The Detection of Gravitational Waves

Academic Lecture Series

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CERN

May 20-24, 1996

LIGO

LIGO-G960139-00-M

Lecture 1

B. Barish

LIGO
Introduction

- Laser Interferometer Gravitational Wave Observatory
  - » **DIRECT** Detection of Gravitational Waves
- Joint Caltech/MIT Project funded by the National Science Foundation
- Under Construction
  - » Two Sites -- Louisiana and Washington

GENERAL RELATIVITY + GRAVITATIONAL WAVES

- General Relativity 'fixes' the problem posed by moving sources of gravitational field.

⇓

- gravitational field (eg curvature of space time)

  does not change instantaneously at arbitrary distances from moving source.

- Analogous to E.M. the 'news' travels at speed of light.

WAVES IN GENERAL RELATIVITY

- Space-time interval ds between points

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

OR

SPECIAL THEORY OF RELATIVITY

$$ds^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu$$

with Minkowski metric

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

μ, ν indices over t, x, y, z

- Same physical concept carried over to General Theory of Relativity

except,

spacetime no longer necessarily flat

- general definition of space-time interval

$$\boxed{ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu}$$

$g_{\mu\nu}$ ← space time curvature in this metric.

- for our purpose, we only need special case of a small perturbation to flat space-time.

$$\boxed{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}}$$

$h_{\mu\nu}$ ← metric perturbation away from Minkowski space.

  - Key physics in $h_{\mu\nu}$
  - In weak field limit, non-linear Einstein equations can be approximated as linear equations

- useful gauge is "**transverse traceless gauge**" [TT gauge]
- in this gauge, coordinates are marked by world lines of free falling test masses
- with this choice, weak field limit of Einstein's field equation becomes a wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right) h_{\mu\nu} = 0$$

- $h_{\mu\nu}$ can take form of plane wave propogating in direction $\hat{k}$ with speed c.

$$h\left(2\pi f t - \vec{k}\cdot\vec{x}\right) \quad \text{with } f = |\vec{k}|/2\pi c$$

Note: speed c due to way space-time brought together in relativity.

- Consider the wave propogating along the z axis

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

transverse and traceless –

∴ can write as sum of two components

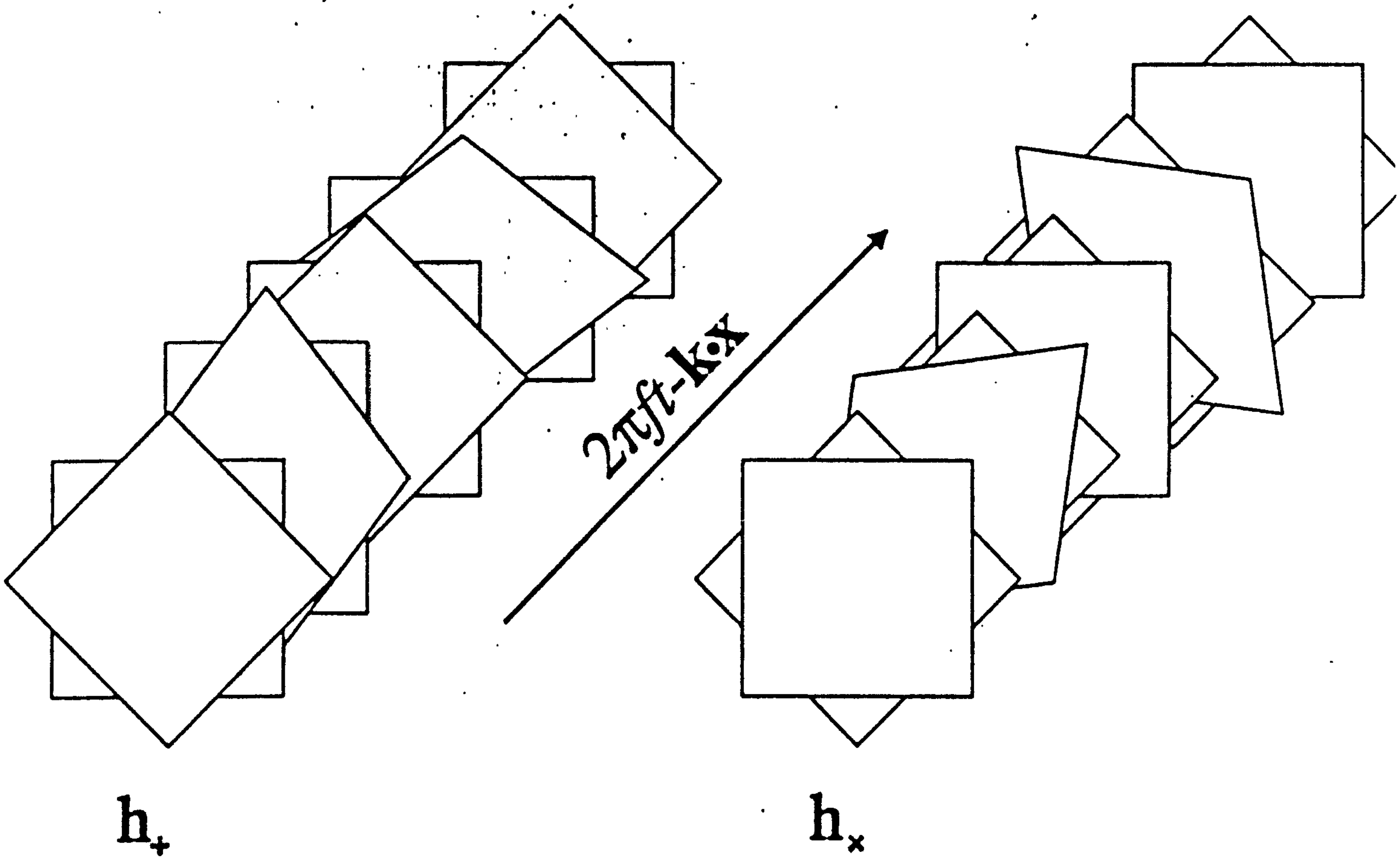
$$h = a\,\hat{h}_+ + b\,\hat{h}_\times$$

two orthoganal polarizations (45°)

$$\hat{h}_+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad h_\times = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Gravitational Waves

Two Polarizations



LIGO

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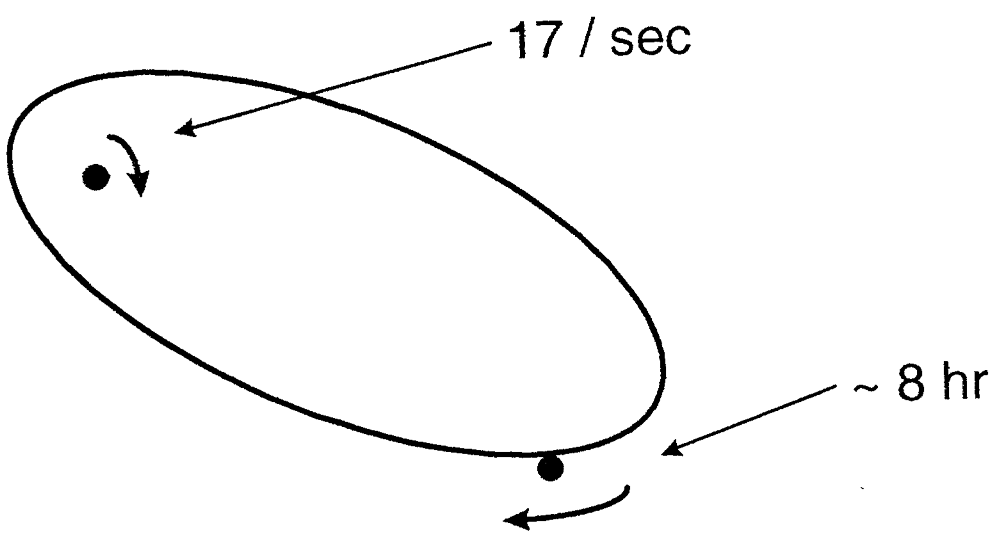
Gravitational vs E.M. Waves

	EM WAVES	GRAV. WAVES
Nature	Oscillation of EM Fields Propagating Through Spacetime	Oscillations of the "fabric" of spacetime
Emission Mechanism	Incoherent superposition of waves from molecules, atoms, particles	Coherent emission by bulk motion of energy
Interaction with Matter	Strong absorption and Scattering	Essentially None!
Frequency Band	$f > 10^7$ Hz	$f < 10^4$ Hz

- Implications
  - Most gravitational sources not seen as electromagnetic (and vice versa)
  - Potential for great surprises
  - Uncertainty in strengths of waves

