Thermal Compensation in Stable Recycling Cavity

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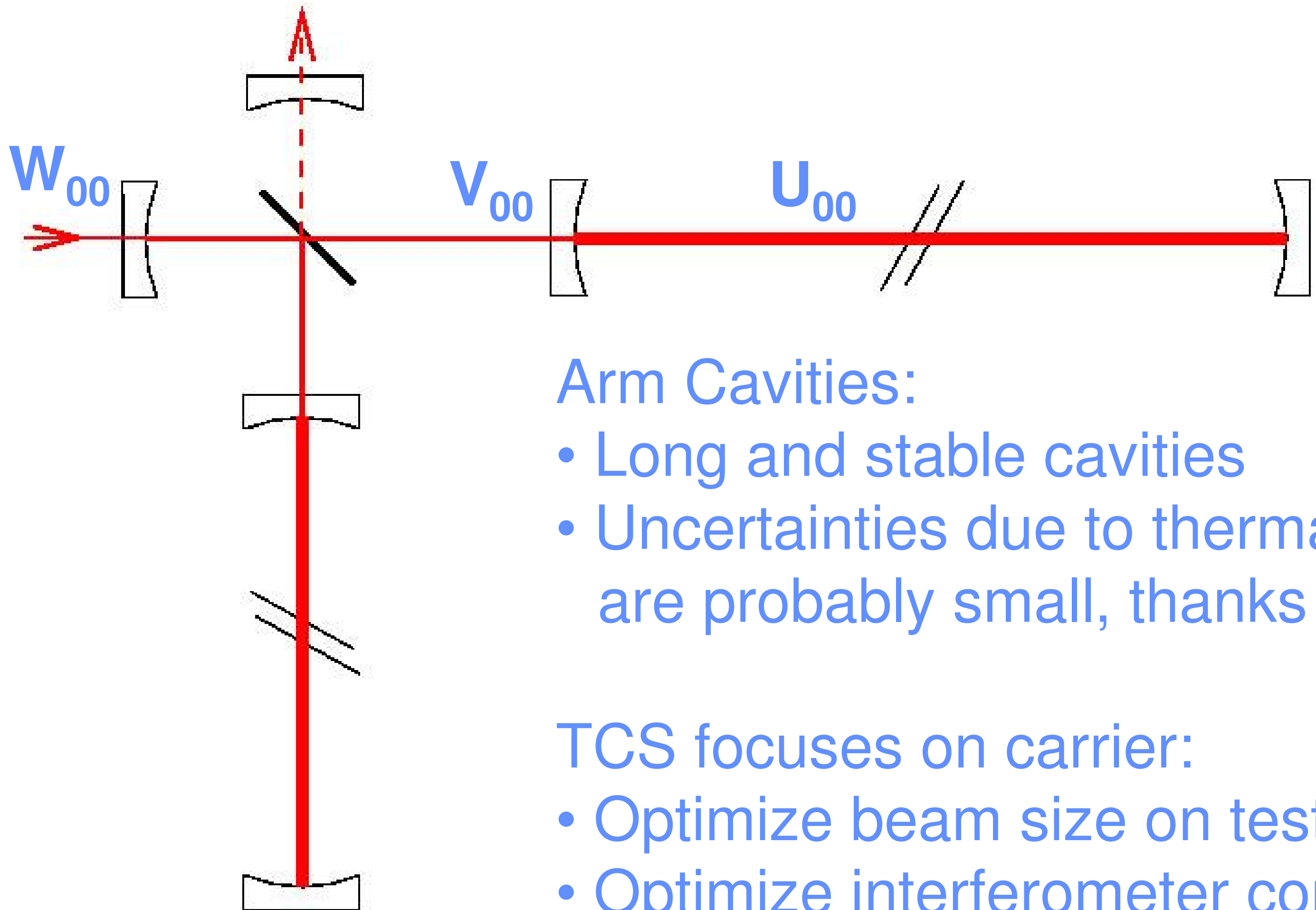
- Summary

Advanced LIGO – arm cavities



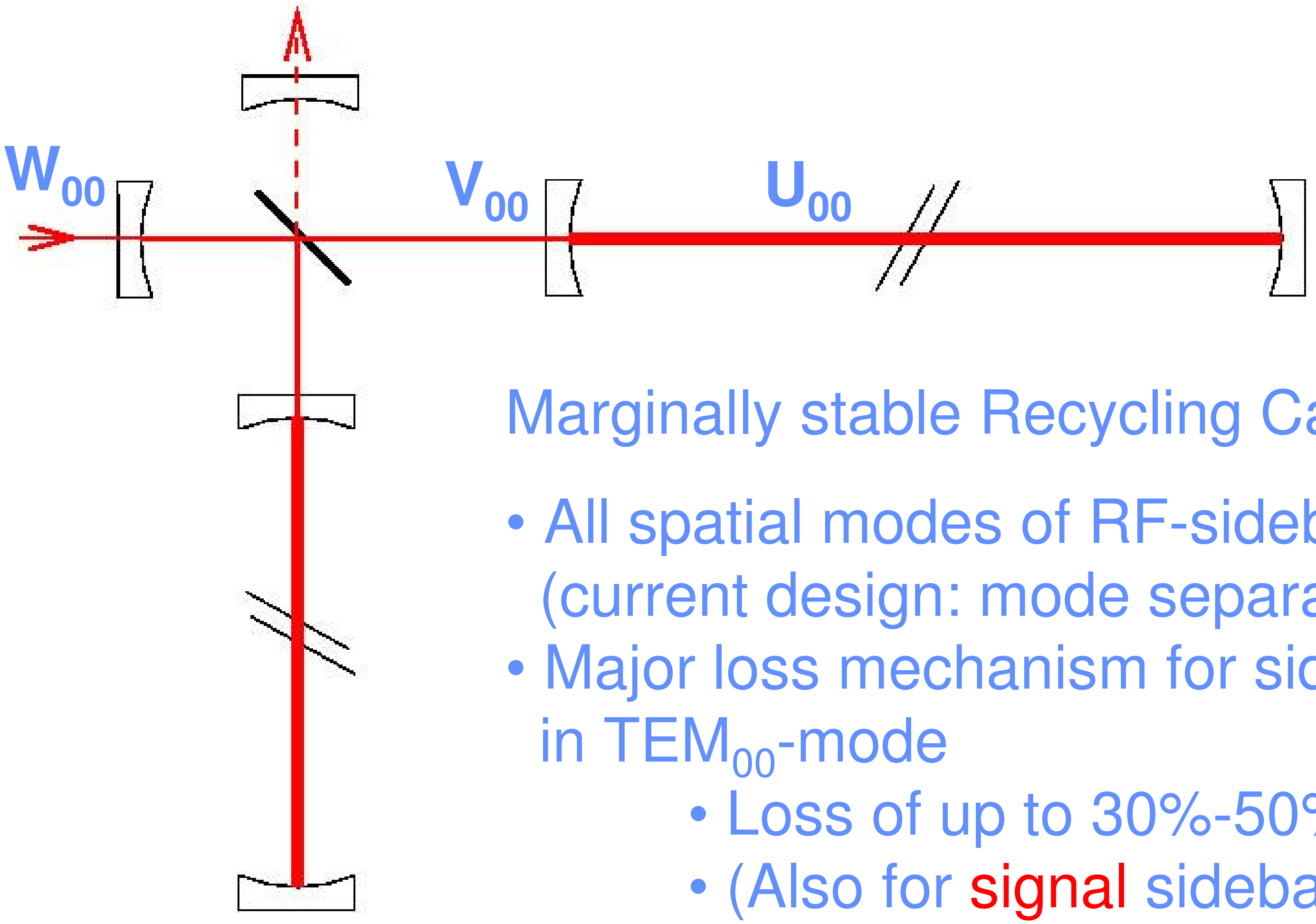
Arm Cavities:

- Long and stable cavities
- Uncertainties due to thermal lensing are probably small, thanks to TCS

TCS focuses on carrier:

- Optimize beam size on test masses
- Optimize interferometer contrast
- Optimize mode matching(?)

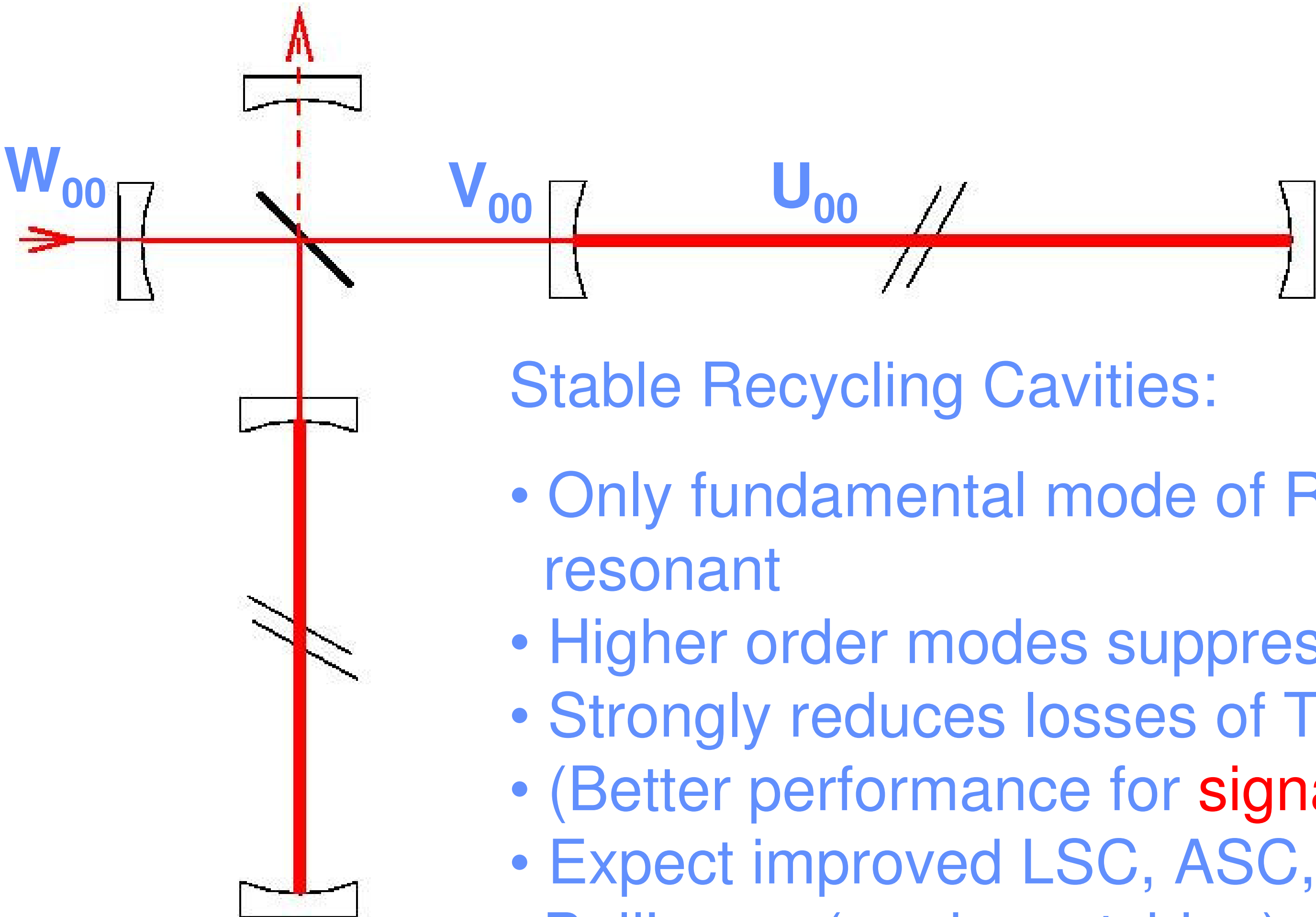
Adv. LIGO
Marginally Stable Recycling Cavities



Marginally stable Recycling Cavities:

- All spatial modes of RF-sidebands resonant (current design: mode separation ≈ 4 kHz)
- Major loss mechanism for sidebands in TEM_{00}-mode
  - Loss of up to 30%-50%
  - (Also for signal sidebands!)
- Impact on LSC and ASC

Adv. LIGO
Stable Recycling Cavities

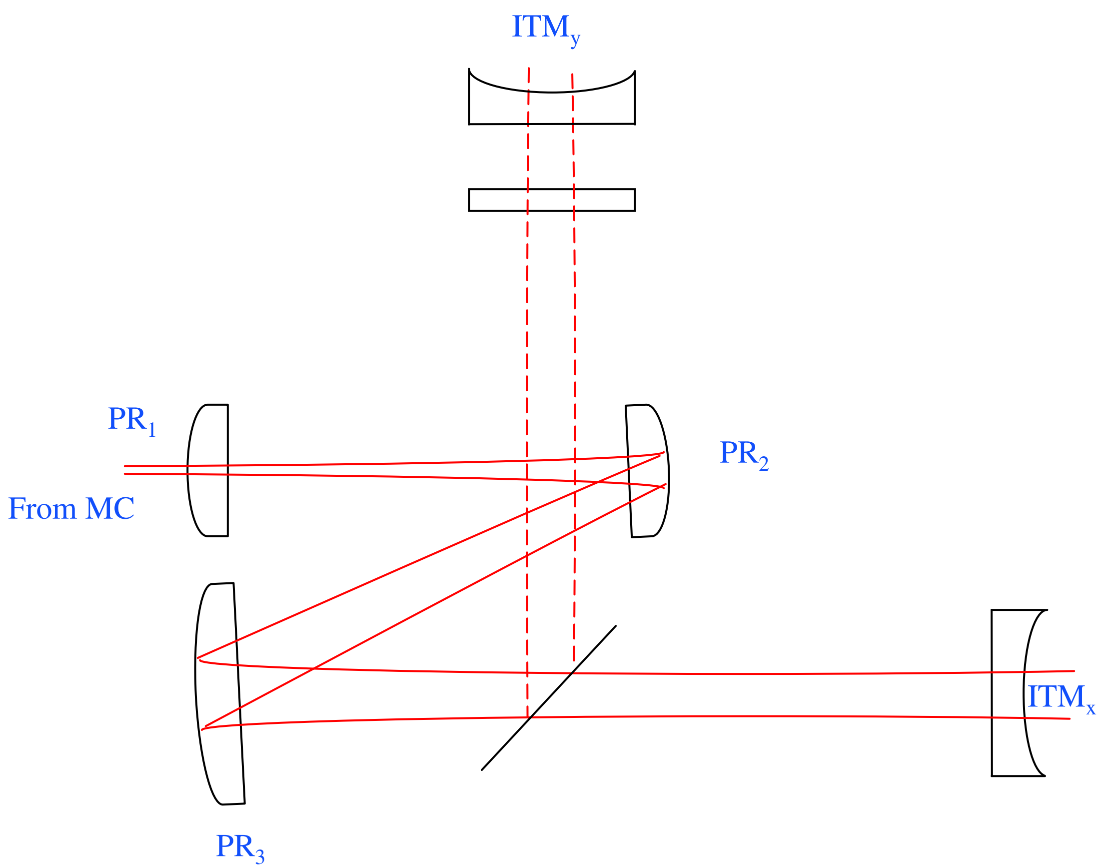


Stable Recycling Cavities:

