Status of Coating Work at Sannio

Giuseppe Castaldi,
Vincenzo Galdi,
Vincenzo Pierro,
Innocenzo M. Pinto,

TWG, University of Sannio at Benevento

The Waves Group
Università degli Studi del Sannio

LSC/VIRGO Joint Meeting,
Amsterdam NL, sept 21-25, 2008

Summary

- INFN funding for 2nd generation (doped Tantala) optimised coating mirror TNI-prototypes approved;

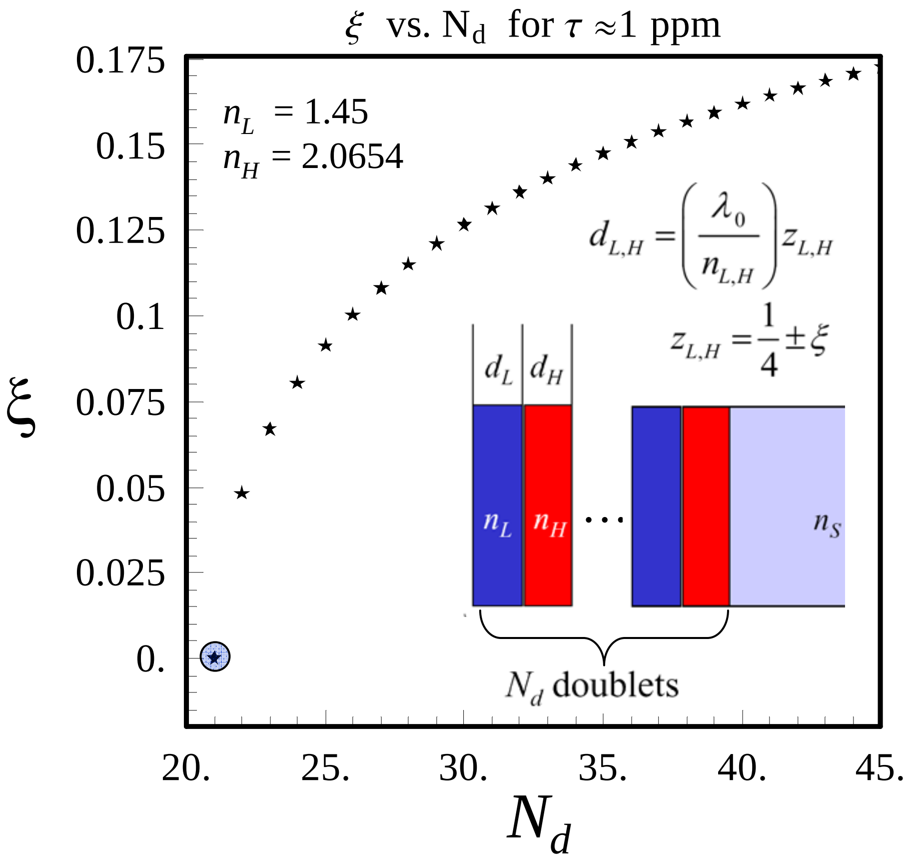
- Impact of thermoelastic/thermorefractive noise canc-ellation on coating optimization (aka, *living with a minus sign*).

INFN Funding Obtained

- INFN funding for 2nd generation (Ti-doped Tantala) optimized-coating mirror prototypes approved by INFN panel-V (September 18, 2008).
  Money (35 KEU) available jan. 2009.