Gravitational Waves
Evidence

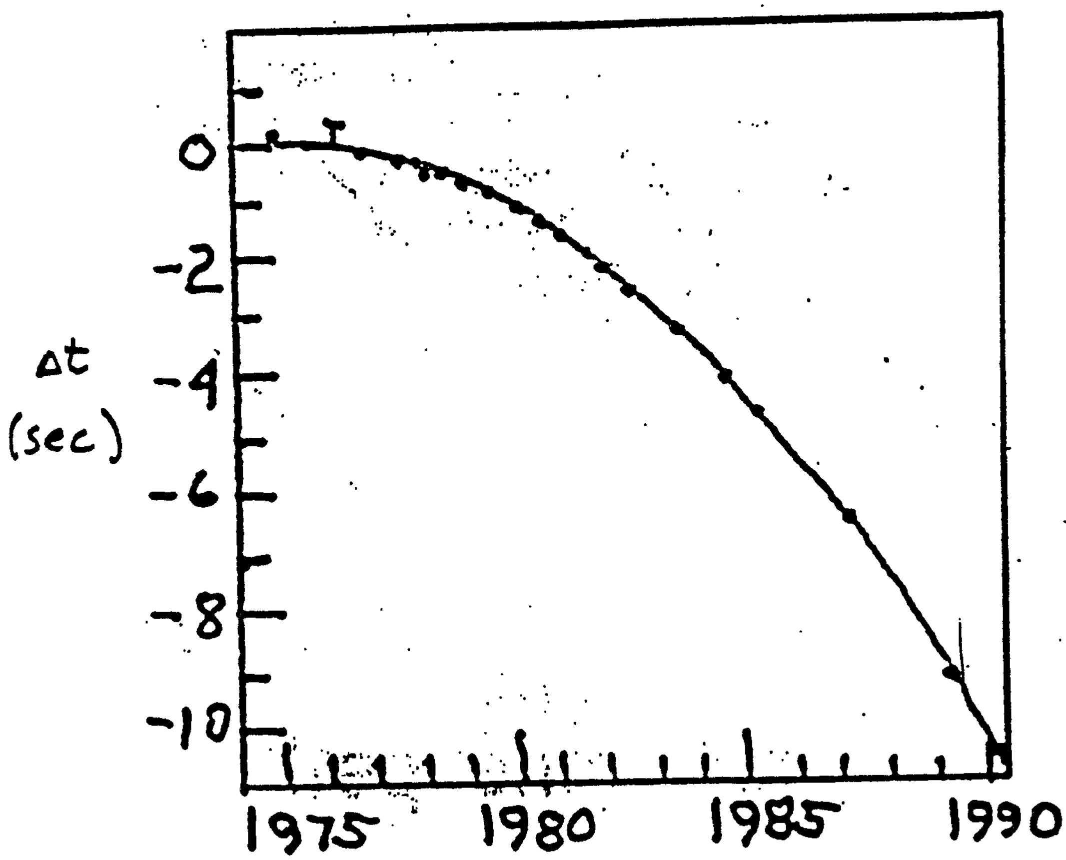
- **Russell Hulse and Joseph Taylor**
- Neutron Binary System
  - » PSR 1913 + 16 -- Timing of Pulsars



Hulse and Taylor

Timing of Orbit

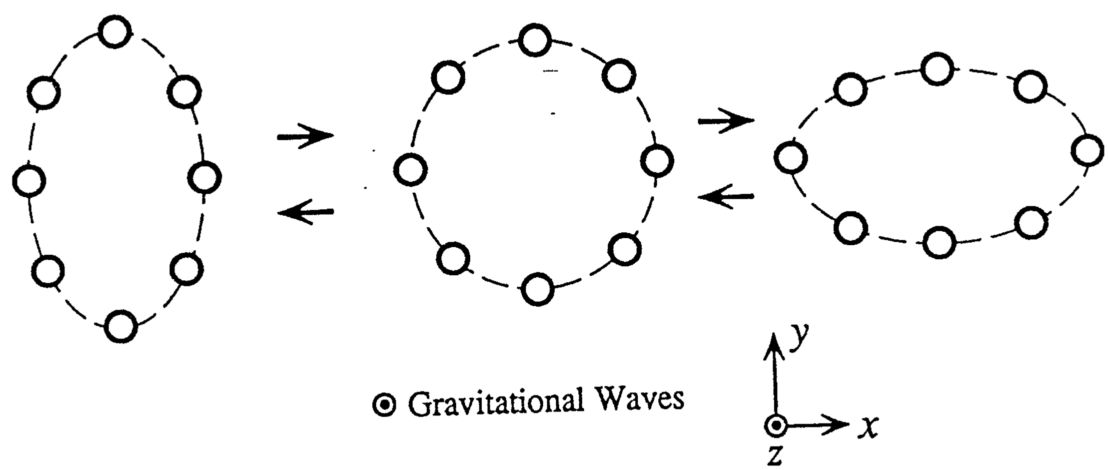
- Speed up 10 sec in 15 years
  - » measured to ~50 µsec accuracy
- Deviation grows quadratically in time



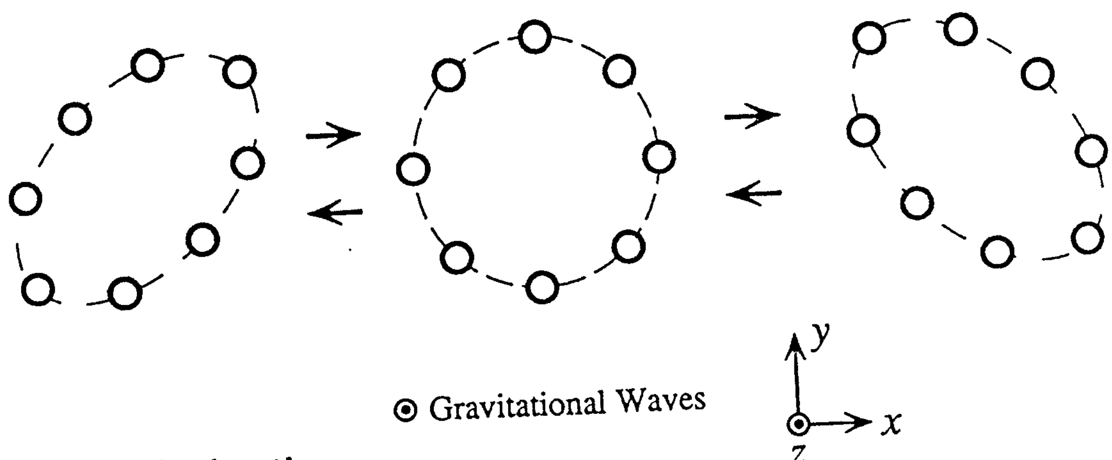
- Due to loss of orbital energy, from emission of gravitational waves

