Searching for Periodic GW Signals in Space- and Ground-Based Detector Data

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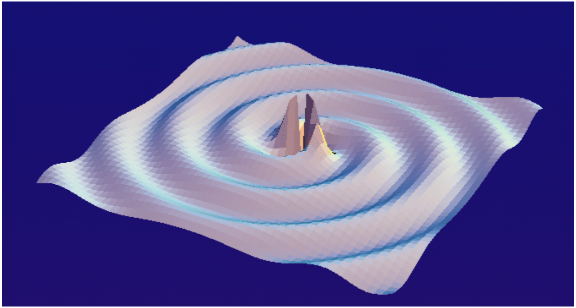
Outline

1. Searches for Gravitational Waves
    - Crash Course in Gravitational Wave Physics
    - Gravitational-Wave Observations & Detectors
    - The Mock LISA Data Challenges
2. Searches for Periodic Gravitational Waves
    - $\mathcal{F}$-Statistic Search Technique
    - AEI $\mathcal{F}$-Statistic Search for White Dwarf Binaries

collaboration with Reinhard Prix & Deepak Khurana

Outline

Motivation



- In Newtonian gravity, force dep on distance btwn objects
- If massive object suddenly moved, grav field at a distance would change instantaneously
- In relativity, no signal can travel faster than light $\longrightarrow$ time-dep grav fields must propagate like light waves

Gravity as Geometry

- Minkowski Spacetime:

$$ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

$$= \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\mathrm{tr}} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = \eta_{\mu\nu} dx^\mu dx^\nu$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\mathrm{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = g_{\mu\nu} dx^\mu dx^\nu$$

Gravitational Wave as Metric Perturbation

- For GW detection, spin-2 “graviton tensor” $h_{\mu\nu}$ is difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

  $$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$

  ($h_{\mu\nu}$ “small” in weak-field regime, e.g. for GW detection)
- E.g. Plane wave propagating in z direction

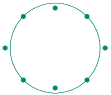
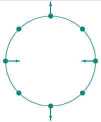
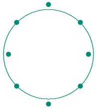
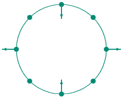
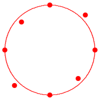
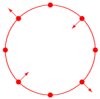
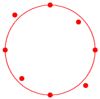
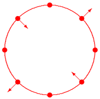
  $$\{h_{\mu\nu}\}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i2\pi f(z-t)}$$

  h_+ and $h_\times$ are amplitudes of “plus” and “cross” pol states.

  $$\overset{\leftrightarrow}{h} = \left[h_+\overset{\leftrightarrow}{e}_+ + h_\times\overset{\leftrightarrow}{e}_\times\right] e^{i2\pi f(\hat{k}\cdot\vec{r}-t)}$$

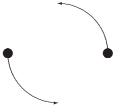
Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

Plus (+) Polarization				
Cross ($\times$) Polarization				

Gravitational Wave Generation

- Generated by moving/oscillating mass distribution
- Lowest multipole is quadrupole
- Classic example: orbiting binary system



  (e.g., Binary Pulsar 1913+16
  – Observed energy loss agrees w/GW prediction)
- Periodic signals with slow freq evolution arise from
  - Early stages of binary evolution
  - Rapidly rotating non-axisymmetric neutron stars

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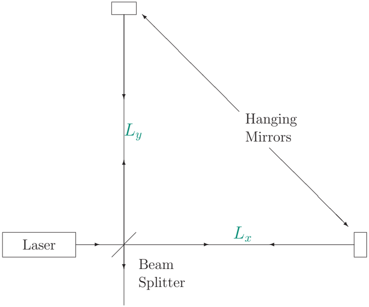
Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



- Measure small change in

$$L_x - L_y = \sqrt{g_{11} L_0^2} - \sqrt{g_{22} L_0^2}$$

$$= \sqrt{(1 + h_{11}) L_0^2} - \sqrt{(1 + h_{22}) L_0^2}$$

$$\approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$$

- More gen,

$$(L_1 - L_2)/L_0 = \overleftrightarrow{h} : \overleftrightarrow{d}$$

with “response tensor”

$$\overleftrightarrow{d} = \frac{\hat{n}_1 \otimes \hat{n}_1 - \hat{n}_2 \otimes \hat{n}_2}{2}$$

(also when $\hat{n}_1$ & $\hat{n}_2$ not $\perp$)

Rogues' Gallery of Ground-Based Interferometers

LIGO Hanford (Wash.)

