Interferometry for Gravity Wave Detection

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Outline of talk

- salient features of GW's for detectors
- how interferometric detectors work
- limitations to sensitivity: physics of and solutions to noise sources

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Nature of Gravity Waves

Assuming General Relativity

- Any GW's impinging on earth are in weak, far field limit

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$$

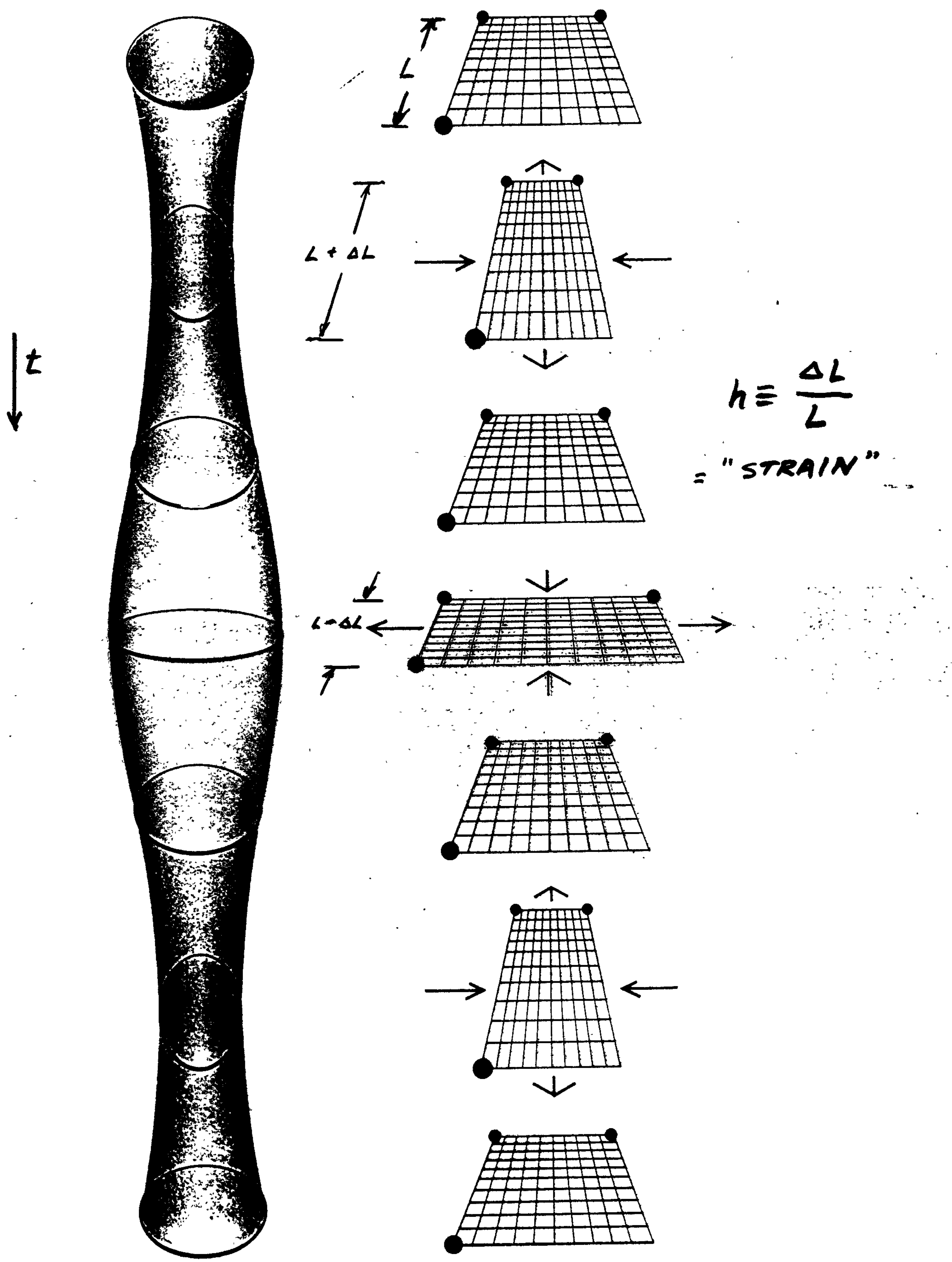
- For particular choice of gauge, *h* satisfies

$$\left[\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] h(x, t) = 0 \ , \quad h_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_{yy} & h_{yz} \\ 0 & 0 & h_{yz} & -h_{yy} \end{bmatrix}$$