Gravitational Waves
Effects

- Displacement of free particles



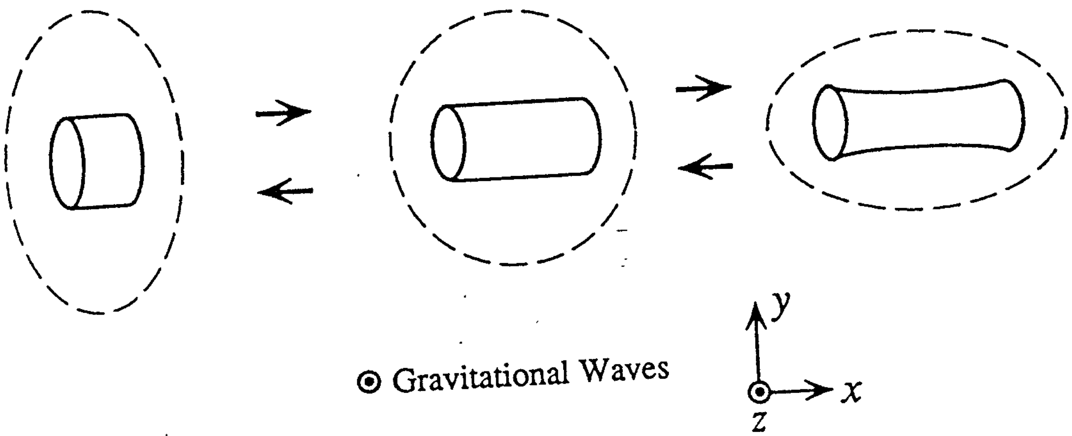
» h_+ polarization



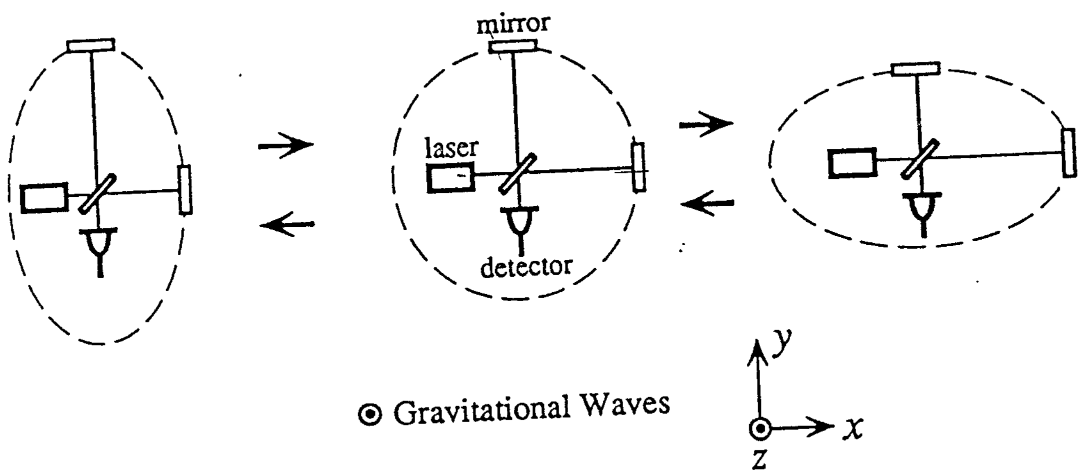
» h_x polarization

Gravitational Waves

Detection



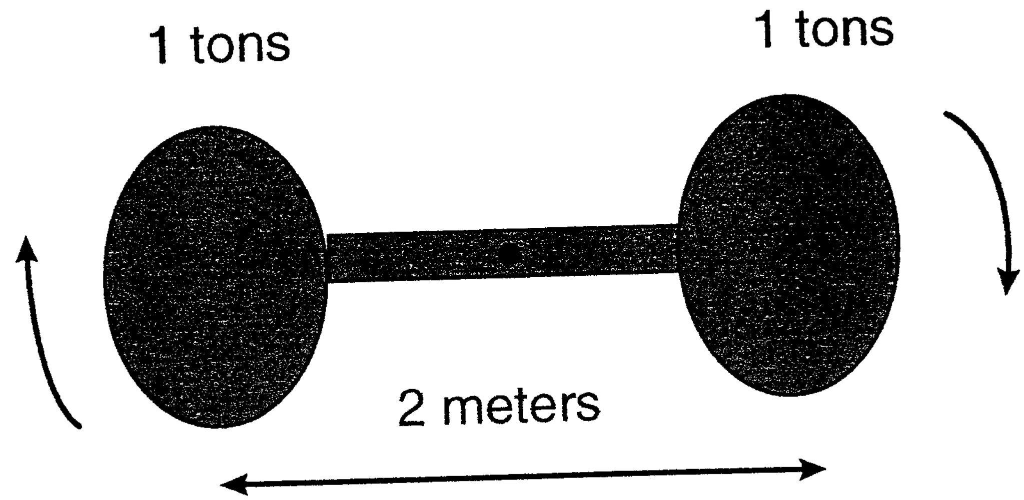
- Bar detector



- Interferometer detector

Laboratory Experiment
(a la Hertz)

Laboratory Dumbbell System



$f_{rot} = 1$ kHz

$h_{lab} = 2.6\ 10^{-33}$ m x 1/R

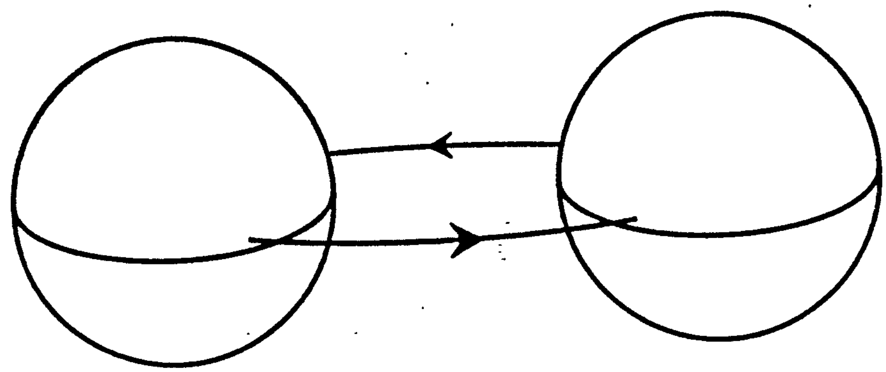
R = detector distance (> 1 wavelength) = 300 km

$\mathbf{h_{lab} = 9\ 10^{-39}}$

This is too weak by about 16 orders of magnitude!

Gravitational Waves

Sources and Detection



- binary star system

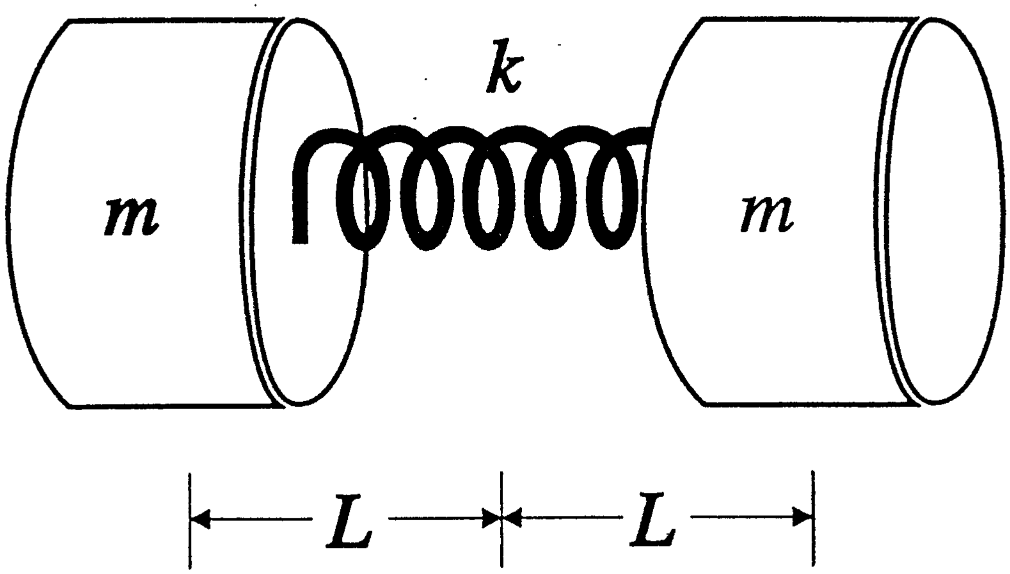
Sources	Frequency	h	Event Rate	Detection
Coalescing Binary Neutron Stars (200 Mpc)	10~1000 Hz	10^{-22}	~3/year	Interferometer +Template
Supernovae (in our Galaxy)	~1 kHz	10^{-18}	~3/century	Interferometer, Resonant
Supernovae (in Virgo)	~1 kHz	10^{-21}	several/year	Interferometer
Generation of Large Black Holes	~1 mHz	10^{-17}	1/year	Interferometer in Space
Pulsars	10~1000 Hz	10^{-25}	periodic	Interferometer, Resonant
Cosmic Strings	10^{-7} Hz	10^{-15}	stochastic	Pulsar Timing

