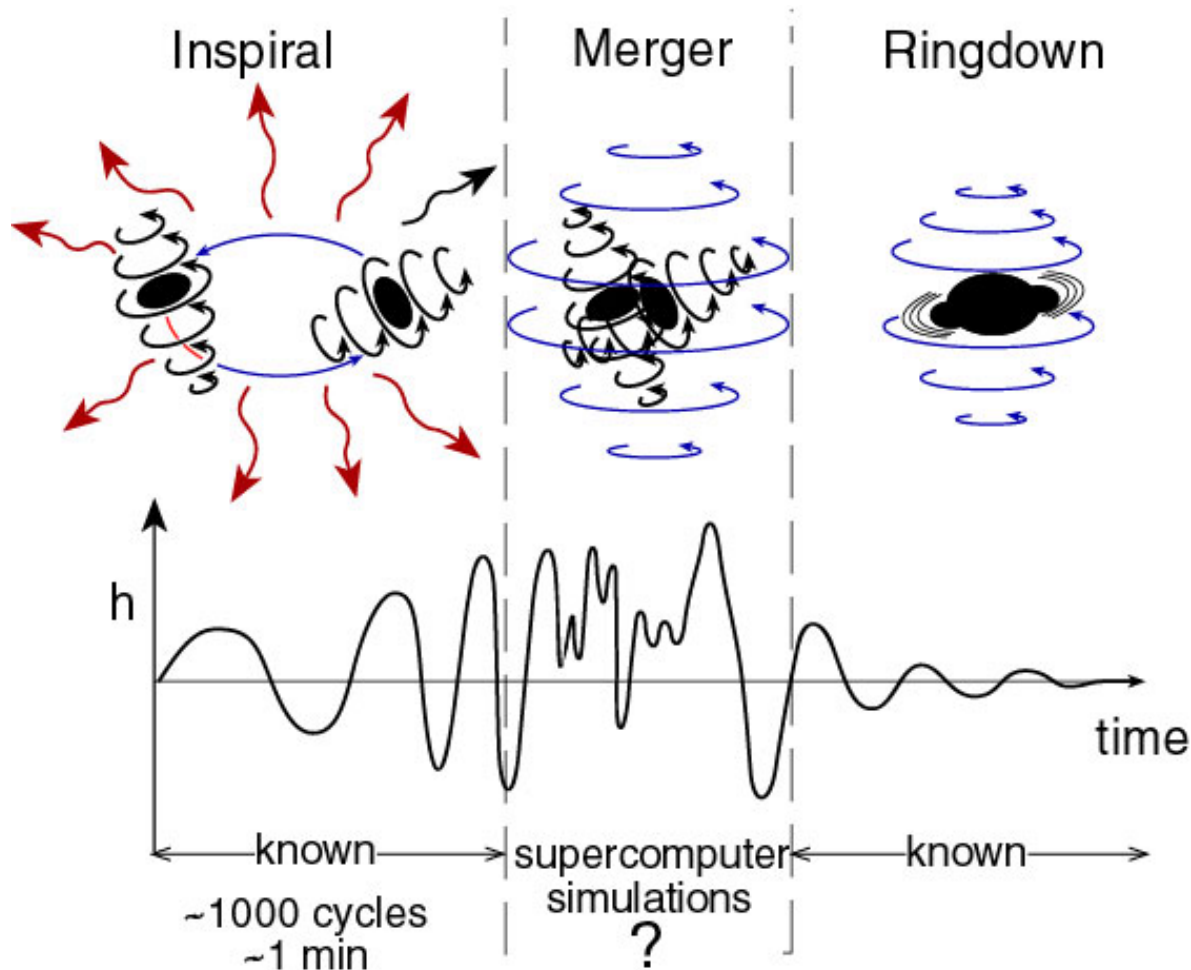


Far from the source the gravitational wave field is described at lowest order by the quadrupole formula which, in the transverse traceless gauge, reads

$$h_{jk}^{TT} = \frac{2G}{c^4 r} \frac{d^2 I_{jk}^{TT}(t-r)}{dt^2}$$

where I_{jk} is the binary quadrupole moment.

