An Introduction to Noise in Frequency Stabilization

Michael J. Martin
University of Colorado at Boulder, JILA

The Sr team:

- **Andrew Ludlow**
- Martin Boyd
- Gretchen Campbell
- Sebastian Blatt
- Jan Thomsen
- Matt Swallows
- Travis Nicholson

Group leader: **Jun Ye**

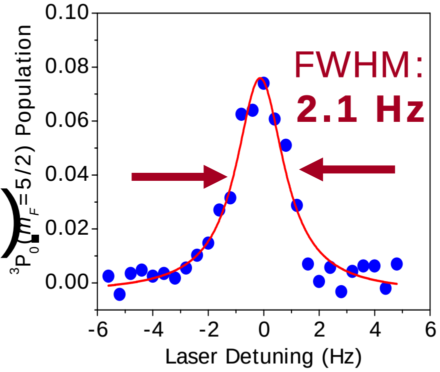
Mirror coatings: **Ramin Lalezari,** Advanced Thin Films

New approaches: **Jeff Kimble,** Caltech

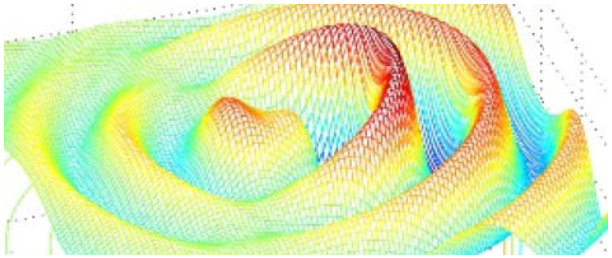
PTB collaborators: **Uwe Sterr, Fritz Riehle**

Motivations

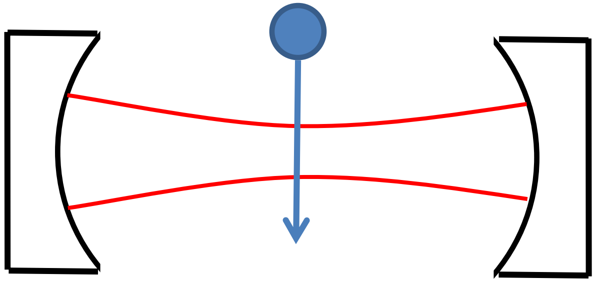
- Optical atomic clocks (spectroscopy).



- Gravity wave detection.

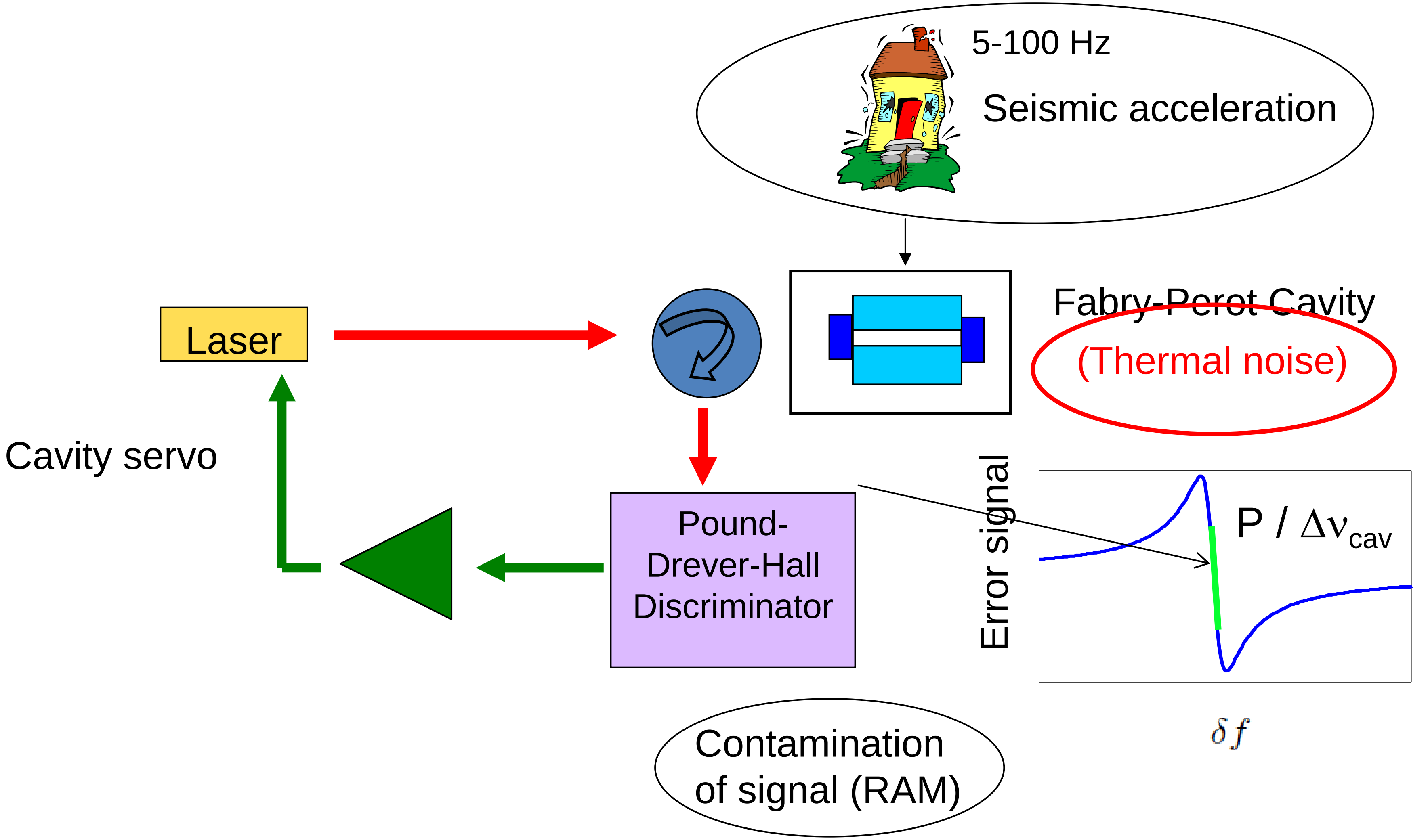


- Quantum Optics.