- sources and detection

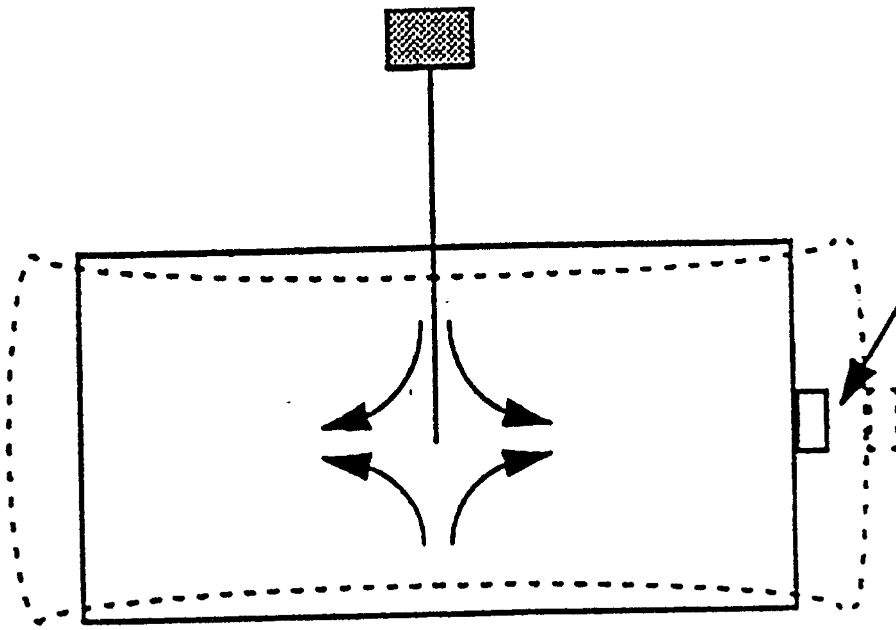
Gravitational Waves

Resonant Bar Detector

- Schematic Version



Gravitational Waves
Resonant Bar Detection



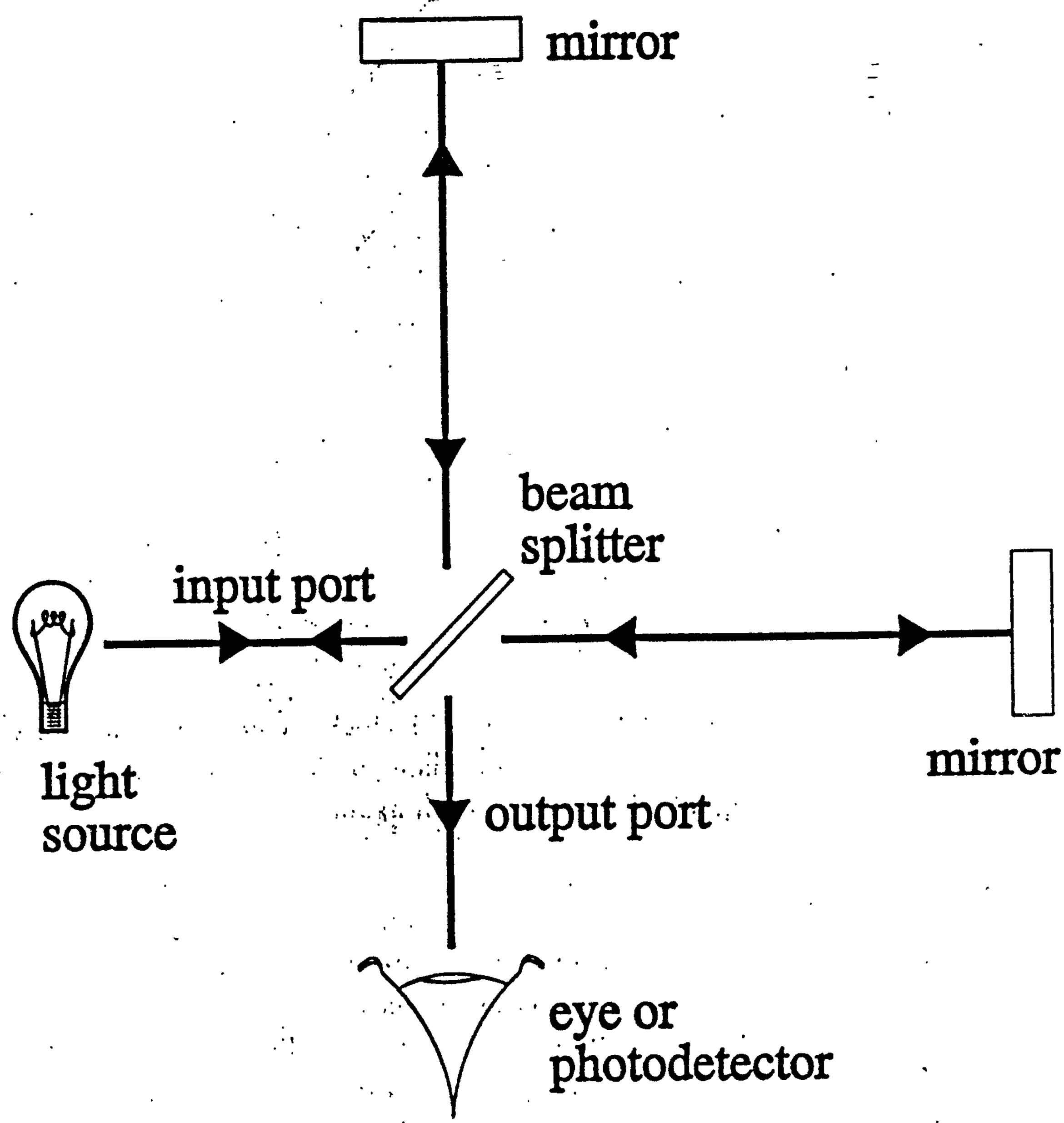
- Bar detector

Group	Antenna	Transducer	Sensitivity (h)
CERN/Rome	Al5056, 2.3ton, 2.6K	Capacitive+SQUID	7×10^{-19}
CERN	Al5056, 2.3ton, 0.1K	Capacitive+SQUID	2×10^{-18}
LSU(USA)	Al5056, 1.1ton, 4.2K	Inductive+SQUID	7×10^{-19}
Stanford	Al6061, 4.8ton, 4.2K	Inductive+SQUID	10^{-18}
UWA(Australia)	Nb, 1.5ton, 5K	RF cavity	9×10^{-19}
ICRR(Japan)	Al5056, 1.7ton, 300K	Laser Transducer	–
KEK(Japan)	Al5056, 1.2ton, 4.2K	Capacitive+FET	4×10^{-22} (60Hz)

- Status of bar detectors

Michelson Interferometer
Schematic Diagram

- Michelson Morley Experiment



The Michelson-Morley Experiment

- (1887) Michelson Interferometer
- Detect an apparent shift in speed of light due to Earth's motion through the 'ether'. (predated theory of relativity)
- **Experiment** –
  - input light $E_0 e^{i(2\pi f t - kx)}$
  - 50/50 beam splitter $r = 1/\sqrt{2}$ $t = i/\sqrt{2}$
  - x axis light $i\left(E_0/\sqrt{2}\right) e^{i(2\pi f t - k_x x)}$
  - y axis light $\left(E_0/\sqrt{2}\right) e^{i(2\pi f t - k_y y)}$

k_x, k_y allow possible different speed of light in the two arms

- Reflection at far mirrors $(\times -1)$ ← multiplies waves
- Light exiting thru output port

$$E_{out} = \left(\frac{i}{2}\right) E_0 e^{i(2\pi f t - 2k_x L_x)} + \left(\frac{i}{2}\right) E_0 e^{i(2\pi f t - 2k_y L_y)}$$

$$E_{out} = i e^{i(2\pi f t - k_x L_x - k_y L_y)} E_0 \cos(k_x L_x - k_y L_y)$$

also

$$E_{refl} = i e^{i(2\pi f t - k_x L_x - k_y L_y)} E_0 \sin(k_x L_x - k_y L_y)$$

(toward lamp)

- Light leaving depends on difference of phase accumulated by light travelling in two arms

Power $\propto E^2 \Rightarrow$

$$P_{out} = \frac{P_{in}}{2} \left(1 + \cos 2(k_x L_x - k_y L_y)\right)$$

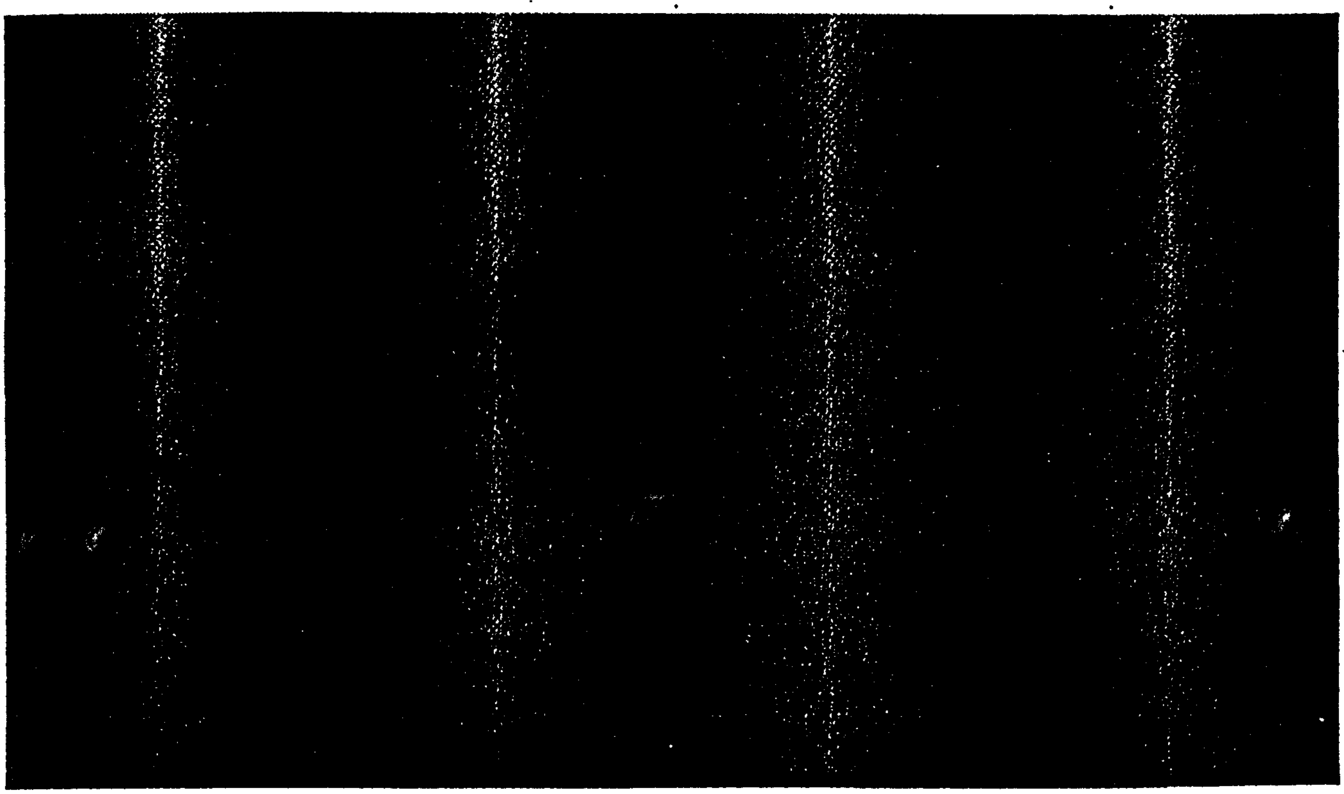
$$P_{out} + P_{refl} = P_{in}$$

- Measure difference in brightness (modern)

Actual – slightly mis-aligned

Michelson Interferometer
Interference Fringes

- Michelson Morley Experiment
  - » Two beams misaligned
- Impressionistic rendering



- Settings arms to same length?
  (hard, but 'coherence' length)

- **sensitivity** of Michelson-Morley

$L_{opt} = 22$ meters (extra mirrors)
Measured shift in fringe $\lambda/20$
$\Downarrow$
$\Delta t = 8 \; 10^{-17}$ sec.