- Proper distance between two particles, $ds^2 = g_{\mu\nu} dx^{\mu} dx^{\mu}$, is perturbed according to

$$\Delta s = \int_0^{y_0} \left| g_{yy} \right|^{1/2} dy \approx \left[1 + \frac{1}{2}h\right] y_0$$

Net effect: variation in proper distance between free masses, proportional to GW amplitude and initial separation $\rightarrow$ tidal force



EFFECTS OF GRAVITATIONAL WAVES are depicted for a rubber tube floating in space (*left*) and for a set of masses in a plane (*right*). Gravitational waves should distort the shape of any region of space they travel through. Here the waves move vertically.

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Simple estimate of magnitude

Lowest order radiation term: quadrupole

- field amplitude proportional to $\ddot{Q}$, 2nd derivative of quadrupole
- or, non-spherical part of kinetic energy $\ddot{Q} \sim M\dot{L}^2 \sim Mv^2 \sim E_{kin}^{ns}$
- dimensional analysis leads to

$$h \sim \frac{G}{c^4}\left(\frac{E_{kin}^{ns}}{r}\right) \sim 10^{-20}\left[\frac{E_{kin}^{ns}}{M}\right]\left[\frac{10Mpc}{r}\right]$$

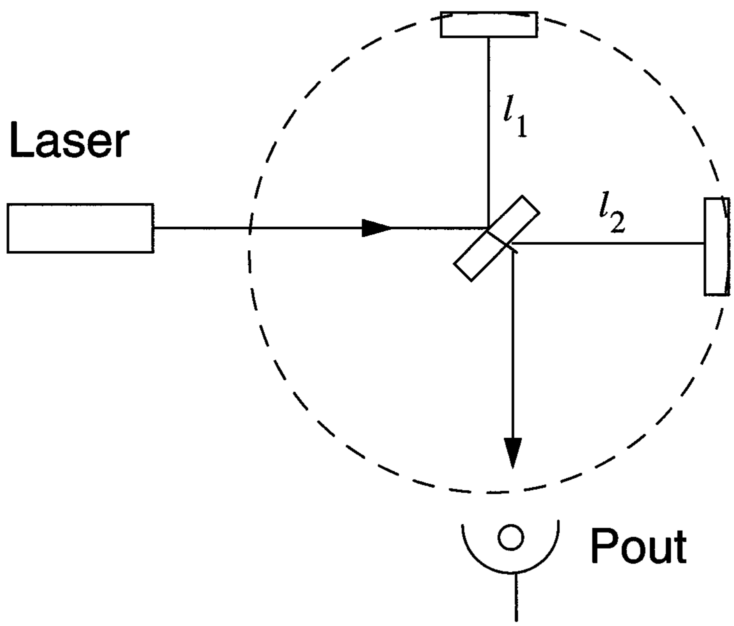
- $h \sim 10^{-17}$ (our galaxy)

  $\sim 10^{-20}$ (Virgo cluster)

  $\sim 10^{-23}$ (cosmological distances)
- Sources
  - compact binary coalescence
  - supernovae
  - spinning neutron stars (pulsars)
  - cosmological: cosmic strings, early universe phase transitions
  - THE UNKNOWN

Basic idea of detection

Allow free test bodies to float inertially in space-time, & continuously monitor the time of flight of light beams bouncing between them to sense changes in their proper separations