- More…

Introduction – Noise sources


5-100 Hz

Seismic acceleration

Fabry-Perot Cavity

(Thermal noise)

Laser

Cavity servo

Pound-Drever-Hall Discriminator

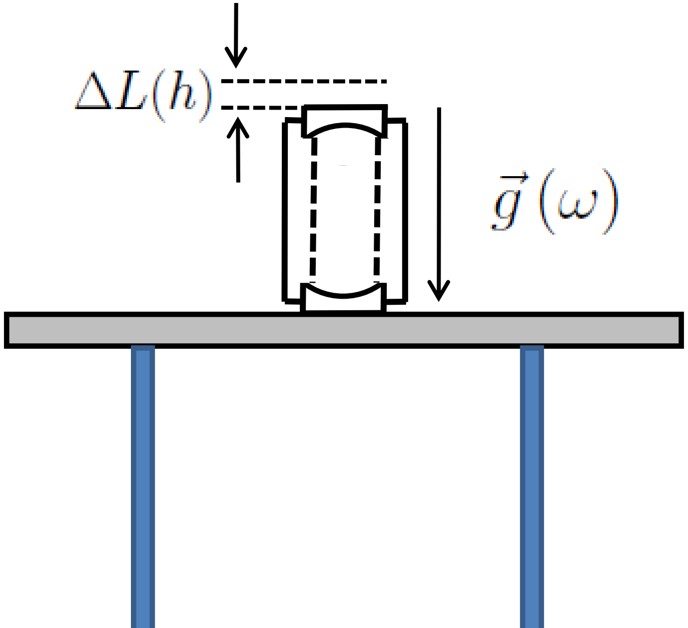
Contamination of signal (RAM)

-For 10 cm ultra-high-finesse cavity, shot noise contributes ~$\mathbf{10^{-16}}$ with 10 μW optical power.

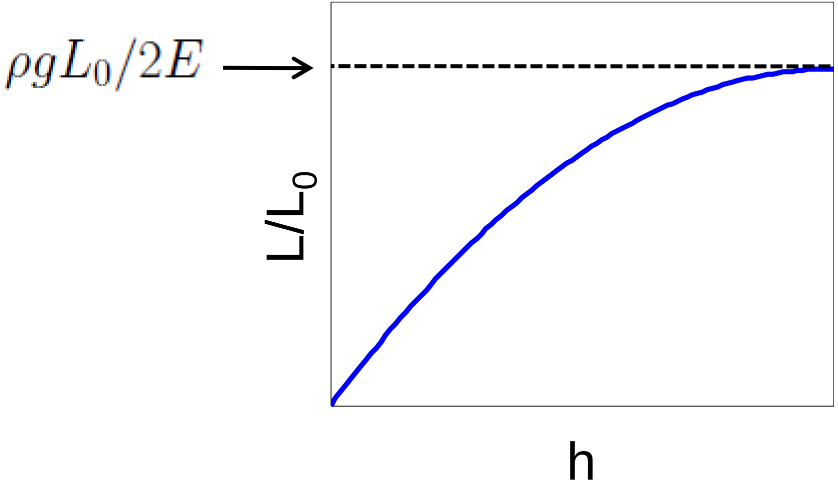
Vibrational Sensitivity

-Low frequency: Cavity subject to uniform acceleration.

-Fractional length change scales linearly with length!



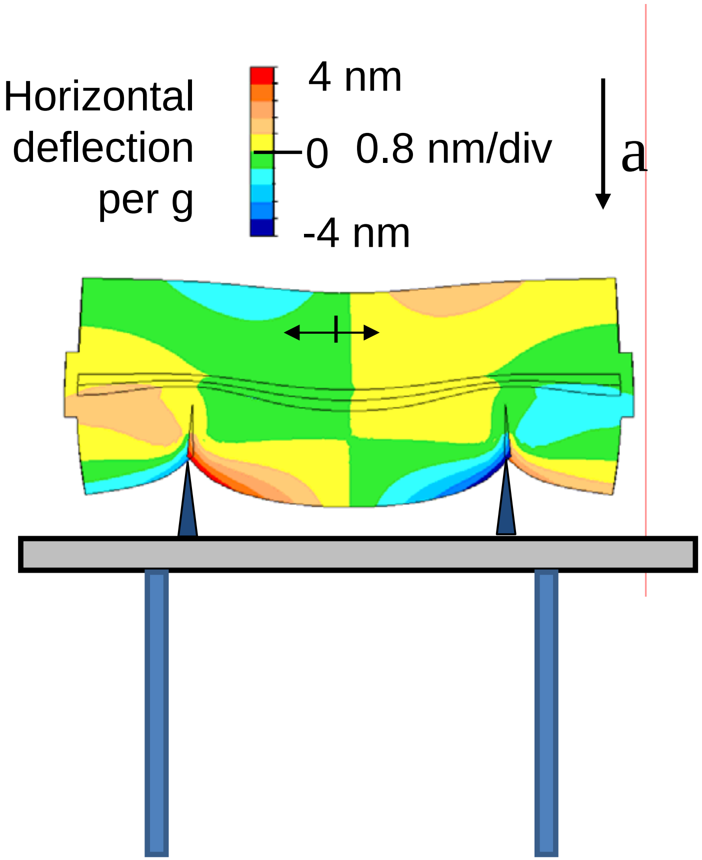
Fractional displacement Vs. height



Chen et. al. *PRA* 74, (20

Vibrational Sensitivity:

Mounting geometry



-Finite element analysis aids in determining vibrational sensitivity

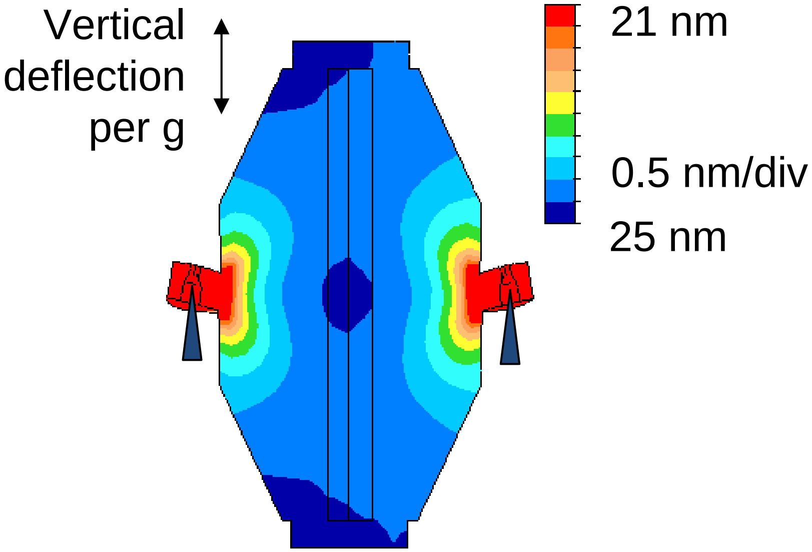
-Eg. for 10cm ULE Cavity, sensitiv is at best 80 kHz/m/s^2 @ 698nm

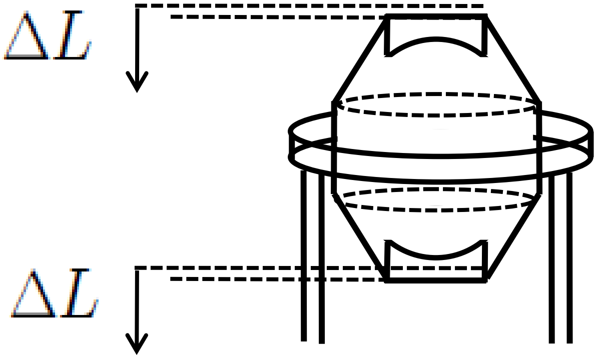
Chen et. al. *PRA* 74, (200

Vibrational Sensitivity:

Mounting geometry

Mid-plane Mounting:





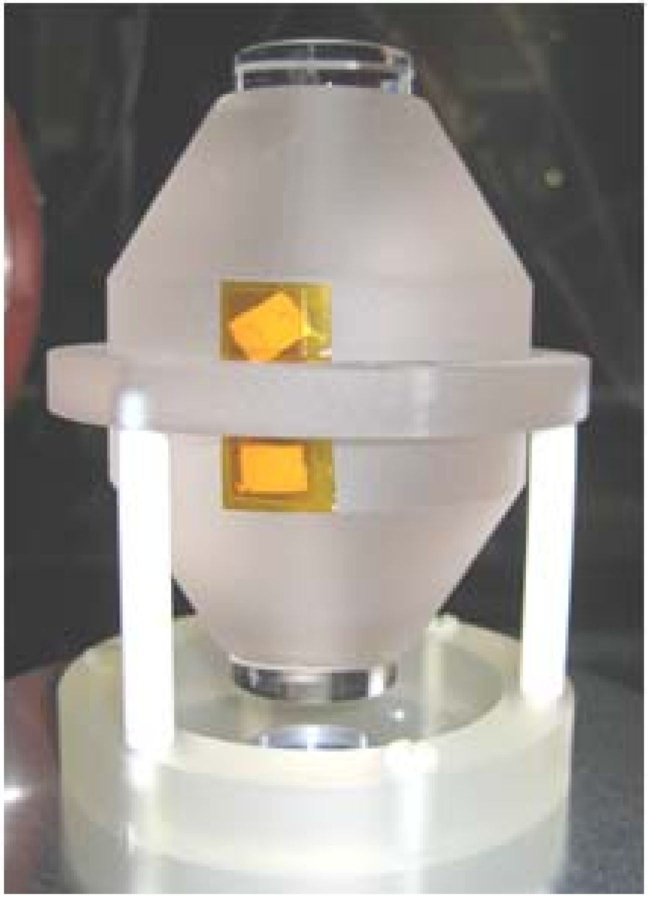
-Differential motion minimized.

-ULE vertical acceleration sensitivity
30 kHz/m/s^2 @ 698 nm.

-Short cavity → vibrational insensitivity.

M. Notcutt et. al. *Optics Letters* 30, (20

The JILA Sr clock laser



-Vibrational contribution:
.1 Hz/rtHz @ 1 Hz.