- Limitation – external vibrations
  (despite mirrors on massive stone slab in pool of mercury)

Lecture 2

B. Barish

Gravitational Wave Detection

- **'gedanken' version of Michelson (1887)**
  - mirrors rest on freely-falling mass
  - 50/50 splitter
  - far end flat mirror

- **History**

(idea) - Gertsenshtein and Pustovoit (1962)

- Weber & Forward 1960's (unpublished)

(1st IFO) - Moss, Miller + Forward (1971)

(LIGO) - Weiss (1972)

Gravitational Wave Signal

- Consider light along $\hat{x}$ axis

$$ds^2 = 0 = g_{\mu\nu}\, dx^\mu dx^\nu$$
$$= (\eta_{\mu\nu} + h_{\mu\nu})\, dx^\mu dx^\nu$$
$$= -c^2 dt^2 + \left(1 + h_{11}(2\pi f t - \vec{k}\cdot\vec{x})\right) dx^2$$

(neighboring space-time events)

time - beam splitter to end of $\hat{x}$ arm

$$\int_0^{\tau_{out}} dt = \frac{1}{c}\int_0^L \sqrt{1 + h_{11}}\, dx \approx \frac{1}{c}\int_0^L \left(1 + \frac{1}{2} h_{11}(2\pi f t - \vec{k}\cdot\vec{x})\right) dx$$

$\Uparrow$

$h \ll 1$ binomial expansion

Round trip →

$$\tau_{rt} = \frac{2L}{c} + \frac{1}{2c}\int_0^L h_{11}(2\pi f t - \vec{k}\cdot\vec{x})\, dx - \frac{1}{2c}\int_L^0 h_{11}(2\pi f t - \vec{k}\cdot\vec{x})\, dx$$

similar for y-arm.

Consider special case,

- sinsusoidal wave in + polarizarition
- frequency = f_{gw}
- amplitude $h_{11} = -h_{22} = h$

If $2\pi f_{gw} \tau_{rt} \ll 1$ can treat the metric perturbation as approximately constant during time wavefront is present in the apparatus

- equal and opposite perturbations to light travel time in two arms
- total travel time difference

$$\boxed{\Delta\tau(t) = h(t)\frac{2L}{c} = h(t)\,\tau_{rt0}}$$

where $\tau_{rt0} \equiv \frac{2L}{c}$.

Comparing travel time to (reduced) period of oscillation of the light gives phase shift

$$\Delta \phi(t) = h(t)\, \tau_{rto} \frac{2\pi c}{\lambda}$$

In words, phase shift between light traveled in the two arms equals a fraction h of the total phase a light beam accumulates as it traverses the apparatus.

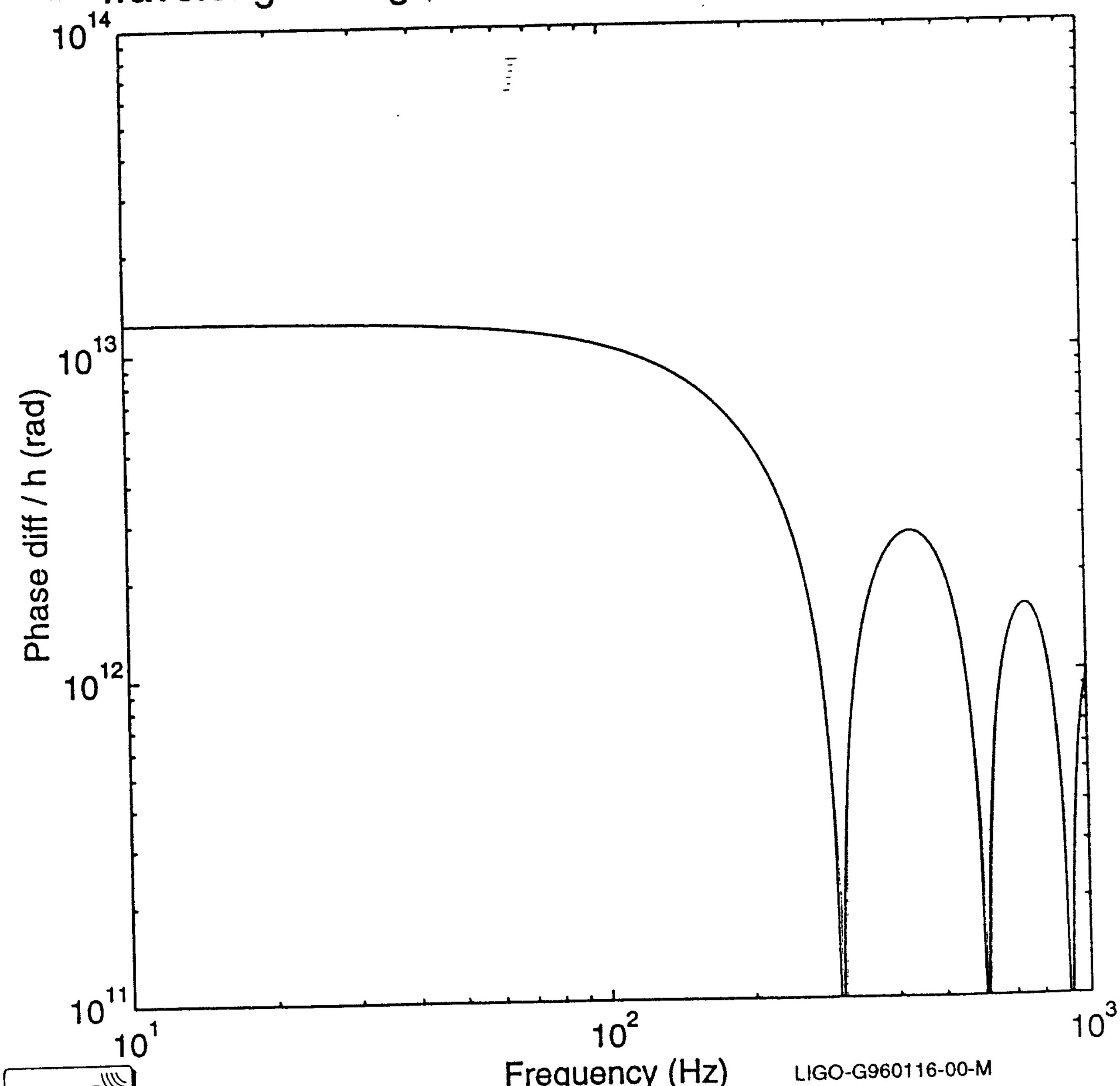
Scaling law won't hold for arbitrarily long arms. (e.g. $2\pi f_{gw} \tau_{rt} \ll 1$ no longer holds)

NO NET MODULATION IF

$$f_{gw} \tau_{rt} = 1$$

Michelson Interferometer
Transfer function

- Example
  - » mirrors are defined at 500 km from beam splitter
  - » wavelength of light = 0.5 microns



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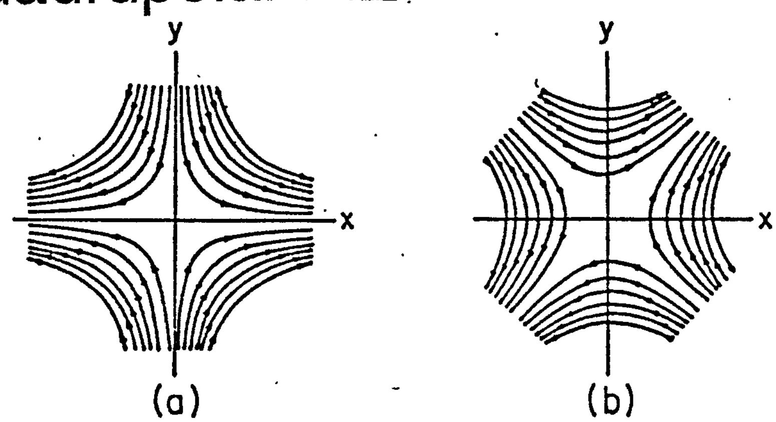
Gravitational Wave
Forces

IF

- Detector Size << Wavelength
  (4 km) (300-30,000km)
  (10 kHz - 10 Hz LIGO)