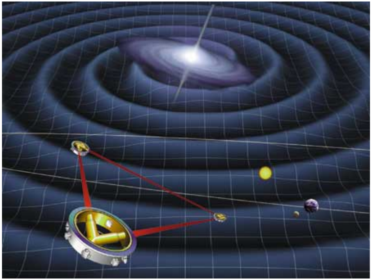
LIGO Livingston (La.)

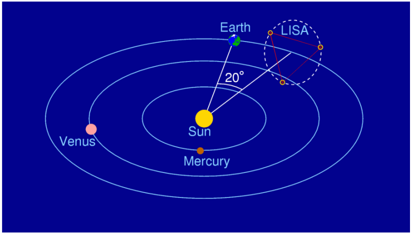
GEO-600 (Germany)

Virgo (Italy)

LISA: Interferometry in Space

- Planned Joint NASA-ESA Mission: to launch 2018 or later
- 3 spacecraft will orbit sun in 5 mio km ▽
  & track each other w/lasers
- Laser phase data combined to simulate IFO:
  "Time-Delay Interferometry" (TDI)

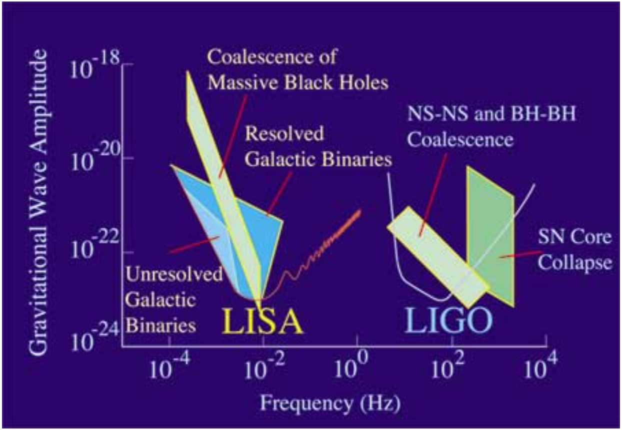




Credits: NASA/JPL; MPI for Gravitational Physics (AEI)/Einstein Online

Differences Between LISA and LIGO

- Diff noise sources & sizes mean diff frequency ranges



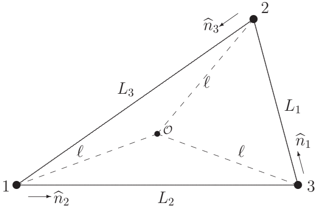
Credit: NASA/JPL

Differences Between LISA and LIGO

- Diff noise sources & sizes mean diff frequency ranges
- LIGO data noise-dominated; can seek one source at a time
  LISA data will contain many strong sources;
  ⟶ must worry about signal extraction
- LISA to observe GWs w/λ comparable to arm length
  ⟶ At higher frequencies, simple IFO picture breaks down
  & response depends on propagation direction

$$\widetilde{X}(f) = \frac{\widetilde{h}(f)}{R(f)} = \frac{\overset{\leftrightarrow}{h}(f) : \overset{\leftrightarrow}{d}(f, \widehat{k})}{R(f)}$$