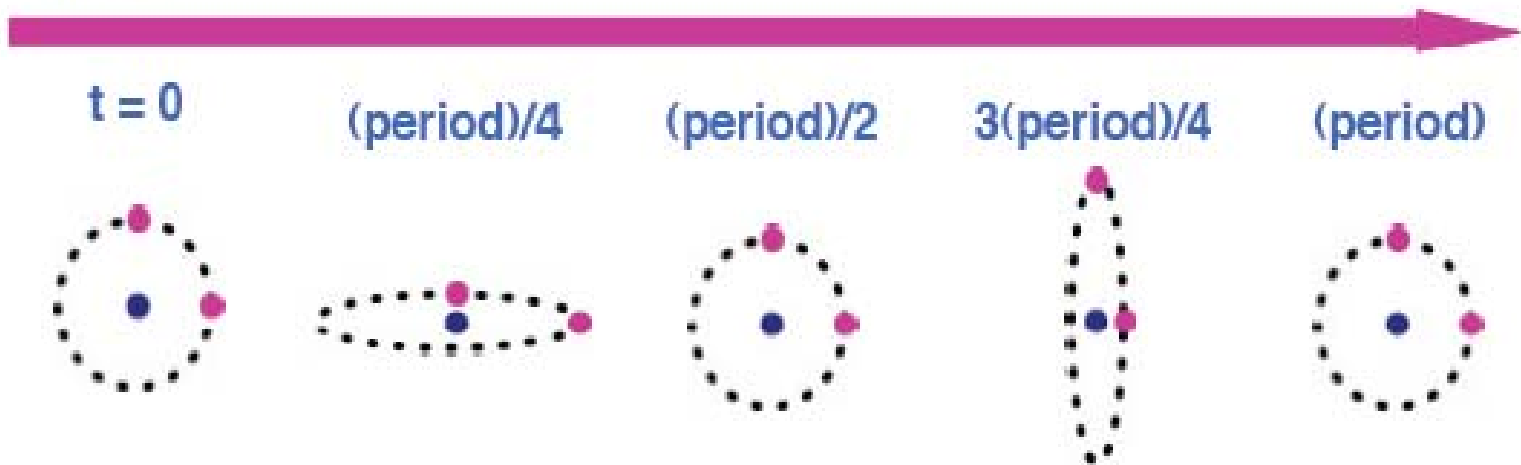
The coalescence of a binary system



The effect of a gravitational wave

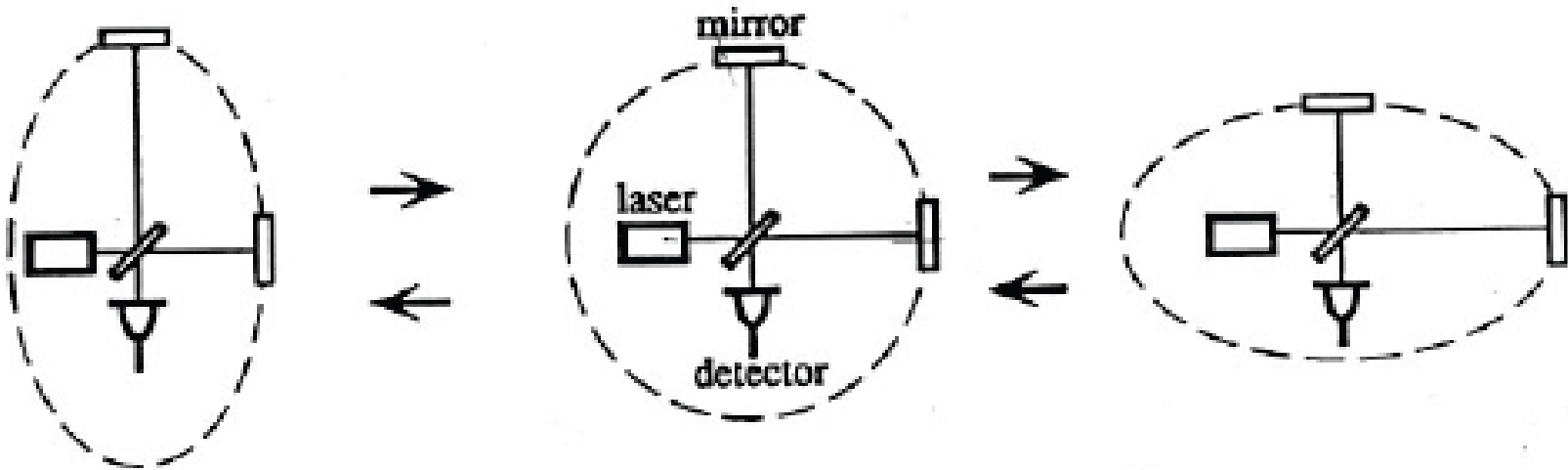
- When a gravitational wave passes through matter it causes the distance between freely falling particles to vary in time. If we consider a ring of freely falling particles, a gravitational wave will induce over a period the motions shown below

Time



What kind of detector do we need?

- Michelson-like **interferometers** can measure with high accuracy the relative displacement of mirrors placed at the ends of the two orthogonal optical paths, by measuring the phase shift of the two recombining laser beams. The time evolution of this phase describes the gravitational wave!

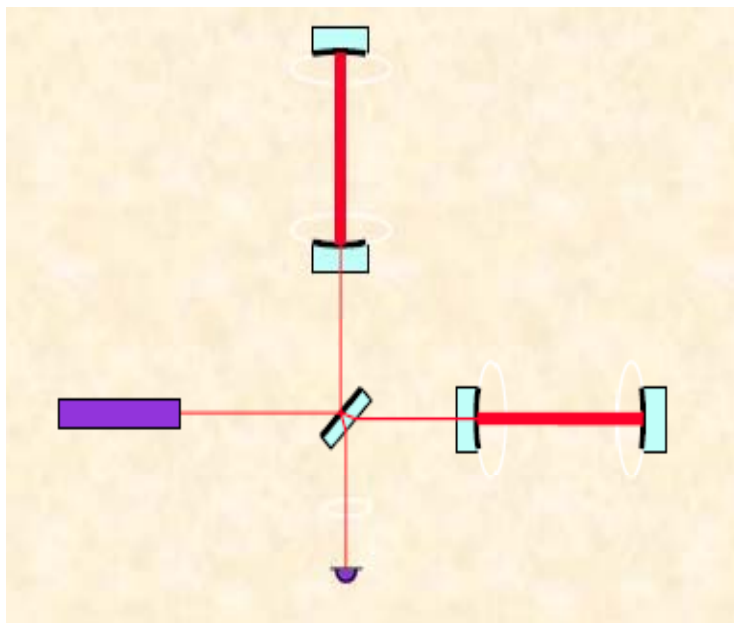


The Challenge!

- The strength of a gravitational wave is measured by the **strain** $h(t) = \Delta L(t)/L$; for a BNS with component masses of $1.4 M_{sun}$ in the VIRGO cluster (~ 18 Mpc) $\longrightarrow h \cong 10^{-21}$
- If the interferometer's arms are 4 Km long (like LIGO!), then the relative displacement between the mirrors will be of the order of $\Delta L = h \times L \cong 10^{-21} \times 4 \cdot 10^3 m \approx 10^{-18} m$
- Therefore the challenge for LIGO is to measure lengths ~ 1000 times smaller than the atomic nucleus!!! In order to increase the interferometer sensitivity we need the phase shift to be as large as possible, therefore we need L to be as large as possible: should we build 100 Km interferometers? It's not necessary! Just fold the light path 100 times!!!