THEN

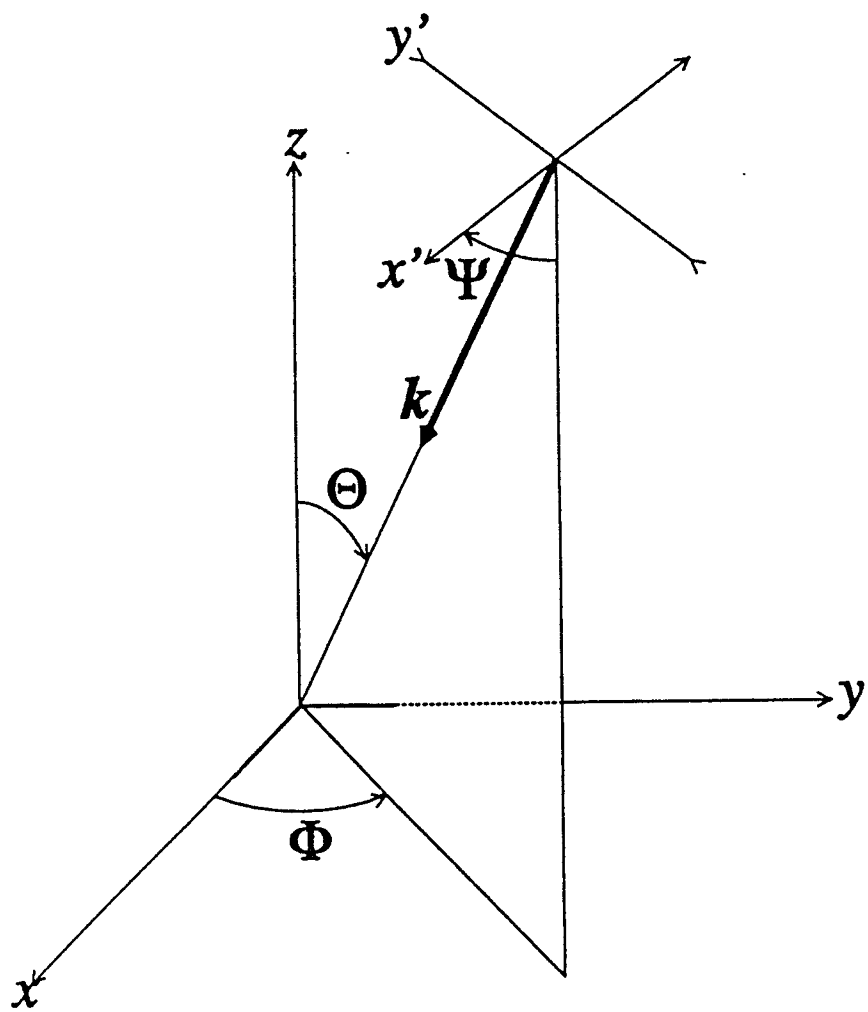
- Free Masses
- Quadrupolar Lines of Force



+ Polarization x Polarization

Gravitational Wave Detector

- Antenna Pattern
  - » coordinate system



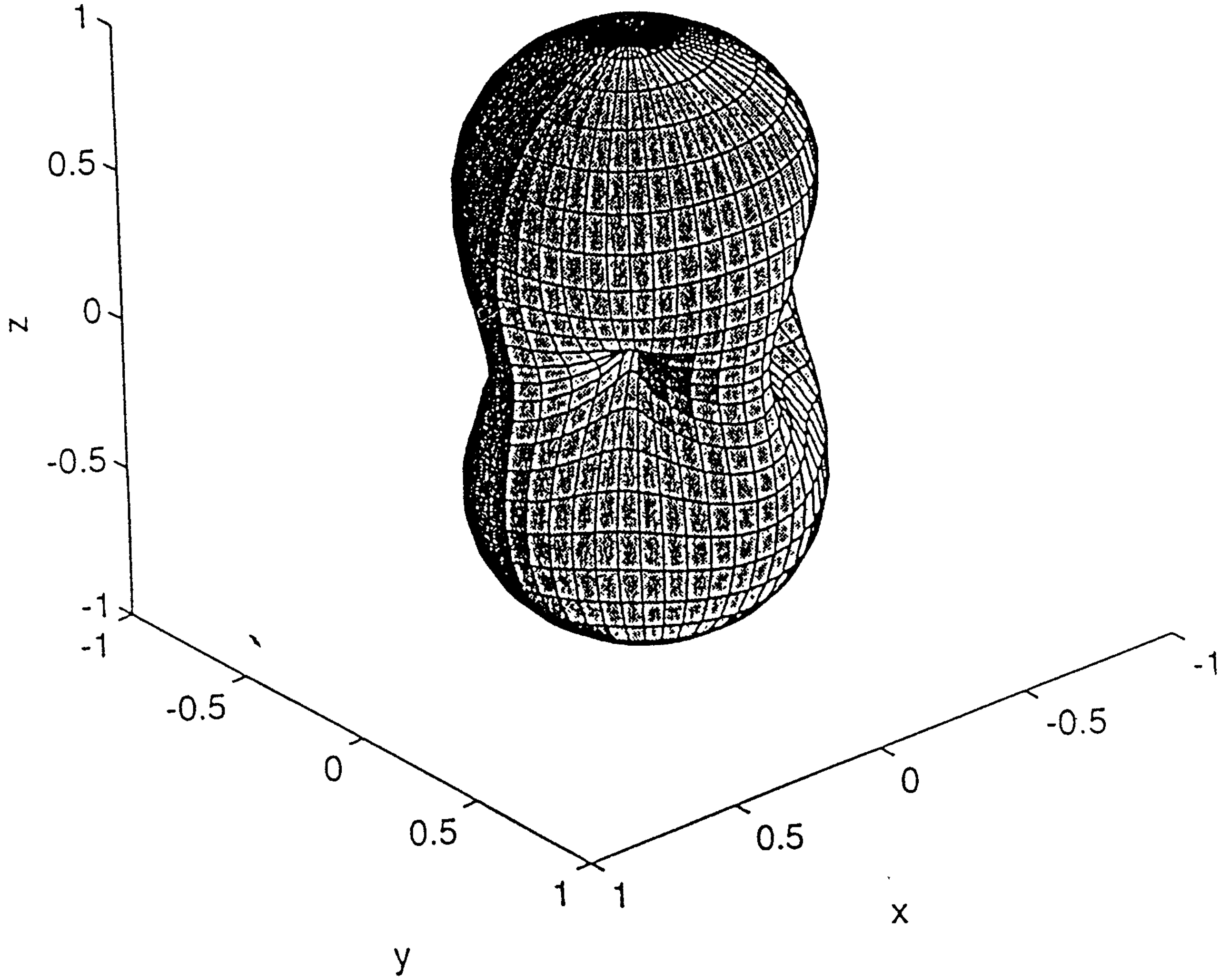
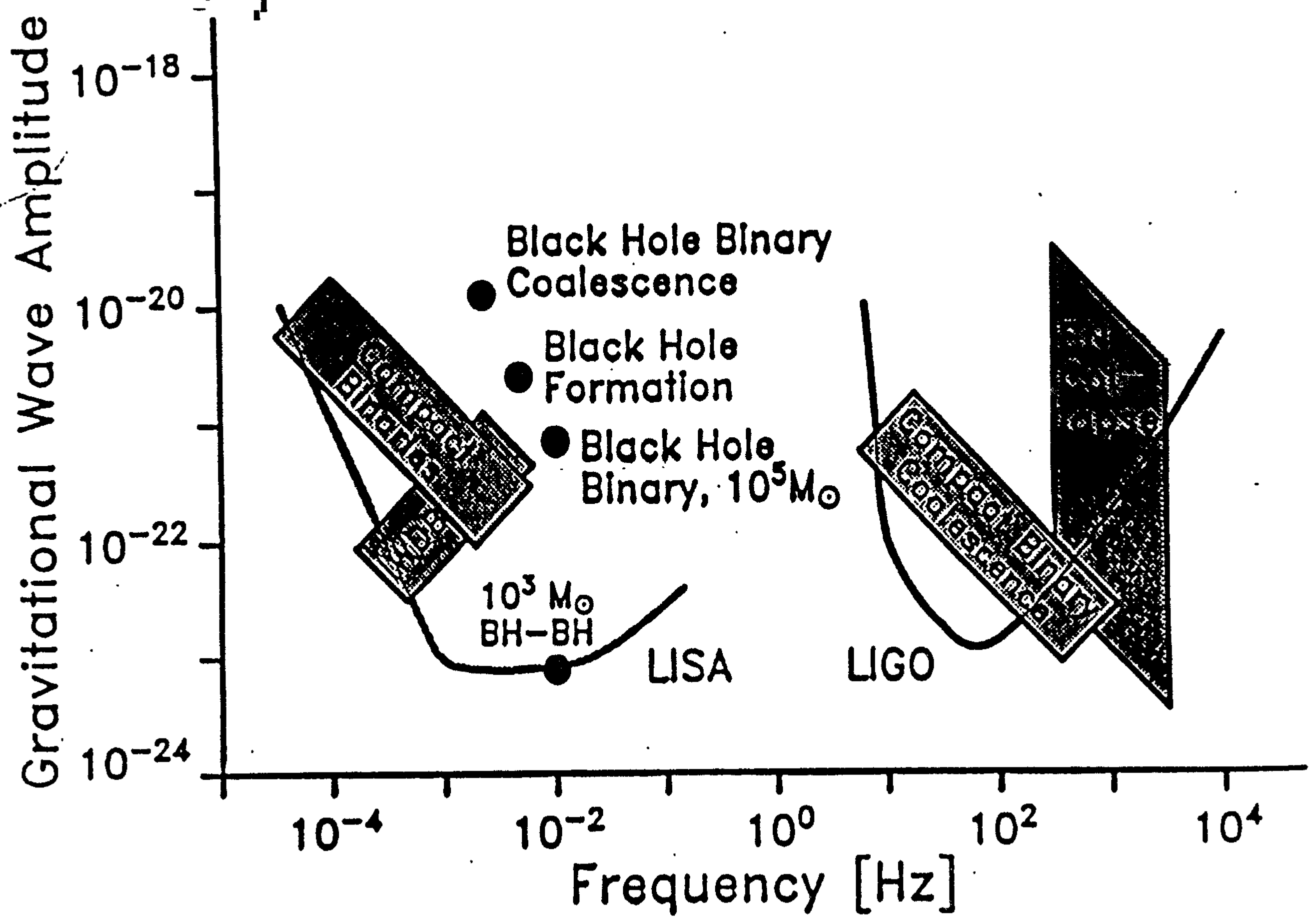