- Only fundamental mode of RF-sidebands resonant
- Higher order modes suppressed
- Strongly reduces losses of TEM_{00}-mode
- (Better performance for signal sidebands)
- Expect improved LSC, ASC, and even Bull's eye (mode matching) signals
- Interferometer will be much easier to understand and debug

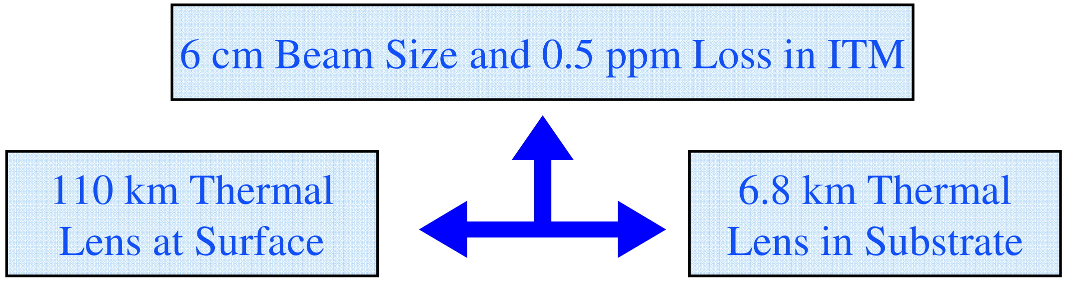
Power Recycling Cavity

Cold State



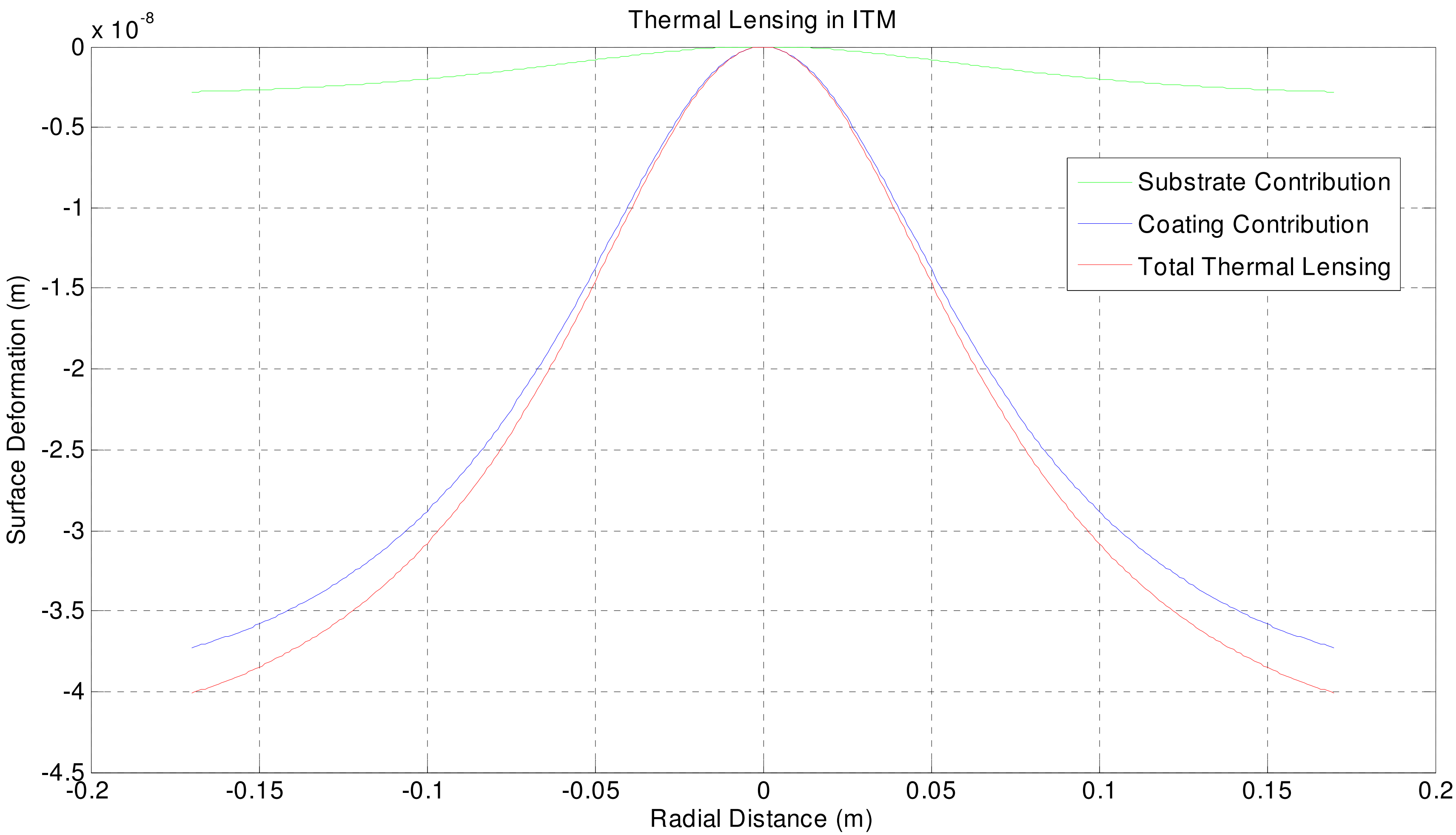
Parameter	Unit	Value
MC Waist	mm	2.113
MC Waist Loc.-PR$_1$	m	2.0
PR$_1$ ROC	m	-128.895
PR$_1$ - PR$_2$	m	16.655
PR$_2$ ROC	m	1.524
Beam Size @ PR$_2$	mm	3.567
PR$_2$ – PR$_3$	m	16.655
PR$_3$ ROC	m	31.384
Beam Size @ PR$_3$	cm	7.23
PR$_3$ –ITM	m	24.995
Beam Size @ ITM	cm	7.10
ITM ROC	m	2037.5
Mode Matching	-	1.0

Hot Operation @ 120 W



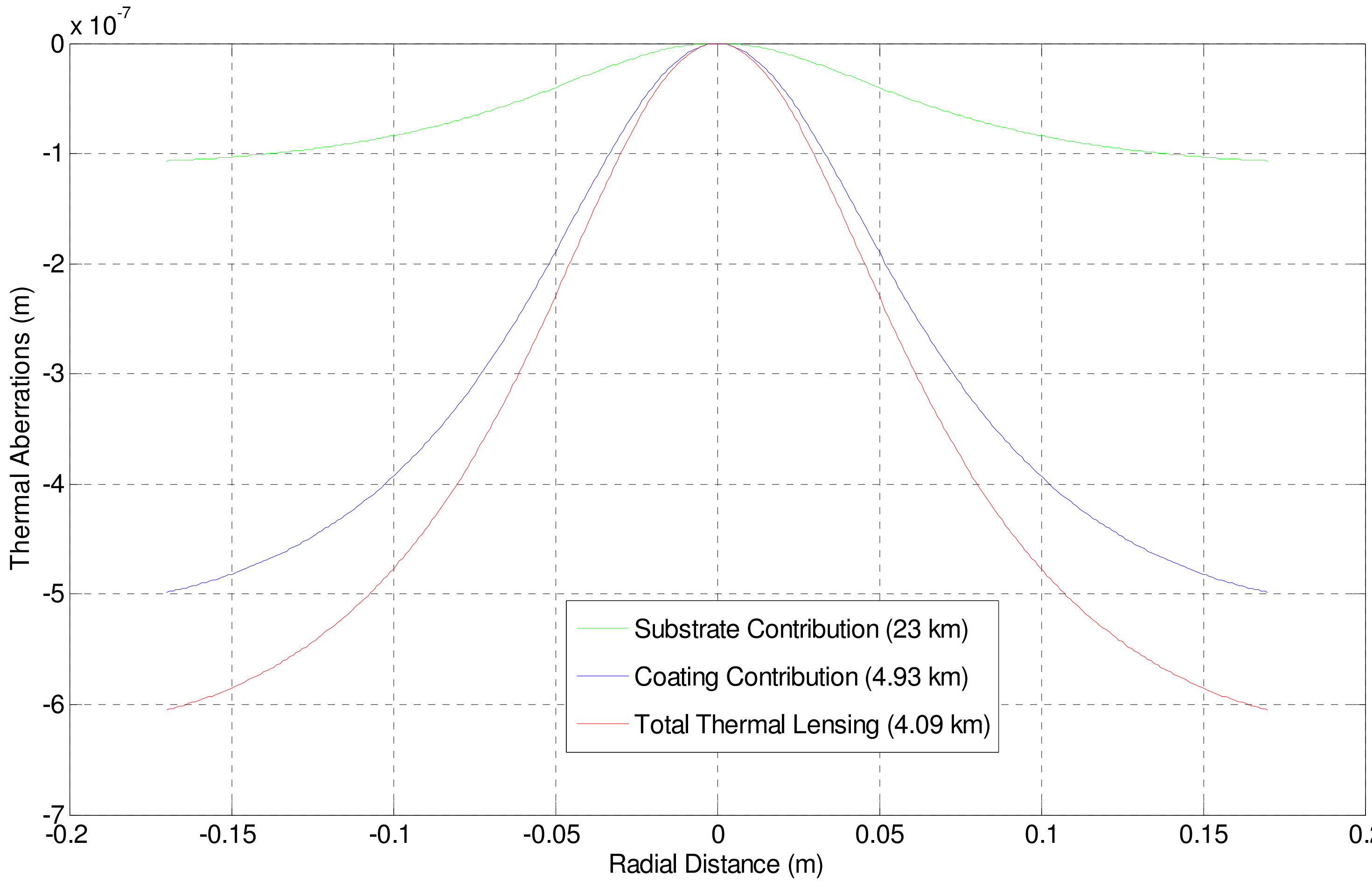
Thermal Lensing in ITM

Surface Deformation (64.76 km Thermal Lens, 12 th Degree Polynomial Fit to H-Vinet Theory)

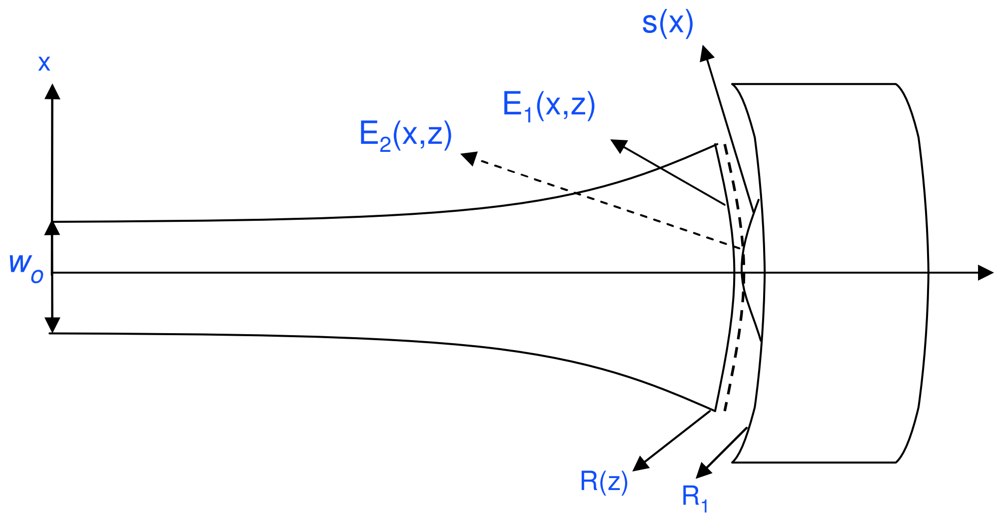


Thermal Lensing in Substrate

Thermal Aberrations (12 th Degree Polynomial Fit to H-Vinet Theory)



Optimized Thermal ROC Estimation



$$E_1(x,z) = \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{\sqrt{w(z,A_{opt})}} e^{-x^2\left[\frac{1}{w^2(z,A_{opt})}+i\frac{\pi}{\lambda R(z)}\right]}$$

$$E_2(x,z) = \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{\sqrt{w(z)}} e^{-x^2\left[\frac{1}{w^2(z)}+i\frac{\pi}{\lambda R(z)}-i\frac{2\pi}{\lambda R_1}+i\frac{\pi}{\lambda_1}\frac{4s(x)}{x^2}\right]}$$

$$E_3(x,z) = \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{\sqrt{w(z)}} e^{-x^2\left[\frac{1}{w^2(z)}+i\frac{\pi}{\lambda R(z)}-i\frac{2\pi}{\lambda R_1}+i\frac{\pi}{\lambda}4A_{opt}\right]}$$