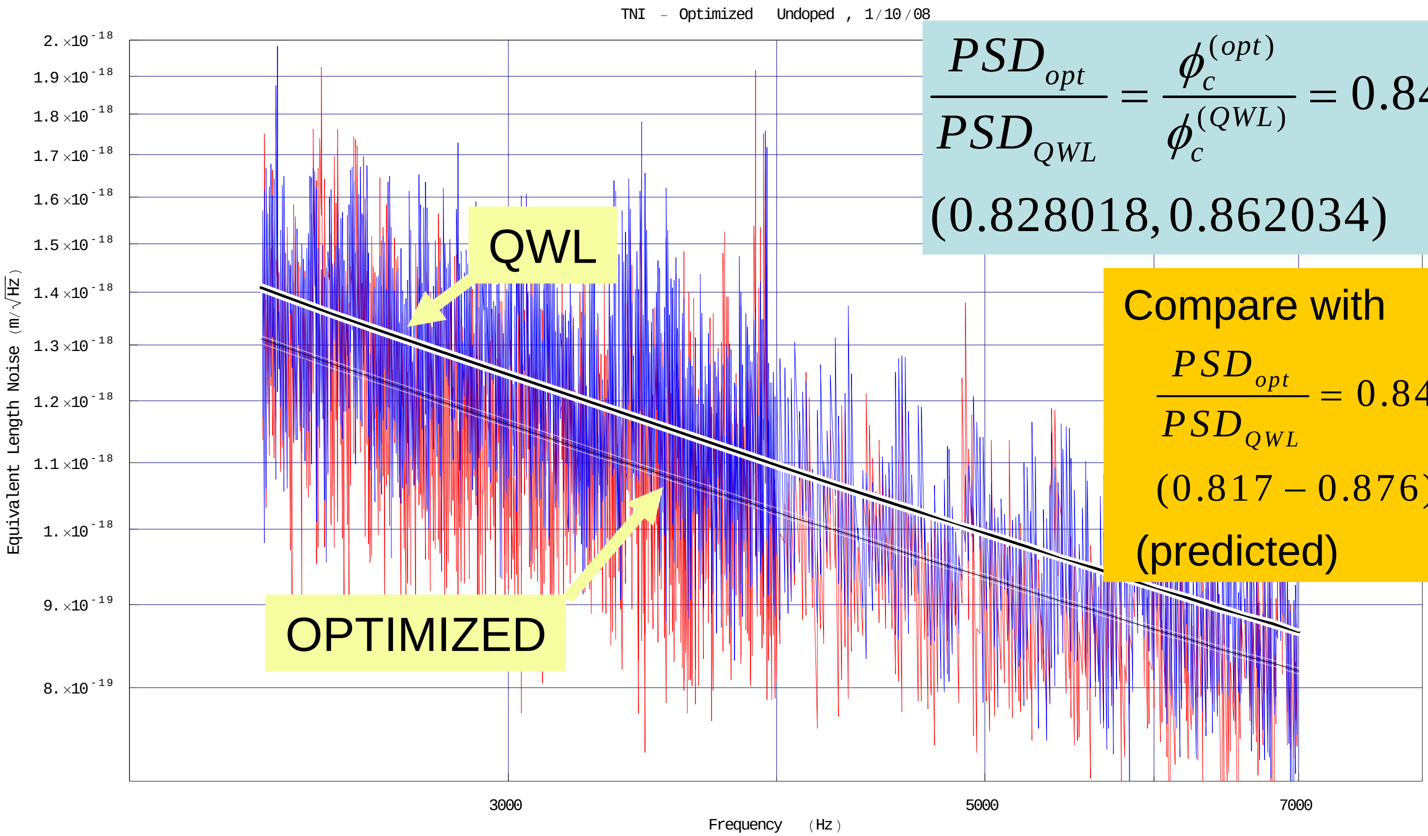
- This funding originally requested for f.y. 2007 was put on hold, waiting for TNI measurements on 1st generation protypes (whose design had been completed in November 2006).
- The well known LMA “bubble” problems delayed prototype delivery. Prototype measurements became available Dec. 2007, and were presented to INFN in Feb. 2008.
- Funding release was then recommended by INFN Referees.

Coating Optimization: Isoreflective SD Coatings (Bragg, Binary)



1st Generation (Plain Tantala) Optimized Coating Prototypes

Courtesy E. Black & A. Villar



$$\frac{PSD_{opt}}{PSD_{QWL}} = \frac{\phi_c^{(opt)}}{\phi_c^{(QWL)}} = 0.844869$$

$$(0.828018, 0.862034)$$

Compare with

$$\frac{PSD_{opt}}{PSD_{QWL}} = 0.843$$

$$(0.817 - 0.876)$$

(predicted)

1st Generation Prototypes. Robustness

γ	N = 16 Plain $\{z_L, z_H\}$ {0.330169, 0.169831}	N = 16 Tweaked $\{z_L, z_H\}$ {0.330169, 0.169831} $\{z_1, z_N\}$ {0.0338288, 0.157079}	N = 17 Plain $\{z_L, z_H\}$ {0.345676, 0.154324}	N = 17 Tweaked $\{z_L, z_H\}$ {0.345676, 0.154324} $\{z_1, z_N\}$ {0.0405062, 0.139003}	N = 18 Plain $\{z_L, z_H\}$ {0.357797, 0.142203}	N = 18 Tweaked $\{z_L, z_H\}$ {0.357797, 0.142203} $\{z_1, z_N\}$ {0.0457719, 0.124762}
10	0.896	0.871	0.901 (+0.005)	0.876 (+0.005)	0.912 (+0.016)	0.886 (+0.015)
7	0.866 (+0.003)	0.847 (+0.004)	0.863	0.843	0.867 (+0.004)	0.847 (0.004)
5	0.842 (+0.012)	0.827 (+0.013)	0.833 (+0.003)	0.817 (+0.003)	0.830	0.814

Table of PSD values relative to QWL design with HWL cap (N=14), for various values of $\gamma = \frac{\phi_H}{\phi_L}\frac{n_L}{n_H}\frac{\left(\frac{Y_H}{Y_S}+\frac{Y_S}{Y_H}\right)}{\left(\frac{Y_L}{Y_S}+\frac{Y_S}{Y_L}\right)}$

$S_x^{(B)} = C[z_L + \gamma z_H]$ (z_L, z_H = layer thicknesses **in units of local wavelength**)

N = number of high/low index layers; optimum (minimum noise) syntheses highlighted in yellow. Numbers in brackets are $\{z_L, z_H\}$ (plain design; first line in tweaked design) and $\{z_1, z_N\}$ (second line in tweaked design).

