# Advances in coherent network searches for gravitational-wave bursts

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# Outline

- Standard Formulation of Coherent Analysis for GWBs
  - detection
  - waveform estimation
  - consistency test (GWBs vs. glitches)
  - source location

- Recent Advances (last 12 months):
  - regularized likelihoods
  - improved consistency tests
  - maximum entropy waveform estimation

# Coherent Analysis for GWBs

“Standard Likelihood” Formulation

# The Basic Problem & Response

- Output of $D \geq 3$ detectors with noise amplitudes $\sigma_i$:
  - Waveforms $h_+(t)$, $h_x(t)$, source direction $\Omega$ all unknown. *How do we find them?*

data — antenna responses ($\Omega$ unknown) — GWB (unknown) — white noise

$$\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_D \end{bmatrix} = \begin{bmatrix} F_1^+(\Omega)/\sigma_1 & F_1^\times(\Omega)/\sigma_1 \\ F_2^+(\Omega)/\sigma_2 & F_2^\times(\Omega)/\sigma_2 \\ \vdots & \vdots \\ F_D^+(\Omega)/\sigma_D & F_D^\times(\Omega)/\sigma_D \end{bmatrix} \begin{bmatrix} h^+ \\ h^\times \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_D \end{bmatrix}$$

- Approach: Treat $\Omega$ and $h_+(t)$, $h_x(t)$ as parameters to be fit by the data.
  - Scan over the sky ($\Omega$).
  - At each sky position construct the least-squares fit to $h_+$, $h_x$ from the data.
  - The amplitude of $h_+$, $h_x$ (SNR) and the quality of the fit ($\chi^2$) determine if a GWB is detected.

# The Modern View

Gursel & Tinto PRD **40** 3884 (1989)
Flanagan & Hughes, PRD **57** 4566 (1998)

Follow formulation by Rakhmanov, gr-qc/0604005. Adopt matrix notation:

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_D \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} F_1^+(\Omega)/\sigma_1 & F_1^\times(\Omega)/\sigma_1 \\ F_2^+(\Omega)/\sigma_2 & F_2^\times(\Omega)/\sigma_2 \\ \vdots & \vdots \\ F_D^+(\Omega)/\sigma_D & F_D^\times(\Omega)/\sigma_D \end{bmatrix} \qquad \mathbf{h} = \begin{bmatrix} h^+ \\ h^\times \end{bmatrix} \qquad \mathbf{n} = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_D \end{bmatrix}$$

Dx1 Dx2 2x1 Dx1

Vector of network data values:

$$\mathbf{d} = \mathbf{F}\mathbf{h} + \mathbf{n}$$

# Wavefrom Estimation by Least-Squares

For trial sky position $\Omega$, compute $\mathbf{F}(\Omega)$ and find best-fit waveform $\mathbf{h}$ that minimizes residual $(\mathbf{d}-\mathbf{F}\mathbf{h})^2$. Simple linear problem!

$$\mathbf{0} = \left.\frac{\partial(\mathbf{d}-\mathbf{F}\mathbf{h})^*(\mathbf{d}-\mathbf{F}\mathbf{h})}{\partial \mathbf{h}^*}\right|_{\mathbf{h}=\hat{\mathbf{h}}}$$

* = conjugate transpose

$$\Rightarrow \quad \hat{\mathbf{h}} = (\mathbf{F}^*\mathbf{F})^{-1}\mathbf{F}^*\mathbf{d}$$