Interferometric GW detectors

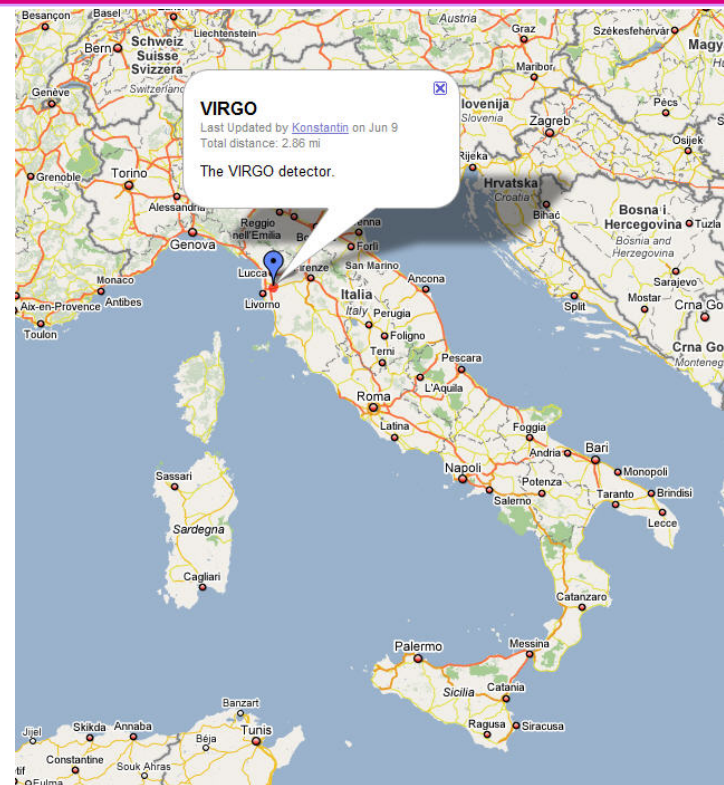
- LIGO/VIRGO interferometers are **Fabry-Perot resonant cavities**; two more mirrors are added which allow to store the light inside the interferometer arms and obtain an effective optical path 200 times longer than a normal Michelson



LIGO and VIRGO locations

LIGO: Two observatories 3002 Km apart, three detectors

- Hanford, WA (2 detectors)
- Livingston, LA (1 detector)



VIRGO: One observatory

- Cascina, Italy

The LIGO Observatory Sites

Interferometers are aligned along the great circle connecting the sites

LIGO Hanford Observatory (LHO)

H1 : 4 km arms

H2 : 2 km arms

10 ms

CALTECH

LIGO Livingston Observatory (LLO)