LISA Response



- LISA spacecraft 1, 2, 3
- Arm lens L_1, L_2, L_3
  all $\approx L = 5\,\text{million km}$
  vary due to GW & orbit
- TDI vars X, Y, Z comb links
  btwn sc to cancel laser noise
- Convert into "strains"
  $\widetilde{h}^X(f) = R(f)\widetilde{X}(f) = \overleftrightarrow{h}(f) : \overleftrightarrow{d}^X(f, \widehat{k})$
  where in long-λ limit
  $\overleftrightarrow{d} \approx \frac{1}{2}(\hat{n}_2 \otimes \hat{n}_2 - \hat{n}_3 \otimes \hat{n}_3)$
  (etc. for Y & Z)

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Mock LISA Data Challenges

- LISA data analysis presents unusual challenges;
  Need to coördinate searches for different types of signals
  Need plan worked out before LISA flies
- LISA International Science Team (LIST) has organized MLDCs to build community expertise
  Extract simulated signals from simulated LISA noise

Challenge	Dates	Results Presented
MLDC1	2006 Jun-Dec	GWDAW 11, Potsdam
MLDC2	2007 Jan-Jun	GR 18 / Amaldi 7, Sydney
MLDC1B	2007 Jul-Dec	GWDAW 12, Boston
MLDC3	2008 Jan-Dec	GWDAW 13, Arecibo

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MLDC1B	2007 Jul-Dec	GWDAW 12, Boston
MLDC3	2008 Jan-~~Dec~~ -2009 Apr	~~GWDAW 13, Arecibo~~ Amaldi 8, NYC?

First Mock LISA Data Challenge

- MLDC1 Results submitted December 2006
- MLDC1B Results submitted December 2007
- Data sets:
  - Challenge 1.1: White Dwarf Binaries: Periodic Sources
  - Challenge 1.2: Super-Massive Black Hole Inspirals
  - Challenge 1.3: Extreme Mass Ratio Inspirals (deadline postponed until MLDC2)
- Entries submitted by ten groups each time
- AEI group of Reinhard Prix & JTW searched for WD binaries w/$\mathcal{F}$-statistic method w/Deepak Khurana for MLDC1B

Second Mock LISA Data Challenge

- Results submitted June 2007
- Data sets:
  - Challenge 1.3: Extreme Mass Ratio Inspirals
  - Challenge 2.1: Galactic Binaries (30 Million)
  - Challenge 2.2: "Whole Enchilada": Galaxy + EMRIs + BHB
- Entries submitted by thirteen groups
- AEI group of Reinhard Prix & JTW
  searched for WD binaries w/$\mathcal{F}$-statistic method
  (improved pipeline to distinguish sources)

Third Mock LISA Data Challenge

- Results due ~~December 2008~~ April 2009
- Data sets:
    - Challenge 3.1: Galactic WDB w/frequency evolution
    - Challenge 3.2: SMBH binary + galaxy
    - Challenge 3.3: EMRIs
    - Challenge 3.4: Bursts
    - Challenge 3.5: Stochastic Background

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Periodic Gravitational Waves

- Searching for sinusoidal signals is easy: Fourier transform $\widetilde{x}(f)$ & look for peaks
- But signal won't be sinusoidal:
  - Motion of detector doppler-shifts signal
  - Change in orientation changes projection $\overleftrightarrow{h} : \overleftrightarrow{d}$
- Signal parameters:
  - 3+ Doppler params $\theta \equiv \{\beta, \lambda, f, \dot{f}, \ldots\}$
    f, $\dot{f}$, etc; sky pos $\equiv$ ecliptic lat β & lon λ
    determine signal templates $\{\overleftrightarrow{h}_\mu(\theta) | \mu = 1 \ldots 4\}$
  - 4 Amplitude params $\mathcal{A}^\mu(\{h_0, \iota, \phi_0, \psi\})$
    GW amp h_0 & init phase ϕ_0; inclination ι & orientation ψ
    Allows factorization ($\sum_{\mu=1}^4$ implicit) $\overleftrightarrow{h} = \mathcal{A}^\mu \overleftrightarrow{h}_\mu(\theta)$

$\mathcal{F}$-Stat Search for Periodic GWs (JKS 1998)