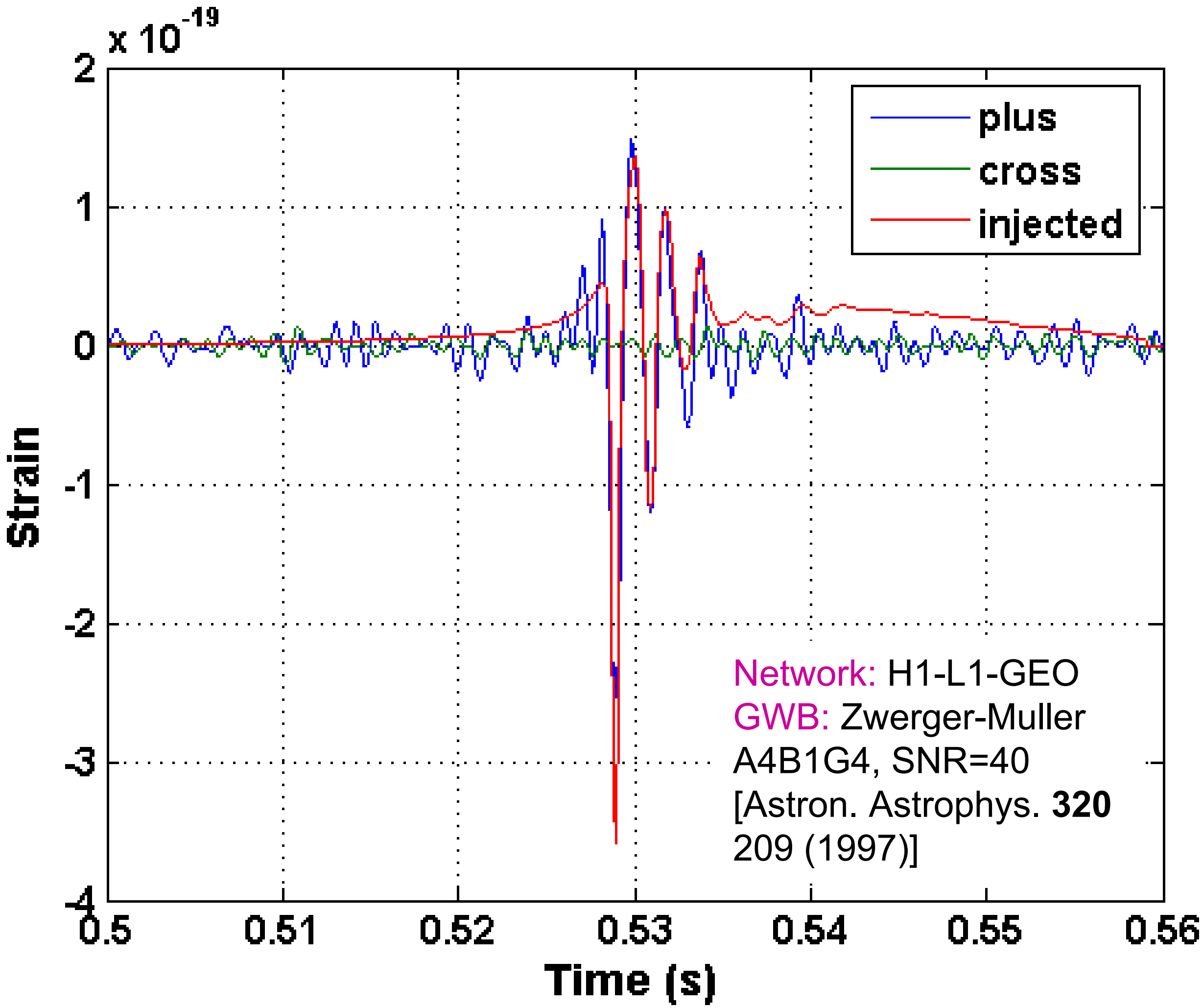
linear best-fit solution for $h_+$, $h_x$

“Moore-Penrose inverse” (2xD matrix) :

$$\mathbf{F}_{\mathbf{MP}}^{-1} := (\mathbf{F}^*\mathbf{F})^{-1}\mathbf{F}^* \qquad \mathbf{F}_{\mathbf{MP}}^{-1}\mathbf{F} = \mathbf{I}$$

$$\hat{\mathbf{h}} = \mathbf{F}_{\mathbf{MP}}^{-1}(\Omega)\,\mathbf{d}$$

# Example:  Supernova GWB Recovery




Recovered signal (blue) is a noisy, band-passed version of injected GWB signal (red)

Injected GWB signal has $h_x = 0$.

Recovered $h_x$ (green) is just noise.

# Detection from Likelihood Ratio

Is **d** due to a GWB (**h**) or Gaussian noise (**h**=0)?

