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Dimension reduction of search parameter manifolds used to detect continuous gravitational wave sources

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1: Introduction

Continuous wave (CW) sources

Continuous gravitational waves (CW) result from quasi-stationary quadrupole sources such as non-axisymmetrically rotating neutron stars.The signals may be then detected by Earth-based laser interferometers, such as those used by LIGO or Virgo, or space based instruments such as the proposed LISA project.

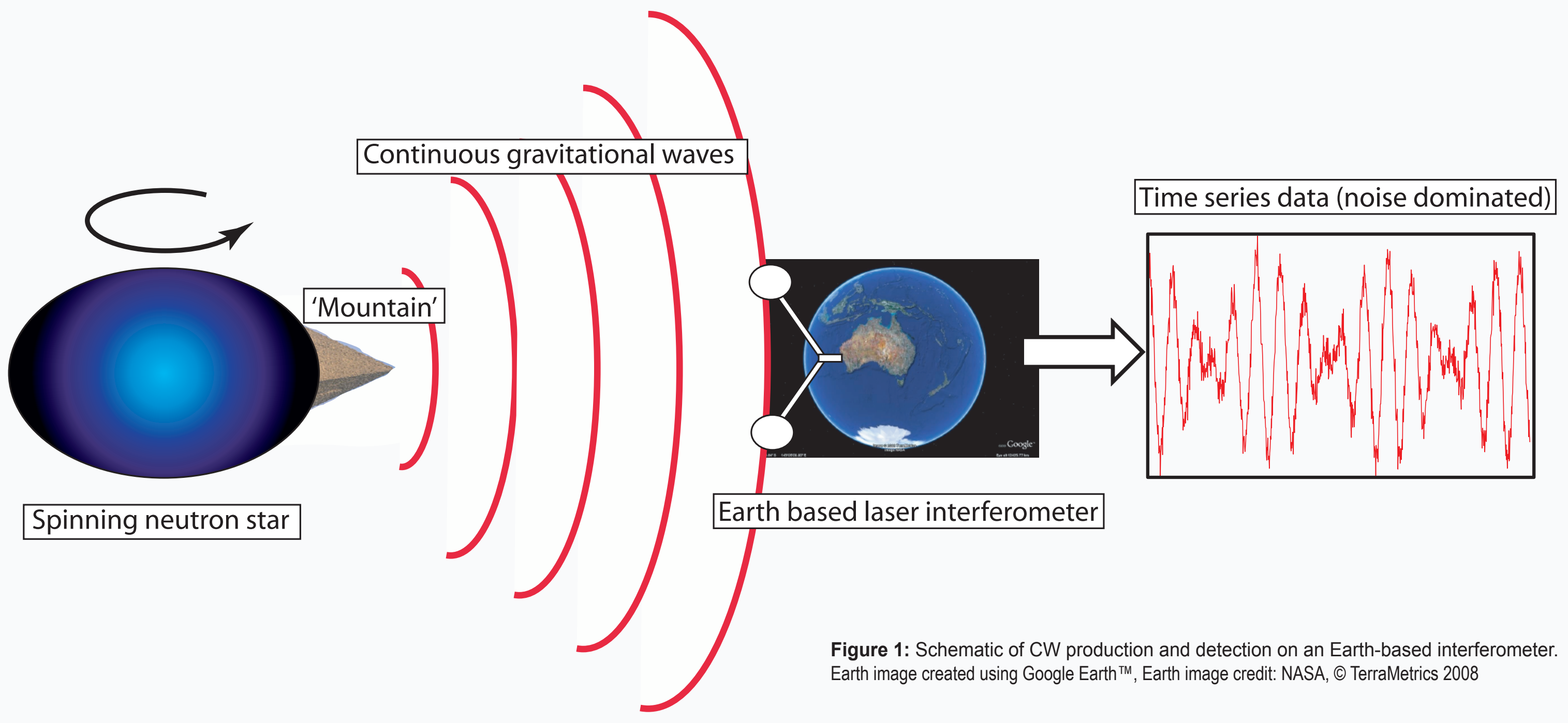

Figure 1: Schematic of CW production and detection on an Earth-based interferometer. Earth image created using Google Earth™, Earth image credit: NASA, © TerraMetrics 2008

However the expected strain magnitude from CW sources is very small: typically $O(10^{-24})$ [1]

Although this is well below the sensitivities of current interferomers, if the dominant noise sources are stationary enough, it is possible to use signal averaging/integration to improve signal-to-noise ratios.
Because of the long integration times, most current CW searches are computationally very expensive.