Calculate Overlap Integral b/w E_2 and E_3 and maximize it w.r.t. the ROC of thermal lens to get the optimal value

The Solution

$$I = 1 - \frac{\pi^2\omega^4(z)}{2\lambda^2}\left[3A_{opt}^2 - 2A_{opt}\sum_{n=2}^{N}\frac{(n+1)A_n\omega^{n-2}}{2^{n-2}} + 2\sum_{n=2}^{N}\frac{nA_n\omega^{n-2}}{2^{n-2}}\sum_{n=2}^{N}\frac{A_n\omega^{n-2}}{2^{n-2}} - \left(\sum_{n=2}^{N}\frac{A_n\omega^{n-2}}{2^{n-2}}\right)^2\right]$$

$$+ i\times\left[\frac{4\pi}{\lambda}\sum_{n=2}^{N}\frac{(n-1)A_n\omega^{n}}{2^{n}} - \frac{A_{opt}\pi\omega^2(z)}{\lambda}\right]$$

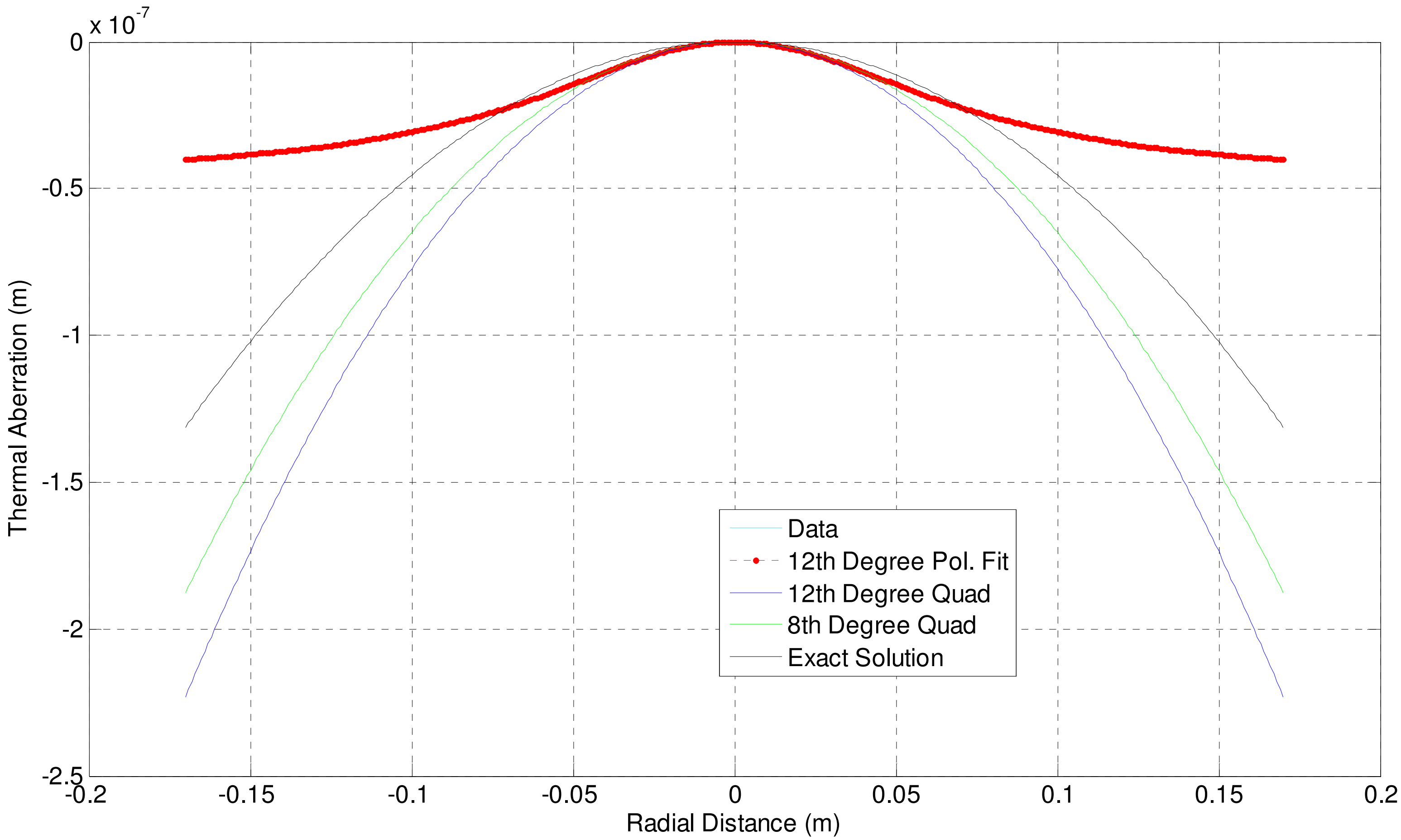
An analytical solution is difficult however a numerical solution can be obtained. The curve is a parabola like shape and the vertex gives you the optimal value.

$$\text{Valid Only for } \left(s(x) - A_{opt}x^2\right) << \lambda$$

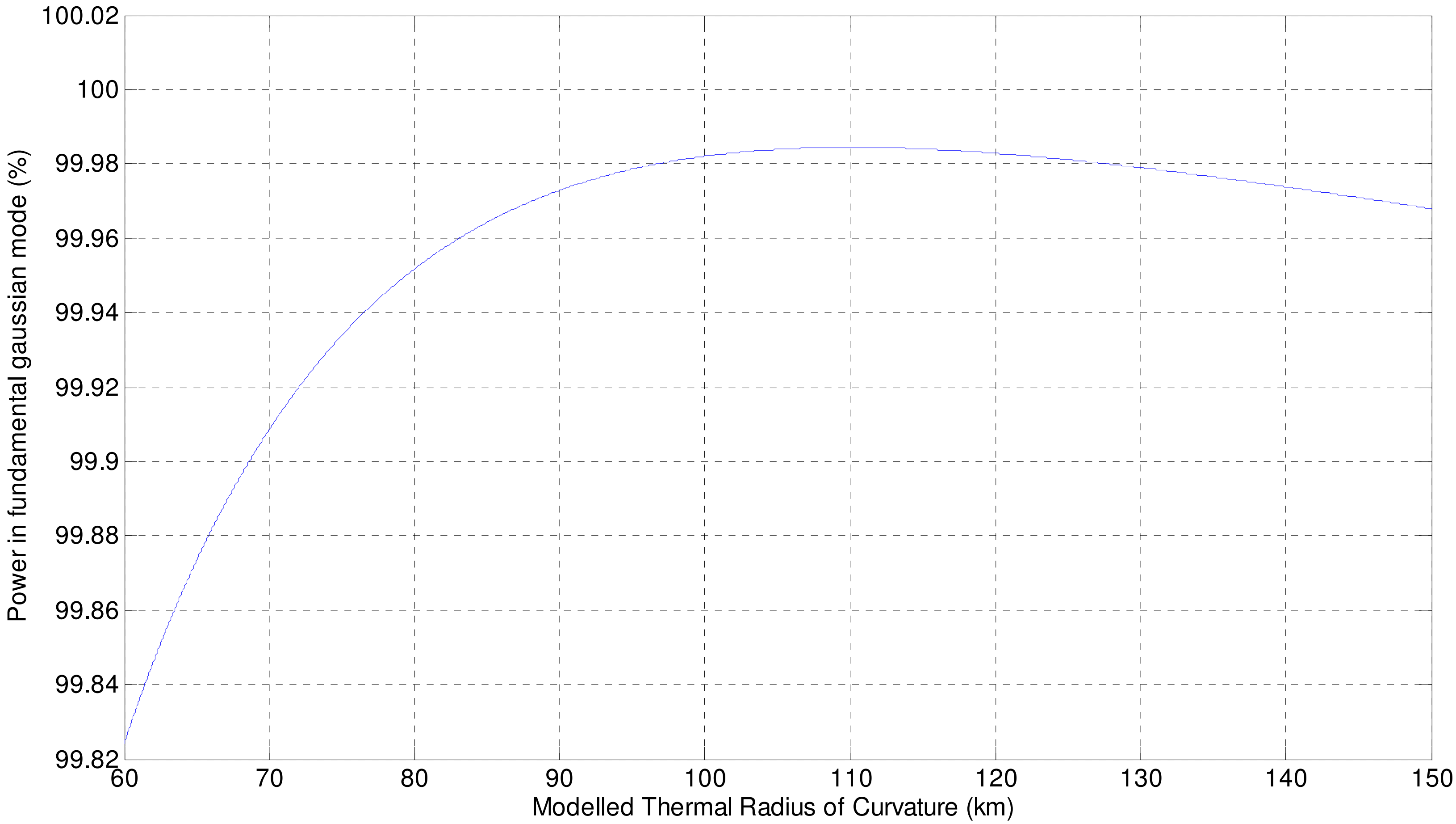
Solution: Numerical Integration

$$I(A) = \left(\frac{2}{\pi}\right)^{1/2}\frac{1}{w(z)}\int_{-\infty}^{\infty} e^{-x^2\left[\frac{2}{w^2(z)}\right]} e^{i\left[\frac{4\pi}{\lambda_1}s(x) - \frac{\pi}{\lambda}4A_{opt}x^2\right]}\,dx$$