The N=17 design yields the minimum degradation (in brackets) compared to optimum design, if γ is allowed to change throughout the interval [5,10].

Thermooptic Noise

When using doped Tantala (with almost halved loss-angle) thermo-optic noise should be included in the total noise budget.

$$S^{(\Delta x)}_{Thermo-optic} = \frac{2^{1/2} k_B T^2}{\pi r_0^2 \sqrt{\omega k_s \rho_s C_s}} \left(\frac{\Delta x_{TO}}{\Delta T} \right)^2, \quad \frac{\Delta x_{TO}}{\Delta T} = \frac{\Delta x_{TE}}{\Delta T} + \frac{\Delta x_{TR}}{\Delta T}$$

For positive $\alpha_{L,H}$ and $\beta_{L,H}$, the thermoelastic & thermorefractive displacements Δx_{TE} and Δx_{TR} have opposite signs. If added coherently, they *cancel in part* [M. Evans, 2008]

How does this result impact on coating optimization ?

Thermooptic Noise Cancellation

Formulas:

$$\left(\frac{\Delta x_{TR}}{\Delta T}\right)$$

[V.B. Braginsky et al., Phys. Lett. A271 (2000) 303;
I.M. Pinto et al., LIGO-T070159-00-Z]

Braginsky & Vyatchanin, ArXiv:Cond-Mat/0302617 v5

$$\left(\frac{\Delta x_{TE}}{\Delta T}\right) = N_d\left(\tilde{\alpha}_L d_L + \tilde{\alpha}_H d_H\right), \quad d_{L,H} = \frac{\lambda_0}{4n_{L,H}}(1 \pm \xi)$$

Fejer et al., PRD 70 (2004) 082003

$$\tilde{\alpha}_{L,H} = 2(1+\sigma_S)\left\{\frac{\alpha_{L,H}}{2(1-\sigma_{L,H})}\left[\frac{1+\sigma_{L,H}}{1+\sigma_S} + (1-2\sigma_S)\frac{Y_{L,H}}{Y_S}\right] - \alpha_S\frac{C_{L,H}}{C_S}\right\}\left(\frac{g(\omega)}{g(0)}\right)^{1/2} C_{fsm}^{1/2}$$

$\equiv \Gamma$

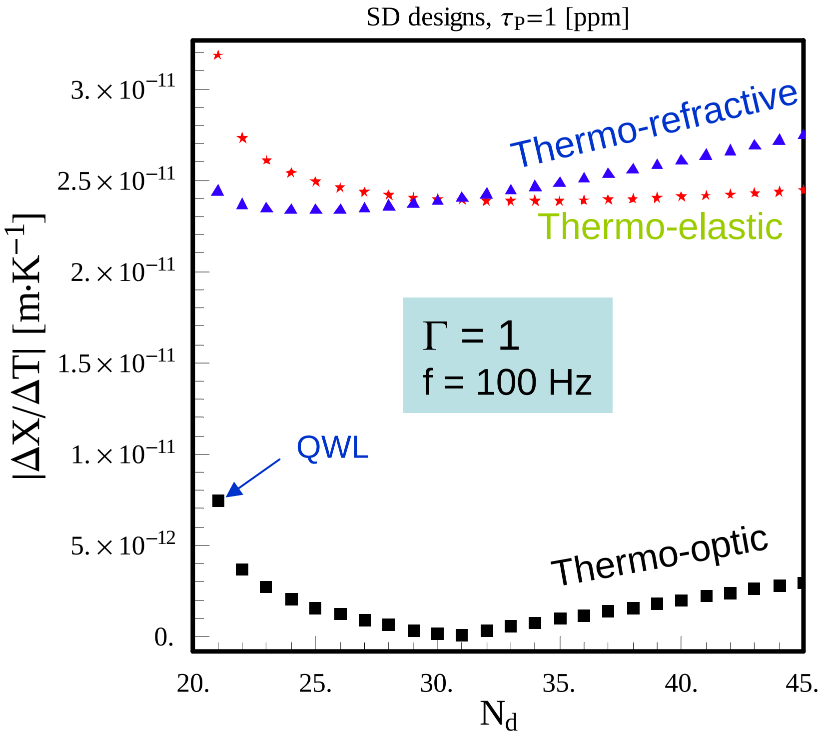
Updates and Checks

- Sign error in thermorefractive term affecting previous version of our optimization code has been fixed. Other minor bugs fixed.

- Vyatchanin calculation of thermoelastic-noise finite-mirror correction factor has been re-checked ab-initio (V. Pierro); MATHEMATICA code written; Code being checked. Parametric study (what if) under way.