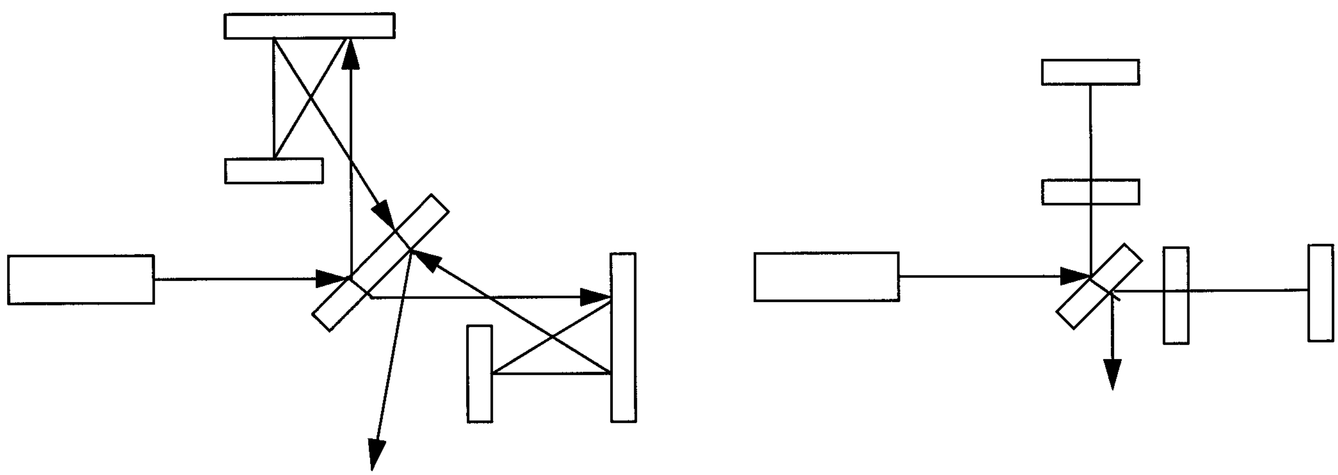


- Laser -illuminated Michelson interferometer is an ideal tool
  - broadband response to GWs of varying frequency, polarization, direction
- Obstacles:
  - free fall not practical
  - small absolute displacement, $\Delta x \sim 10^{-18}$ m, 10^{-12} of laser λ

More on interferometry

Interaction time with the GW

- signal δl, $\delta\varphi$ grows as length of interferometer L grows, in the limit where $L \gg \lambda_{GW}$. $\rightarrow$ L up to about 100 km
- not practical to make 100km straight path, so fold it; for the same δl, increase $\delta\varphi$ by optical storage:



- Delay line
  - conceptually simple, but requires large mirrors
- Fabry-Perot
  - compact, but imposes modes, resonance constraints on system
- 1 msec storage time for initial ground-based system
  - gives optimum sensitivity around 100 Hz

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Detector frequency response

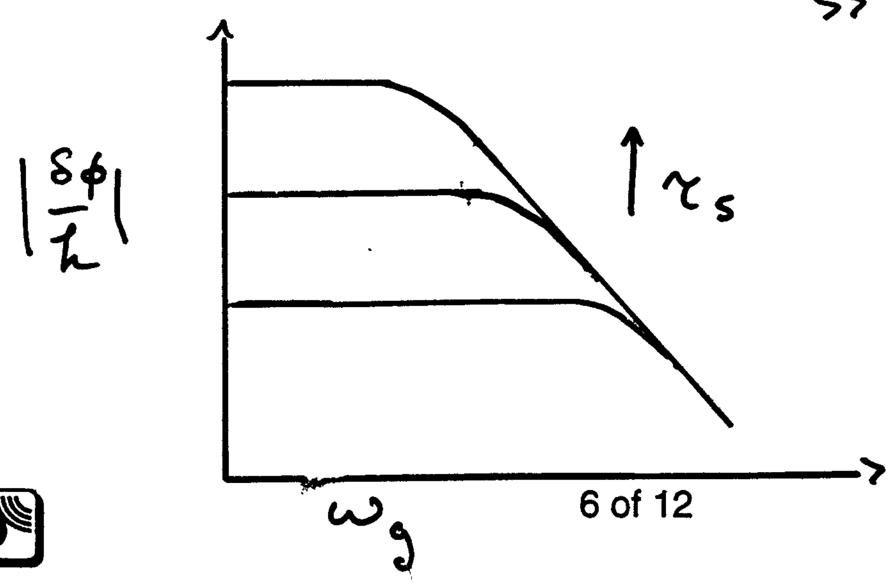
When storage time is longer than half the GW period, the phase shift built up during one half-cycled is reduced during the next half-cycle