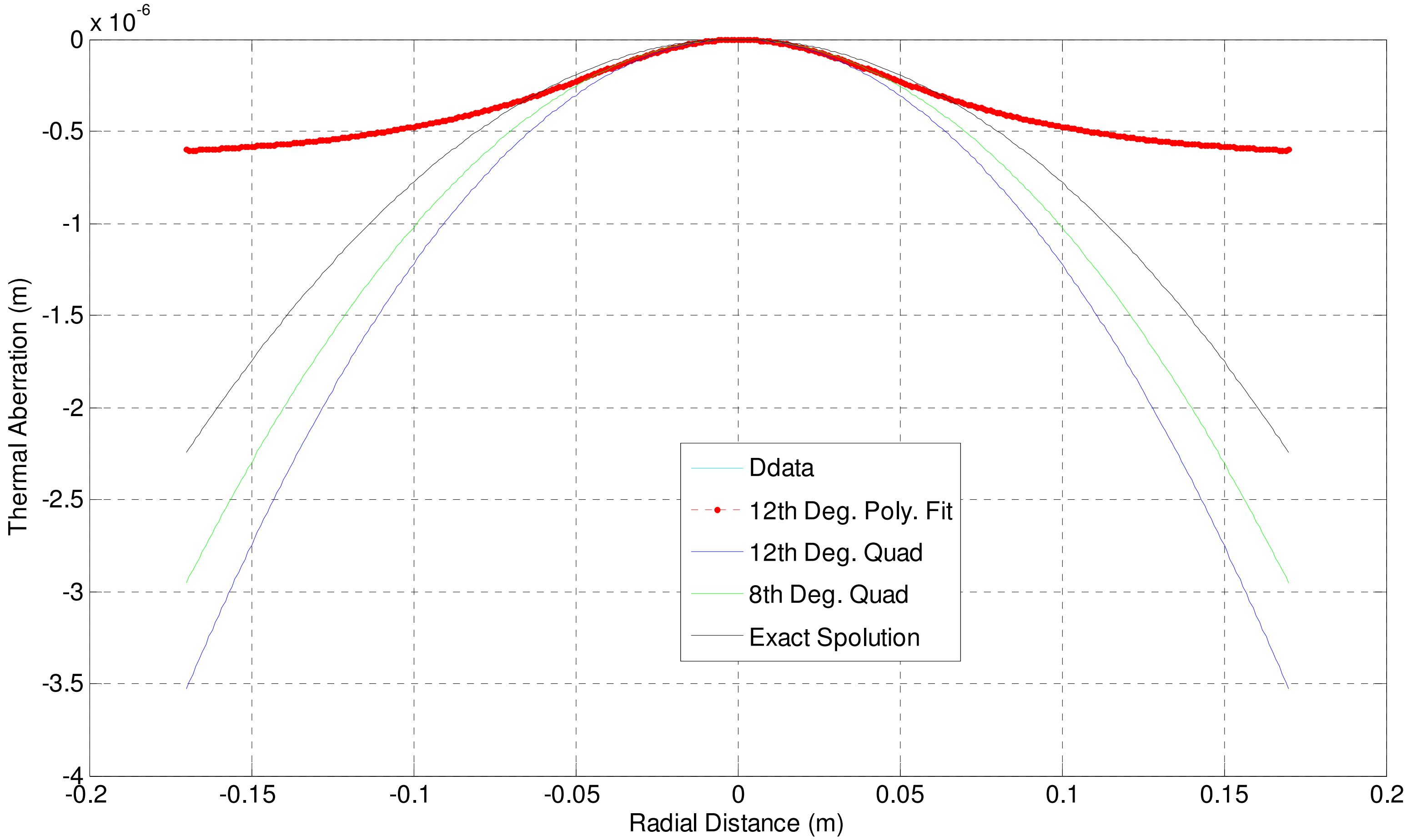
Various Approximations for ITM



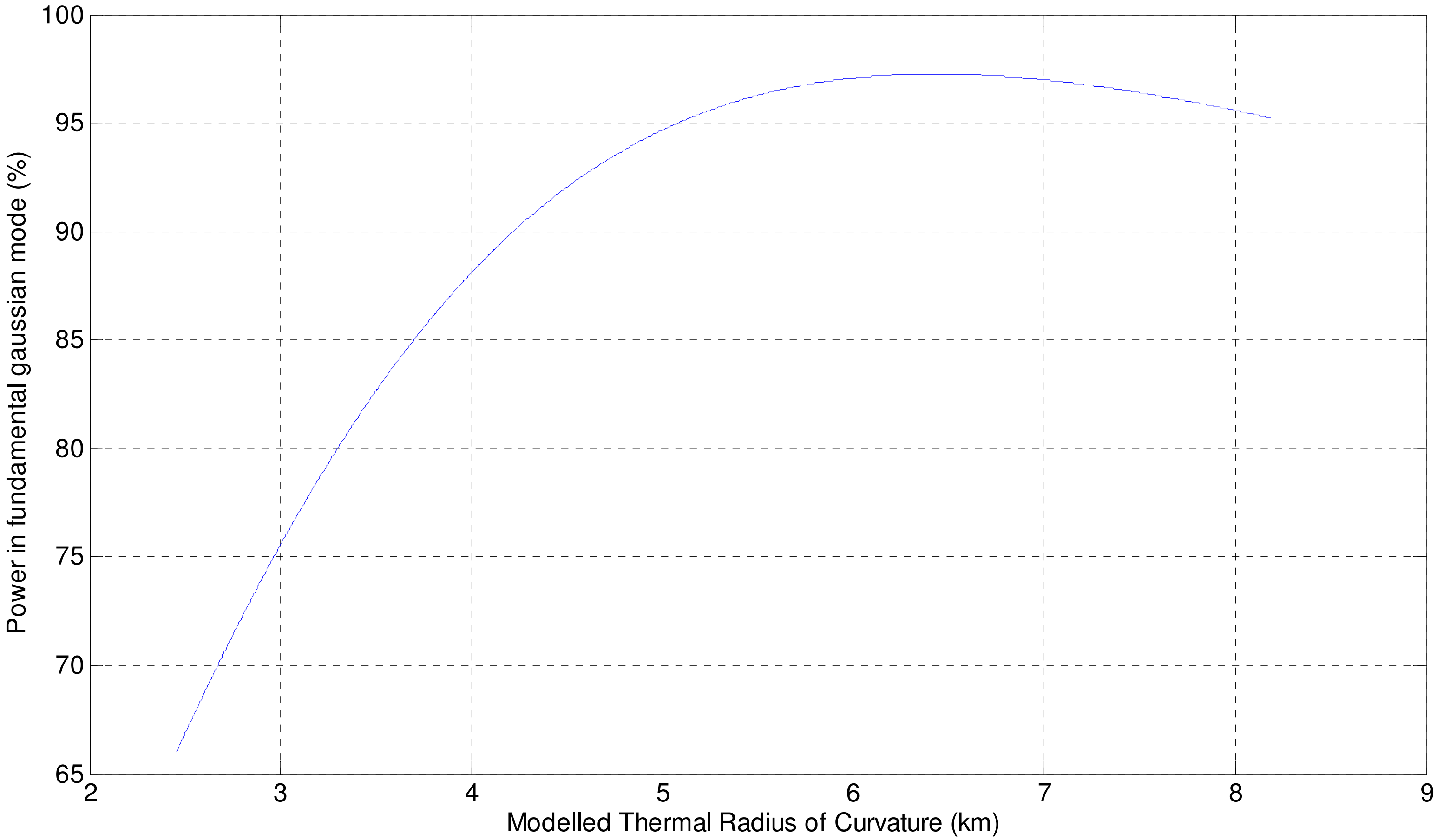
Overlap Integral (1-D)



Various Approximations for Substrate



Overlap Integral (1-D)



Comparison of ROC Change Estimation

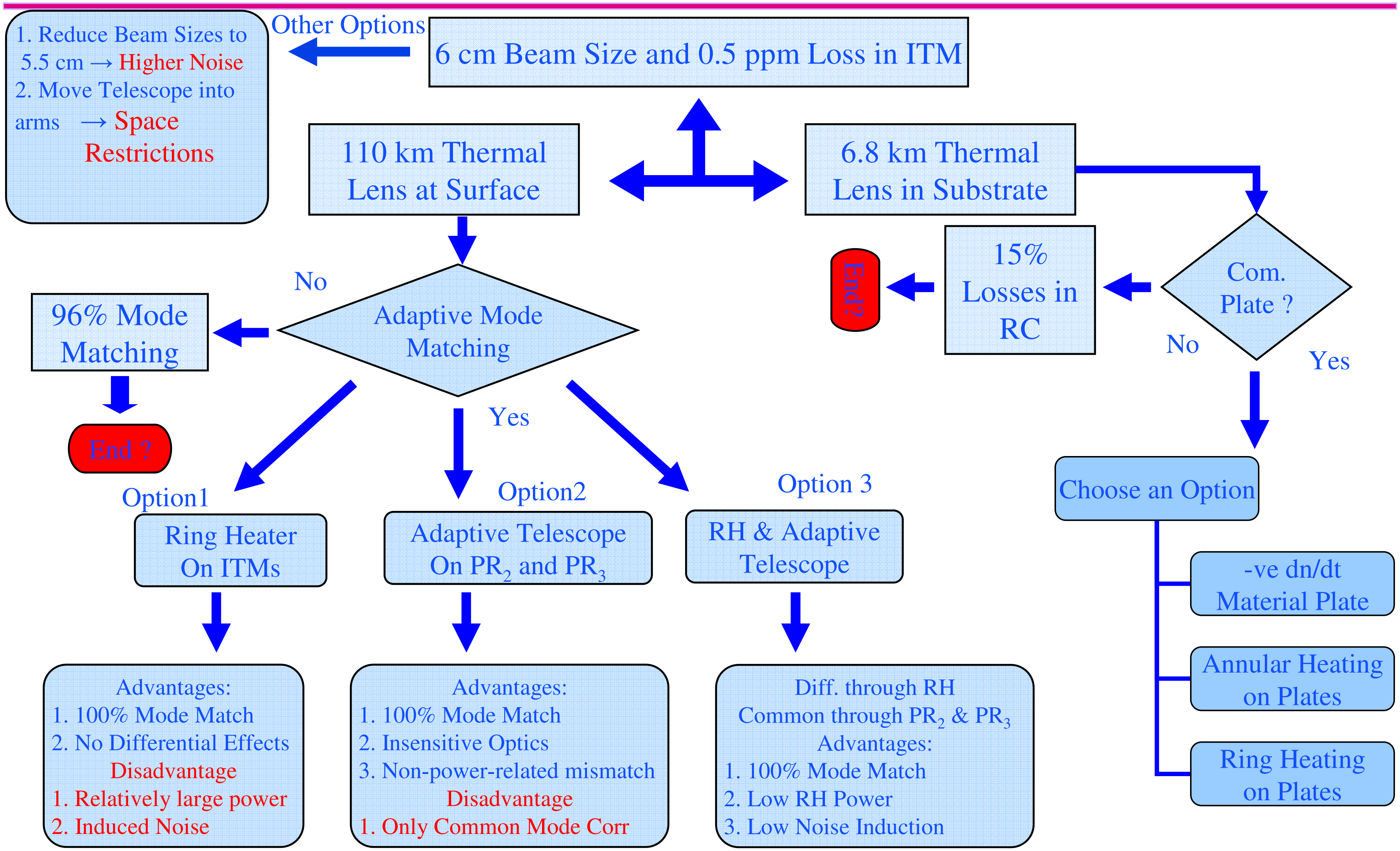
Thermal Lens in ITM

Method	*Thermal ROC(Km)*	ROC_{cold} (m)	Cold Beam Size (cm)	Losses %
A_2 of 12^{th} Pol.	64.768	2012.46	9.29	0.48
A_2 of 8^{th} Pol.	77.013	2022.45	8.04	0.23
Exact Sol.	110.10	2038.54	7.05	0.06

Thermal Lens in Substrate

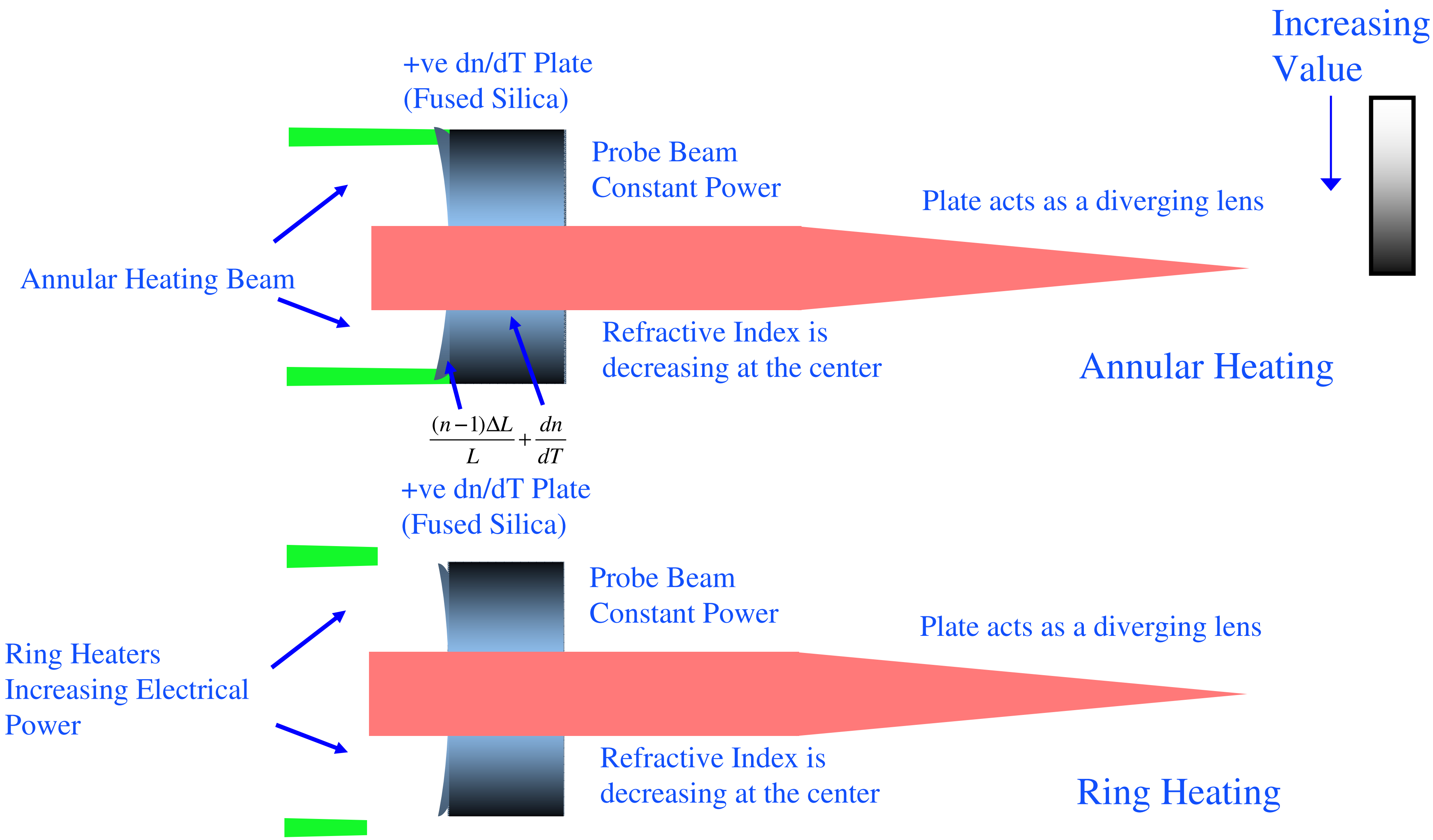
Method	*Thermal ROC(Km)*	Losses %
A_2 of 12^{th} Pol.	4.091	37.43
A_2 of 8^{th} Pol.	4.890	21.29
Exact Sol.	6.42	10.52