- Measured strain (= noise + signal) is
  $x(t;\mathcal{A},\theta) = n(t) + \mathcal{A}^\mu h_\mu(t;\theta)$
  $n(t)$ & $h_\mu(t;\theta) = \overleftrightarrow{h}_\mu : \overleftrightarrow{d}$ depend on detector, $\mathcal{A}$ does not
- Jaranowski, Królak, Schutz 1998: Log-likelihood ratio

$$-\int \frac{|\widetilde{x}(f) - \mathcal{A}^\mu \widetilde{h}_\mu(f)|^2}{S_n(f)} df + \int \frac{|\widetilde{x}(f)|^2}{S_n(f)} df = -\mathcal{A}^\mu \mathcal{M}_{\mu\nu} \mathcal{A}^\nu + 2\mathcal{A}^\mu x_\mu$$

data “vector” $x_\mu = \int \frac{\widetilde{h}_\mu(f)^* \widetilde{x}(f)}{S_n(f)} df$

“metric” $\mathcal{M}_{\mu\nu} = \int \frac{\widetilde{h}_\mu(f)^* \widetilde{h}_\nu(f)}{S_n(f)} df$

$\mathcal{F}$-Stat Search for Periodic GWs (JKS 1998)

- Measured strain (= noise + signal) is
  $x(t;\mathcal{A},\theta) = n(t) + \mathcal{A}^\mu h_\mu(t;\theta)$
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$$-\int \frac{|\widetilde{x}(f) - \mathcal{A}^\mu \widetilde{h}_\mu(f)|^2}{S_n(f)} df + \int \frac{|\widetilde{x}(f)|^2}{S_n(f)} df = -\mathcal{A}^\mu \mathcal{M}_{\mu\nu} \mathcal{A}^\nu + 2\mathcal{A}^\mu x_\mu$$

  quadratic in $\mathcal{A}$; maximize analytically
- log-likelihood maximized by amplitude parameters
  $\mathcal{A}^\mu_{\text{MLE}} = \mathcal{M}^{\mu\nu} x_\nu$; max value is $2\mathcal{F} = x_\mu \mathcal{M}^{\mu\nu} x_\nu$
- $\mathcal{F}$-stat search technique:
  - Make a grid of doppler params θ (freq & sky pos)
  - For each choice of θ, calculate $2\mathcal{F}$ from data
  - High values are candidate sources w/amp params $\mathcal{A}_{\text{MLE}}$

Currently the basis of LIGO searches for spinning neutron stars

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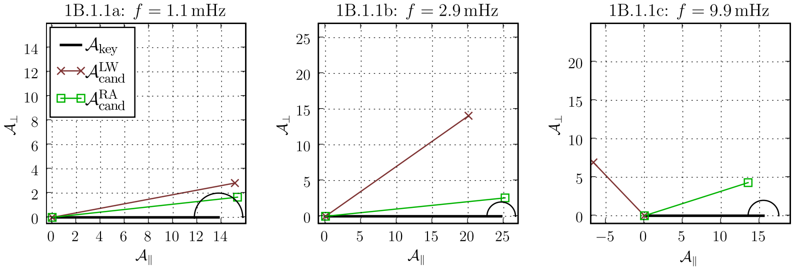
Prix/Whelan/Khurana MLDC Searches

- Re-use LIGO (LAL/LALApps) $\mathcal{F}$-stat code where possible
  Some extensions to handle e.g., LISA response
- Search for individual signals using
  hierarchically refined Doppler param grid
- Papers (Prix/Whelan/Khurana) and presentations:
  - MLDC1: P&W CQG **24**, S565 (2007) (arXiv:0707.0128)
    Used LW limit $R^{\text{LW}}(f)$ & $\overleftrightarrow{d}^{\text{LW}}$
  - MLDC1B: W,P&K CQG **25**, 184029 (2008) (arXiv:0805.1972)
    Used RA response $R^{\text{RA}}(f)$ & $\overleftrightarrow{d}^{\text{RA}}(f,\widehat{k})$
  - MLDC2, Amaldi Poster LIGO-G070462-00-Z
  - Longer paper, incl MLDC2: W,P&K on arXiv soon!