- To find response of a particular interferometer configuration to gw's of frequency f, consider that gw-induced modulation of optical phase produces *sidebands* on the light, at

$$\omega_0 - \omega_g \ , \ \omega_0 + \omega_g$$

- Calculate response of optical system to all three frequencies to determine frequency response to GWs
- For a Michelson with Fabry-Perot cavities in the arms, this leads to

$$\left|\frac{\delta\phi}{h}\right| = \frac{4\omega_0\tau_s}{\sqrt{1+(2\omega_g\tau_s)^2}} \xrightarrow[\omega_g\tau_s \gg 1]{} \frac{2\omega_0}{\omega_g}$$



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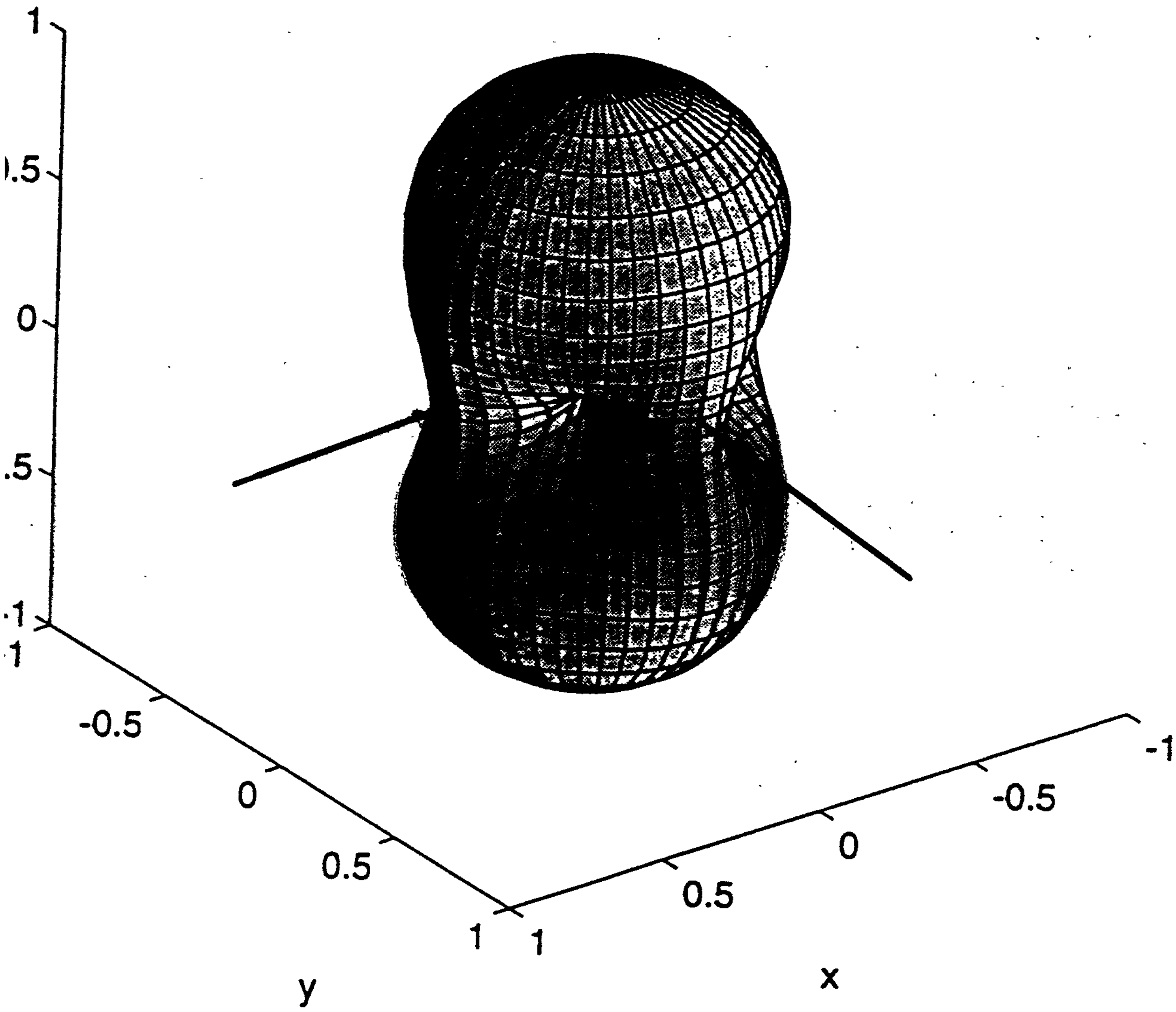
ANGULAR RESPONSE