L1 : 4 km arms

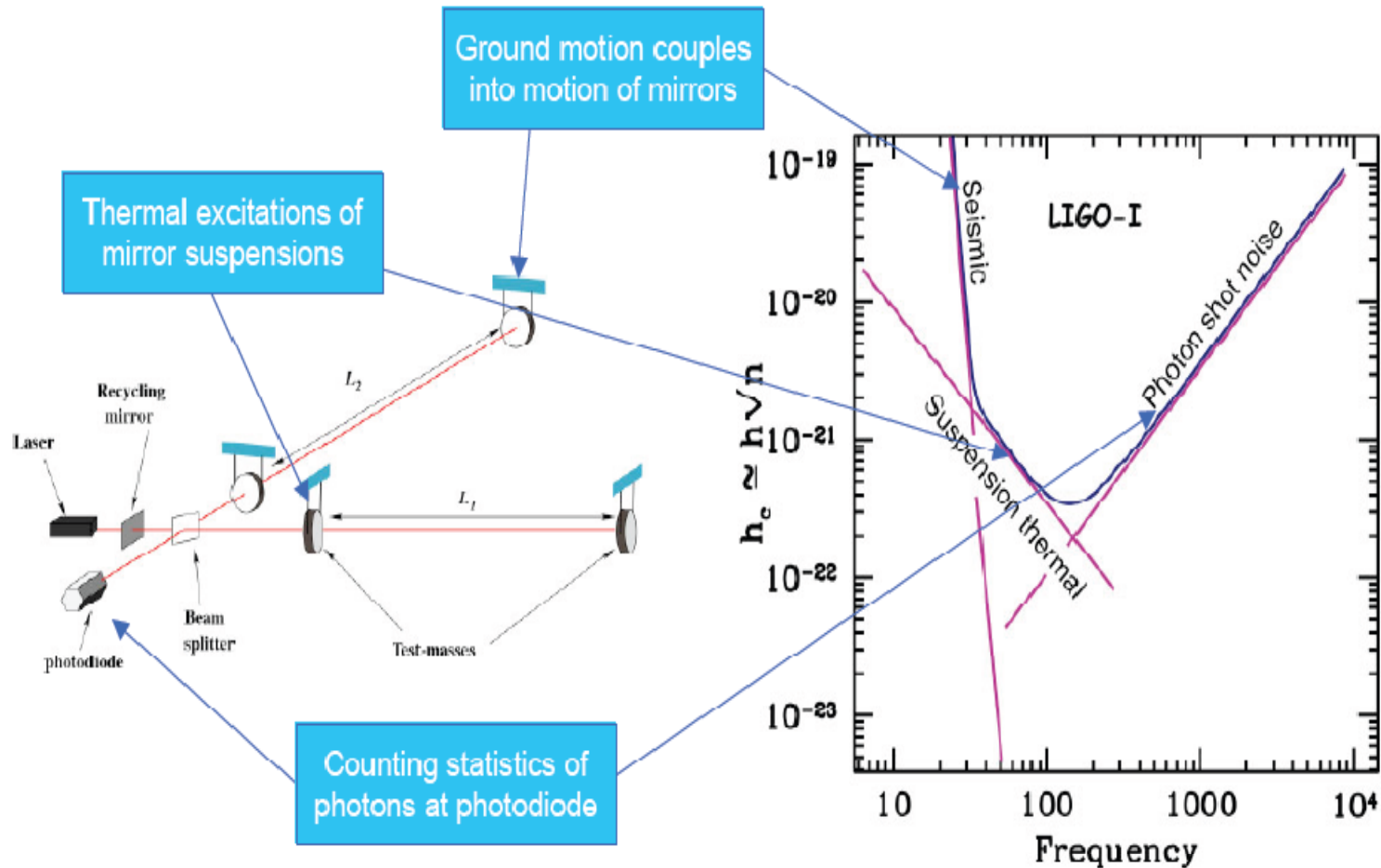
MIT



Noise sources in LIGO

- LIGO has formidable isolating and noise-suppressing systems, both passive (mechanical attenuators) and active (feedbacks), however noise still dominates the interferometer's output
- We have two main types of noise: **stationary noise** and **transient noise**
- Stationary noise is always present and is intrinsic to the interferometer mechanical system and electronics
- Conversely transient noise is due to random events which may happen in the interferometer's surroundings, such as earthquakes, temporary malfunctioning of the electronics, trucks/airplanes passing by...

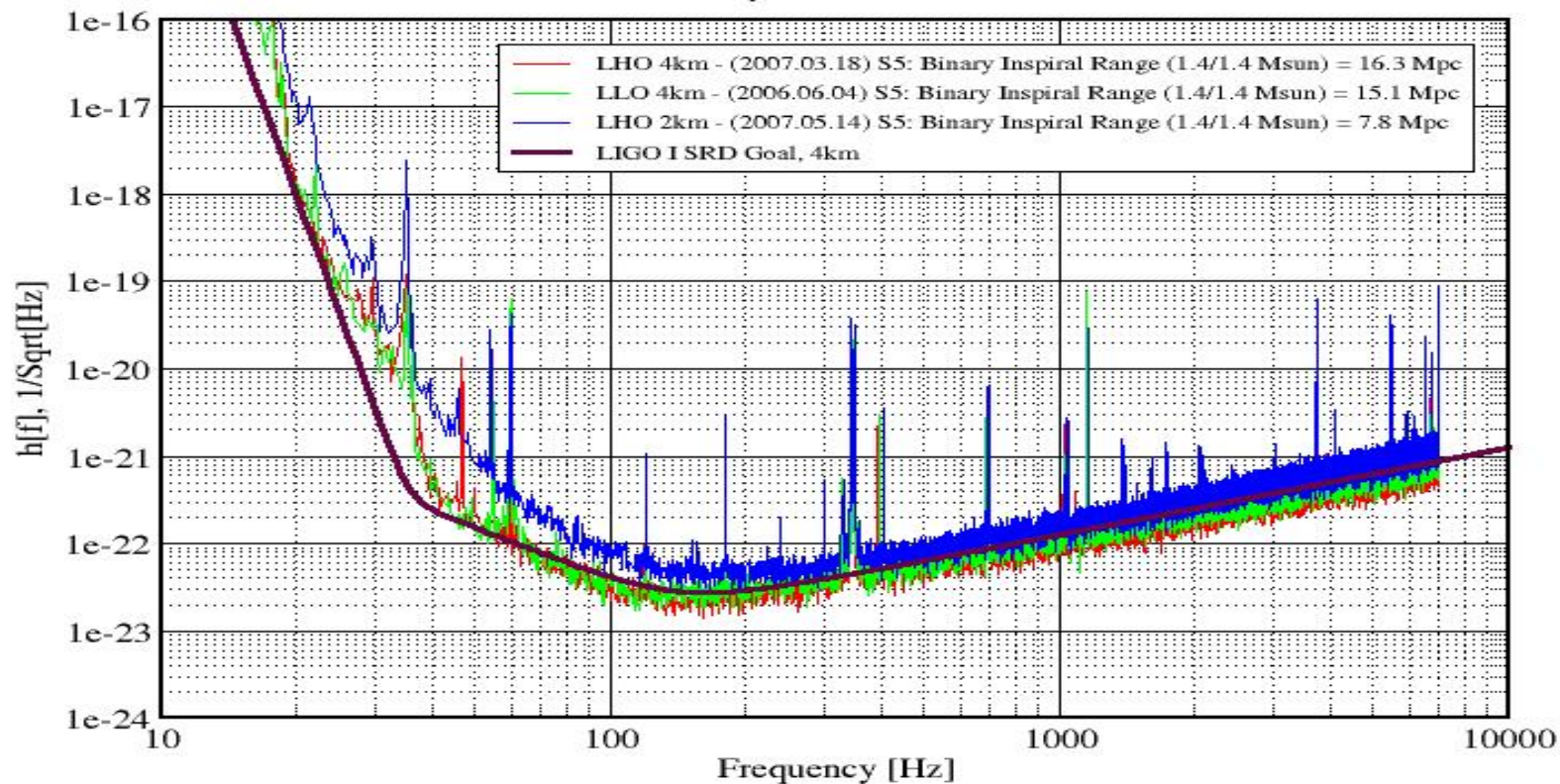
Noise sources in LIGO



Current LIGO sensitivity

Strain Sensitivity of the LIGO Interferometers

S5 Performance - May 2007 LIGO-G070366-00-E



Matched filtering

How do we recover gravitational waves from the data stream?

- We construct theoretical models of the waveform called **templates** which depend on few parameters: masses and spins of the compact objects, orbital angular momentum, direction and orientation angles of the binary with respect to the detector, initial phase and time of arrival
- We evaluate the convolution of the the data stream **s** with the template **h** and maximize over the parameters to obtain the **signal-to-noise ratio (SNR)** time series which is a measure of how much the data stream “looks like” the template for every value of the time of arrival t

SNR

The SNR is defined as

$$\rho(t) = \max_{\text{params}} \left(\langle s, h \rangle(t) / \sqrt{\langle h, h \rangle} \right)$$

where the inner product $\langle s, h \rangle$ is given by

$$\langle s, h \rangle(t) = \int_{f_{\min}}^{f_{\max}} \frac{\tilde{s}^*(f) \tilde{h}(f)}{S_n(f)} e^{2\pi i f t} df$$

Inspiral Pipeline - I

- We decide that we have a **trigger** at time t_0 if the SNR at that time is bigger than a chosen threshold $\rho(t_0) \geq \rho_0$

Is it all so easy???