-Finesse = 250,000

-Spacer and substrate material: ULE$\left(\phi = 1.6 \times 10^{-5}\right)$

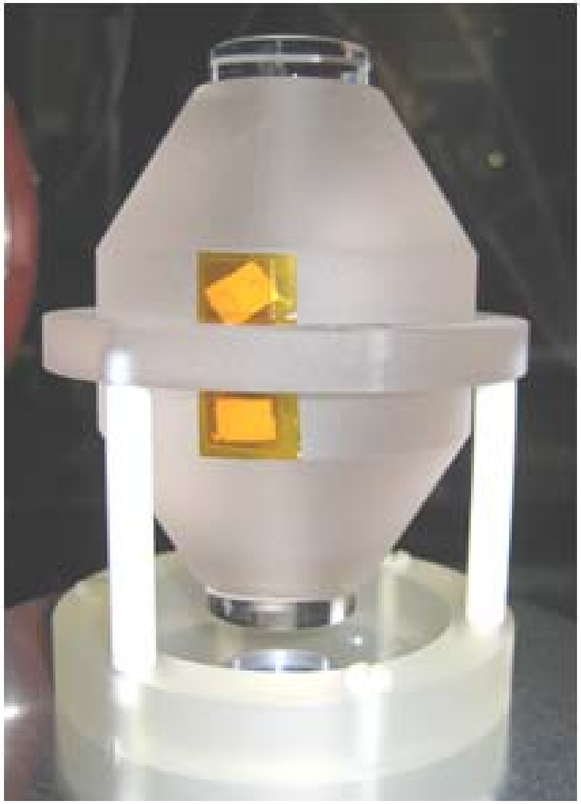
-Linear drift: <1 Hz/sec

-Spacer length = 7cm

-**System occupies < 1 m^3**

[*]Notcutt et. al *PRA* 73, (200

The JILA Sr clock laser



-Mirror technology:

- 40 layers of Ta_2O_5 and SiO_2 ¼ wave dielectric stacks $\left(\phi = 4 \times 10^{-4}\right)$ *.

-Deposited via ion beam sputtering.

-Surface flatness and reflectivity support finesse of 250,000.

-Substrate and coating thermal noise limited.

*Numata et. al. *PRL* 93, 200

Thermal noise

-With internal dissipation comes thermally driven fluctuation

$$S_\xi = \frac{4k_BT}{\omega^2} Re\left[Y_\xi\left(\omega\right)\right] \quad \left[\frac{[\xi]^2}{Hz}\right]$$ (in 1 Hz BW)

-Most significant thermal contributions typically from mirror substrate and coating, **not cavity spacer**.

-Thermoelastic noise small for low CTE materials.

-Generalized coordinate—Gaussian-weighted surface displacement:

$$x\left(t\right) = \int \left|\psi_{00}\left(\vec{r},t\right)\right|^2 y\left(\vec{r},t\right) d^2\vec{r}$$

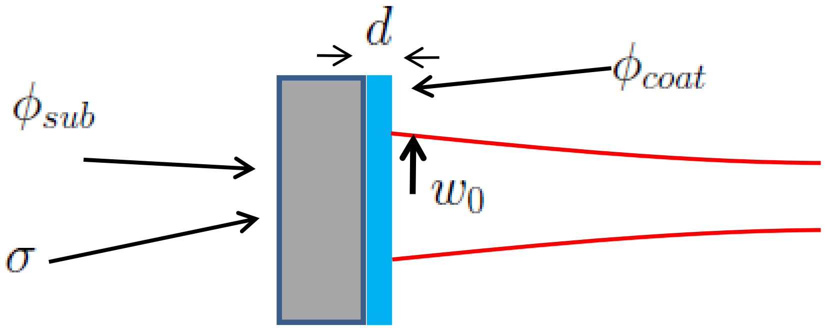
Callen and Greene. *PR* 86 (195

A. Gillespie and F. Raab, *PRD* (19

Y. Levin. *Physical Review D* 57, 659

Mirror Thermal noise

Complex (lossy) Young's modulus: $E(\omega) = E_0\left[1 + i\phi(\omega)\right]$



$$S_{x,\,mirror} \simeq \frac{4k_BT}{\omega}\frac{1-\sigma^2}{\sqrt{\pi}Ew_0}\phi_{sub}$$

