Searching for gravitational waves from spinning binaries in LIGO data using a Physical Template family

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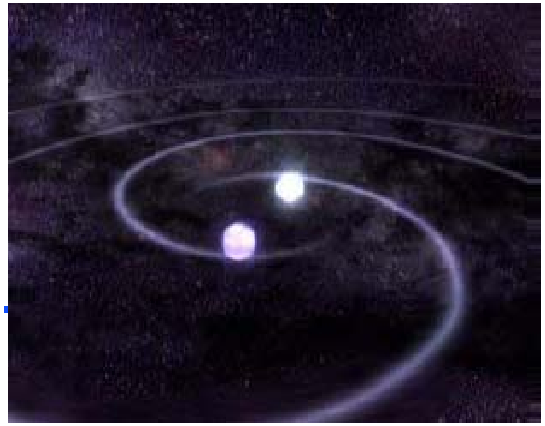
The theory behind gravitational waves

Einstein's equations admit wave-like solutions for non-static space-times for which the metric can be written as $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$

- In particular, spinning compact binaries such as neutron-star (BNS) and black-hole (BBH) binaries are expected to emit energy in the form of GW's during their coalescence phase.



Far from the source the gravitational wave field is described at lowest order by the quadrupole formula which, in the transverse traceless gauge, reads

$$h_{jk}^{TT} = \frac{2G}{c^4 r} \frac{d^2 I_{jk}^{TT}(t-r)}{dt^2}$$

where I_{jk} is the binary quadrupole moment.

Matched filtering

The SNR is defined as

$$\rho(t) = \max_{params}\left(\langle s,h\rangle(t)\big/\sqrt{\langle h,h\rangle}\right)$$

where the inner product $\langle s,h\rangle$ is given by

$$\langle s,h\rangle(t) = \int_{f_{\min}}^{f_{\max}} \frac{\tilde{s}^*(f)\tilde{h}(f)}{S_n(f)} e^{2\pi i f t}\,df$$

Dynamical evolution of spinning binaries

- Very complicated, due to Spin-Spin and Spin-Orbit couplings which cause **precession** of the orbital angular momentum $\hat{L}_N$ and therefore **modulations** in the phase and amplitude of the GWs emitted.
- Waveforms depend on a relatively large number of parameters (15 for 2 spins, 17 for eccentric orbits) to be maximized over in matched filtering:
- **intrinsic** parameters typically influence the shape of the waveform ➡ a different template is needed for every point in the intrinsic parameter space (template bank)
- **extrinsic** parameters can be searched over quickly via analytical or numerical maximization

Single spin binaries

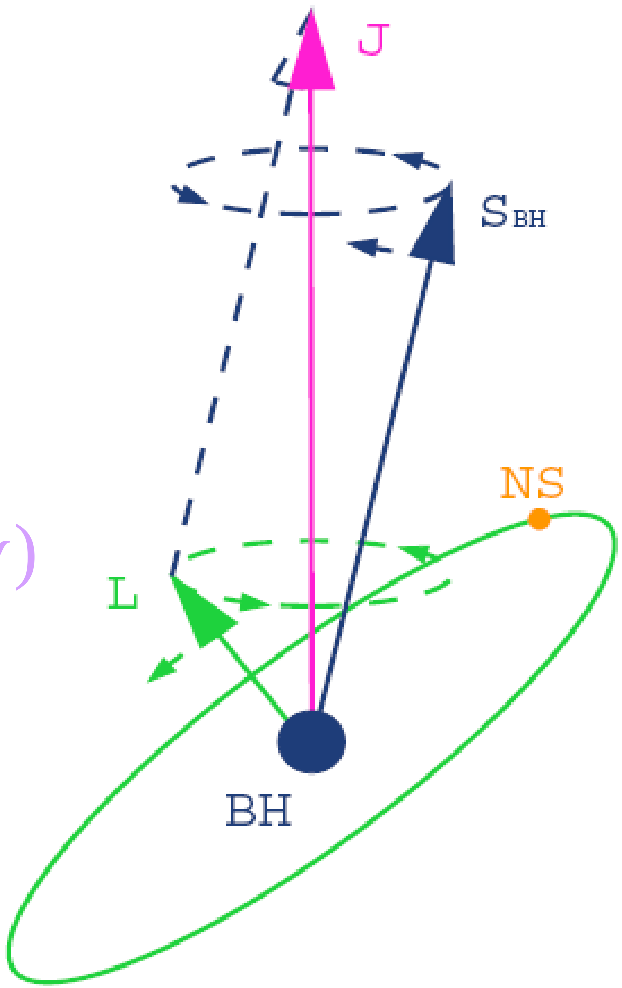
The detector response to GWs emitted by binaries (BBH or BH-NS) with a single spinning BH depends on 11 parameters.

Upon defining $\chi_1 \equiv S_{BH}/m_{BH}^2, \ \kappa_1 \equiv \hat{L}_N \cdot S_{BH}$ we have:

- 4 extrinsic parameters $(\Theta, \varphi, \Phi_0, t_0)$
- 7 intrinsic parameters $(M, \eta, \chi_1, \kappa_1, \phi, \theta, \psi)$

(Θ, φ) specify the direction to the detector in the source frame.

(ϕ, θ, ψ) specify the orientation of the detector with respect to the radiation frame.



Physical Template Families

Searching over more than n=3-4 intrinsic parameters is too computationally expensive for matched filtering (need n-dimensional template bank).