- Accuracy of generalized Braginsky's formula (β_{eff} for non QWL coatings) has been tested OK against brute-force (transfer matrix) calculation (drafting a LIGO-T report).

Isoreflective Coatings: TO Noise Cancellation

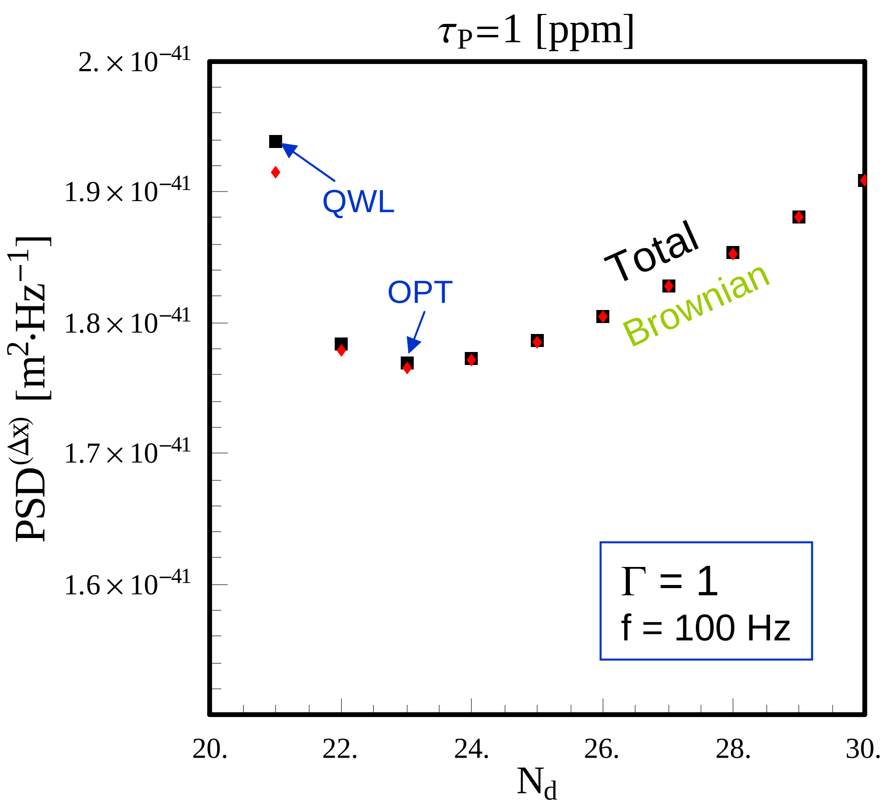


Doped Tantala

$$\alpha_H = 5.0 \cdot 10^{-6}, \beta_H = 6.0 \cdot 10^{-5}$$

$$\alpha_L = 5.1 \cdot 10^{-7}, \beta_L = 1.5 \cdot 10^{-5}$$

Optimization still Worth ?



Doped Tantala

$$\alpha_H = 5.0\cdot10^{-6}, \beta_H = 6.0\cdot10^{-5}$$
$$\alpha_L = 5.1\cdot10^{-7}, \beta_L = 1.5\cdot10^{-5}$$

Optimal design : $\{Nd = 23,\ \xi = 0.0688726\}$

$PSD^{(opt)} / PSD^{(QWL)} = 0.912$

event rate boost = 1.147

Finite Mirror Correction

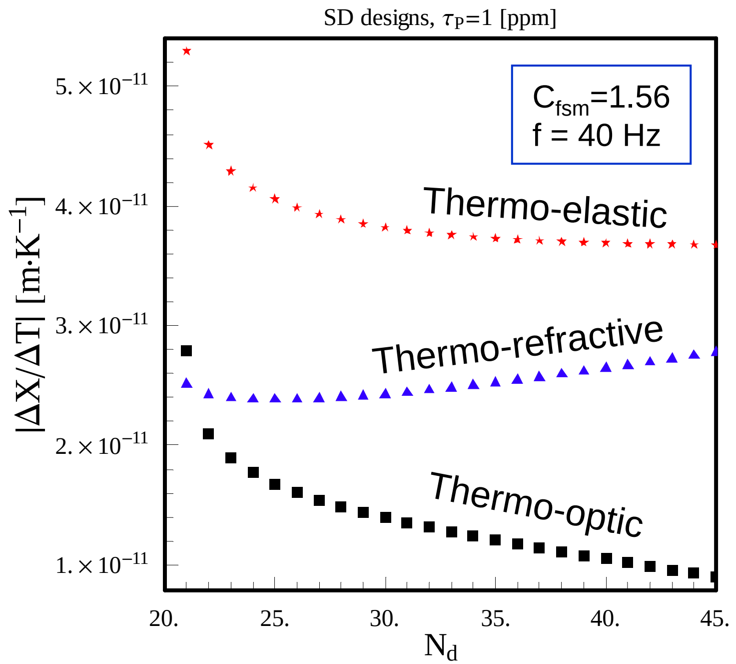
[Braginsky & Vyatchanin, ArXiv:Cond-Mat/0302617 v5]