Search parameter manifolds

To aid computation, the 'true' or 'exact' manifold of the parameter space of interest may be approximated by a measurement manifold. The optimal spacing may be estimated from sampling/information theory via e.g. the Fisher information matrix [2, 3].

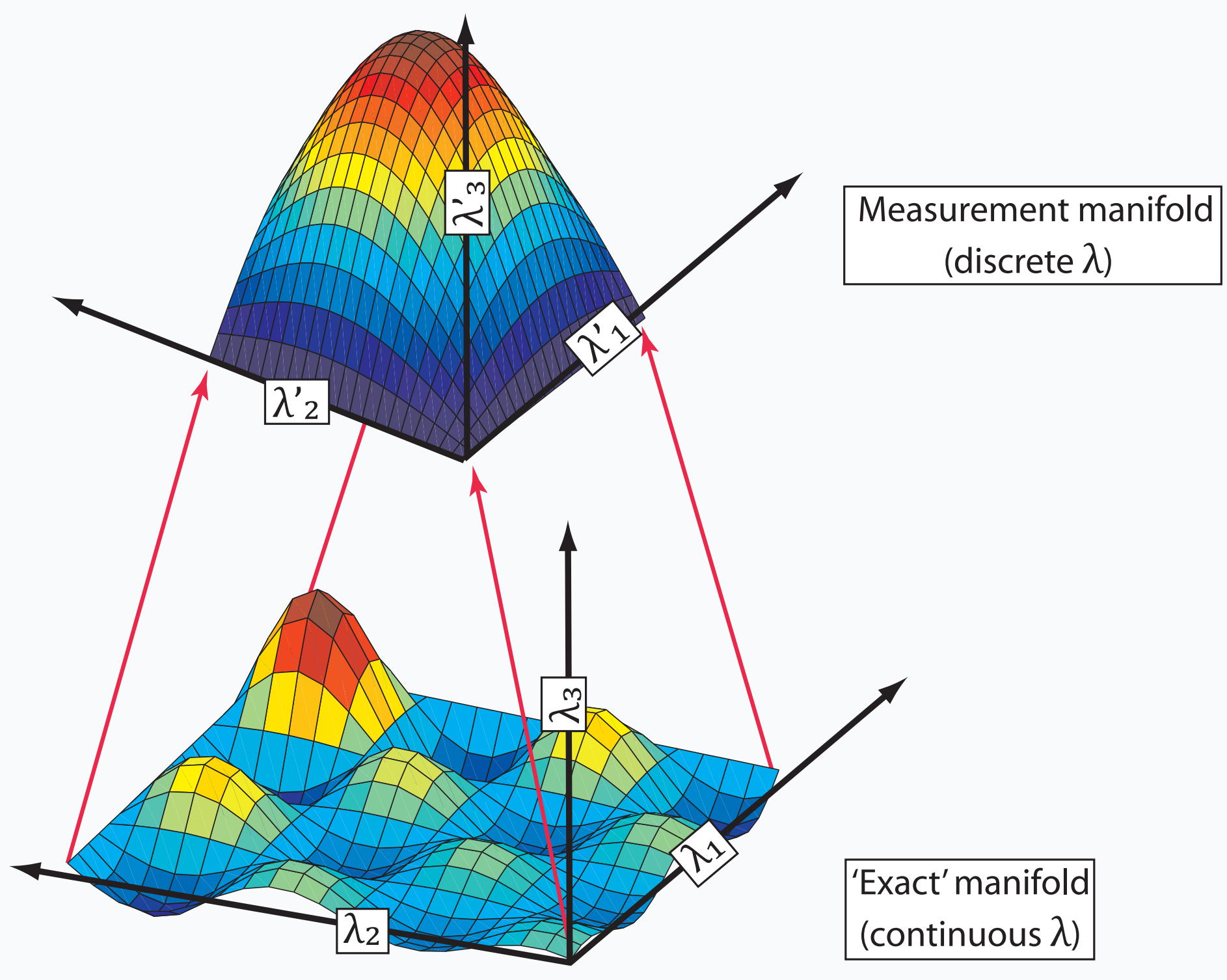

Figure 2: Schematic of relationship between the measurement manifold and the 'exact' manifold.

The metric for the search parameter manifold may be given by [3]:

$$g_{ij}(\lambda) = -\frac{1}{2}\left(\frac{\partial^2 \mathrm{M}}{\partial\Delta\lambda^i \partial\Delta\lambda^j}\right)_{\Delta\lambda^k=0}$$

Where M is the 'ambiguity function': the fraction of the optimal signal-to-noise ratio with parameters λ to that of a 'nearby' set of parameters λ+Δλ

Application: Cassiopeia A (a young supernova remnant)

Cassiopeia A is the remnant of a very recent (~300 yr) supernova.
The position of the Central Compact Object (CCO) is well known due to x-ray observations from Chandra.
Although the CCO is likely to be a neutron star produced in the core collapse of the supernova, no pulsed radio or other electromagnetic signals have yet been detected, making it an interesting candidate for a CW search [4].

Properties of the CCO in Cassiopeia A :
1: Angular position well known from Chandra x-ray studies,
2: It is relatively young and therefore, if it were a neutron star, undergoing significant braking processes. Therefore the spin-down (time-dependent rotational frequency) parameters are probably a strong function (viz. to second order) of time.