Figure 2.7 The sensitivity, as a function of direction, of an interferometric gravitational wave detector to unpolarized gravitational waves. The interferometer arms are oriented along the x and y axes.

Astrophysical Sources

Frequency Range

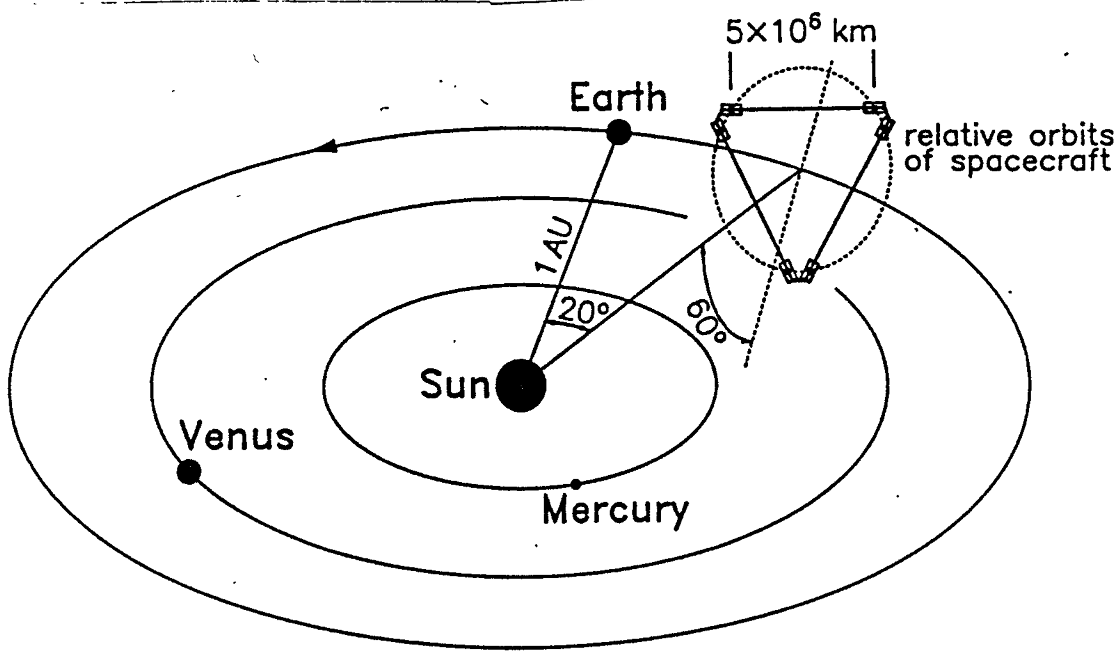
- Electromagnetic Waves - ~ 20 orders of magnnitude (ULF radio -> HE γ rays)
- Gravitational Waves - ~ 10 orders of magnitude
- Combination of terrestrial and space experiments



Gravitational Waves

Space Experiment

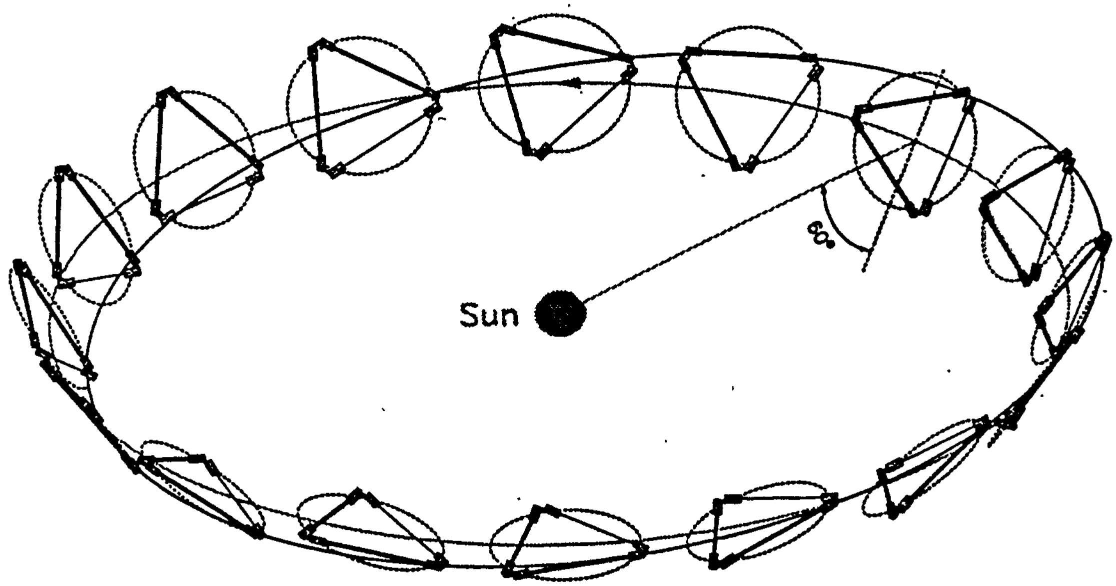
- LISA - Laser Interferometer Space Antenna
  - » six spacecraft in triangle (four needed)
  - » pair at each vertex



LISA

Annual Revolution

- 60 degree half opening angle
- 'tumbling' allows determination of position of source and polarization of wave



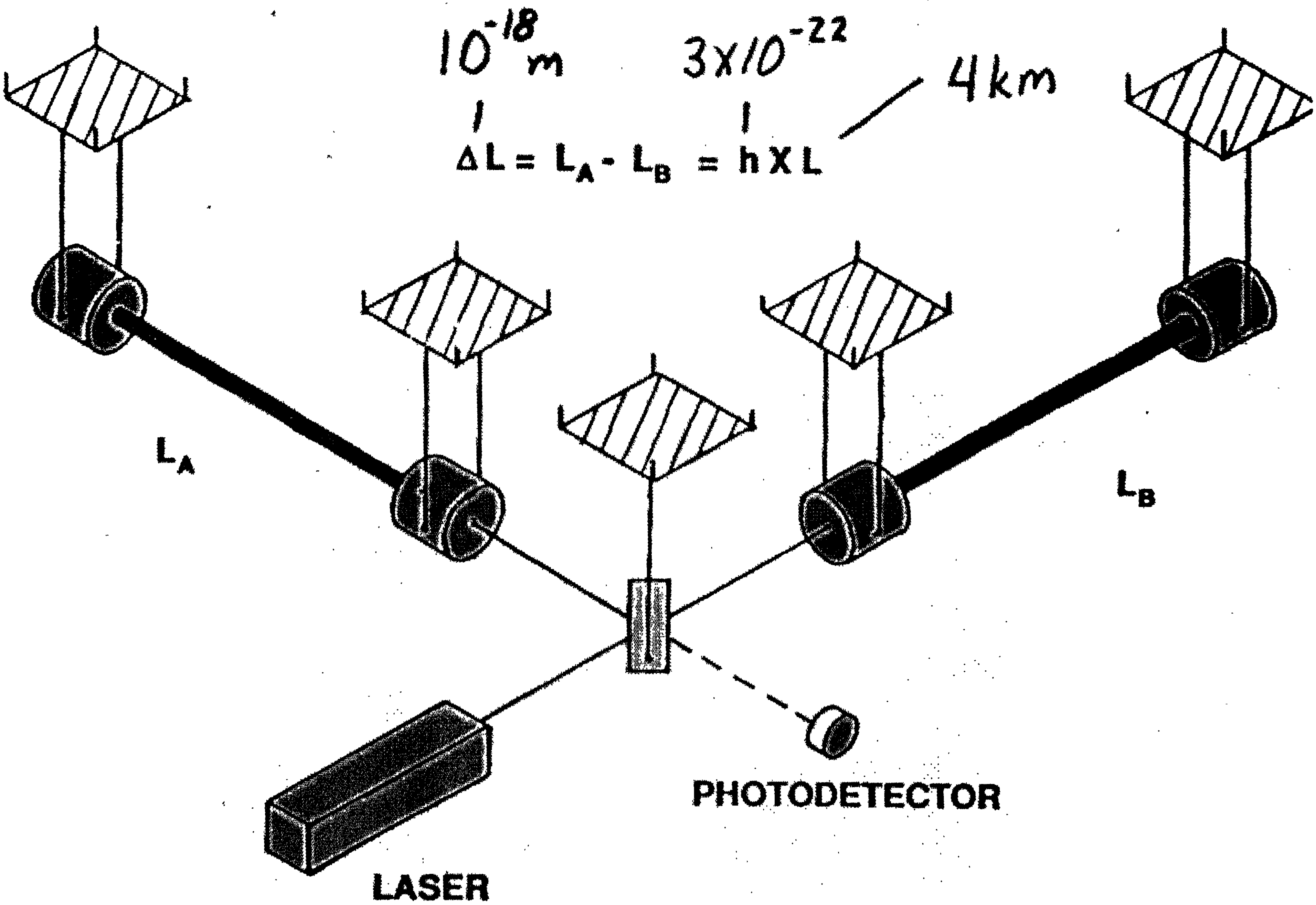
Gravitational Waves
International Effort