Challenge 1(B).1.1: Isolated Binaries

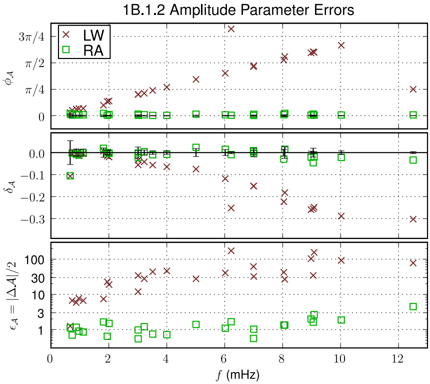

1B.1.1a: $f = 1.1\,\mathrm{mHz}$ 1B.1.1b: $f = 2.9\,\mathrm{mHz}$ 1B.1.1c: $f = 9.9\,\mathrm{mHz}$

	f (mHz)	Δf(nHz) LW	 RA	ϕ_{sky}(mrad) LW	 RA	ϵ_θ LW	 RA
a	1.1	−0.7	−0.7	61.9	46.1	0.5	0.3
b	2.9	0.9	0.9	12.3	7.7	1.1	0.9
c	9.9	1.8	1.8	5.1	7.5	0.4	0.5

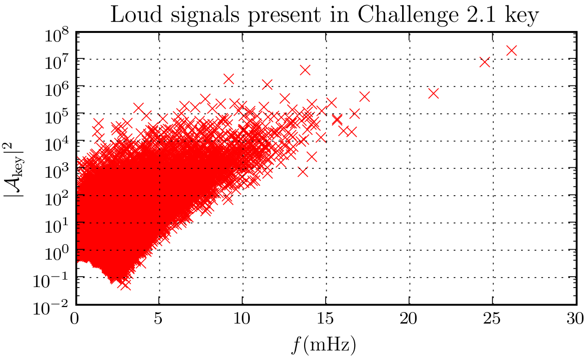
Good sky position even w/long-wavelength response
Rigid adiabatic response needed to get amp params

Challenge 1(B).1.2: Verification Binaries

25 verification binaries w/known dop params; amp params well fit if RA response used.

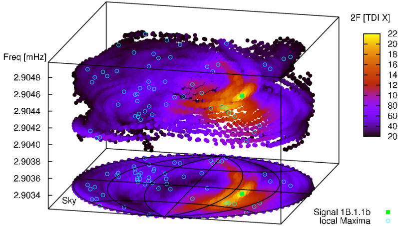


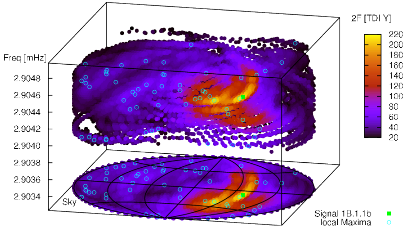
Galactic Binaries Injected in MLDC2

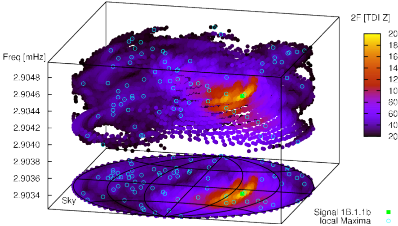


Challenge 2.1 has 26 million galactic WD binaries, of which 59401 designated as “bright” sources

Secondary Maxima (Orientation of the Vorlon)