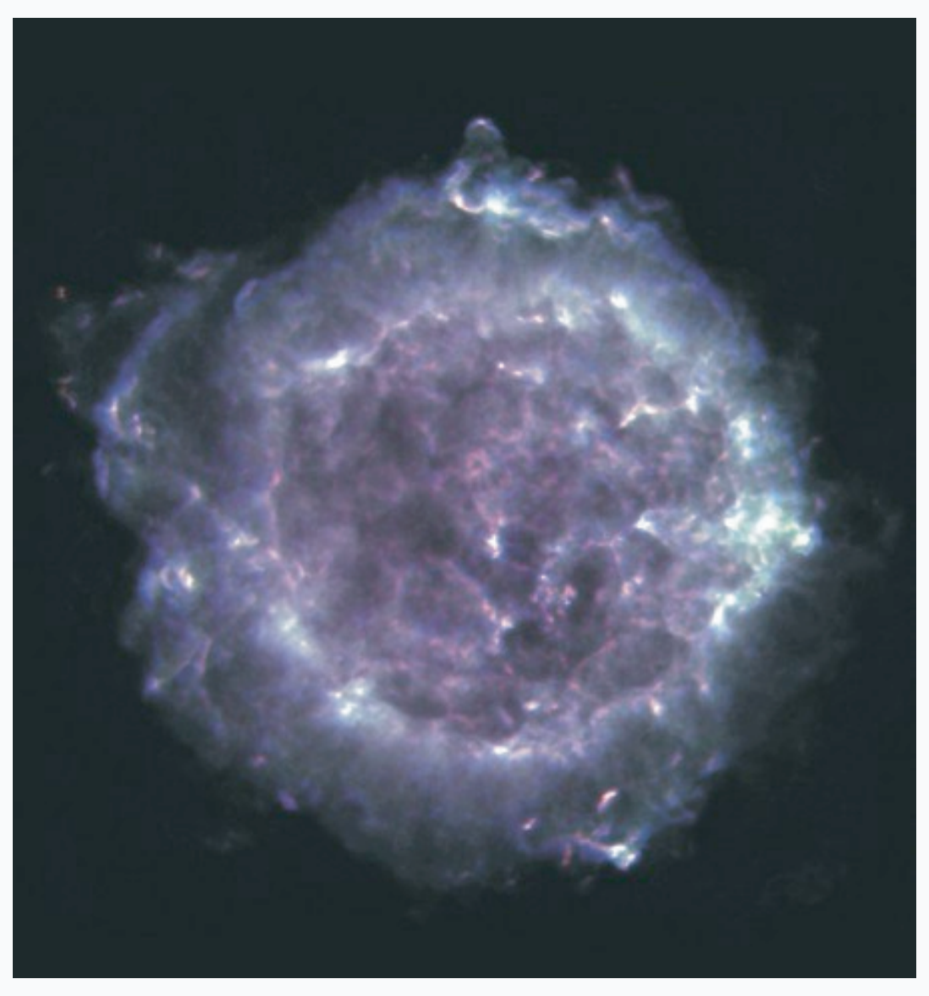

Figure 3: Radio image of Cassiopeia A. Note the CCO at the centre. Image credit: National Radio Astronomy Observatory/ Associated Universities, Inc./National Science Foundation.

So here the measurement manifold is three dimensional:
First search parameter: $\lambda_1 = f$;
Second search parameter: $\lambda_2 = \dot{f}$;
Third search parameter: $\lambda_3 = \ddot{f}$

2: Dimension reduction

Principal Component Analysis (PCA)

Exploratory Principal Component Analysis (PCA) is a common data analysis technique used to reduce the dimensionality of large data sets displaying a high degree of covariance or 'clustering'. The original data set undergoes a linear transformation whereby the axes of the new co-ordinate system are aligned such that the maximum variance is parallel to the first axis (principal component), the second highest variance lies along the second principal axis, etc.

This is achieved by finding the principal eigenvalues of the covariance matrix, **X**, and then applying criteria to the eigenvalues to optimise the dimensionality of the data space. Here we make use of the Kaiser rule: if the magnitude of any eigenvalue is less than one, the associated dimension is removed from the data set.