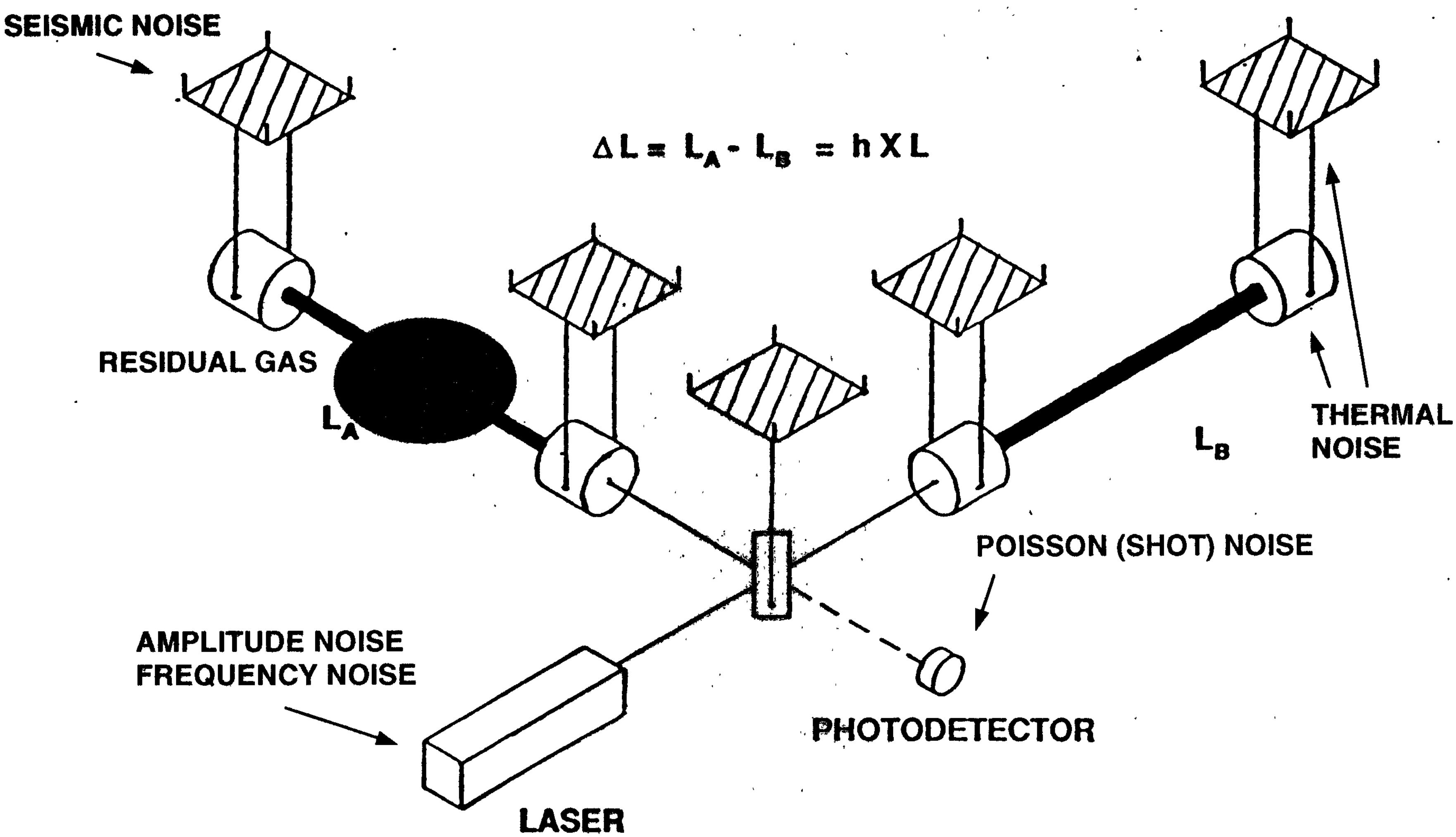
Limitations: types of noise

Sensitivity to GW depends on conversion of *h* to ϕ, *and* on ability to measure ϕ

Noise sources:

- natural division into
  - sensing noise (generally optical sources)
  - displacement noise (forces on test masses)
- fundamental vs. technical
- statistical characteristics:
  - stationary noise