- Of course not, because the data stream is dominated by **noise** and high SNR's could correspond to non-stationary noise transients (or “glitches”)!
 - How do we discard noise artifacts?

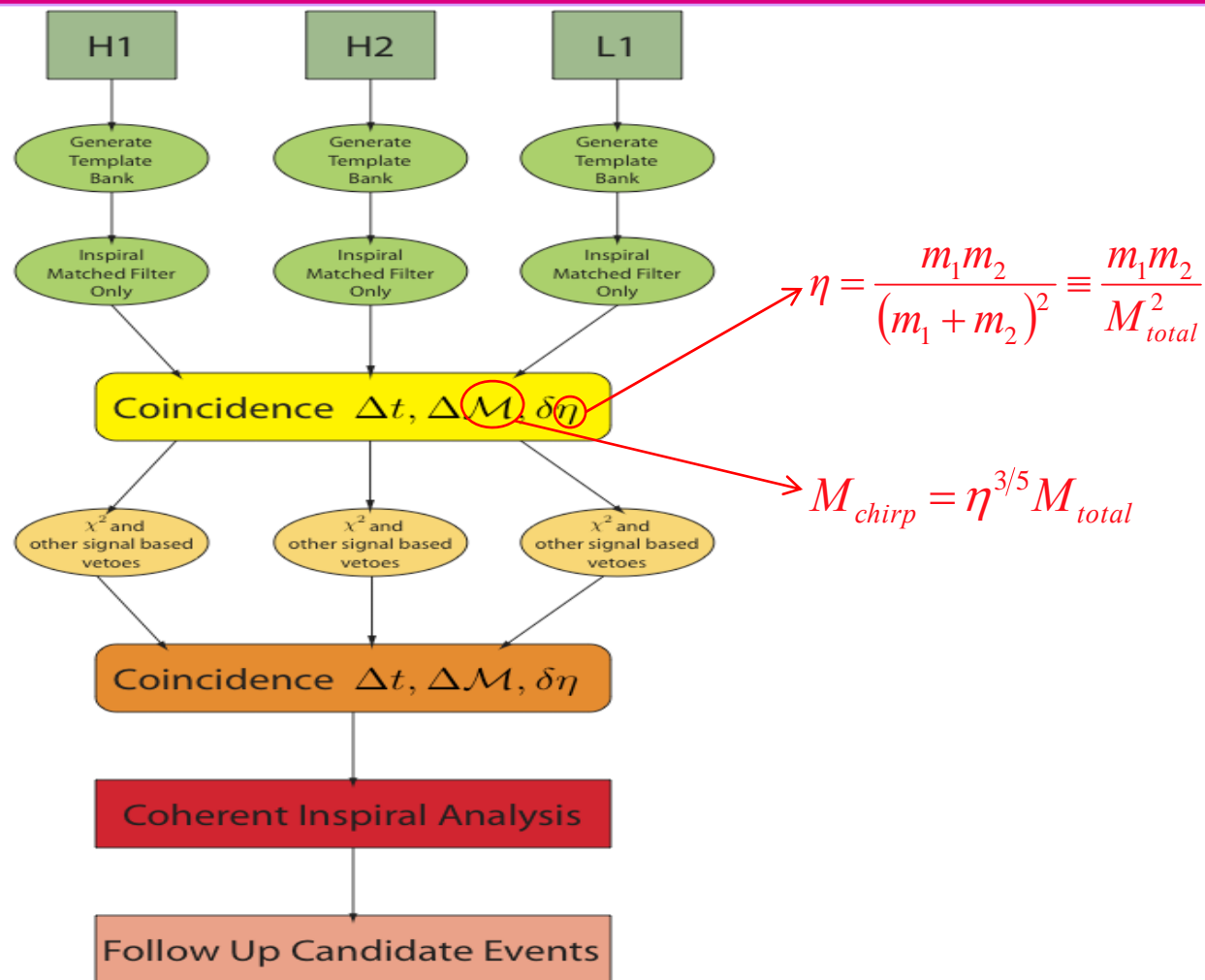
- We remember we have 3 interferometers! If an event is a real gravitational wave it should show up in all the instruments, so we look for **coincident triggers**, i.e. triggers which appear in at least two instruments within the light or, equivalently, the gravitational-wave travel time.

Inspiral Pipeline - II

So, have we found a gravitational wave?

- No, we just increased our confidence that the trigger can be a real event; the matched filtering-coincidence scheme alone is not capable of ruling out all noise triggers that can cause accidental coincidences (background triggers)
- We therefore perform consistency checks (like χ^2 and r^2 vetoes) and if the trigger survives all these than we say that we have an **event candidate**
- Finally the candidate goes through the detection checklist (which is a “manual” follow-up) and only at the end of this we can state that we have a **detection**, i.e. what we think is a real gravitational wave

Pipeline flux diagram



Dynamical evolution of spinning binaries

- Very complicated, due to Spin-Spin and Spin-Orbit couplings which cause **precession** of the orbital angular momentum L_N and therefore **modulations** in the phase and amplitude of the GWs emitted.
- Waveforms depend on a relatively large number of parameters (**15** for 2 spins, **17** for eccentric orbits) to be maximized over in matched filtering:
- **intrinsic** parameters typically influence the shape of the waveform **➡** a different template is needed for every point in the intrinsic parameter space (template bank)
- **extrinsic** parameters can be searched over quickly via analytical or numerical maximization