Detection statistic: threshold on maximum of the *likelihood ratio*

$$\mathbf{L} \equiv \log \frac{\mathbf{P}(\mathbf{d}\,|\,\mathbf{h})}{\mathbf{P}(\mathbf{d}\,|\,\mathbf{0})} = \frac{1}{2}\underbrace{\mathbf{d}^*\mathbf{d}}_{} - \frac{1}{2}\underbrace{(\mathbf{d}-\mathbf{Fh})^*(\mathbf{d}-\mathbf{Fh})}_{}$$

"total energy" in original data

"null energy" after subtracting **h**

Maximum value of likelihood is attained for $\mathbf{h} = \hat{\mathbf{h}}$

$$\mathbf{L}_{\mathbf{max}} = \mathbf{L}(\hat{\mathbf{h}}) = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{d}^*\mathbf{F}\mathbf{F}_{\mathbf{MP}}^{\mathbf{-1}}\mathbf{d} = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{E}_{\mathbf{total}} - \mathbf{E}_{\mathbf{null}})$$

detection if $\mathbf{L}_{\mathbf{max}}$ > threshold

Flanagan & Hughes, PRD **57** 4566 (1998)
Anderson *et al*. PRD **63** 042003 (2001)

# Consistency Test: GWB vs. Glitch

Wen & Schutz, CQG **22** S1321 (2005)
Ajith, Hewitson & Heng, gr-qc/0604004 (2006)

Consistency: Is the transient a true GWB or a noise "glitch"? If a GWB, then residual data should be pure Gaussian noise ⇒ energy is $\chi^2$ distributed:

$$E_{null} \equiv \left(d - F\hat{h}\right)^{*}\left(d - F\hat{h}\right) \sim \chi^2\left(\left[D-2\right]N\right)$$

true GWB ➡

$$\left\langle E_{null} \right\rangle \approx \left[D-2\right]N \left[1 \pm O\left(\frac{1}{\sqrt{\left[D-2\right]N}}\right)\right]$$

If $E_{null}$ >> [D-2] N then reject event as noise "glitch".

# Source Location

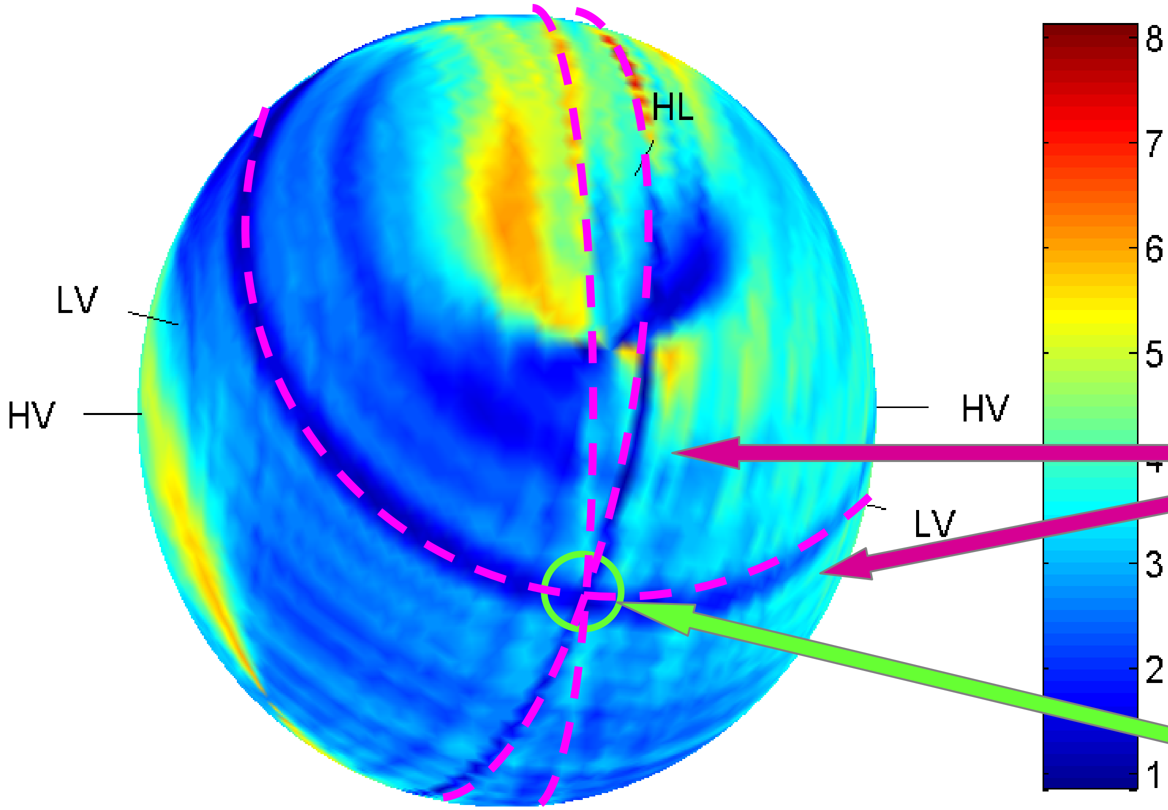
Source location: Usually we do not know the direction of the GWB source *a priori* (with exceptions: supernova, GRB, …)