  - quasi-stationary noise, changes on time scales of seconds
  - impulsive noise: time scale of GW events or shorter
  - periodic 'noise'
- length scaling
- frequency dependence

SCHEMATIC INTERFEROMETRIC DETECTOR



"DISPLACEMENT" NOISE

"READOUT" NOISE

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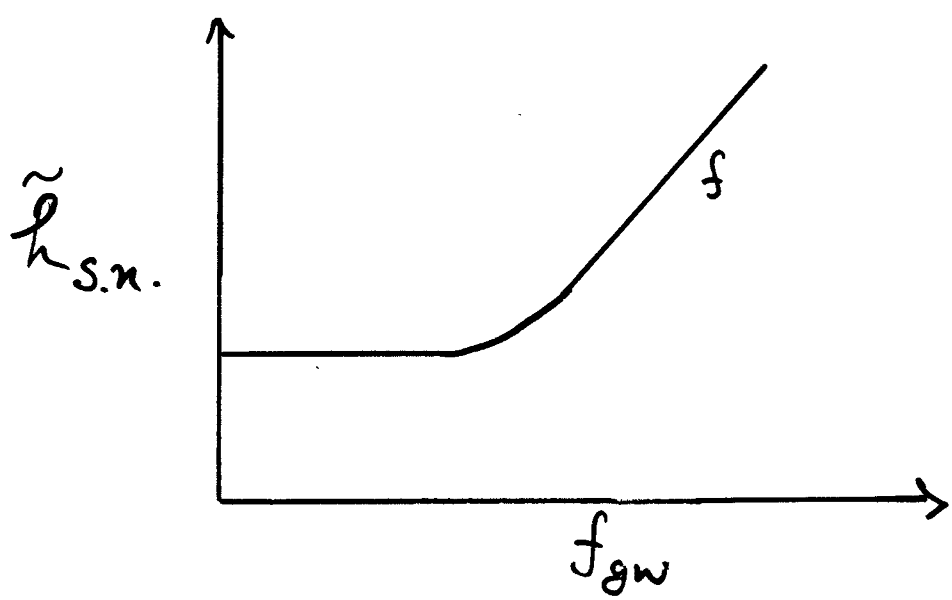
Fundamental limits