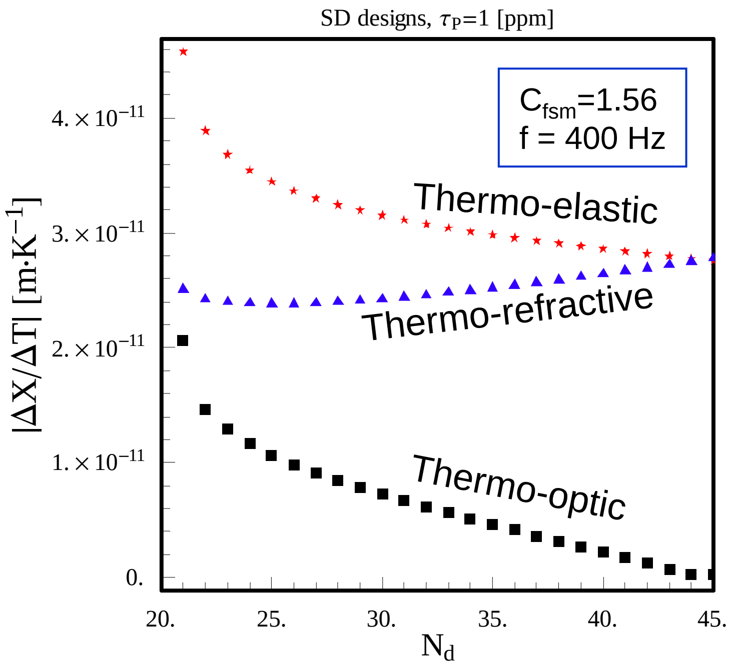
R_{sub}=0.194 m, H_{sub}=0.115 m, r_{beam} = 0.06 m

Doped Tantala

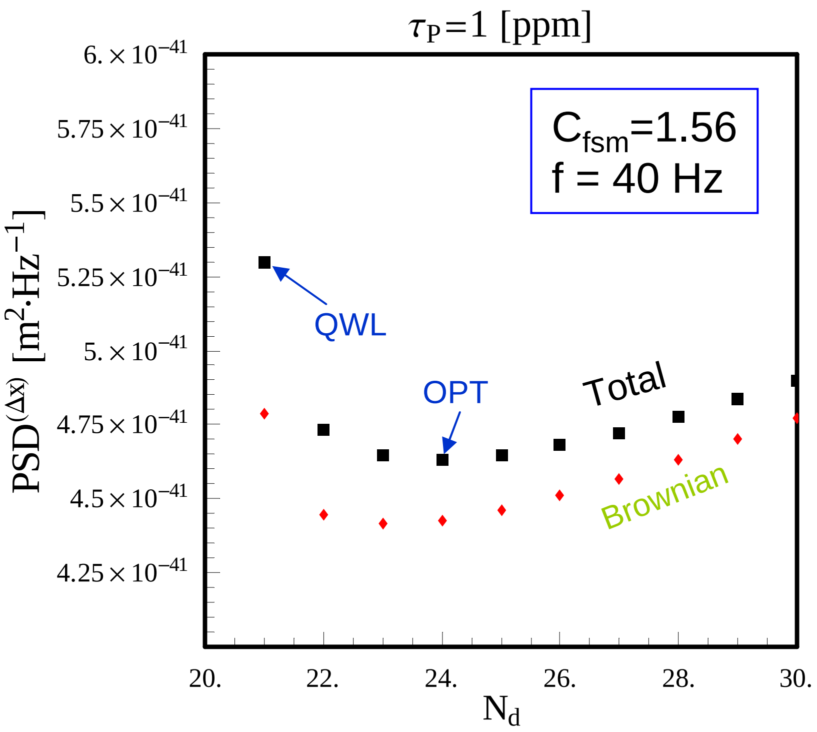
$\alpha_H = 5.0 \cdot 10^{-6}, \beta_H = 6.0 \cdot 10^{-5}$