Single spin binaries

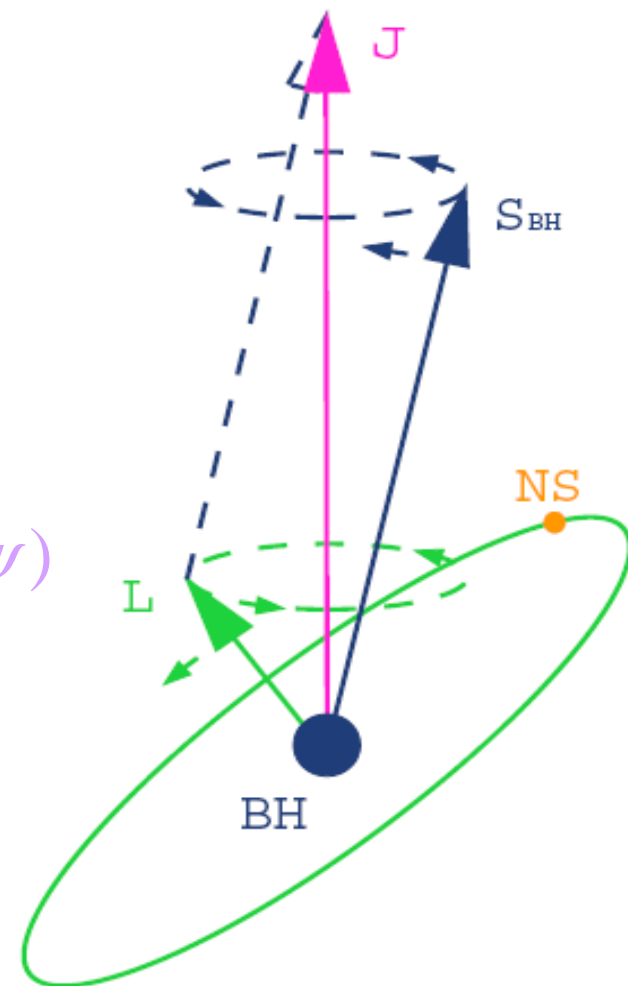
The detector response to GWs emitted by binaries (BBH or BH-NS) with a single spinning BH depends on **11** parameters.

Upon defining $\chi_1 \equiv S_{BH} / m_{BH}^2$, $\kappa_1 \equiv \hat{L}_N \cdot S_{BH}$ we have:

- 4 extrinsic parameters $(\Theta, \varphi, \Phi_0, t_0)$
- 7 intrinsic parameters $(M, \eta, \chi_1, \kappa_1, \phi, \theta, \psi)$

(Θ, φ) specify the direction to the detector in the source frame.

(ϕ, θ, ψ) specify the orientation of the detector with respect to the radiation frame.





Matched-filtering search: DTF vs PTF

Searching over more than $n=3-4$ intrinsic parameters is too computationally expensive (need n -dimensional template bank)



Detection Template Families (DTF):

- Waveforms are described by a smaller number of phenomenological **non-physical** parameters.
- Good for the purpose of detection **but** not for parameters estimation.
- Relations with physical parameters not well defined 
a larger parameter space is searched over 
higher false alarm rate!

Physical Template Families

Through an optimal choice of the **reference frame** and a **reparametrization** it is possible to both **reduce** the overall number of parameters and to **convert** some intrinsic parameters to extrinsic so that we can still use exact waveforms for matched filtering



Physical Template Families (PTF):

- Slightly slower but reliable for parameter estimation
- The parameters being searched over are actual **physical** parameters of the binary system



Lower false alarm rate

PTF - Parameter reduction

Buonanno, Chen, Vallisneri – Physical Review D67, 104025 (2003)

Pan, Buonanno, Chen, Vallisneri - Physical Review D 69, 104017 (2004)

In the **precessing convention** the orientation angles (θ, ϕ, ψ) become **extrinsic** parameters and the interferometer response to gravitational waves assumes a particularly compact form

$$h = -\frac{2\mu}{D} \frac{M}{r} \underbrace{\left([\mathbf{T}_+]_{ij} F_+ + [\mathbf{T}_\times]_{ij} F_\times \right)}_{\text{P-factor: detector projection}} \underbrace{\left([\mathbf{e}_+]^{ij} \cos 2(\Phi + \Phi_0) + [\mathbf{e}_\times]^{ij} \sin 2(\Phi + \Phi_0) \right)}_{\text{Q-factor: wave generation}}$$

The dependence on (θ, ϕ, ψ) is only through the antenna patterns F_+ and F_\times which are orthogonal, so we can rewrite them in terms of only **one** angle $\alpha[\theta, \phi, \psi]$ as

$$\begin{Bmatrix} F_+ \\ F_\times \end{Bmatrix} \equiv \sqrt{F_+^2 + F_\times^2} \begin{Bmatrix} \cos \alpha \\ \sin \alpha \end{Bmatrix} \equiv F \begin{Bmatrix} \cos \alpha \\ \sin \alpha \end{Bmatrix} = \begin{Bmatrix} \cos \alpha \\ \sin \alpha \end{Bmatrix}$$

where template normalization allows to set $F=1$.

Reparametrization

Introducing a convenient basis the template can eventually be written as a linear combination in a 5-D vector space

$$h = \sum_{I=1}^5 P^I [\Theta, \varphi; \alpha] Q^I [M, \eta, \chi_1, \kappa_1; \Phi_0, t_0; t]$$

and now we are left with 5 extrinsic parameters and only **4 intrinsic parameters** $M, \eta, \chi_1, \kappa_1$, all contained in Q^I .