Shot or Poisson noise

- intensity at ifo output is a function of arm phase difference:

$$P_{out} = \frac{P_{in}}{2}(1 - \cos[\delta\varphi])$$

- maximum slope: $\frac{dP}{d\delta\varphi} = \frac{P_{in}}{2}$
- uncertainty in intensity due to counting statistics: $\delta\tilde{P}_{out} = \sqrt{2h\nu P}$
- can solve for equivalent strain: $h_{shot} = \frac{1}{|\delta\varphi/h|}\sqrt{\frac{2h\nu}{P_{in}}}$
- Note: scaling with $1/\sqrt{P_{in}}$; gives requirement for laser power



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Quantum noise

Radiation Pressure

- quantum-limited intensity fluctuations anti-correlated in two arms
  - results from vacuum fluctuations entering output port
- photons exert a time varying force, with spectral density

$$\tilde{f} = \sqrt{\frac{2\pi h P_{in}}{c\lambda}}$$

- results in opposite displacements of EACH of the masses:

$$\tilde{x}(f) = \frac{1}{mf^2}\sqrt{\frac{hP_{in}}{8\pi^3 c\lambda}}$$, or strain $$h = \frac{\delta l}{l} = \frac{2\tilde{x}}{L}$$

- NOTE: scaling with $\sqrt{P_{in}}$, and with interferometer arm length L

total readout, or quantum noise

- quadrature sum $h_q = \left(h^2_{shot} + h^2_{rad\ press}\right)^{1/2}$
- frequency dependence according to ifo configuration, but
- always a minimum for a given frequency as a function of Power
- for simple Michelson, $P_{opt} = \pi c \lambda m f^2$; later limitation, not now

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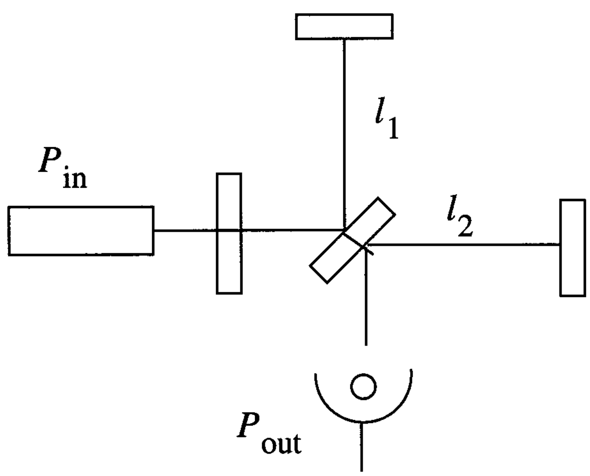
Lowering shot noise: recycling

Problem: insufficient laser power

- sensitivity goal requires shot noise from ~100 W on beamsplitter
- suitable lasers produce ~10 W, ~5W at the ifo input

Solution: make a resonant cavity out of the interferometer and an additional mirror at the input



- ifo output port is operated at the 'dark fringe'
- then nearly all the input power is reflected back towards laser (when optical losses are low)
- factor or ~30 increase in effective power possible
- improves the shot noise limited sensitivity by $\sqrt{\text{gain}} \approx$ 5-6

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Displacement noise

Fundamental

- Thermal noise: mechanical systems excited by thermal environment

  ›› nearly all of each mode's kT's worth of thermal energy is at the resonant peak, but small fraction is distributed in frequency

  ›› lower mechanical loss → lower thermal motion out-of-resonance

  ›› Below resonance: internal modes of test masses

  ›› Above resonance: pendulum suspension

- Gravity gradients

  ›› time varying mass distributions in the vicinity of the test masses produce fluctuating forces on them