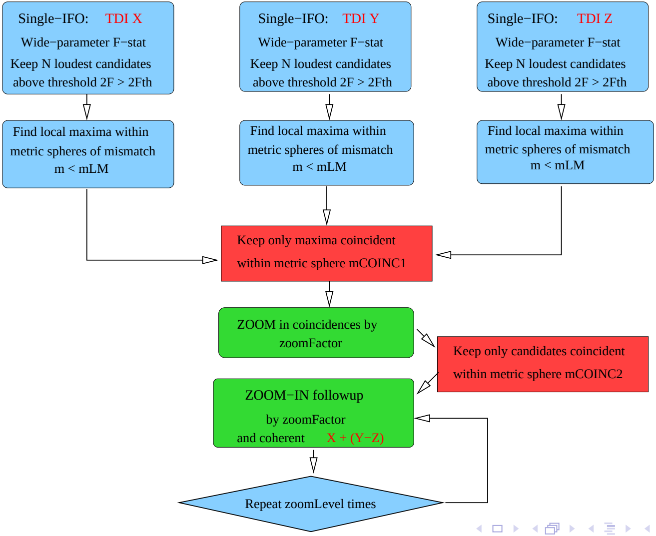






- Spurious local $2\mathcal{F}$ maxima
- 2ndary maxima in diff places in diff TDI vars
- Our pipeline requires coïncidence

Pipeline for Prix/Whelan/Khurana MLDC Searches



Single−IFO: TDI X
Wide−parameter F−stat
Keep N loudest candidates above threshold 2F > 2Fth

Single−IFO: TDI Y
Wide−parameter F−stat
Keep N loudest candidates above threshold 2F > 2Fth

Single−IFO: TDI Z
Wide−parameter F−stat
Keep N loudest candidates above threshold 2F > 2Fth

Find local maxima within metric spheres of mismatch m < mLM

Find local maxima within metric spheres of mismatch m < mLM

Find local maxima within metric spheres of mismatch m < mLM

Keep only maxima coincident within metric sphere mCOINC1

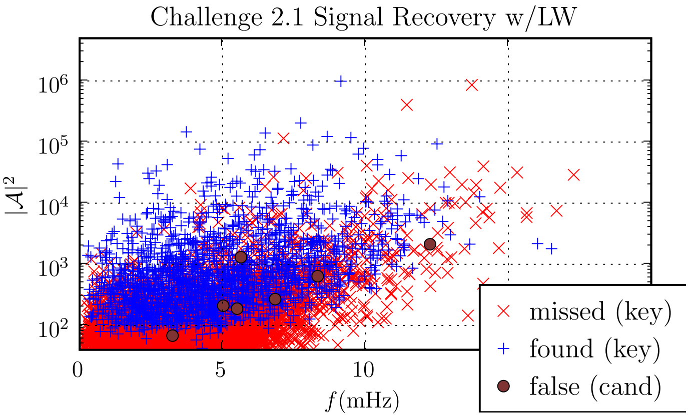
ZOOM in coincidences by zoomFactor

Keep only candidates coincident within metric sphere mCOINC2

ZOOM−IN followup by zoomFactor and coherent X + (Y−Z)

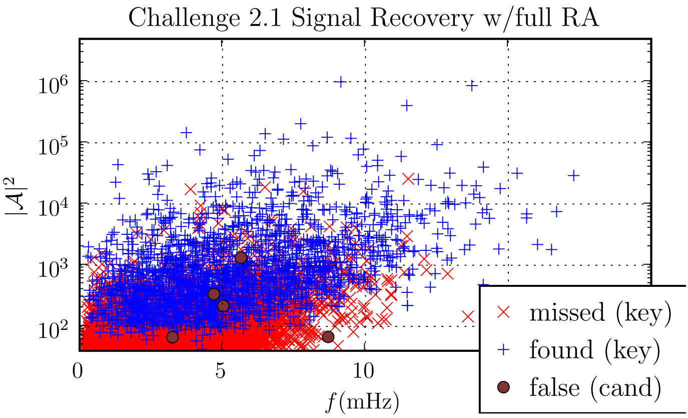
Repeat zoomLevel times

Overview of Galactic Signals Recovered (LW)



Found many signals, but still missed some bright ones (especially at higher *f*), using long-wavelength response

Overview of Galactic Signals Recovered (RA)