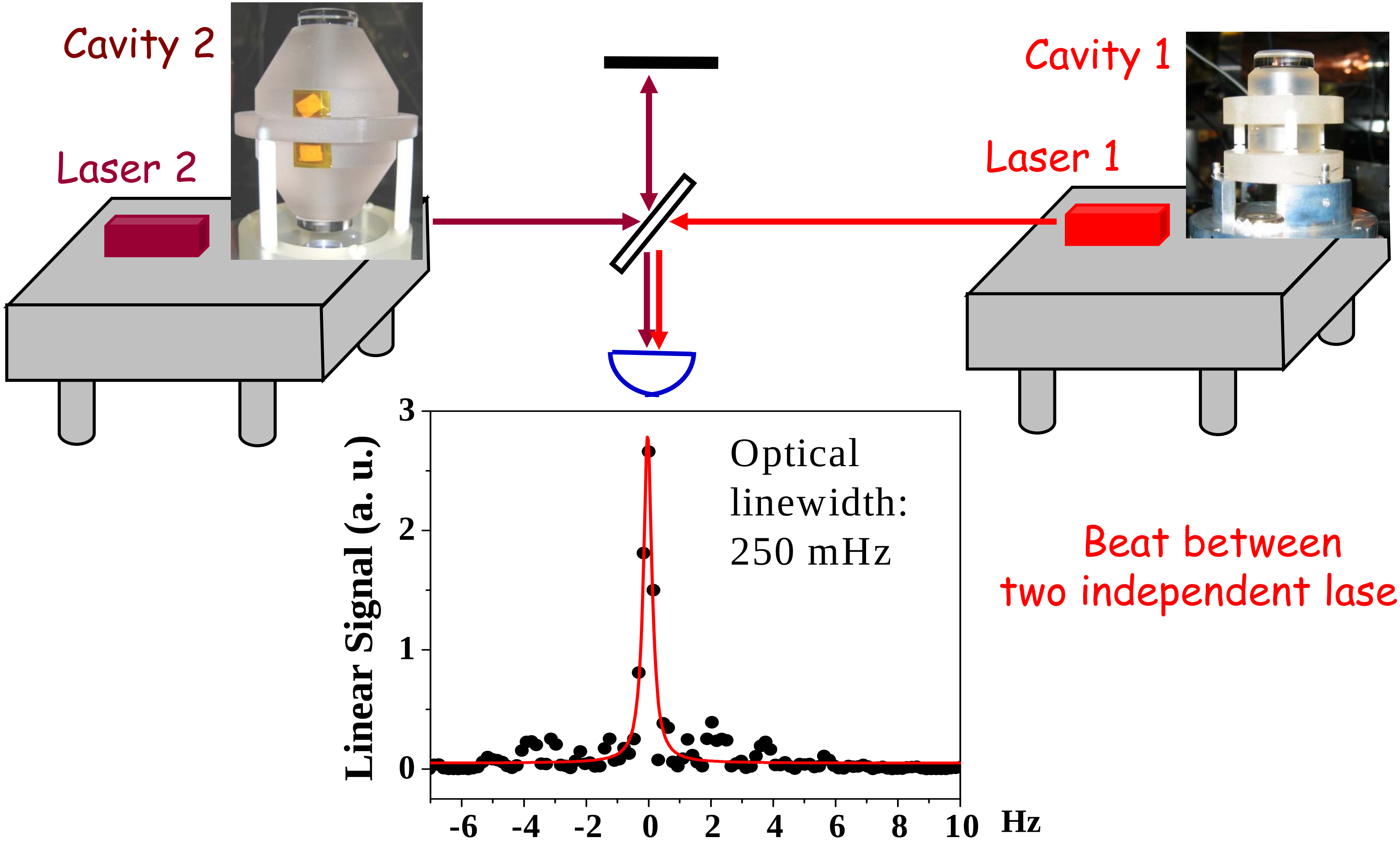
-Fractional change scales inversely with length!

-ULE cavity has 1.5 times more substrate noise than coating noise.

K. Numata et. al. *PRL* 93, (2004).
G. M. Harry et. al. *Class. Quant. Grav.* 19, (2002).

Long-term coherence (>1 s) across the visible spectrum

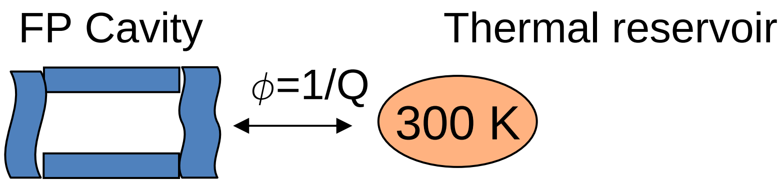
Notcutt *et al*., *Opt. Lett*. 30, (2005).
Ludlow *et al*., *PRL* 96, (2006).



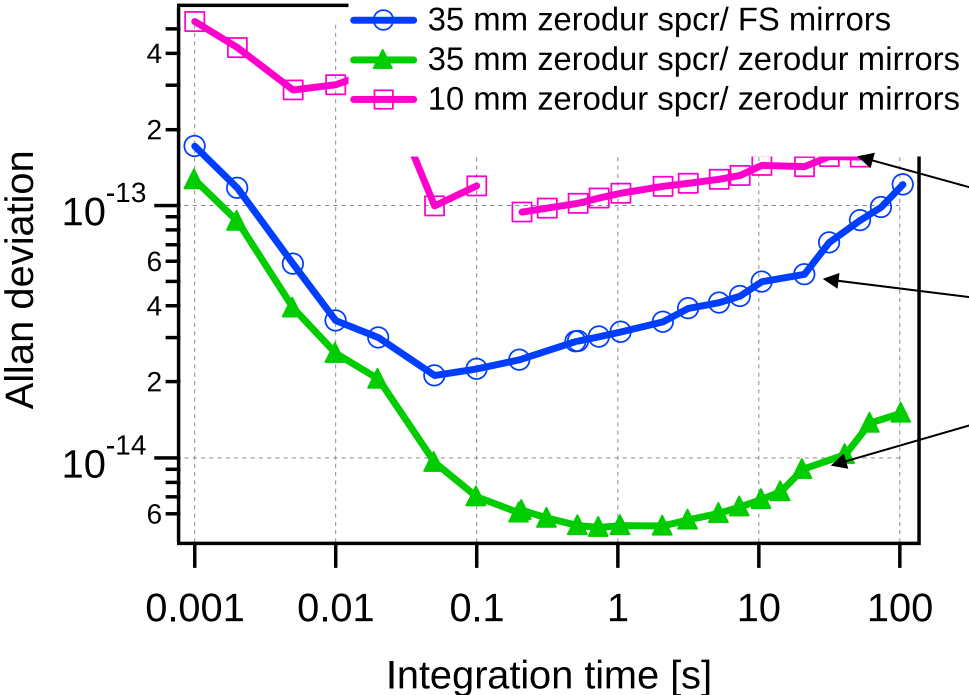
Beat between
two independent lasers

Frequency fluctuations from thermal noise in Fabry-Perot cavities

Notcutt et. al *PRA* 73, (2006)