Simplest solution: Moore-Penrose inverse depends on sky position:

$$\mathbf{F}_{\mathbf{MP}}^{-1} = \mathbf{F}_{\mathbf{MP}}^{-1}(\mathbf{\Omega})$$

Test over grid of sky positions. Estimate $\Omega$ as sky position with lowest $\chi^2$.

# Example: Supernova GWB




$\chi^2$ / DOF consistency with a GWB as a function of direction for a simulated supernova (~1 kpc)

Interference fringes from combining signal in two detectors.

True source location:
– intersection of fringes
– $\chi^2$ / DOF ~ 1

GWB: Dimmelmeier et al. A1B3G3 waveform, Astron. Astrophys. 393 523 (2002) , SNR = 20
Network: H1-L1-Virgo, design sensitivity

# Pros and Cons

- This standard approach is known as the "maximum likelihood" or "null stream" formalism.

- Very powerful:
  – Can detect, distinguish from noise, locate, and extract GWB waveform with no *a priori* knowledge of the waveform!

- Standard approach also has significant weaknesses:
  1. Need 3 detector sites at a minimum to fit out 2 waveforms!
  2. Very expensive on data (squanders statistics). Use up 2 detectors just fitting $h_+$, $h_x$. *(More on next slide.)*
  3. Can break down at some sky positions & frequencies (**F** becomes singular, so $\mathbf{F}_{\mathbf{MP}}^{-1}$ does not exist).

# Cost in Statistical Power compared to Templated Searches

Consistency: If a GWB, then residual energy should be $\chi^2$ distributed:

$$\langle E_{null} \rangle \equiv \frac{1}{2}\left(d - F\hat{h}\right)^* \left(d - F\hat{h}\right) \sim \chi^2\left(\underbrace{[D-2]N}\right)$$

D: number of detectors ~ 3

N: number of data samples per detector ~ 100

[D-2]N, *not* DN: Lose 2 data streams to make best-fit $h_+$, $h_x$. Very expensive loss of data and loss of statistical power for the consistency test!

Compare to, e.g., matched filter for binary neutron-star inspiral signal:

- Templates have only 2 parameters to be fit to the data (mass of each star).
- Consistency test: 3N-2 instead of N degrees of freedom.

**Not a replacement for templated searches (if you have a good template)!**

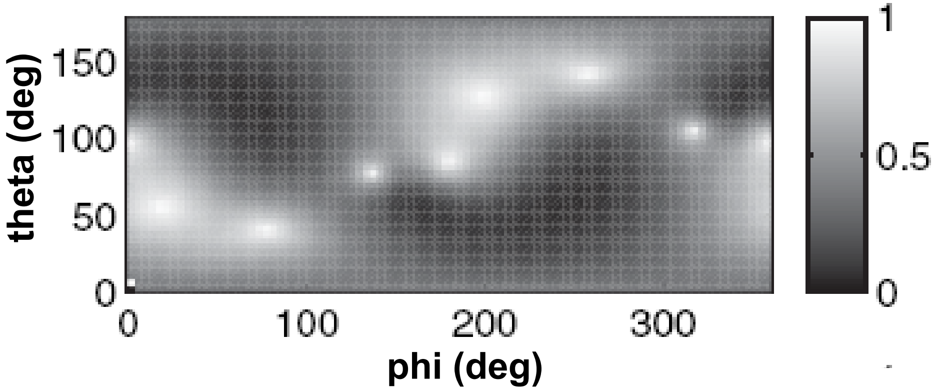
# Post-Modernism

- Over the past year, several groups have rediscovered the maximum likelihood formalism and have extended and improved it.
- Advances on all fronts of coherent analyses:
  - detection
  - consistency / veto
  - source location (Wen's talk)
  - waveform extraction
- Also some amelioration of weaknesses on previous slide.
- Rest of talk: walk through examples from each area.