Hot Operation @ 120 W

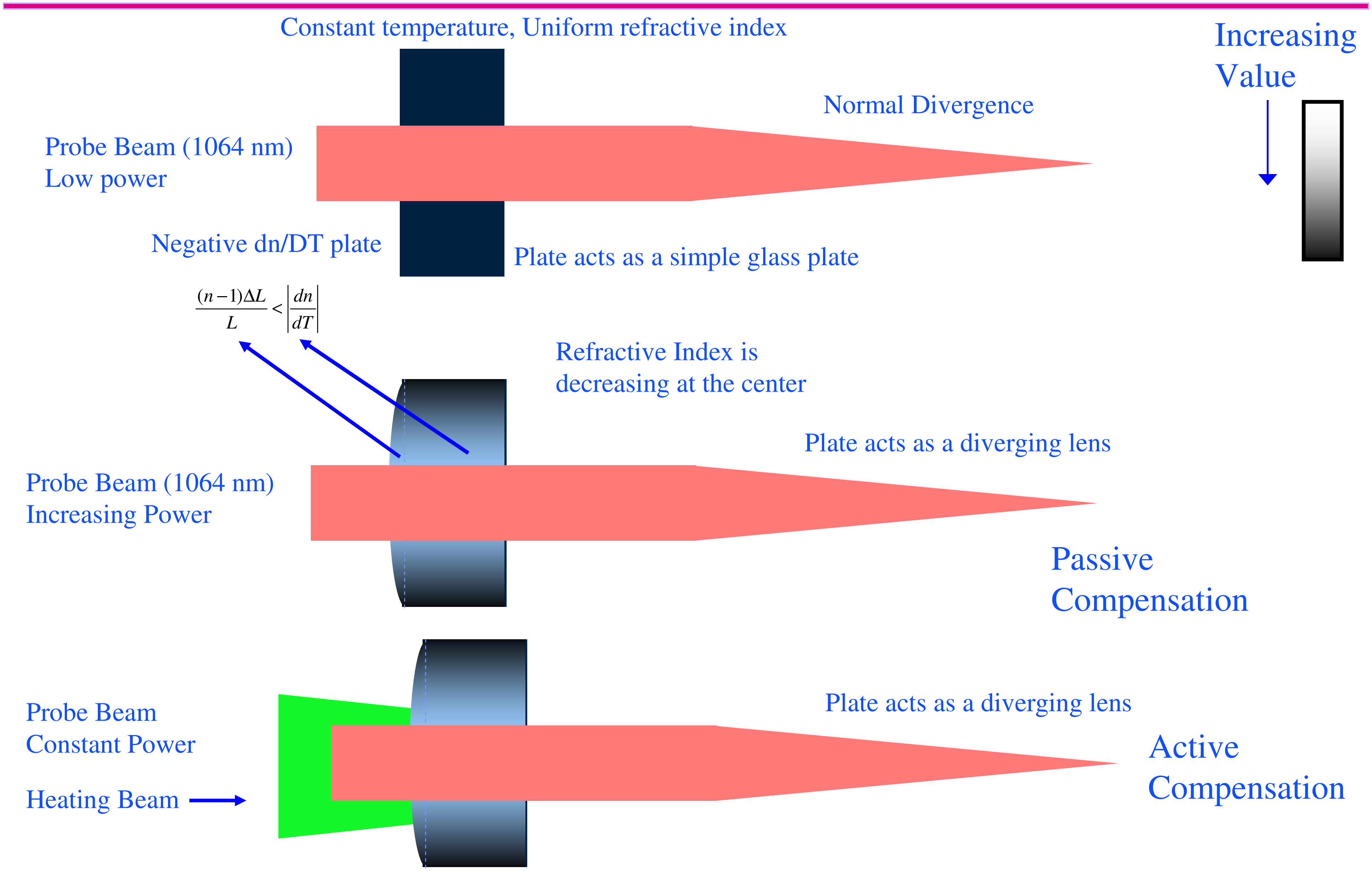


Compensation Plate for Substrate Compensation

Annular & Ring Heater Compensation



Negative dn/dT Compensation



Option 1: Negative dn/dT

- Active Compensation
- Requirements:
  - » CO_2 laser system for surface heating
  - » Availability in Large Size
  - » Purity/homogeneity
- Advantages:
  - » Works without any doubt, trick is use same beam size as ITM
  - » Requires very low power (< 2W)
  - » Highly Efficient/Adaptive
- Disadvantages:
  - » New material
  - » A lot of Data and tests needed
  - » Requires coating on both surfaces

- Passive Compensation
- Requirements:
  - » Availability in Large Size
  - » Purity/homogeneity
  - » Exact Size absorption values
- Advantages:
  - » No laser needed
  - » Highly Efficient
  - » Analogous to compensation in Faraday Rotator
- Disadvantages:
  - » New material
  - » A lot of Data and tests needed
  - » Less Adaptive
  - » Requires coating on both sides

Does such a material Exists ??

Combination of Two is essential due to substrate heating at 1064 nm

Potassium Bromide, Initial Choice

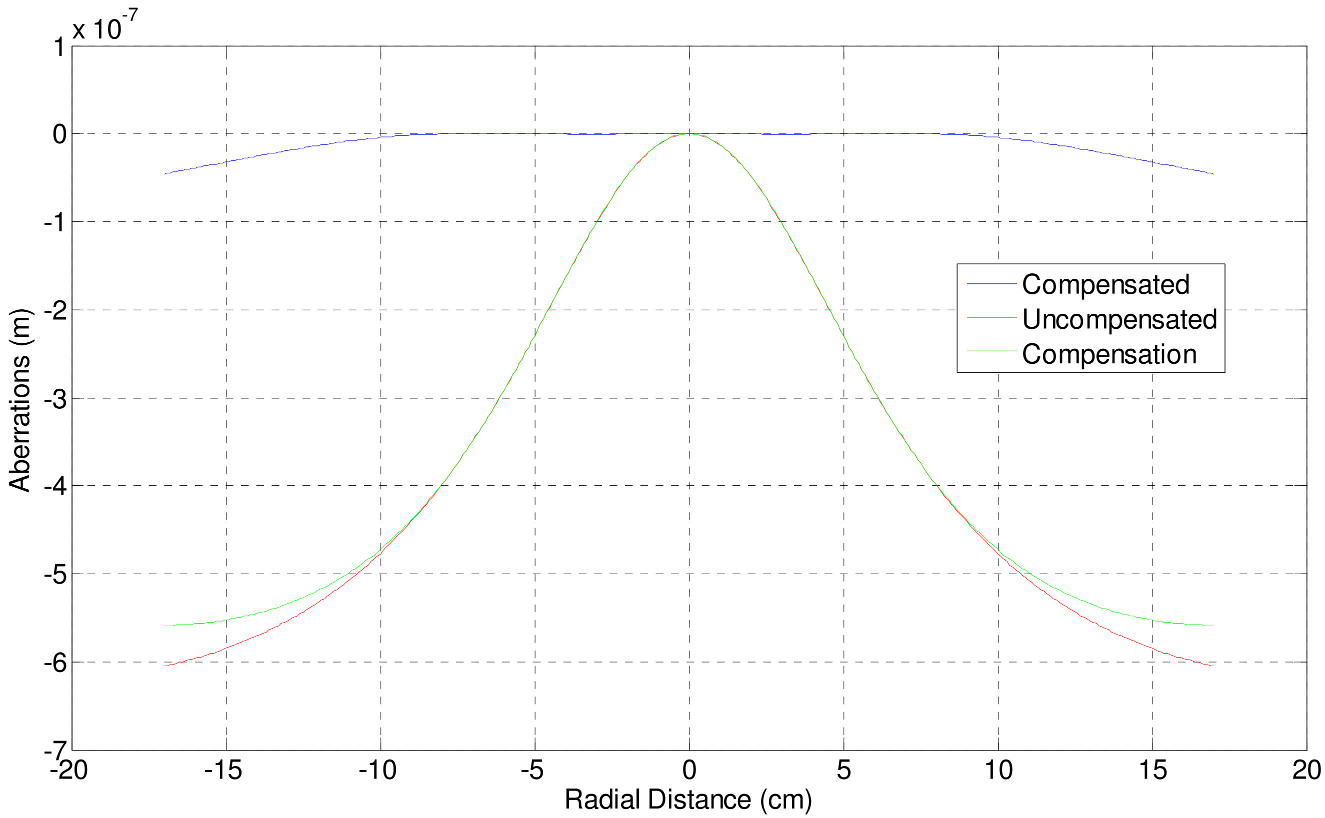
- Important Properties (KBr):
- $n = 1.5441$ @ 1064 nm
- $(dn/dT-(n-1)\alpha_t = -17.43\times10^{-6}\ /^0C$
- Absorption = 0.002 cm^{-1}
- $K_t =$ 4.816 W m^{-1} K^{-1} @ 319K

Product Name:	Potassium Bromide (KBr)
Transmission Range:	0.23 to 25 µm
Refractive Index:	1.527 at 10 µm
Reflection Loss:	8.3% at 10 µm
Absorption Coefficient:	1.6 x 10[illegible] at 3.8 µm
Reststrahlen Peak:	77.6 µm
dN/dT:	-40.83 x 10^{-6}/°C
dN/du:	4.2 µm
Density:	2.753 g/cc
Melting Point:	730°C
Thermal Conductivity:	4.816 W m^{-1} K^{-1} @ 319K
Thermal Expansion:	43 x 10^{-6} /°K @300K
Hardness:	Knoop 7 in <100> with 200g indenter
Specific Heat Capacity:	435 J Kg^{-1} K^{-1}
Dialectric Constant:	4.9 @ 1MHz
Youngs Modulus (E):	26.8 GPa
Shear Modulus (G):	5.08 GPa
Bulk Modulus (K):	15.03 GPa
Elastic Coefficients:	C11=34.5 C12=5.4 C44=5.08
Apparent Elastic Limit:	1.1 MPa (160psi)
Poisson Ratio:	0.203
Solubility:	53.48g/100g water at 273K
Molecular Weight:	119.01
Class/Structure:	Cubic FCC, NaCl, Fm3m, (100) cleavage

- Important Properties (Fused Silica):
- $n = 1.4497$ @ 1064 nm
- $dn/dT = 8.7\times10^{-6}\ /^0C$
- $\alpha_t = 5.5\times10^{-7}\ /^0C$
- Absorption = 2×10^{-6} cm^{-1}
- $K_t =$ 1.37 W m^{-1} K^{-1}

- Normally used in IR Spectrometers
- Available in large sizes
- Crystalline in nature
- Soluble in water

A Typical Example
6.42 km Thermal Lens in Substrate



- Crystal Thickness = 2.05 mm
- Coating Absorption = 2.0 W
- Uncompensated Losses = 11%
- After Compensation = 0.001%

Other Options for Compensation

- Option 2: Ring Heaters
- General Comments
  - Easy to control electrical power
  - Sensitive to geometry
  - Requires relatively high power
  - Compensating a lens of 6.8 km in substrate might be too much for ring heaters
  - Increases the temperature

Further Considerations for Experts

- Annular Heating
- General Comments
  - Established technique
  - Probably will require less power as compared to Ring Heaters
  - Silica can be used as compensation plates
  - No new material is required
  - Still needs an extra laser and related control like negative *dn*/*dT*
  - Less efficient than negative *dn*/*dT* method

Surface Thermal Lensing

1. Highly variable due to coating variations
2. Differential Variations can be severe
3. Changes the mode in the arm cavity and mode matching can decrease to 96% without correction
4. Can cause contrast defects

Adaptive Mode Matching

Recall:

Ring Heater On ITMs

Advantages:
1. 100% Mode Match
2. No Differential Effects

Disadvantage
1. Relatively large power
2. Induced Noise

Adaptive Telescope On PR_2 and PR_3

Advantages:
1. 100% Mode Match
2. Insensitive Optics
3. Non-power-related mismatch

Disadvantage
1. Only Common Mode Corr

RH & Adaptive Telescope

Diff. through RH
Common through PR_2 & PR_3

Advantages:
1. 100% Mode Match
2. Low RH Power
3. Low Noise Induction

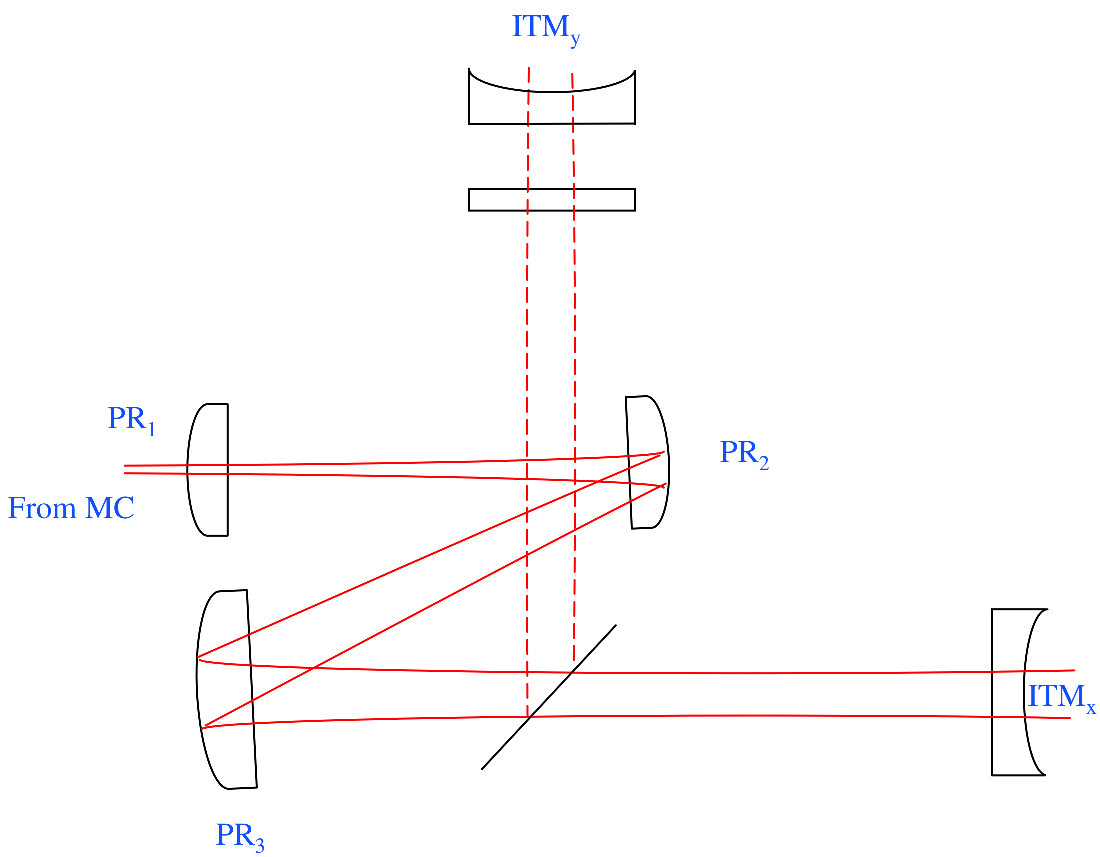
Proposed Scheme

Procedure
1. Use RH on ITMs to keep the ROC on both ITMs same.
2. Use Adaptive correction on PR_2 and PR_3.

Assumptions
1. We have selected a base value of 0.5 ppm as coating absorption
2. Any deviation more than this, has to be corrected by RH. (Probably requires very low power).
3. If absorption is low, we have to apply some correction in the hot case.