	Loss angle ϕ
Zerodur	3e-4
Fused Silica	1e-6
Coating	4e-4



35mm -> 10mm long spacer

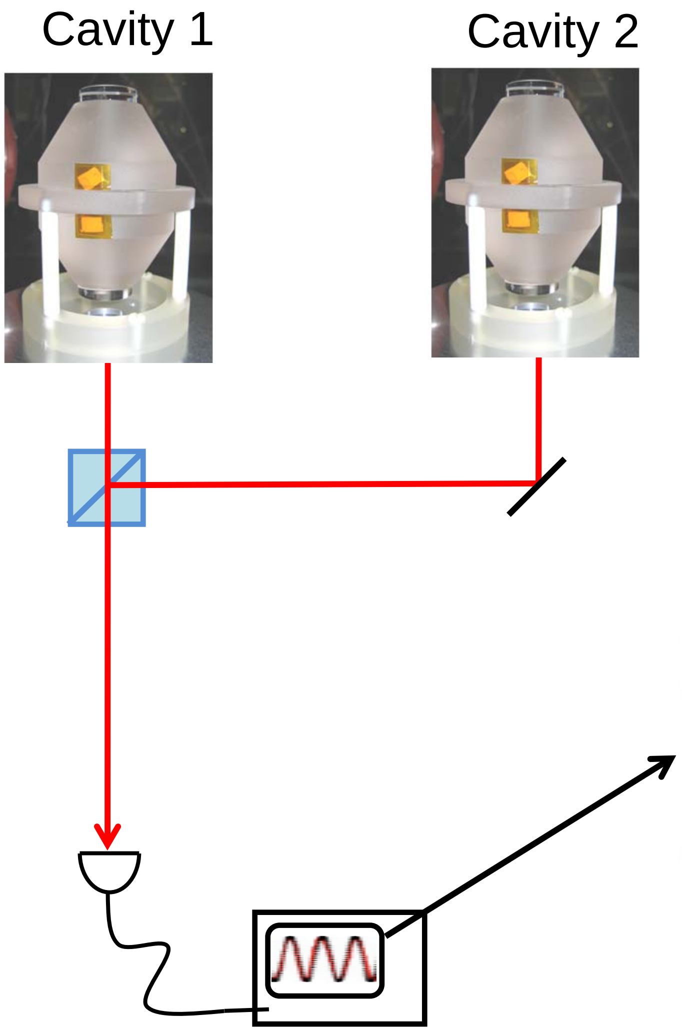
FS mirrors -> Zerodur mirrors

Measured/calculated ~ 1.8-2.6
K Numata, PRL **93,** 250602 (2004)

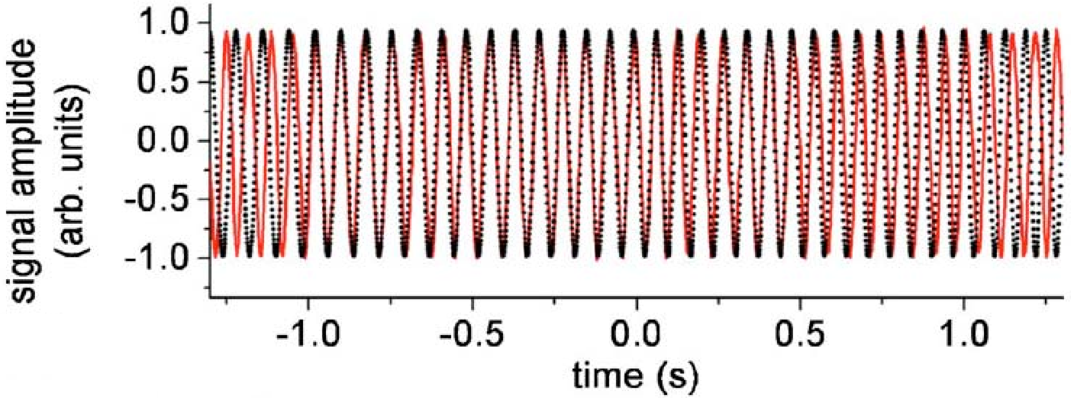
Comparison of results & theory

Case	Calculated Allan-dev.	Measured Lowest Point A-dev	Ratio Measured/Calc	Measured Displacement Noise (m/rtHz)
FS Subs	2.8×10^{-15}	5.0×10^{-15}	1.8	
Zerodur Subs	1.3×10^{-14}	2.0×10^{-14}	1.6	1.0×10^{-15}
698 nm cavity	9.4×10^{-15}	2.5×10^{-14}	2.7	1.2×10^{-15}
10 mm w/ Zer	3.8×0^{-14}	1.0×10^{-13}	2.7	1.0×10^{-15}
10mm w/ FS	7.1×10^{-15}	3.0×10^{-14}	4.2	2.7×10^{-16}