# Breakdown of standard approach

Moore-Penrose inverse can be singular (ill-conditioned) for sky positions where network has poor sensitivity to one or both GW polarizations.

Alignment factor $\varepsilon$ for LIGO-GEO-Virgo network




Klimenko et al., PRD **72** 122002 (2005): can choose polarization gauge ("dominant polarization frame") such that

$$\mathbf{F}_{\mathbf{MP}}^{-1} = \frac{\mathbf{1}}{\mathbf{g}} \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \frac{\mathbf{1}}{\boldsymbol{\varepsilon}} \end{pmatrix} \mathbf{F}^{*}$$

For some $\Omega$, $\varepsilon(\Omega) << 1$. Estimated waveform for that polarization becomes noise dominated:

$$\hat{\mathbf{h}} \equiv \mathbf{F}_{\mathbf{MP}}^{-1}\mathbf{d} = \mathbf{h} + \mathbf{F}_{\mathbf{MP}}^{-1}\mathbf{n}$$

**~n/**$\varepsilon$ for one polarization

# Regularization Schemes

- Breakdown of Moore-Penrose inverse explored in several recent papers:
  - Klimenko, Mohanty, Rakhmanov, & Mitselmakher: PRD **72** 122002 (2005), J. Phys. Conf. Ser. **32** 12 (2006), gr-qc/0601076
  - Rakhmanov gr-qc/0604005
- Key advance: Regularization of Moore-Penrose inverse.
  - Effectively impose penalty factor for large values of $h_+$, $h_x$.
  - Important side benefit: allows application to 2-detector networks.

# One Example: Constraint Likelihood

- Klimenko, Mohanty, Rakhmanov, & Mitselmakher: PRD **72** 122002 (2005), J. Phys. Conf. Ser. **32** 12 (2006).
- In dominant polarization frame:

$$L_{max} = L_{max,1} + L_{max,2}$$

$$\langle L_{max,1} \rangle \approx \frac{1}{2}(g\, h_1^2 + n^2)$$

$$\langle L_{max,2} \rangle \approx \frac{1}{2}(\varepsilon\, g\, h_2^2 + n^2)$$

small GWB contribution

full noise contribution

# Constraint Likelihood

- Constraint likelihood: Lower weighting of less sensitive polarization "by hand".

$$L_{max} \Rightarrow L_{max,1}$$ "hard constraint"

$$L_{max} \Rightarrow L_{max,1} + \varepsilon L_{max,2}$$ "soft constraint"

$$L_{max,hard} \approx \frac{1}{2}(g h_1^2 + n^2)$$

zero signal and noise contribution from second polarization

$$L_{max,soft} \approx \frac{1}{2}(g h_1^2 + \varepsilon^2 g h_2^2 + [1+\varepsilon] n^2)$$