Rigid adiabatic response improves signal recovery
Loudest “misses” now found

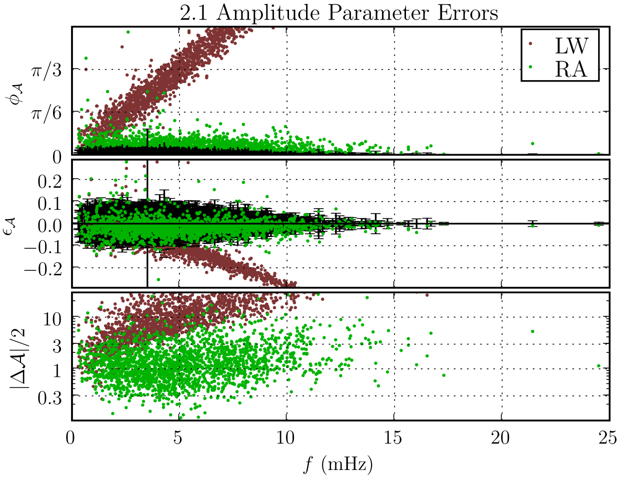
Statistics of Galactic Signals Recovered

Focus on sources w/expected $2\mathcal{F} > 40$

Freqs	Signals ($\lvert\mathcal{A}\rvert^2 > 40$)	Found		False	
		LW	RA	LW	RA
0–5 mHz	4443	982	1025	1	2
5–10 mHz	1966	652	822	5	3
10–15 mHz	163	68	133	1	0
15–20 mHz	7	2	7	0	0
20–27 mHz	3	0	2	2	0
Total	6582	1704	1989	9	5

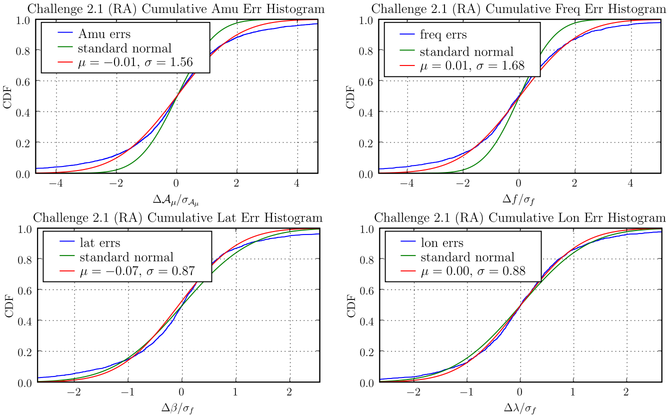
Improved response improves efficiency
Find $\sim$ 10% as many signals as Crowder et al;
limitation of reductionist approach?

Amplitude Accuracy w/RA



Amp param errors comparible to statistical expectations

Cumulative Histograms: Err/Sigma



Challenge 2.1 (RA) Cumulative Amu Err Histogram

Challenge 2.1 (RA) Cumulative Freq Err Histogram

Challenge 2.1 (RA) Cumulative Lat Err Histogram

Challenge 2.1 (RA) Cumulative Lon Err Histogram

Current Status and Future Directions

- MLDC3 has $\dot{f}$; underlying code already handles this,
  still playing with pipeline & infrastructure
- Higher dim param space may require more sophisticated bank
  but note only need ~ 10 templates in $\dot{f}$ direction
- Currently only use Doppler params for coïnc;
  Could also use Amplitude param information
- Could try to subtract bright signals & iterate
  or at least produce our own residuals

Conclusions

- Mock LISA Data Challenge Searches
(Prix, JTW, Khurana):
$\mathcal{F}$-statistic method to find doppler-shifted periodic signals
applied to mock LISA data
- Had to model LISA response beyond long-λ limit
to get accurate amplitude param recovery
& find higher-frequency signals
- Weaker signals can be mistaken for secondary maxima
partially overcome by coïncidence condition
Probably need signal subtraction to go further
- Currently working on MLDC3 including $\dot{f}$