JILA Clock laser frequency noise

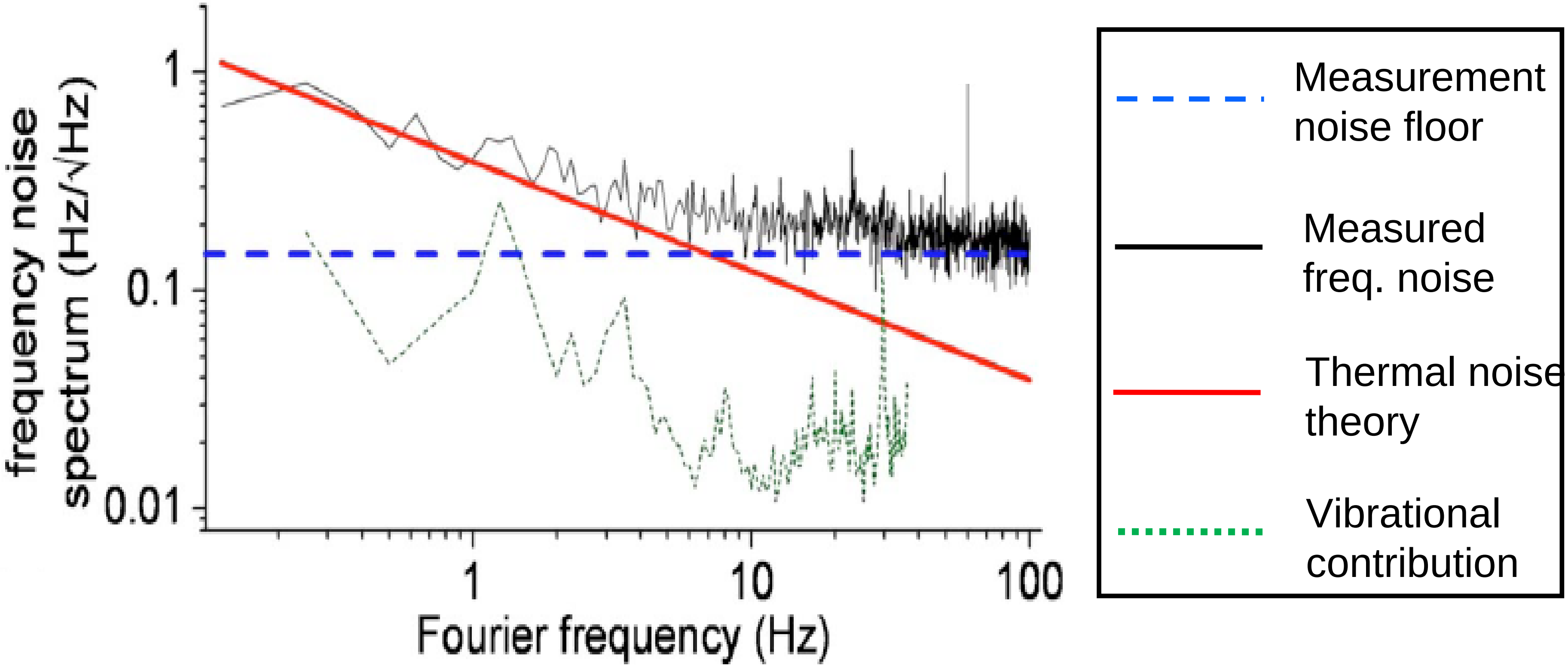


Data (dotted) and chirped sine fit (solid)



A. D. Ludlow et. al. *Opt. Lett.* 32, 641

Frequency noise S.D.

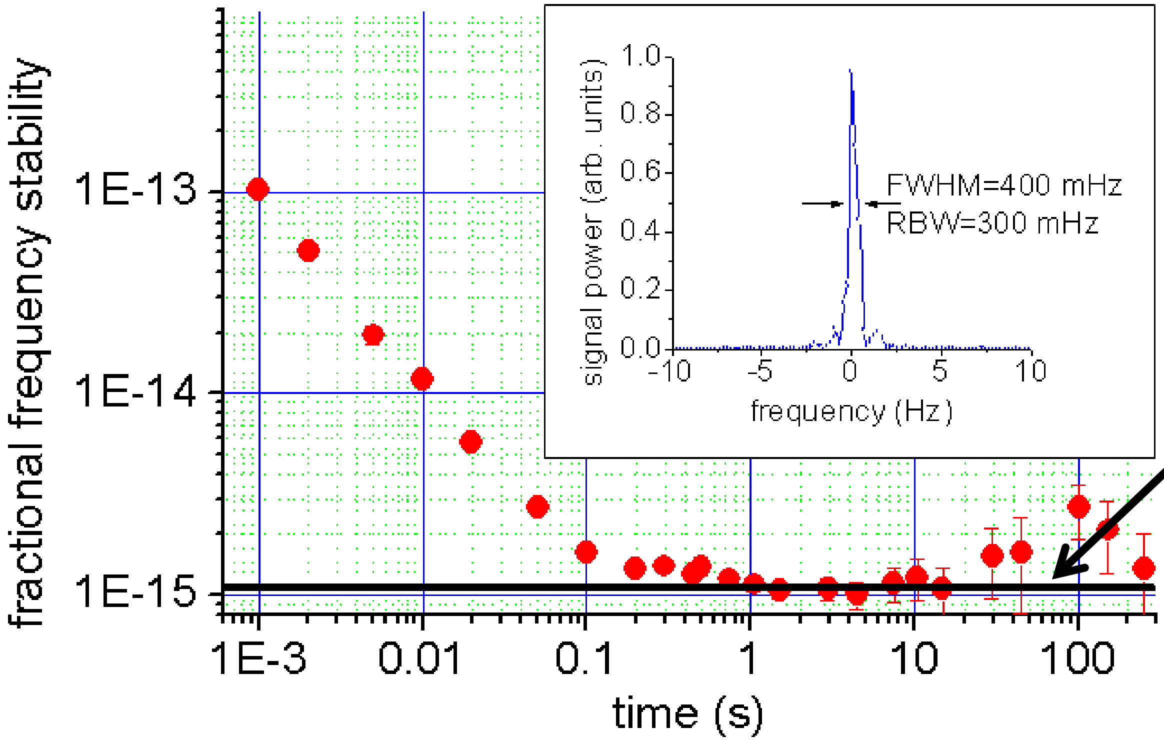


Ludlow *et al.*, Opt. Lett. 32, 641 (2007)

Thermal Noise

-Can relate fractional frequency stability (Allan Deviation) to frequency noise spectral density with 1/f behavior:

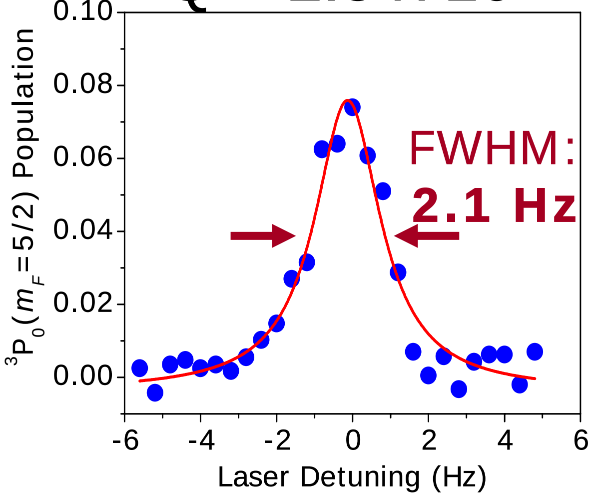
$$\frac{\delta\nu(\tau)}{\nu_0} = \frac{\sqrt{2\ln 2}\sqrt{f S_f(f)}}{\nu_0}$$

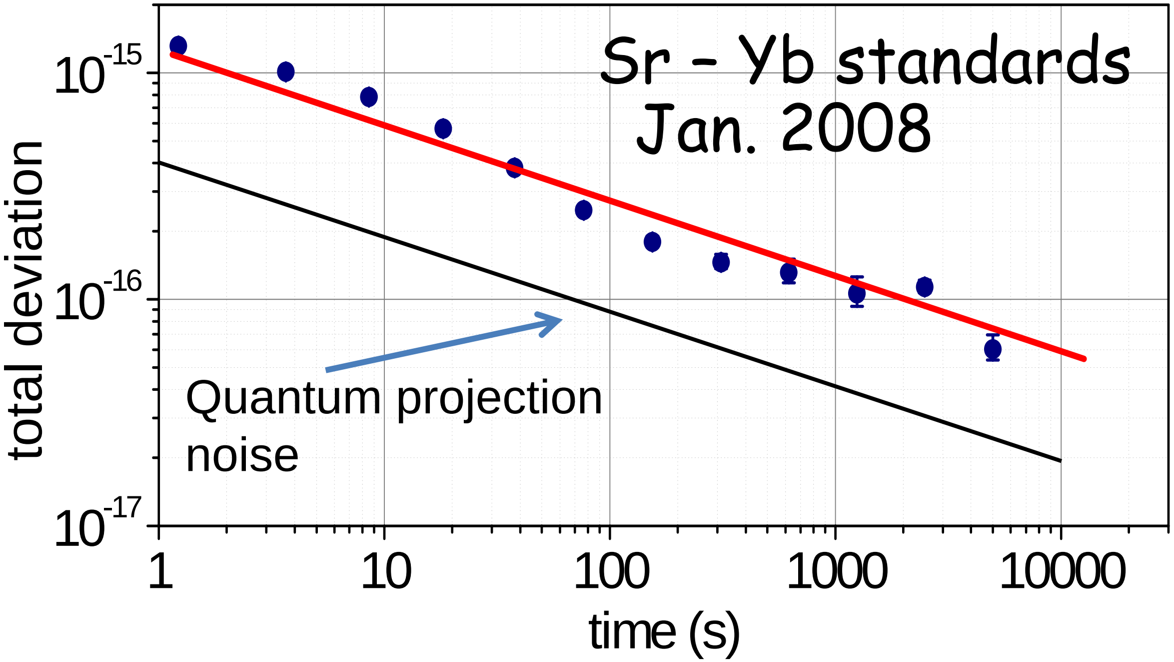


Thermal Noise: ——

Record high precision and stability

Q ~ 2.5 x 10^{14}





- Single trace without averaging
- Estimated instability

$$\sigma(\tau) \sim 2\times10^{-15}/\sqrt{\tau}$$

Ludlow *et al.*, Science **in press** (2008)

Boyd *et al.*, Science **314**, 1430 (2006)

Ludlow *et al.*, Opt. Lett. **32**, 641 (2007)

QPN on the horizon

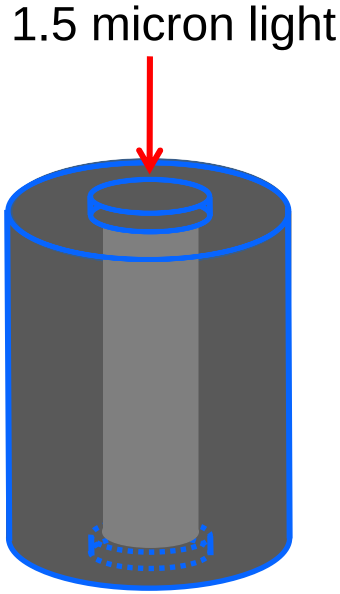
-To reach the quantum projection potential we mus
overcome the dominant thermal noise.

-Higher substrate Q

-Lower temperature

-**For the future:** Si cavity

- $Q \sim 10^8$
- Thermal coefficient = 0 @ 120 K
- 150 GPa Young's modulus



Conclusions

-Designs trade off between vibrational and thermal sensitivity.

- -Longer cavities→ better thermal insensitivity, greater vibrational sensitivity.

-Thermal noise in substrate/coatings is currently largest noise source.

- -Higher Q substrate can reduce this contribution.
- -Possible coating noise cancellation schemes? (Jeff Kim
- -Lower temperatures help, but only as $\sqrt{T}$.

-New Silicon design will improve both vibrational insensitivity and increase material Q.

The Sr team:

- **Andrew Ludlow**
- Martin Boyd
- Gretchen Campbell
- Sebastian Blatt
- Jan Thomsen
- Matt Swallows
- Travis Nicholson

Group leader: Jun Ye

Mirror coatings: Ramin Lalezari, Advanced Thin Films

New approaches: Jeff Kimble, Caltech

PTB collaborators: Uwe Sterr, Fritz Riehle