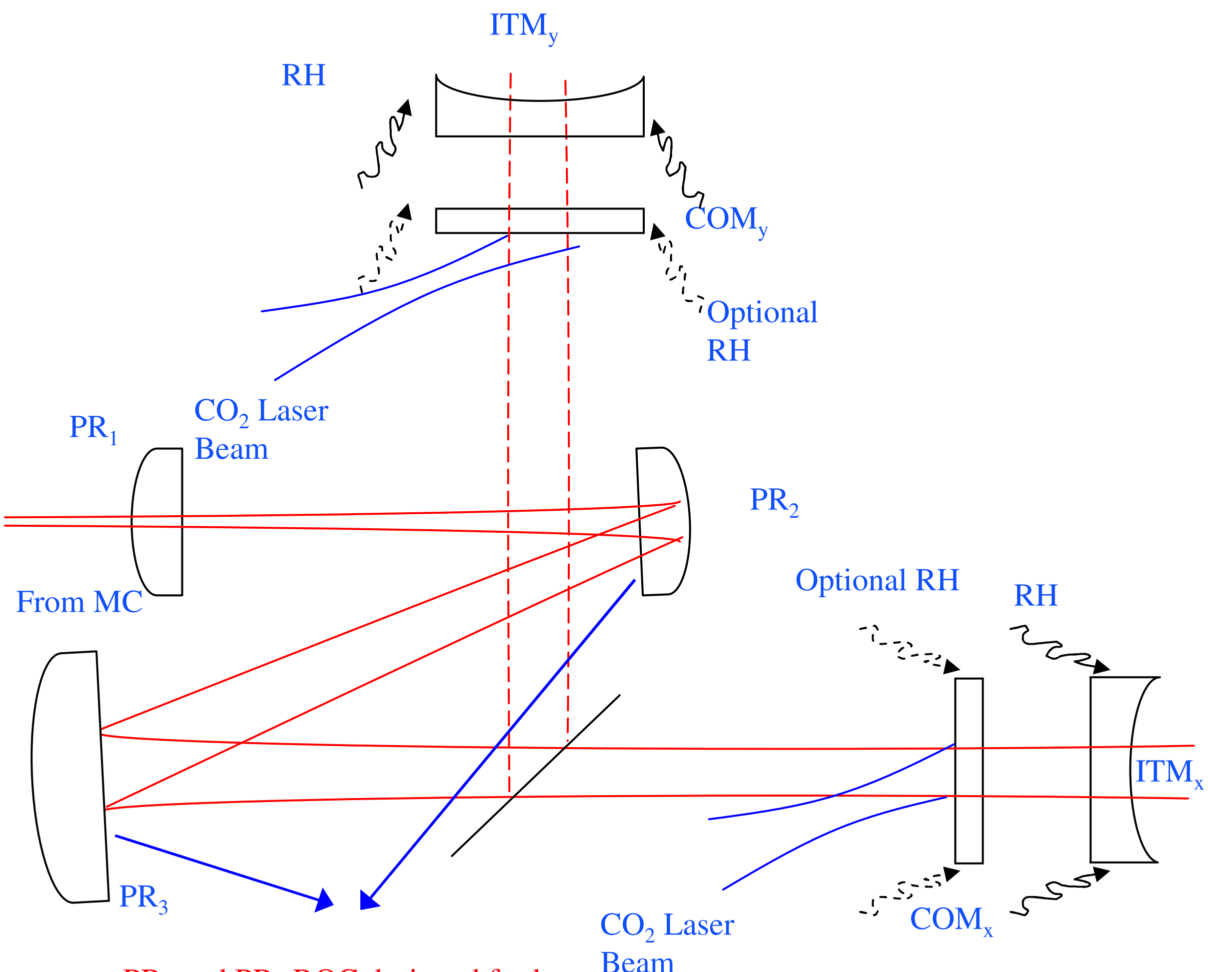
Power Recycling cavity

Cold State



Parameter	Unit	Value
MC Waist	mm	2.113
MC Waist Loc.-PR_3	m	2.0
PR_1 ROC	m	-128.895
PR_1 - PR_2	m	16.655
PR_2 ROC	m	1.524
Beam Size @ PR_2	mm	3.567
PR_2 – PR_3	m	16.655
PR_3 ROC	m	31.384
Beam Size @ PR_3	cm	7.23
PR_3 –ITM	m	24.995
Beam Size @ ITM	cm	7.10
ITM ROC	m	2037.5
Mode Matching	-	1.0

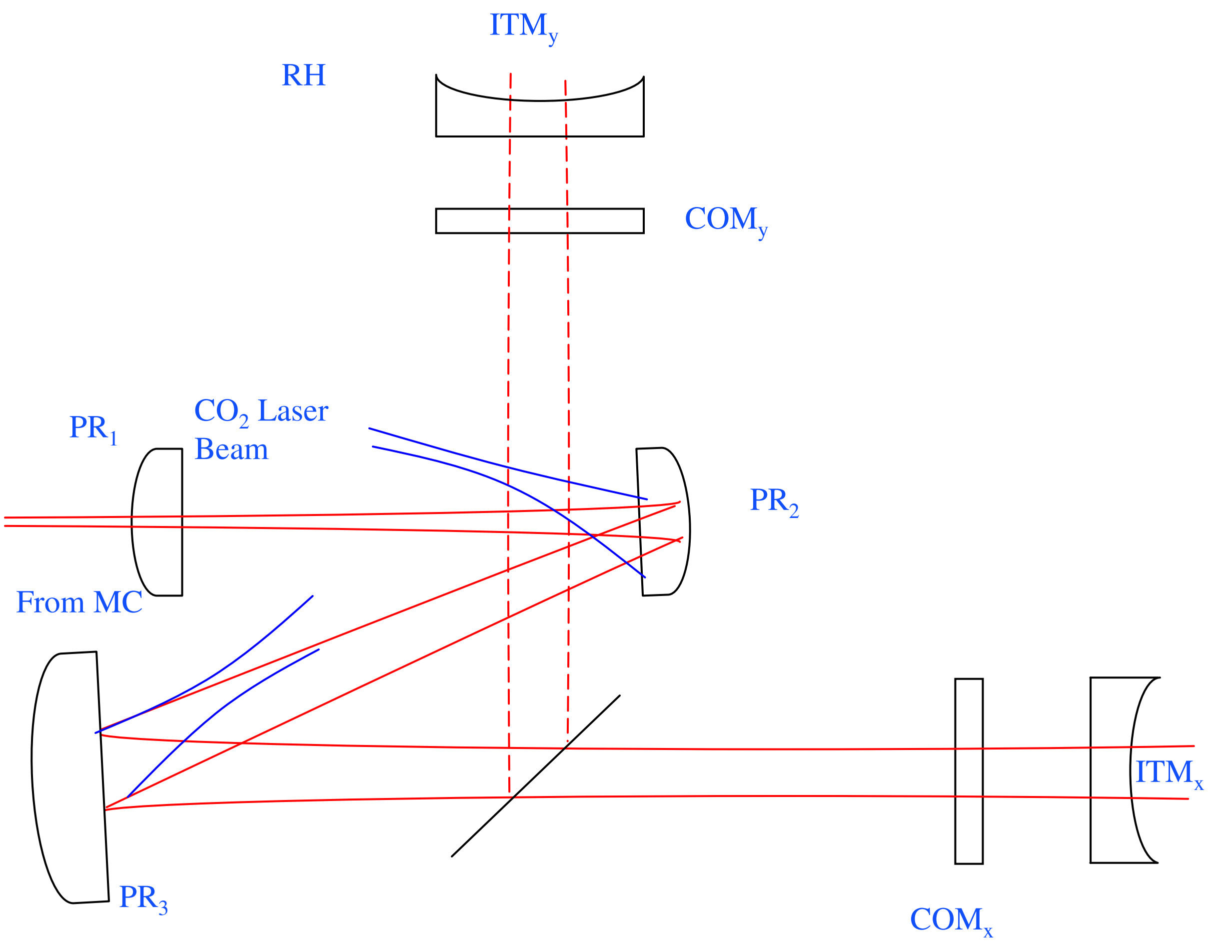
Compensation Scheme for Adaptive Mode Matching – Hot Case



PR$_2$ and PR$_3$ ROC designed for hot value with 110 km thermal lens at ITM surface
(i.e., 0.5 ppm, ITM ROC 2076 m)

Parameter	Unit	Hot Value
MC Waist	mm	2.113
MC Waist Loc.-PR$_1$	m	2.0
PR$_1$ ROC	m	-128.895
PR$_1$ - PR$_2$	m	16.655
PR$_2$ ROC	m	1.783
Beam Size @ PR$_2$	mm	3.567
PR$_2$ – PR$_3$	m	16.655
PR$_3$ ROC	m	31.1217
Beam Size @ PR$_3$	cm	6.09
PR$_3$ –ITM	m	24.995
Beam Size @ ITM	cm	6.00
ITM ROC	m	2076.0
Mode Matching	-	1.0

Compensation Scheme for Adaptive Mode Matching- Cold Case



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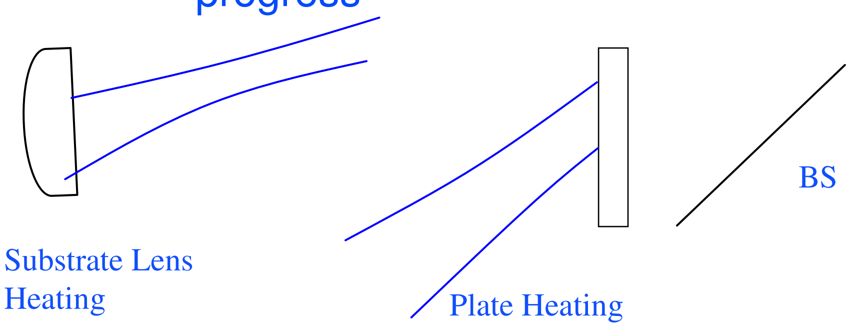
PR$_3$ ROC	m	31.384
Beam Size @ PR$_3$	cm	7.23
PR$_3$ –ITM	m	24.995
Beam Size @ ITM	cm	7.10
ITM ROC	m	2037.5
Mode Matching	-	1.0

Correction @ PR$_2$	m	10.49
Correction @ PR$_3$	Km	3.72
Beam Size @ PR$_2$	mm	3.567
Beam Size @ PR$_3$	cm	7.10

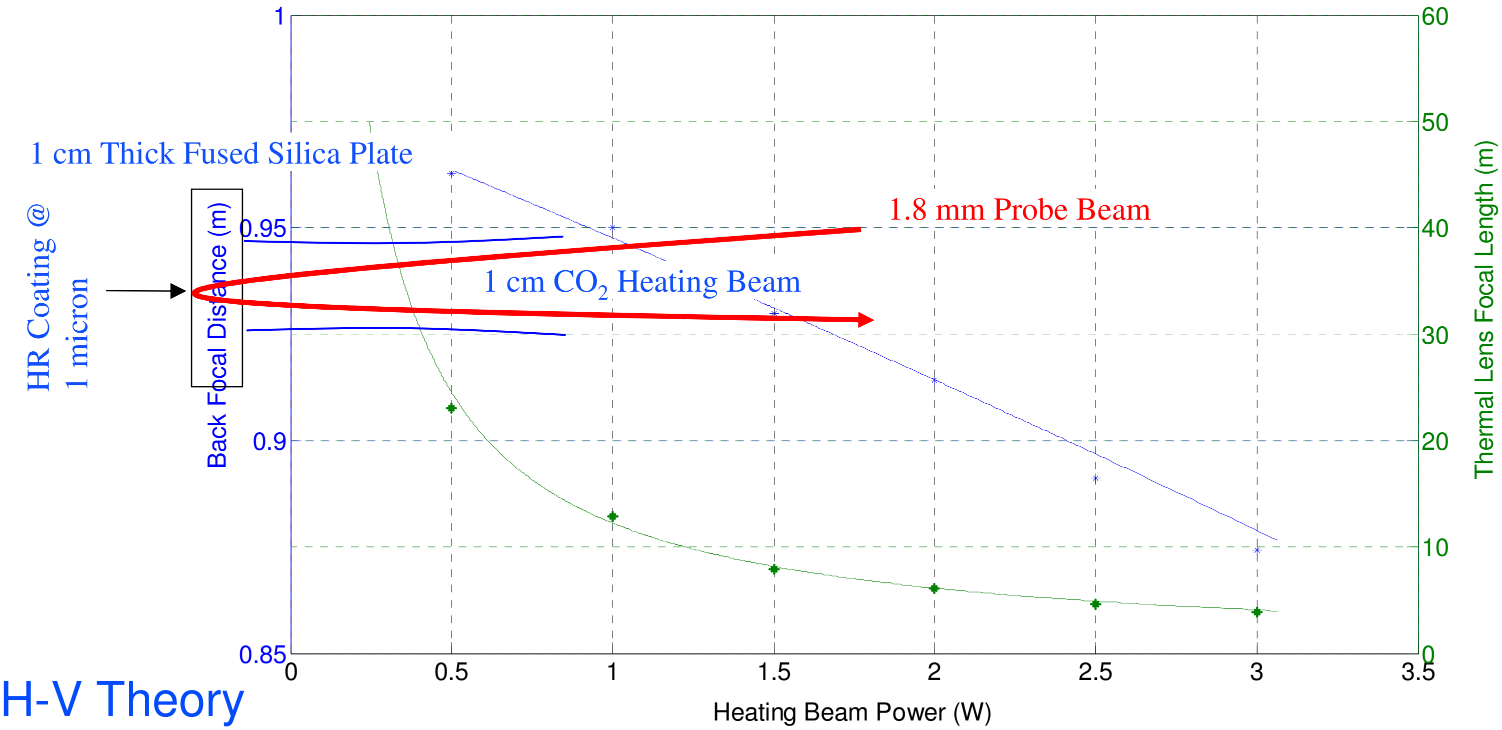
Details of Adaptive Correction
PR Cavity Designed for Hot Case

- Correction @ PR_2
  - » Requires a 10 m converging lens at 3.5 mm as we move from hot to cold case
  - » Easily achievable by using CO_2 heating
  - » Experimental demonstration in progress