$\alpha_L = 5.1 \cdot 10^{-7}, \beta_L = 1.5 \cdot 10^{-5}$





Finite Mirror Correction



Doped Tantala

$\alpha_H = 5.0\cdot10^{-6}, \beta_H = 6.0\cdot10^{-5}$

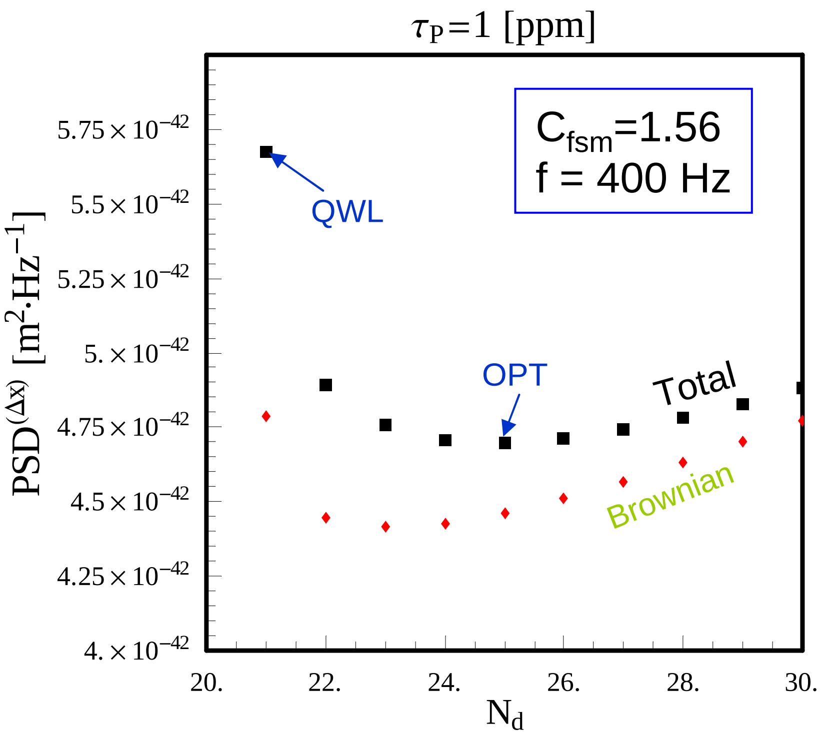
$\alpha_L = 5.1\cdot10^{-7}, \beta_L = 1.5\cdot10^{-5}$

Optimal design : $\{Nd = 24,\ \xi = 0.0827096\}$

$PSD^{(opt)} / PSD^{(QWL)} = 0.874$

event rate boost = 1.223

Finite Mirror Correction



Doped Tantala

$\alpha_H = 5.0 \cdot 10^{-6}, \beta_H = 6.0 \cdot 10^{-5}$

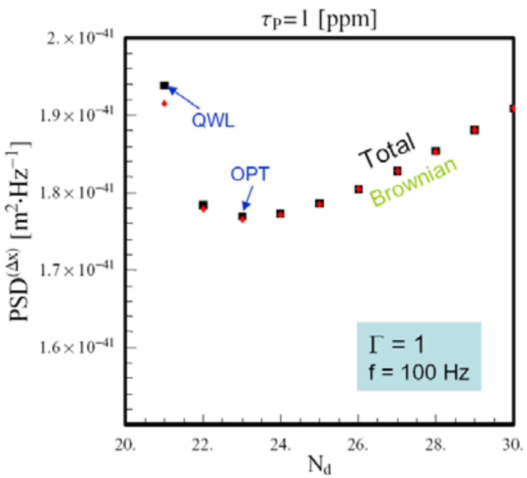
$\alpha_L = 5.1 \cdot 10^{-7}, \beta_L = 1.5 \cdot 10^{-5}$

Optimal design : $\{Nd = 25,\ \xi = 0.0937243\}$

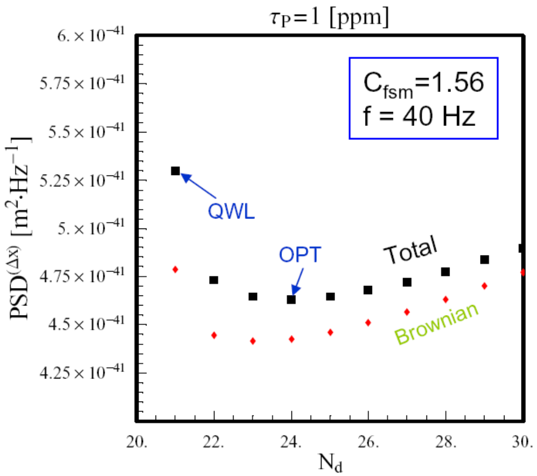
$PSD^{(opt)} / PSD^{(QWL)} = 0.827$

event rate boost = 1.327

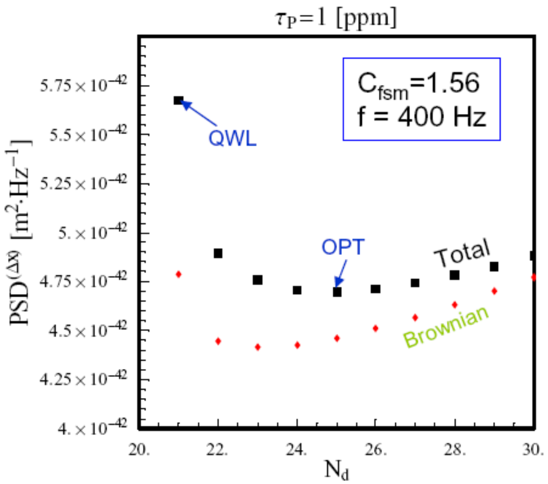
Finite Mirror Correction



Optimal design : $\{Nd = 23,\ \xi = 0.0688726\}$
$PSD^{(opt)} / PSD^{(QWL)} = 0.912$
event rate boost = 1.147