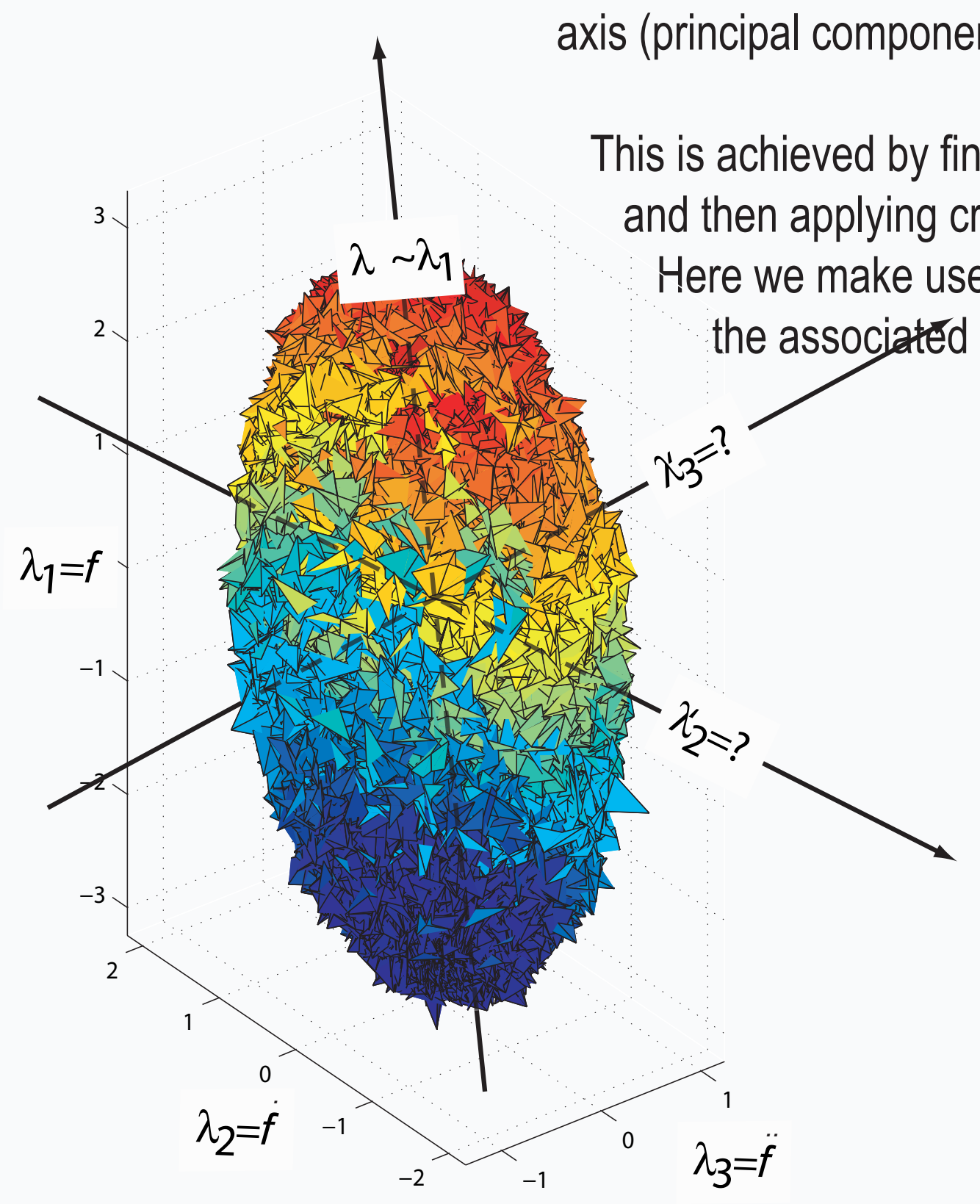

Figure 4: Schematic of noisy surface embedded in three dimensional parameter space. Here the parameters are three significant spin-down parameters of a neutron star in a directed CW search.

Recipe

1: Obtain GW data
2: Coarse sample
3: Standardise variance
4: PCA algorithm (covariance matrix **X**: $\boldsymbol{\lambda}'_i = \mathbf{X}\,\boldsymbol{\lambda}_i$)
5: Reduce data set (values of principal eigenvalues)
6: Perform CW search
7: Map back to original axes, $\boldsymbol{\lambda}'_i \rightarrow \boldsymbol{\lambda}_i$