- Correction @ PR_3
  - » Requires a 3.4 km diverging lens at 7.1 cm beam size as we move from cold to hot case
  - » Proposed locations are substrate of PR_3 or separate plate before beam splitter
  - » No higher order losses in the hot state
  - » 10% higher order losses in the cold state
  - » Cold state losses can be decreased by using CO_2 beam of larger size

Experimental Demonstration

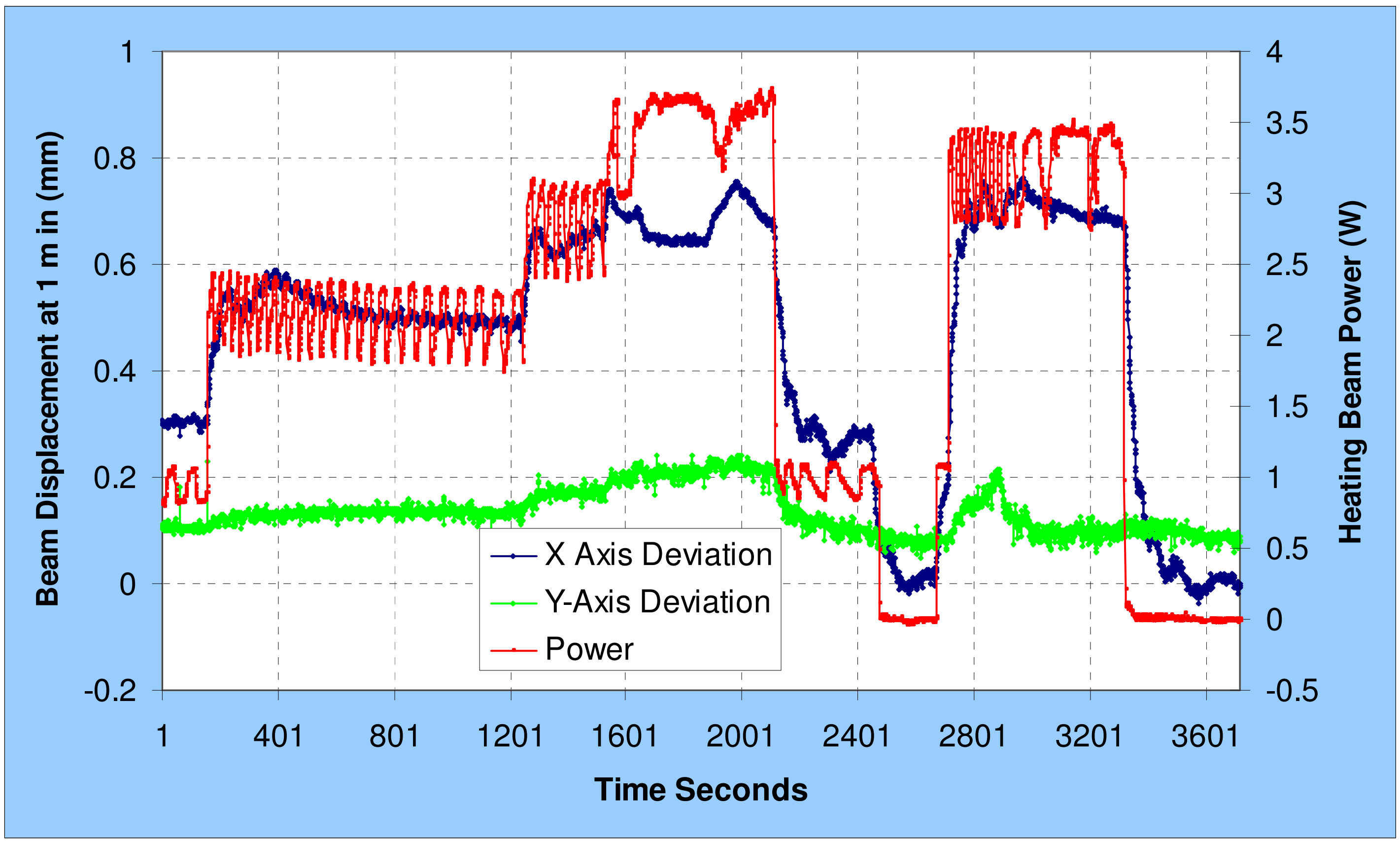


Summary

- Solid Line is H-V Theory
- Dots are experimental data
- 10 m focal length lens at 1.2 W of surface heating at 1.8 mm probe beam and 1.0 cm heating beam

Problems/Concerns

- CO_2 beam power stability
- 'Beam Walking' Problem
- Beam Pointing Stability
- Geometrical Considerations