Now introducing $Q_0^I \equiv Q^I (\Phi_0 = 0)$, $Q_{\pi/2}^I \equiv Q^I (\Phi_0 = \pi/4)$

we can factor out the initial phase to get

$$h = \sum_I P^I [Q_0^I \cos(2\Phi_0) + Q_{\pi/2}^I \sin(2\Phi_0)]$$

The maximization of the overlap over Φ_0 is algebraic

$$\langle s, \hat{h} \rangle = \frac{\langle s, h \rangle}{\sqrt{\langle h, h \rangle}} \xrightarrow{\max \Phi_0} \sqrt{\frac{\sum_{I,J} P^I P^J A^{IJ}}{\sum_{I,J} P^I P^J B^{IJ}}}$$

where

$$A^{IJ} \equiv \langle s, Q_0^I \rangle_{t_0} \langle s, Q_0^J \rangle_{t_0} + \langle s, Q_{\pi/2}^I \rangle_{t_0} \langle s, Q_{\pi/2}^J \rangle_{t_0}$$

$$B^{IJ} \equiv \langle Q_0^I, Q_0^J \rangle$$

We could now maximize over the **5** P^I , however they are not free parameters (depend on only **3** angles) and they must satisfy two **constraints** : $P_I P_J B^{IJ} = 1$, $\det P_{ij} = 0$

So the maximization of the overlap is actually constrained to a 3-dimensional physical submanifold $P^I(\Theta, \varphi, \alpha)$.

Two-stage search scheme

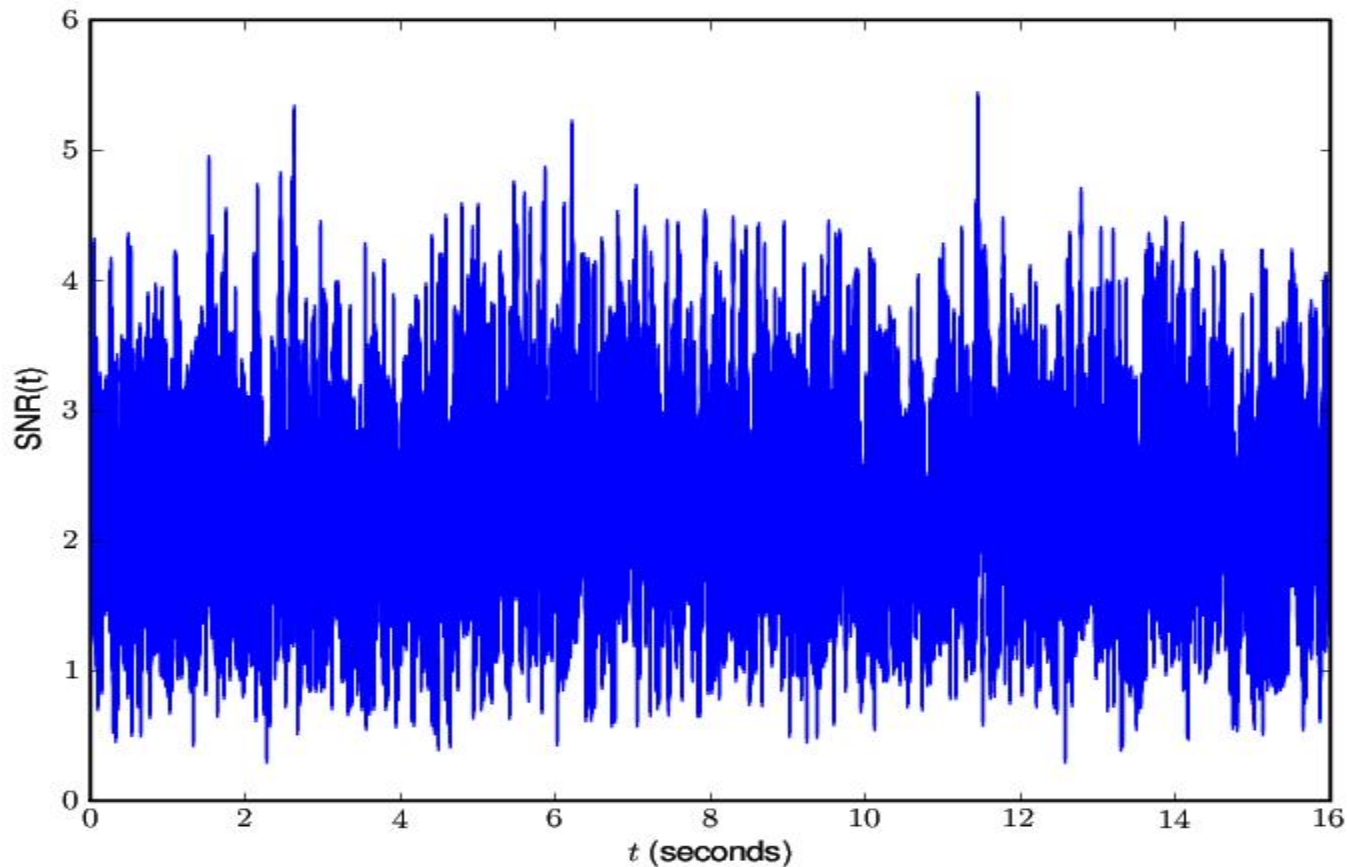
The constrained maximization over $(\Theta, \varphi, \alpha)$ is still too computationally expensive, so we build a two-stage search;

1. **Unconstrained** analytical maximum ρ' over the new extrinsic parameters P^I as if they were free parameters
2. Full constrained maximization procedure only for the values of t_0 for which ρ' rises above a given threshold ρ'^*