very small GWB contribution from second polarization

reduced noise contribution from second polarization

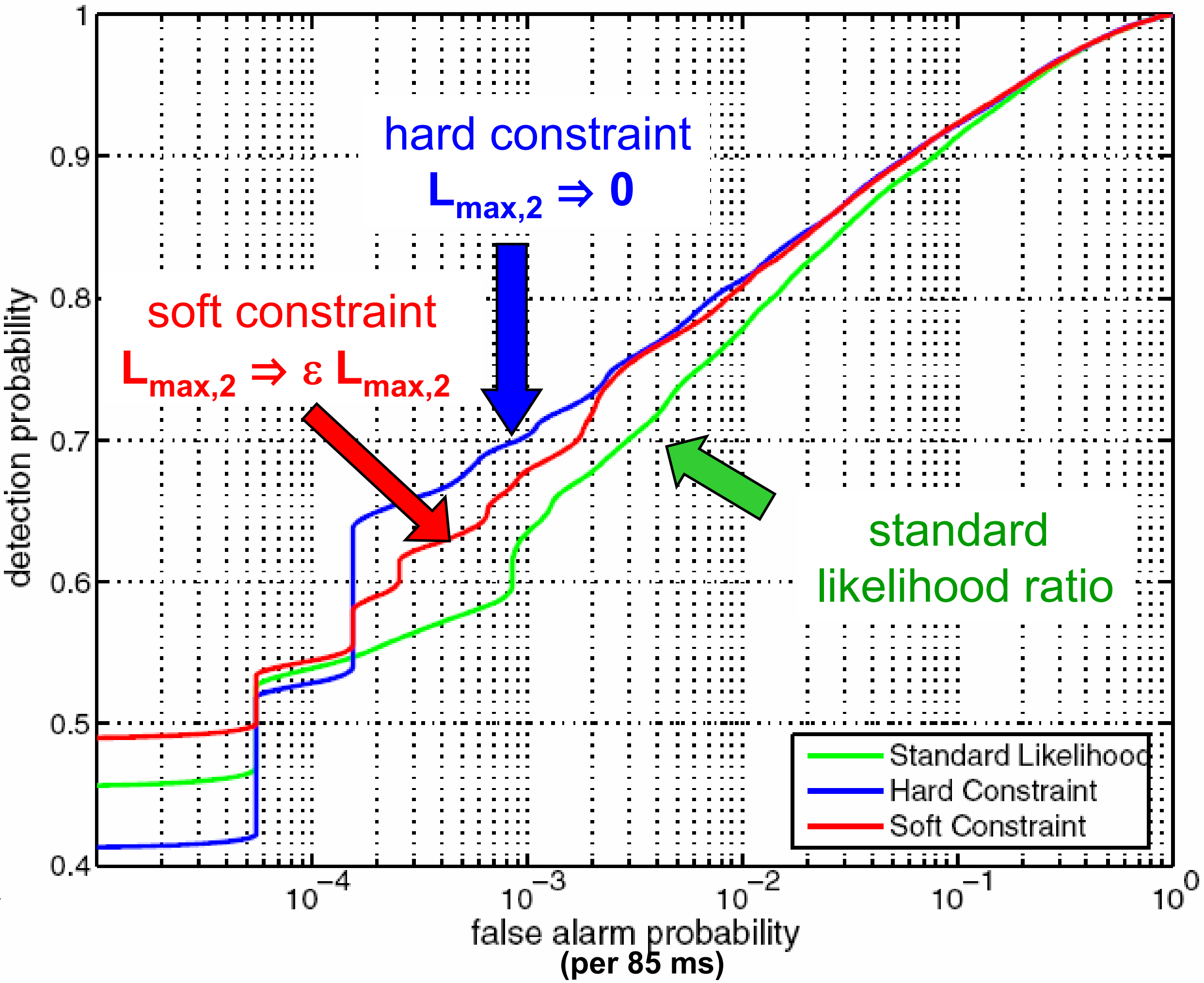
# Example: ROC for Detecting Black-Hole Mergers (again)

From Klimenko et al., PRD **72** 122002 (2005)

Injected signal:
"Lazarus" black-hole merger, SNR=6.9 [Baker et al., PRD **65** 124012 (2002)]

Network:
H1-L1-GEO (white noise approximation)

Constraint likelihoods have better detection efficiency than standard likelihood for some false alarm rates.



# Improved Consistency / Veto Test

- Real interferometers have noise glitches.
- A $\chi^2$ test can be fooled by, e.g., calibration errors (GWB not exactly subtracted out, so $\chi^2 > 1$), or weak glitches (so $\chi^2 \sim 1$).
- Chatterji, Lazzarini, Stein, Sutton, Searle, & Tinto, grqc/0605002 proposed a robust consistency test.
  - Compare energy in residual (the $\chi^2$) to that expected for *uncorrelated* glitches.