- Techniques
  - » Resonant Bar Detectors (LSU, Rome, etc)
    - – narrow band
  - » Large Scale Interferometers
    - – broad band

- International Interferometer Effort
  - » U.S. -- LIGO (Two Sites)
    - – Caltech & MIT (Wash and Louisiana)
  - » Europe -- VIRGO (One Site)
    - – French and Italian (near Pisa)
  - » Smaller efforts
    - – Germany, Japan, Australia

- Time Scale (Interferometers)
  - » Approximately year 2000

SCHEMATIC INTERFEROMETRIC DETECTOR



LIGO

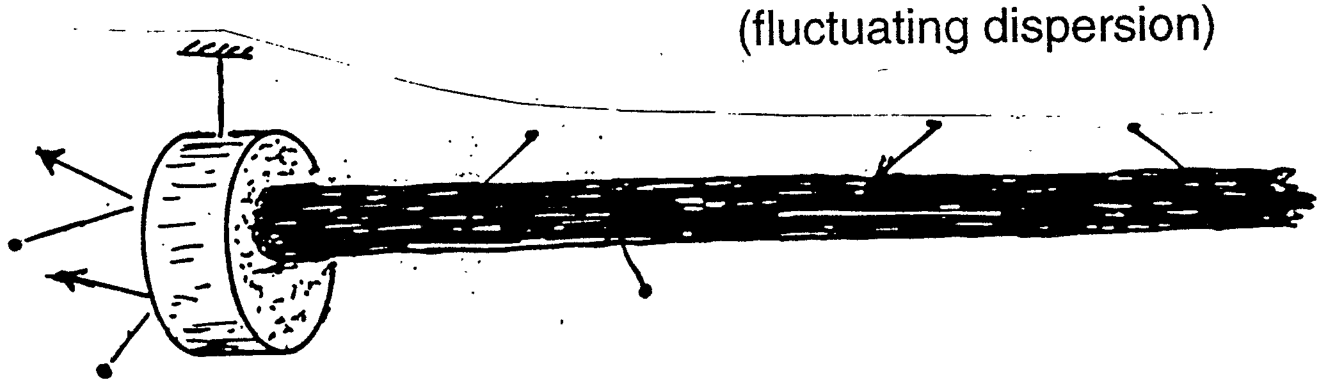
Achieving 10^{-18} m Sensitivity

How is it possible????

- Air molecules:

  Buffer mirrors

  Buffet light beam (fluctuating dispersion)



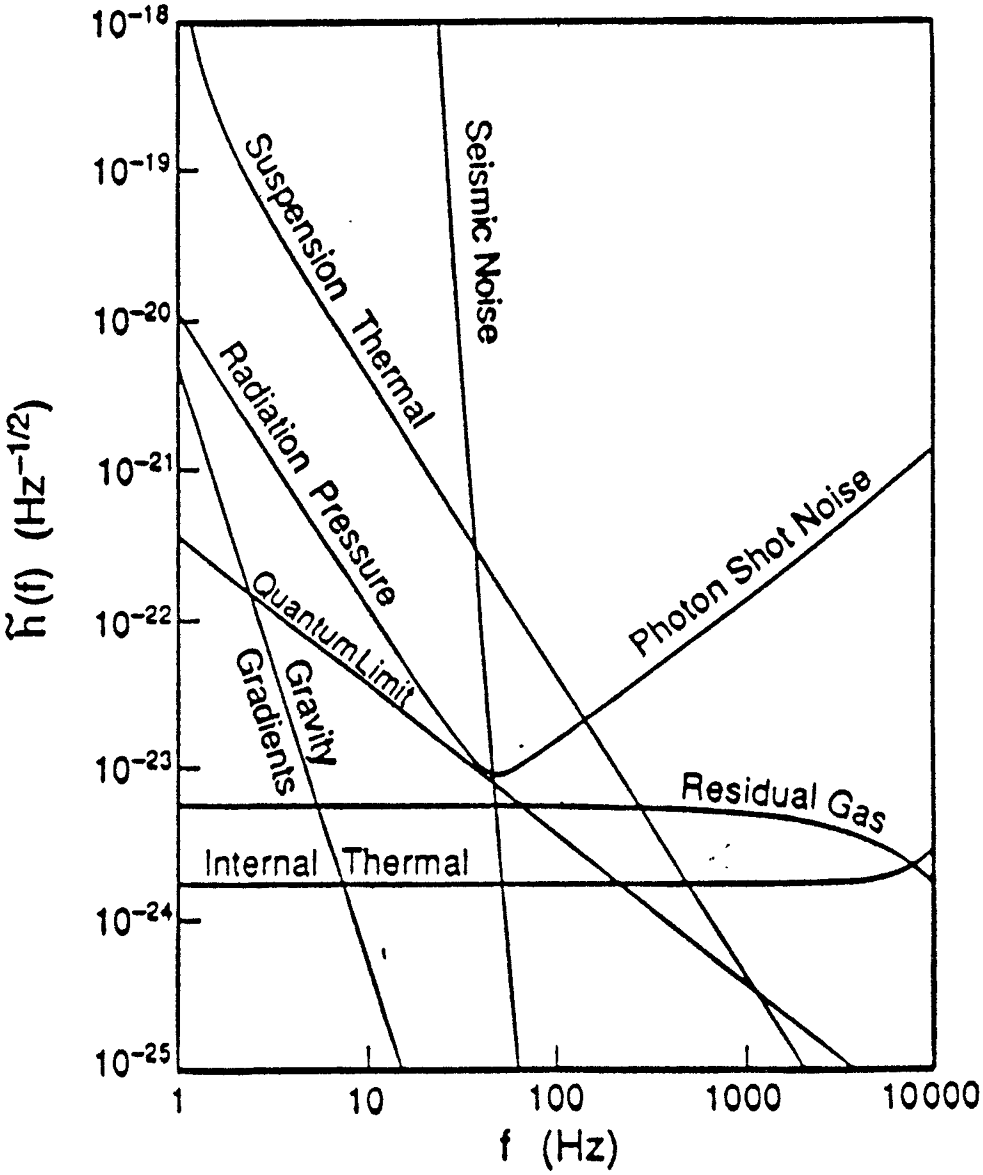
  » **Mirrors and light beam must be in vacuum**

- Mirror's atoms vibrate (thermal noise)
  - » light beam feels 10^{18} atoms
  - » atoms vibrate fast: ~10^{13} Hz
  - » beam measures slow variables: ~ 100 Hz

- Earth vibrates and shakes mirrors
  - » **anti-vibration suspension**
  - » **quiet environment**

Noise Budget For First LIGO Detectors

- 5 Watt Laser
- Mirror Losses 50 ppm
- Recycling Factor of 30
- 10 kg Test Masses
- Suspension Q=10^7



LIGO

Gravitational Wave Generation

- analogous to EM waves
- expressed in terms of retarded potential
- simplest to work with approximation
  - multiple expansion
  - ok if $r_{source}/\lambda \ll 1$

(size of source much smaller than wavelength)

- EM ⇒ radiation field from time variation of electric dipole moment

$$\vec{E} = \frac{1}{Rc^2}\left(\ddot{d} \times \hat{n} \right) \times \hat{n})$$

R = distance from source to observer

$\hat{n}$ = unit vector source to observer

$\vec{d}$ = electric dipole moment

$$\vec{d} = \int dV \, \rho_q(r) \, r$$

ρ_q = charge density

integrate over source

Next term,

magnetic dipole

electric quadrupole

(weaker by r_{source}/λ)