Optimal design : $\{Nd = 24,\ \xi = 0.0827096\}$
$PSD^{(opt)} / PSD^{(QWL)} = 0.874$
event rate boost = 1.223



Optimal design : $\{Nd = 25,\ \xi = 0.0937243\}$
$PSD^{(opt)} / PSD^{(QWL)} = 0.827$
event rate boost = 1.327

Robust (conservative) design possible (here, $N_d=24$) yielding nearly maximal noise reduction irrespective of actual value of C_{fsm} and f

Sign of $\beta_{L,H}$

A simple model

Loentz-Lorenz factor

$\phi = P^{-1} dP/dT$, $P \equiv$ mean polarizability

Volume expansion coefficient

$$\frac{dn}{dT} = \frac{(n^2-1)(n^2+2)}{6n}(\phi - 3\alpha)$$

Linear expansion coefficient

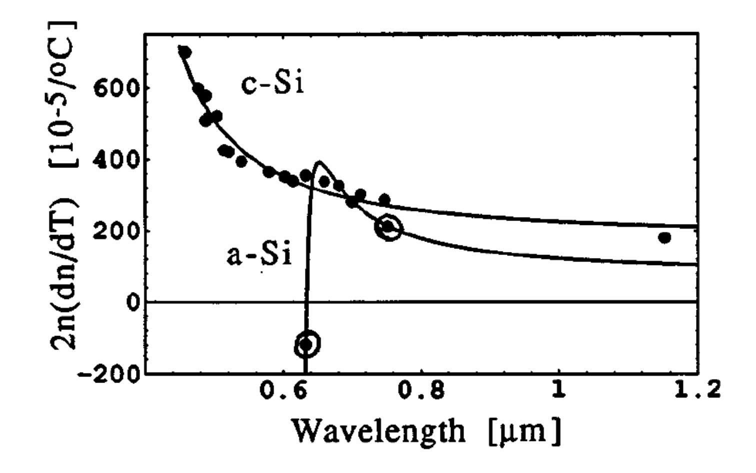
[L. Prod'homme, Phys. Chem Glasses, 1 (1960) 119]

More sophisticated models account for the wavelength dependence of the polarizability, and hence of the sign of dn/dT …

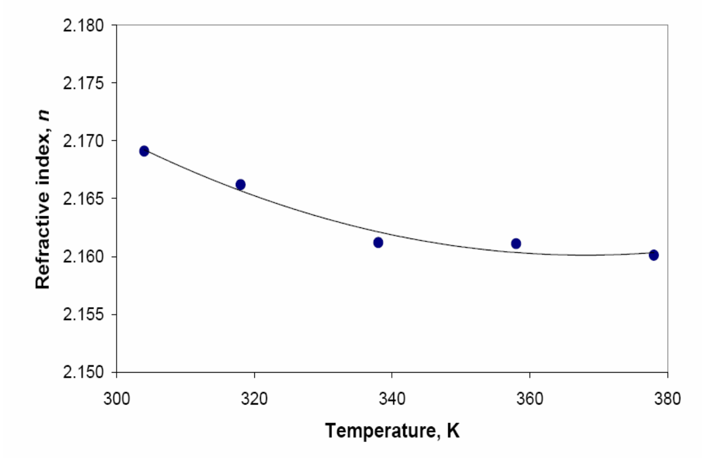
[G. Ghosh, Optics Letters, 19 (1994) 1391]

Sign depends on whether $\phi \gtrless 3\alpha$

Sign of $\beta_{L,H}$



Wavelength dependent sign of *dn/dT* in amorphous Silicon [G. Ghosh Appl. Phys. Lett., 66 (1995) 3570]