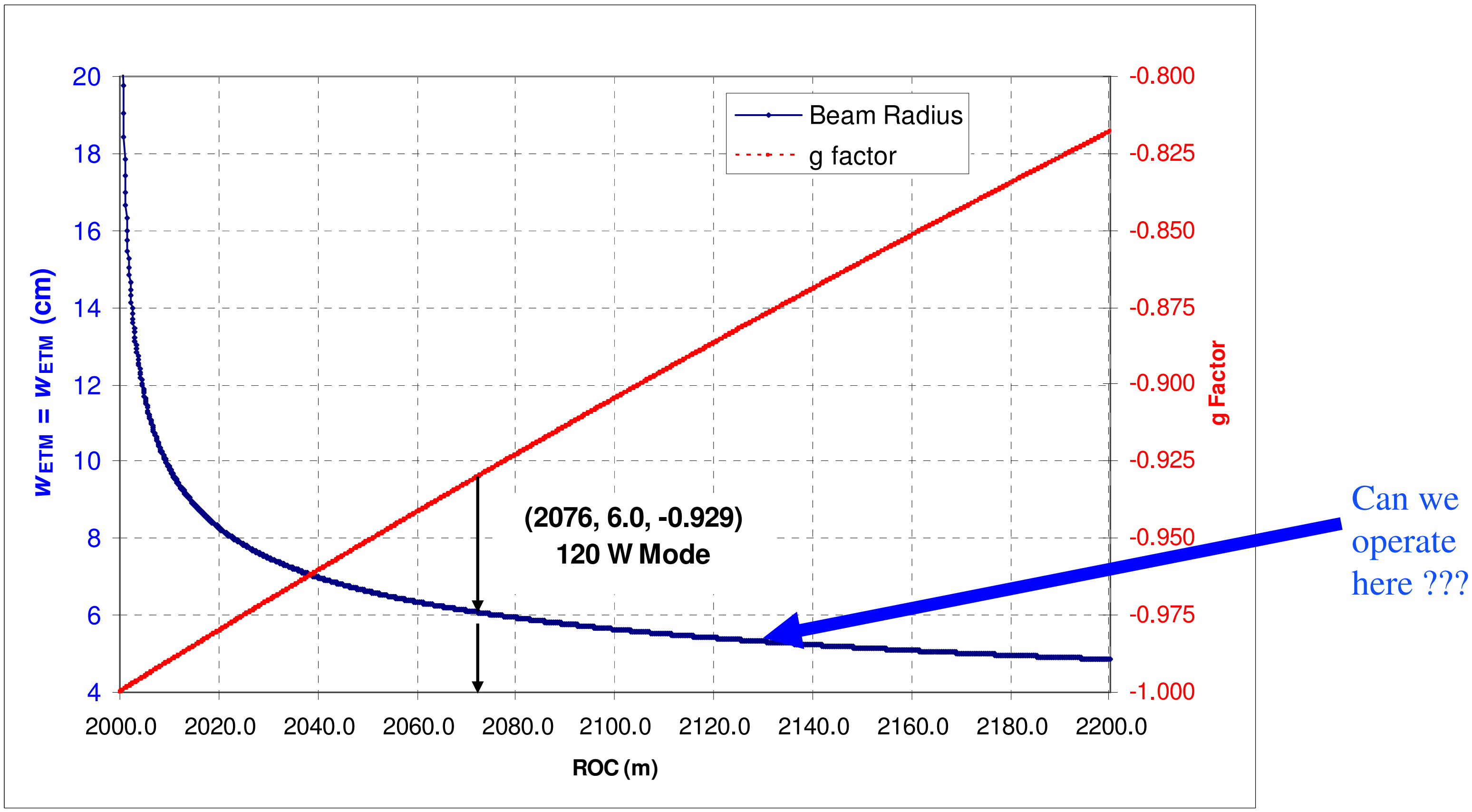


Inherently ‘Athermal’ Cavity

Reduce the Beam Size

Increase the ROC

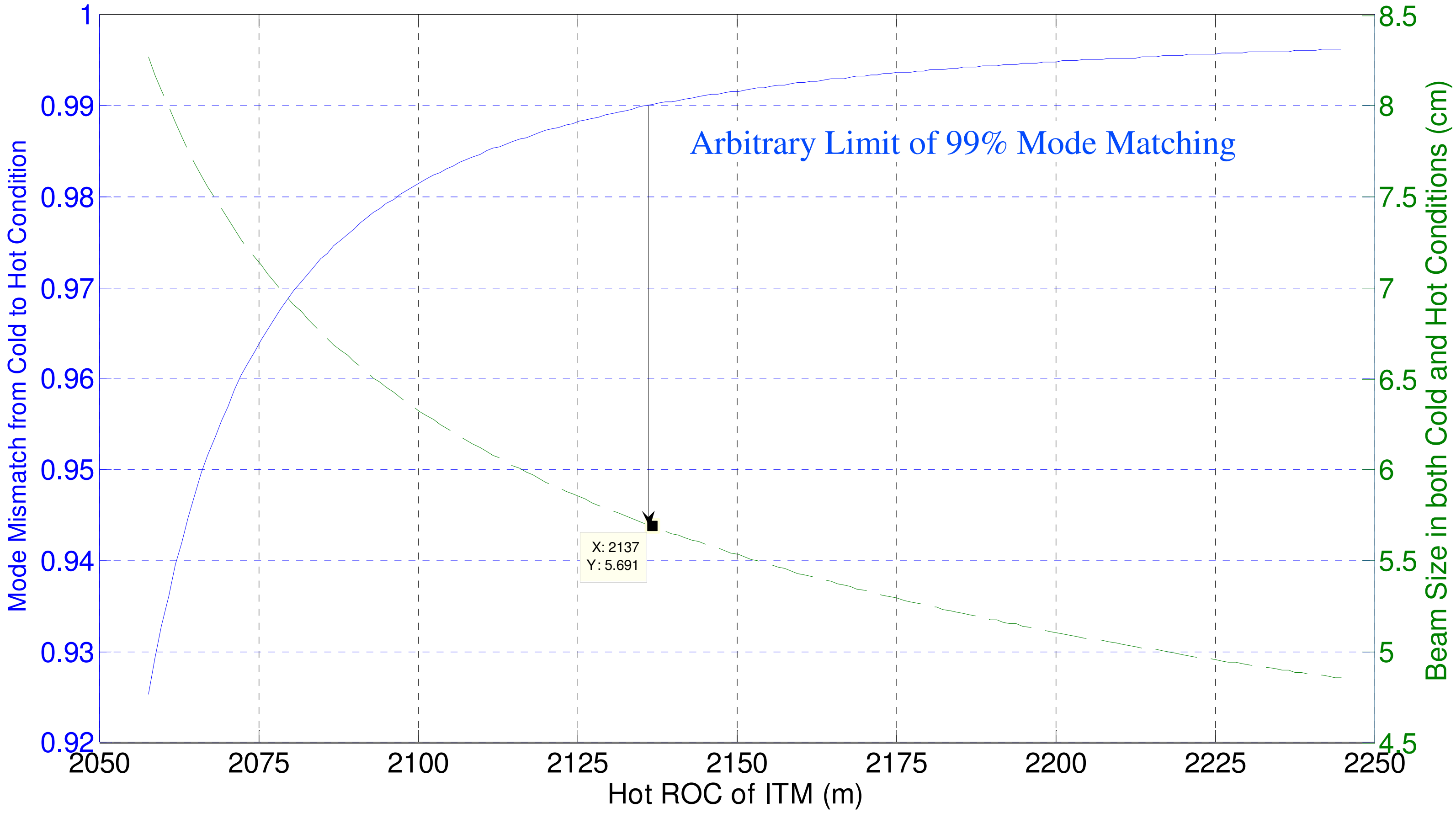
Reduced beam Size Operation



- No Correction Required
- Thermally Stable
- No differential problems

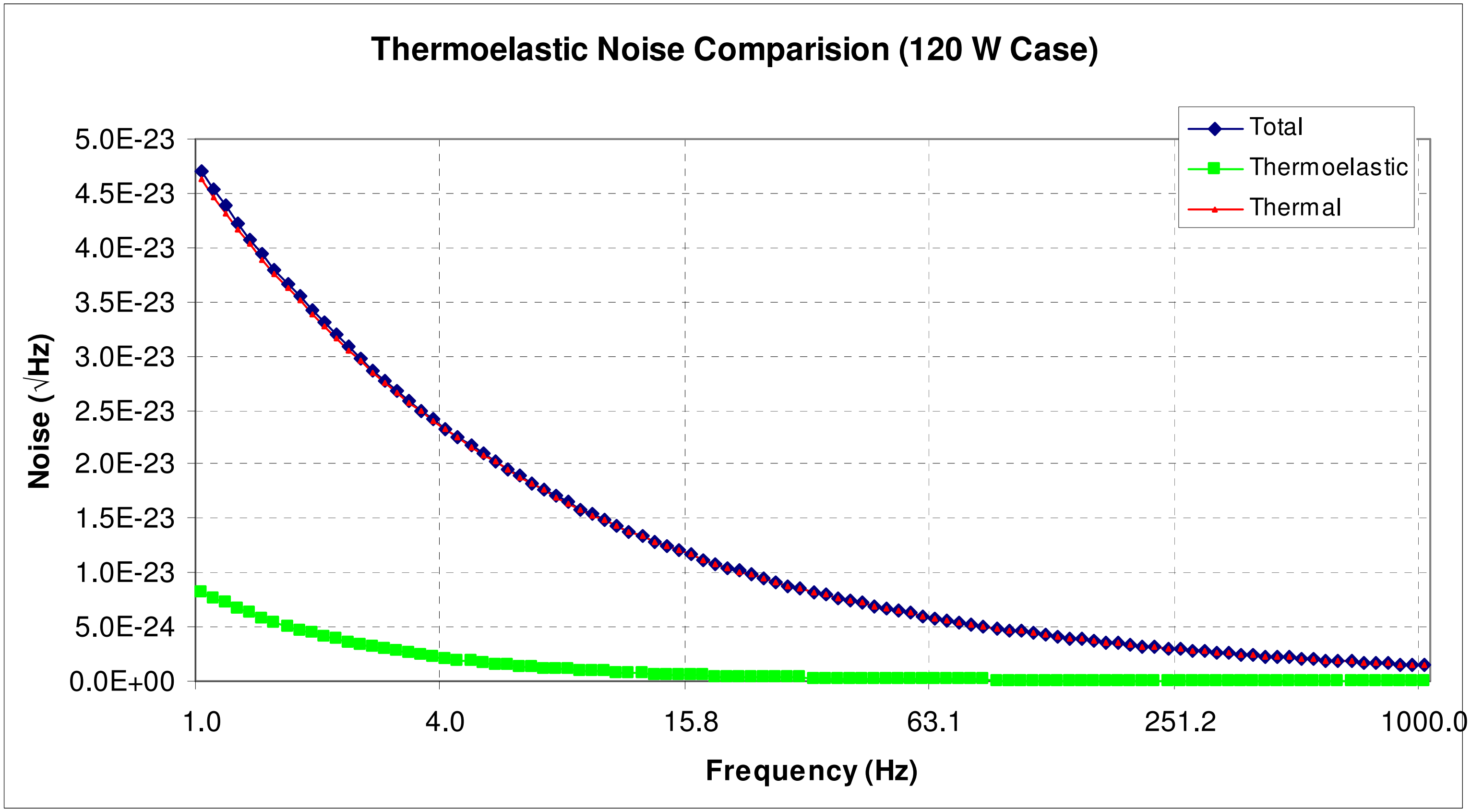
- Increased Noise
- Reduces Sensitivity
- Deviates from 6.0 cm magical beam size value ??

Thermally Insensitive Operation at Reduced Beam Size



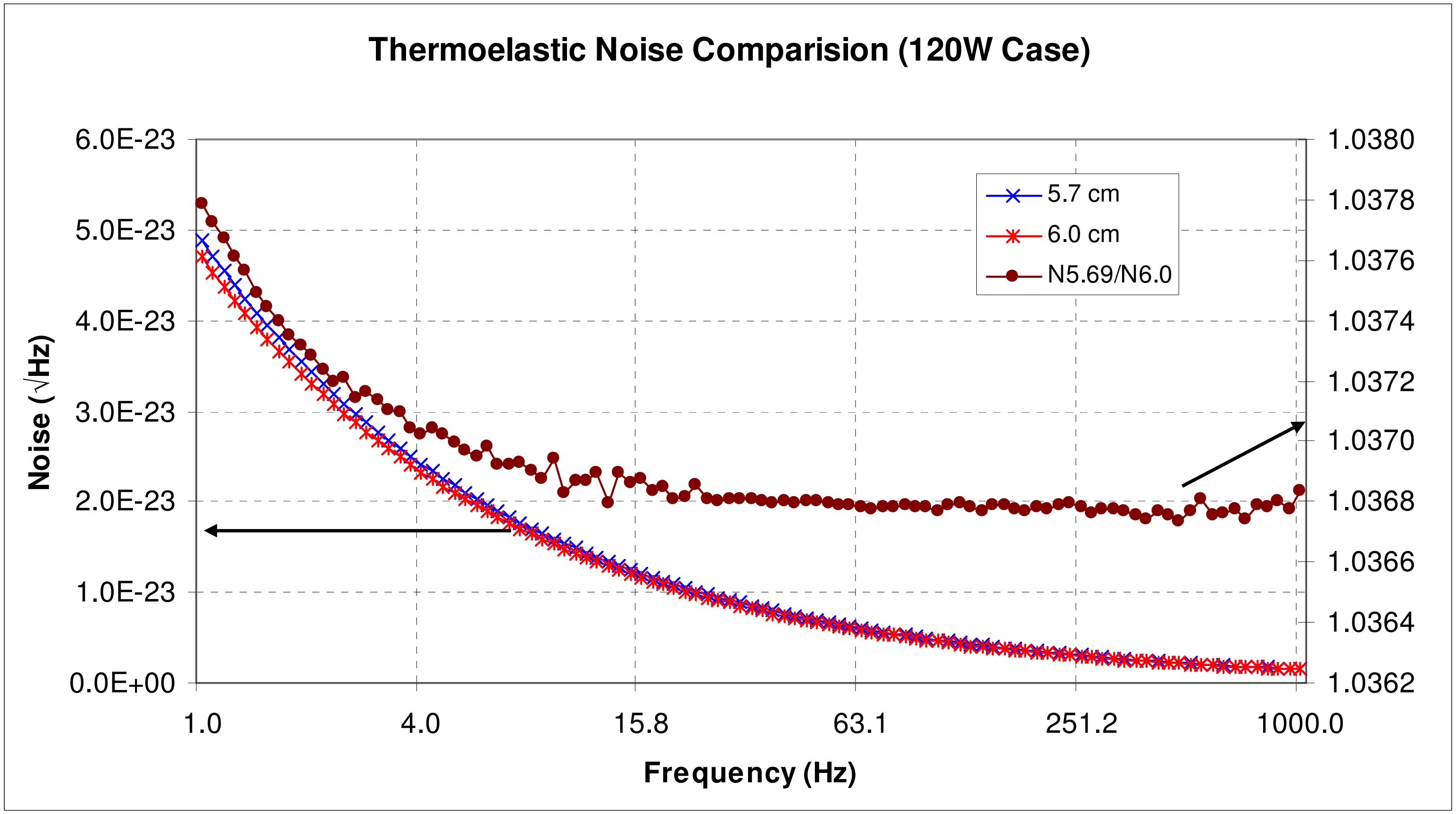
- Hot ROC = 2137 m, Cold ROC = 2096.27m
- Beam Size = 5.7 cm
- No Correction Required
- More Stable

Thermal Noise Considerations

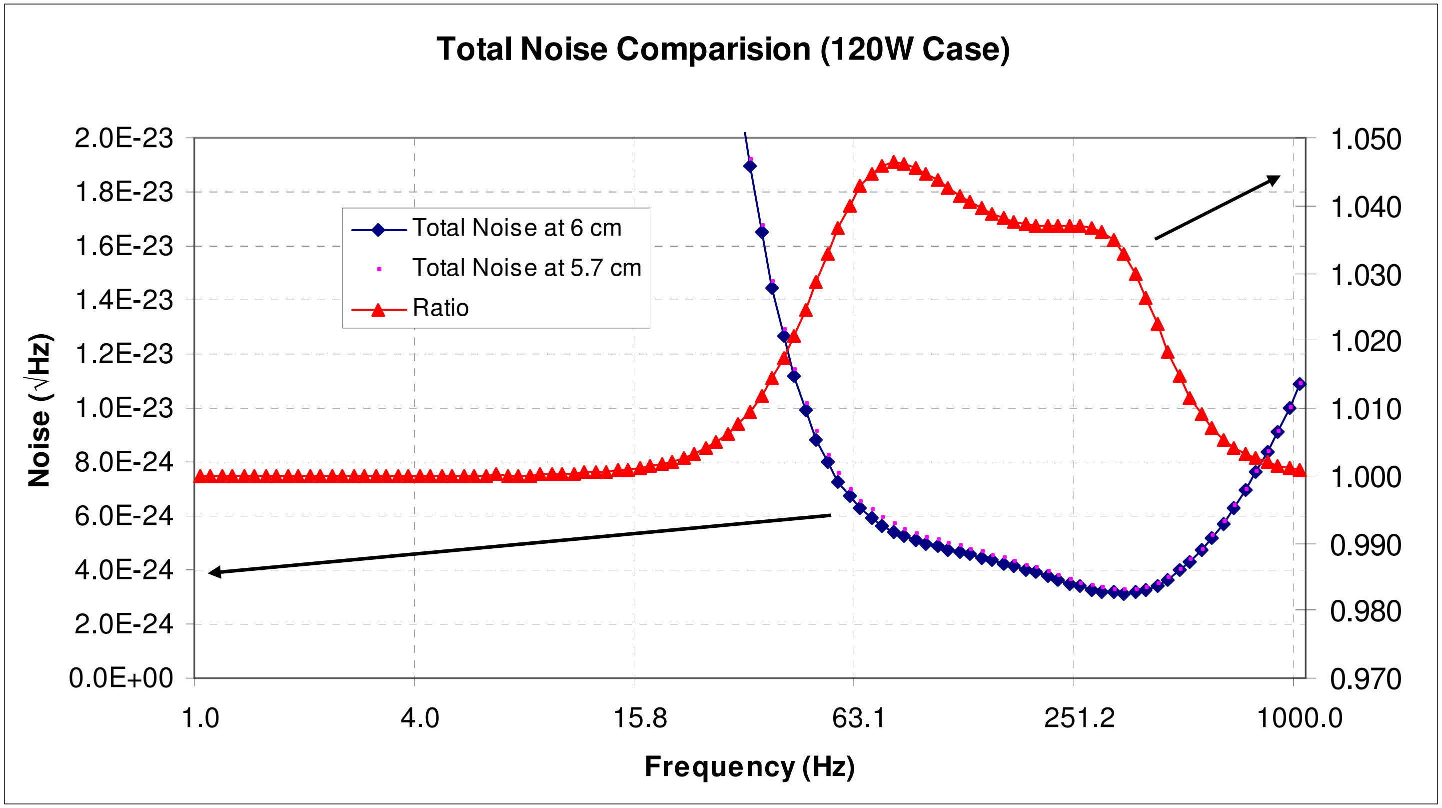


1. Thermoelastic noise scales as $1/(\text{beam size})^{3/2}$, not thermal noise
2. Thermoelastic noise is a minor contributor to the total thermal noise
3. Total Noise does not scale as $1/(\text{beam size})^{3/2}$ of beam size

Comparison of Thermoelastic Noise



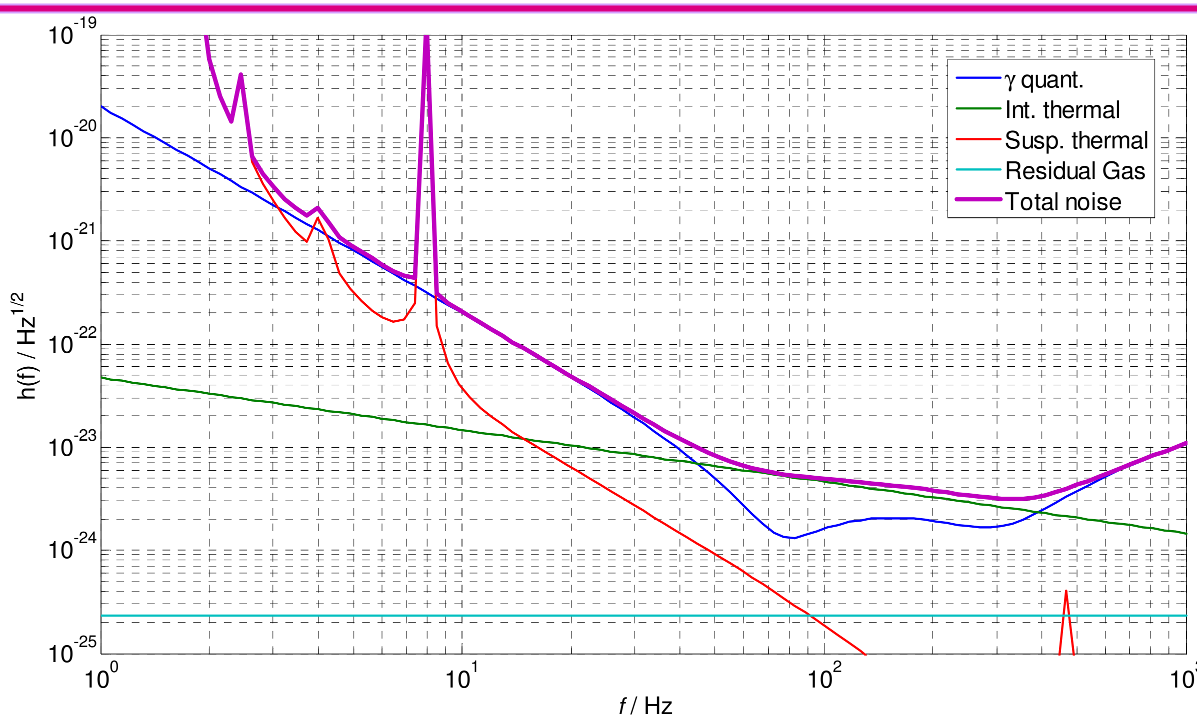
Comparison of Total Noise



Maximum hit of 4.5 % at 64 Hz

Is it acceptable??

Familiar Bench Noise Curve



Hardly any difference in Log Scale

5.7 cm Beam Size Details:

- Finesse: 1238.07
- Power Recycling Factor: 16.83
- Power on beam splitter: 2019.75
- NS Binary Inspiral range: 126.24 Mpc
- Stochastic signal sensitivity: 6.836e-009/h^2

6.0 cm Beam Size Details:

- Finesse: 1238.07
- Power Recycling Factor: 16.83
- Power on beam splitter: 2019.75
- NS Binary Inspiral range: 130.61 Mpc
- Stochastic signal sensitivity: 6.682e-009/h^2

Can we sacrifice a little sensitivity ??

Summary/Recommendations