However…

Through an optimal choice of the **reference frame** and a **reparametrization** we can:

- **reduce** the overall number of parameters
- **convert** some intrinsic parameters to extrinsic

⬇

Physical Template Families (PTF)

PTF - Parameter reduction

Buonanno, Chen, Vallisneri – Physical Review D67, 104025 (2003)
Pan, Buonanno, Chen, Vallisneri - Physical Review D 69, 104017 (2004)

In the precessing convention the orientation angles (θ,ϕ,ψ) become extrinsic parameters and the interferometer response to gravitational waves assumes a particularly compact form

$$h = -\frac{2\mu}{D}\frac{M}{r}\underbrace{\left([\mathbf{T}_+]_{ij}F_+ + [\mathbf{T}_\times]_{ij}F_\times\right)}_{\text{P-factor: detector projection}}\underbrace{\left([\mathbf{e}_+]^{ij}\cos 2(\Phi+\Phi_0) + [\mathbf{e}_\times]^{ij}\sin 2(\Phi+\Phi_0)\right)}_{\text{Q-factor: wave generation}}$$

The dependence on (θ,ϕ,ψ) is only through the antenna patterns F_+ and $F_\times$ which are orthogonal, so we can rewrite them in terms of only one angle $\alpha[\theta,\phi,\psi]$ as

$$\begin{Bmatrix} F_+ \\ F_\times \end{Bmatrix} \equiv \sqrt{F_+^2 + F_\times^2}\begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} \equiv F\begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} = \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix}$$

where template normalization allows to set F=1.

Reparametrization

Introducing a convenient basis the template can evantually be written as a linear combination in a 5-D vector space

$$h = \sum_{I=1}^{5} P^I[\Theta, \varphi; \alpha] Q^I[M, \eta, \chi_1, \kappa_1; \Phi_0, t_0; t]$$

and now we are left with 5 extrinsic parameters and only **4 intrinsic parameters** $M, \eta, \chi_1, \kappa_1$, all contained in Q^I.

Now introducing $Q_0^I \equiv Q^I(\Phi_0 = 0)$, $Q_{\pi/2}^I \equiv Q^I(\Phi_0 = \pi/4)$

we can factor out the initial phase to get

$$h = \sum_I P^I [Q_0^I \cos(2\Phi_0) + Q_{\pi/2}^I \sin(2\Phi_0)]$$

Maximization of the overlap ρ over the extrinsic parameters

The maximization of the overlap over Φ_0 is algebraic

$$\left\langle s,\hat{h}\right\rangle = \frac{\langle s,h\rangle}{\sqrt{\langle h,h\rangle}} \xrightarrow{\max \Phi_0} \sqrt{\frac{\sum_{I,J} P^I P^J A^{IJ}}{\sum_{I,J} P^I P^J B^{IJ}}}$$

where

$$A^{IJ} \equiv \langle s,Q_0^I\rangle_{t_0} \langle s,Q_0^J\rangle_{t_0} + \langle s,Q_{\pi/2}^I\rangle_{t_0} \langle s,Q_{\pi/2}^J\rangle_{t_0}$$

$$B^{IJ} \equiv \langle Q_0^I,Q_0^J\rangle$$

We could now maximize over the 5 P^I, however they are not free parameters (depend on only 3 angles) and they must satisfy two constraints : $P_I P_J B^{IJ} = 1,\ \det P_{ij} = 0$

So the maximization of the overlap is actually constrained to a 3-dimensional physical submanifold $P^I(\Theta,\varphi,\alpha)$.

Two-stage search scheme

The constrained maximization over $(\Theta, \varphi, \alpha)$ is still too computationally expensive, so we build a two-stage search;

1. **Unconstrained** analytical maximum ρ' over the new extrinsic parameters P^I as if they were free parameters
2. Full constrained maximization procedure only for the values of t_0 for which ρ' rises above a given threshold ρ'^* .

$$\rho' = \max_{P_I} \sqrt{\frac{P_I P_J A^{IJ}}{P_I P_J B^{IJ}}} = \sqrt{\max eigv\,[\mathbf{AB}^{-1}]}$$

The location of the approximated maximum provides also good initial guesses for Θ and φ .

Status of the PTF search: what's done

- The PTF unconstrained matched-filter engine has been implemented in C and tested as a standalone code
- The filter has then been integrated in the LIGO analysis pipeline and tested on simulated data
- First tests showed that the filter was effective but slow ⇒ the original SNR formula has been further simplified and expressed in algebraic form using geometrical properties of the matrices A and B, obtaining a filter 2.5 times faster
- The first preliminary tests on real S5 data are being performed and results are very promising!

Status of the PTF search: to do

In order to perform a full search on S5 data we need:

- A metric g_{BC} in the intrinsic parameter space is needed to lay out templates in a sensible way: we want a bank with the smallest possible number of templates with the desired minimal match $\zeta_{\min}$

$$MM = \min_{\lambda^A} \max_{\lambda^{A'}} \left\langle \hat{h}(\lambda^A), \hat{h}(\lambda^{A'}) \right\rangle \geq \zeta_{\min}$$

$$g_{BC} \Delta\lambda^B \Delta\lambda^C \equiv \delta\left[\lambda^A, \lambda^{A'}\right] \equiv 1 - \left\langle \hat{h}(\lambda^A), \hat{h}(\lambda^{A'}) \right\rangle$$

- A template placement code to generate template banks whose efficiency needs to be tested against real data+injections
- Coincidence/vetoing parameters need to be tuned so as not to exceed the desired number of accidental background triggers (time slides) while being able to recover injections

Conclusions

- PTF templates are promising tools for searching for gravitational waves from spinning binaries in S5 LIGO data
- After the time-consuming implementation of the filter and its integration in the inspiral pipeline, progress toward the planning of a final search on S5 is being made much faster
- Even if non-spinning templates turn out to be as effective as PTF in recovering injections, PTF can still be used for a triggered search or for follow-up and parameter estimation
- Hopefully we will make a detection!