Case study: synthesised neutron star spin-down

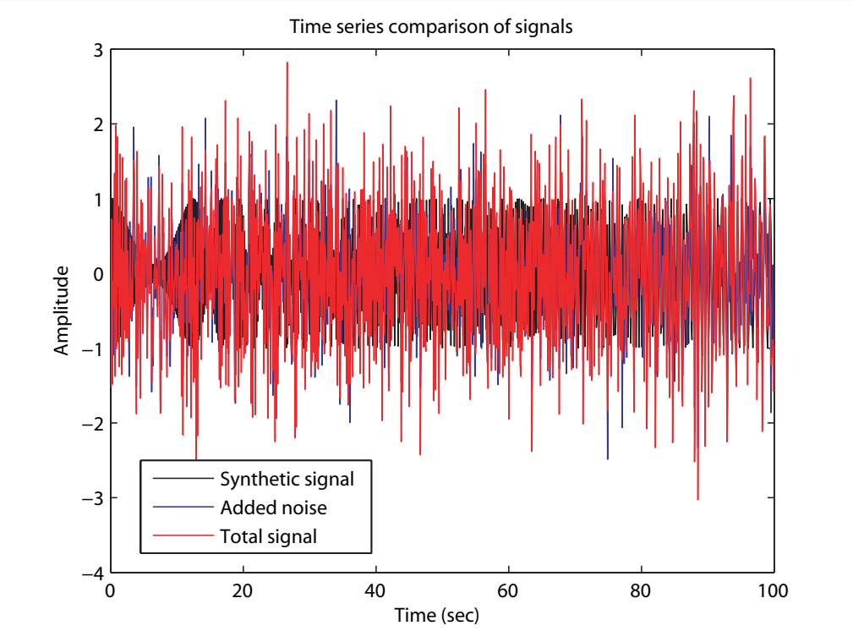

Figure 5: Time-series of synthesised CW signals with added Gaussian noise

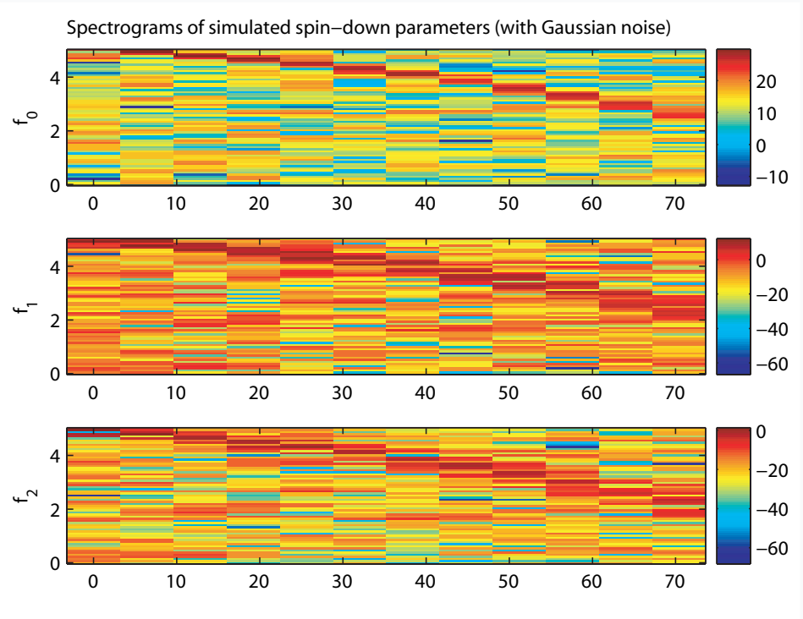

Figure 6: Spectrograms of the spin-down parameters of a synthetic signal with Gausian noise added, resulting from the time series in Figure 5.

1: Synthetic spin-down test function, e.g. $z = \cos(\frac{1}{\pi}(100t - 0.08t^2 - 0.002t^3))$
2: Add Gaussian noise (here, mean and variance equal to the time averaged synthetic signal)
3: Run PCA algorithm, possibly with standardised variables: $z' = \frac{z}{\mathrm{var}(z)}$

Output

Covariance matrices:

$$\mathbf{X} = \begin{matrix} 0.0019 & 0.1001 & 0.9950 \\ -0.0500 & -0.9937 & 0.1001 \\ 0.9987 & -0.0500 & 0.0031 \end{matrix}$$

$$\mathbf{X}_{\mathrm{std}} = \begin{matrix} -0.5365 & 0.7747 & -0.3347 \\ 0.6237 & 0.0967 & -0.7757 \\ -0.5685 & -0.6249 & -0.5350 \end{matrix}$$

Percentage variance explained by eigenvalues:
99.4%, 0.06%, 0.00%

Percentage variance explained by eigenvalues (with standardisation):
83.43%%, 15.35%, 1.22%

According to the Kaiser rule, the optimum number of dimensions is: 1

3: Conclusion and future work

Dimension reduction of multidimensional CW searches, via principal component analysis, may dramatically reduce some of the computational expense, although it has yet to be applied to real data

PCA introduces computational overheads if variance evenly distributed amongst dimensions

Most effective for a priori large dimensional parameter spaces with variances unevenly distributed amongst the parameters

Most interesting features ('information') may be more appropriately analysed with higher order moments (skew, kurtosis etc.) ---may have to modify PCA algorithm

References

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[2] Sathyaprakash, B. S. and Dhurandhar, S. V.:"Choice of filters for the detection of gravitational waves from coalescing binaries," *Phys. Rev. D* **44** (12): 3819-3834 (1991)

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