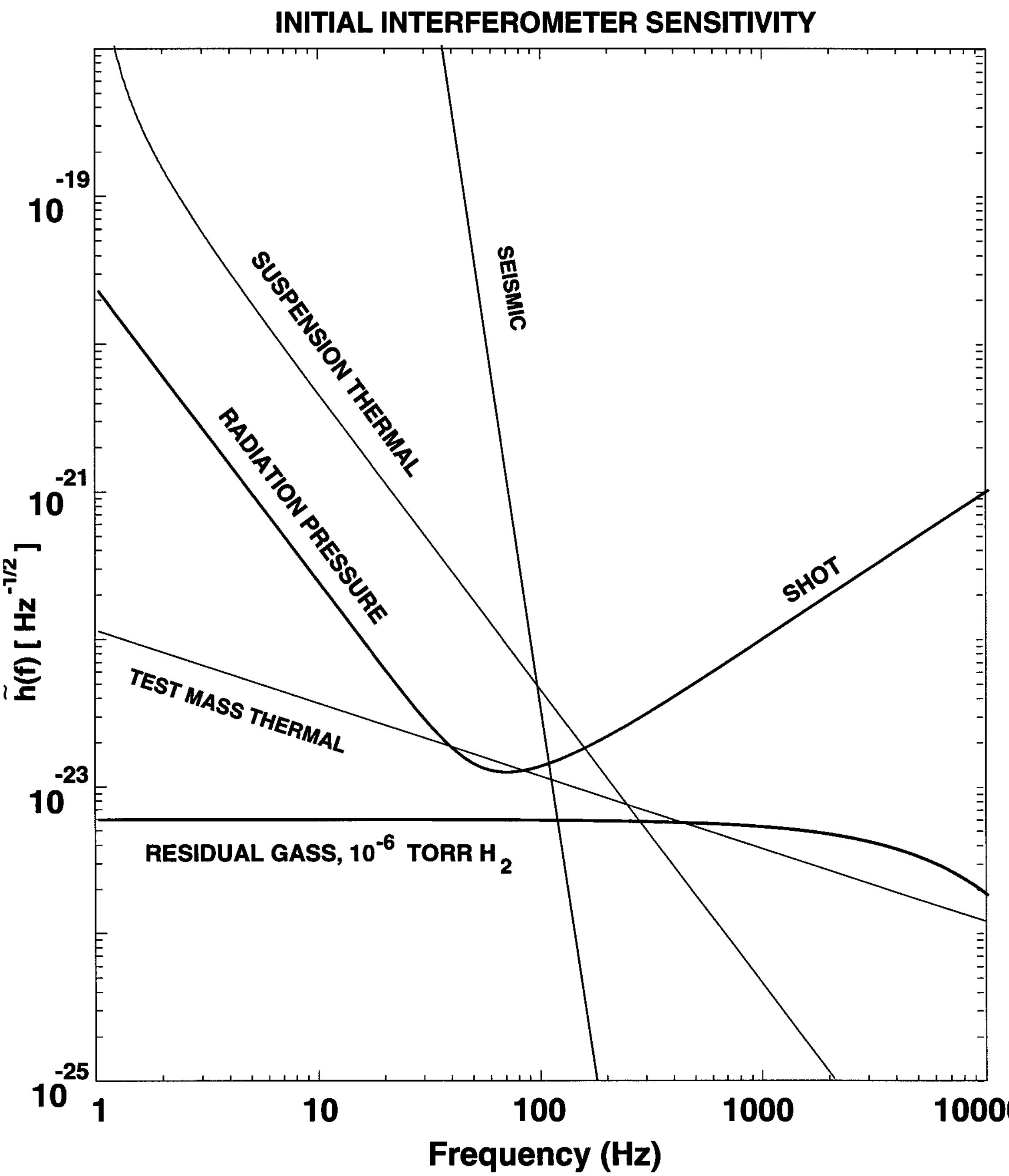
  ›› surface seismic compressional waves, weather, moving objects (humans!)

  ›› places limit on lowest frequencies detectable by ground based detectors of a few Hertz

Both these noise sources scale with arm length L

Thermal noise leads to LIGO 4km length

Initial LIGO sensitivity



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