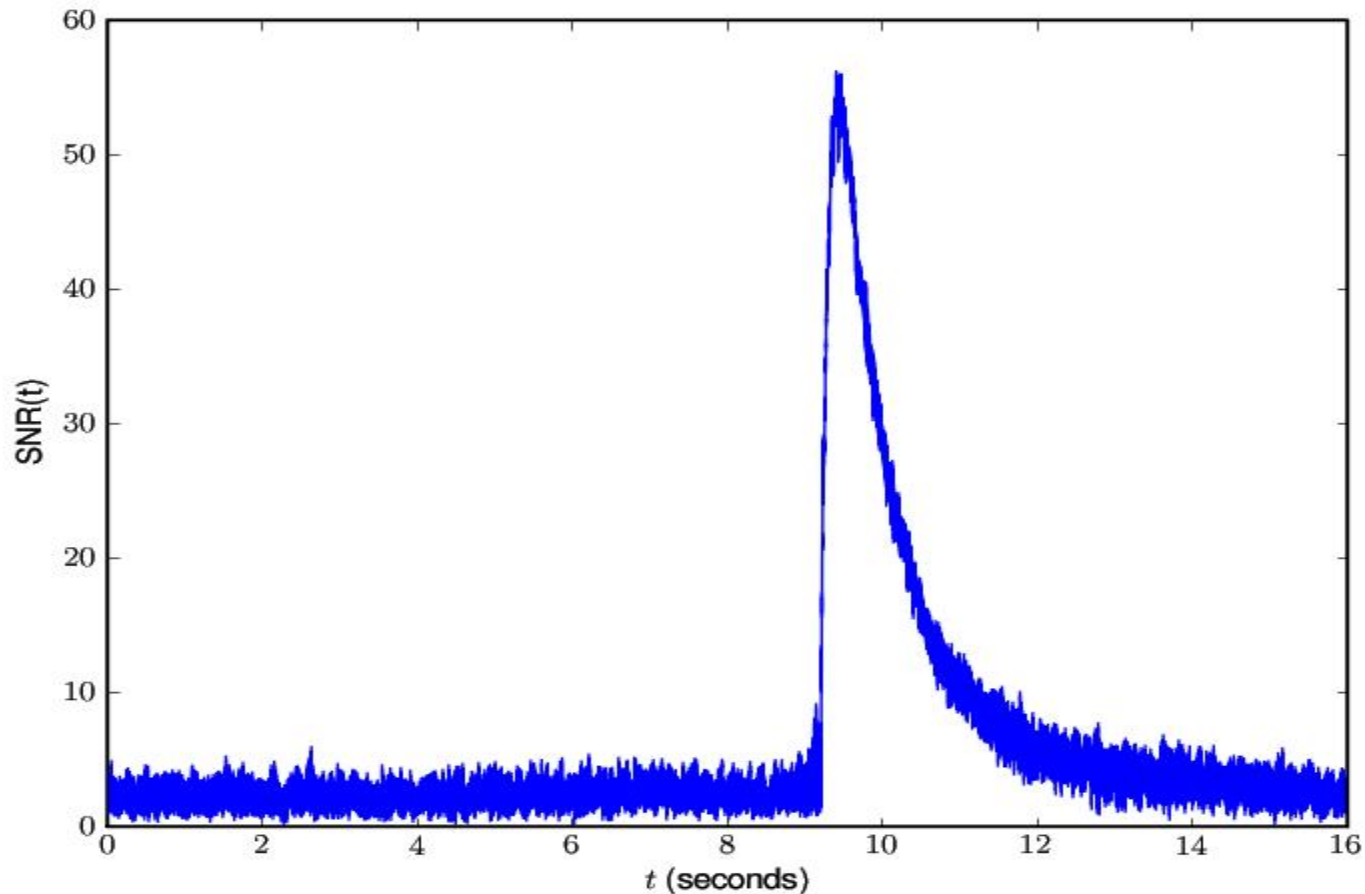
$$\rho' = \max_{P_I} \sqrt{\frac{P_I P_J A^{IJ}}{P_I P_J B^{IJ}}} = \sqrt{\max \text{eigv} [\mathbf{A}\mathbf{B}^{-1}]}$$

The location of the approximated maximum provides also good initial guesses for Θ and φ .


Detection statistic for pure stationary Gaussian noise




Detection statistic with a simulated signal present



Status of LIGO and LSC searches

- LIGO fifth science run S5 has been completed in October 2007 collecting one year of coincident data. LIGO detectors are undergoing an upgrade process in preparation for S6 
- Enhanced LIGO
- Several analysis are being performed on S5 data to search for Gw's from different binary systems
- Simulations show that non-spinning templates are effective in recovering spinning injections and have lower false-alarm rate than DTF, so DTF will most likely not be used to search S5 data
- First numerical relativity simulations of binary coalescence have become available over the past year, showing good agreement with PN waveforms up to merger: our templates are good!

Status of the PTF search: what's done

- The PTF unconstrained matched-filter engine has been implemented in C and tested as a standalone code
- The filter has then been integrated in the LIGO analysis pipeline and tested on simulated data
- First tests showed that the filter was **effective** but **slow** 
the original SNR formula has been further simplified and expressed in algebraic form using geometrical properties of the matrices A and B, obtaining a filter **2.5 times faster**
- The first preliminary tests on real S5 data are being performed

Status of the PTF search: to do

In order to perform a full search on S5 data we need:

- A **metric** g_{BC} in the intrinsic parameter space is needed to lay out templates in a sensible way: we want a bank with the smallest possible number of templates with the desired minimal match

ζ_{\min}

$$MM = \min_{\lambda^A} \max_{\lambda^{A'}} \langle \hat{h}(\lambda^A), \hat{h}(\lambda^{A'}) \rangle \geq \zeta_{\min}$$

$$g_{BC} \Delta \lambda^B \Delta \lambda^C \equiv \delta[\lambda^A, \lambda^{A'}] \equiv 1 - \langle \hat{h}(\lambda^A), \hat{h}(\lambda^{A'}) \rangle$$

- With a metric we can construct a **template bank**, whose efficiency needs to be tested against real data+injections
- Coincidence/vetoing parameters need to be **tuned** so as not to exceed the desired number of accidental background triggers (**time slides**) while being able to recover injections

Conclusions

- PTF templates are promising tools for searching for gravitational waves from spinning binaries in S5 LIGO data
- After the time-consuming implementation of the filter and its integration in the inspiral pipeline, progress toward the planning of a final search on S5 is being made much faster
- Even if non-spinning templates turn out to be as effective as PTF in recovering injections, PTF can still be used for a triggered search or for follow-up and parameter estimation
- Hopefully we will make a detection!