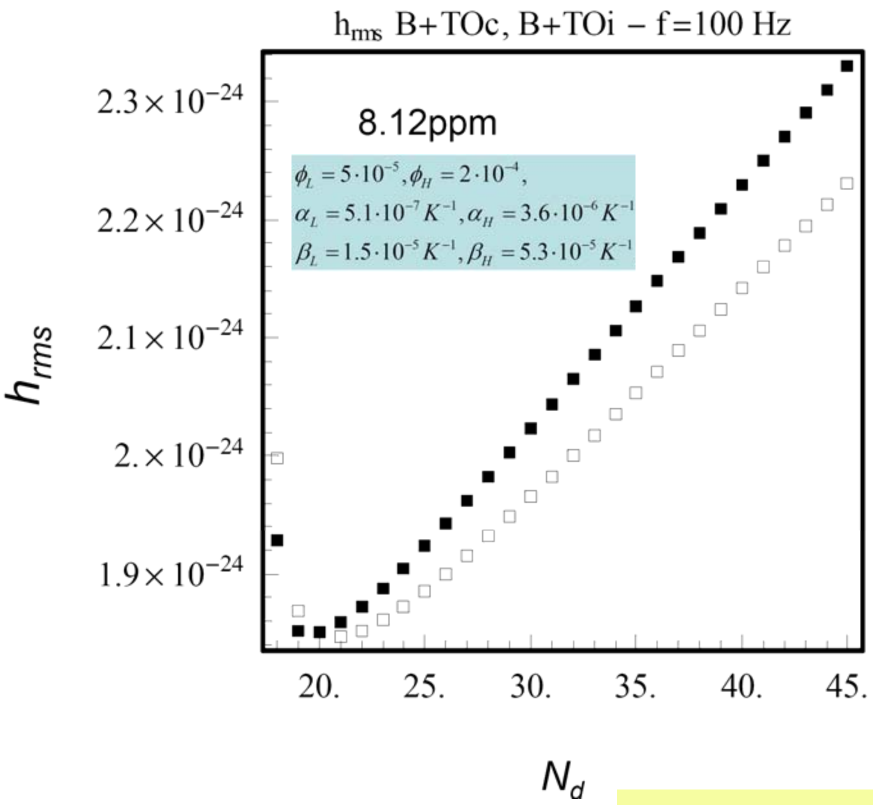


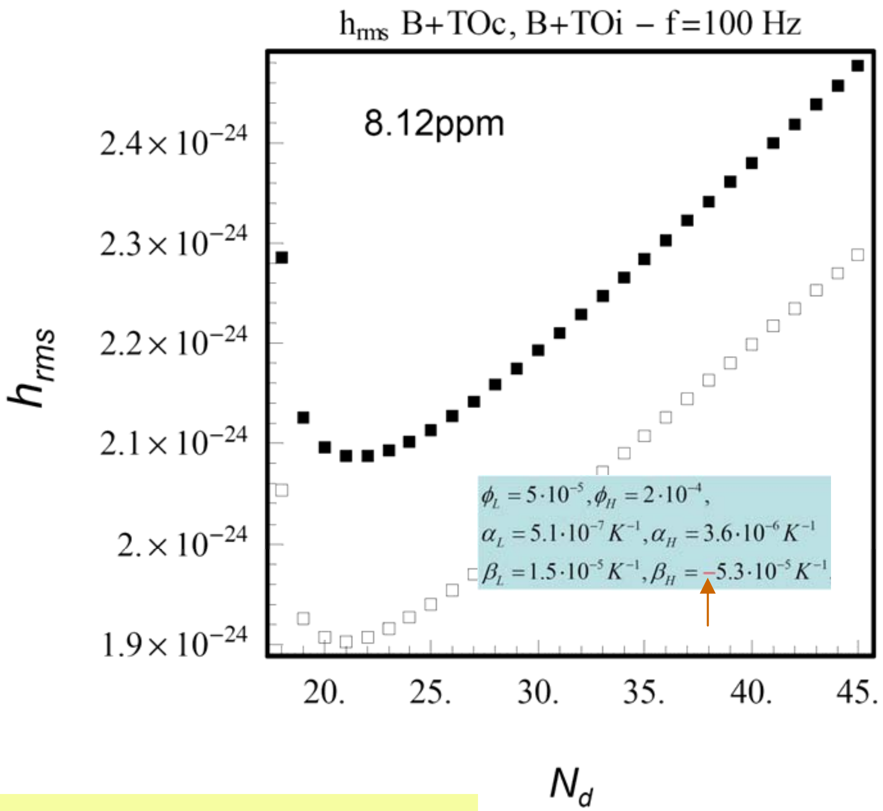
Temperature dependence of *n.* Titanium dioxide film at 632nm [H. Xie et al., SIMTech Reports, 9 (2008) 29]

- dn/dT in SiO_2–TiO_2 films with different TiO_2 concentrations at various annealing temperatures found to change from positive to negative as TiO_2 concentration increases [H. Hirota et al., Jpn. J. Appl. Phys., 44 (2005) 1009]

and more…

Flipping the β_H Sign





Both cases, $N_d = 21$ nearly-optimal

Tentative Conclusions

- Partial cancellation between the thermoelastic and thermorefractive noise occurs if $\alpha_{L,H} > 0$ and $\beta_{L,H} > 0$.
- The QWL design *does not* yield the best cancellation;
- Uncertainties in the values of the thermoelastic and thermorefractive parameters and inclusion of the finite mirror correction factor affect the amount of noise reduction achievable through optimization. A worst-case event-rate boost $\sim$15% may be expected (doped Tantala).
- A robust (conservative) design seems possible, in view of the *broad* minimum exhibited by the PSD vs N_d curves, yielding close-to-maximal noise reduction over a broad range of material parameters and substrate geometries.