# How much cancellation is enough?

- Look at energy in residual data:

projection is function of $F^{+,\times}(\Omega)$, $\sigma(f)$

$$\mathbf{E}_{\mathbf{null}} = \left(\mathbf{d} - \mathbf{F}\hat{\mathbf{h}}\right)^2 = \begin{bmatrix} d_1^* & d_2^* & \cdots & d_D^* \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & & Q_{1D} \\ Q_{21} & Q_{22} & & Q_{2D} \\ & & \ddots & \vdots \\ Q_{D1} & Q_{D2} & \cdots & Q_{DD} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_D \end{bmatrix}$$

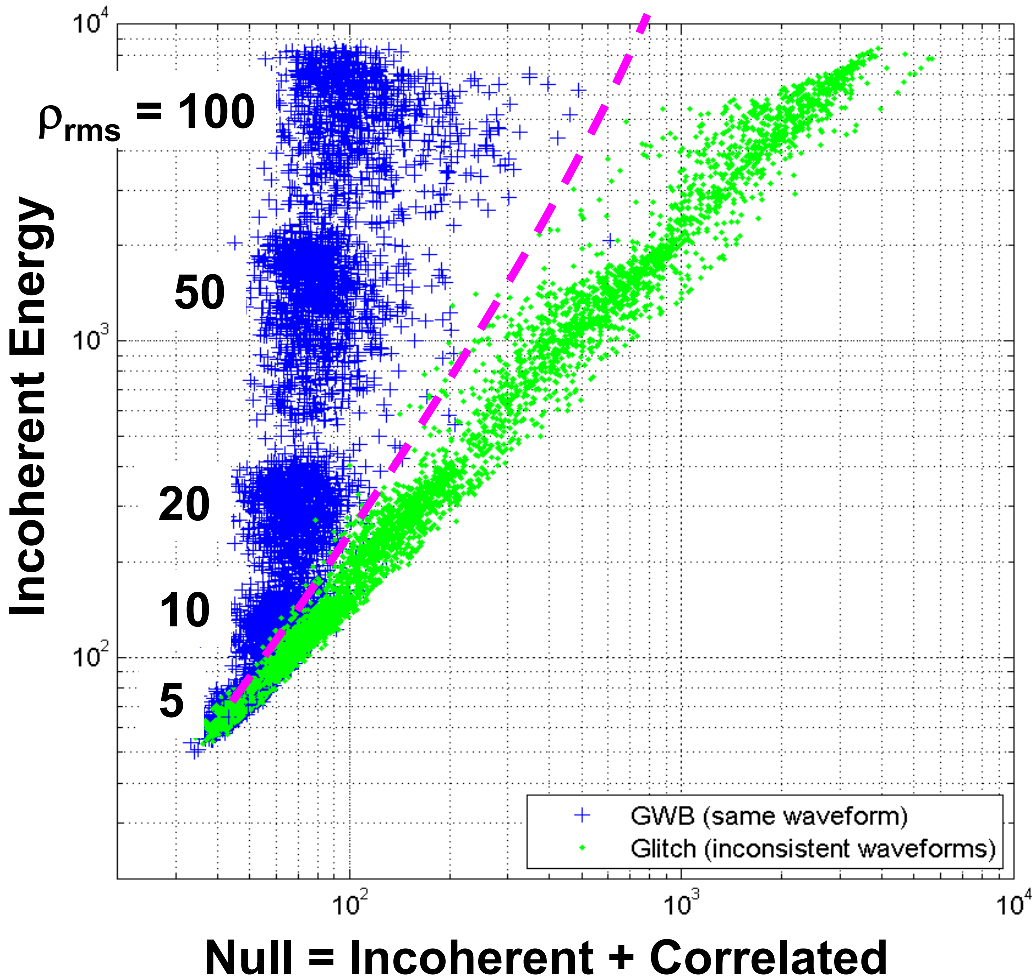
cross-correlation terms
"correlated energy"
glitch: ~ 0
GWB: = -1 x incoherent energy

auto-correlation terms
"incoherent energy"

$$\text{cancellation measure} \equiv \frac{\text{null energy } \mathbf{E}_{\mathbf{null}}}{\text{incoherent energy } \mathbf{E}_{\mathbf{inc}}} = \begin{cases} \ll \mathbf{1} & \text{GWB} \\ \boldsymbol{\sim 1} & \text{glitch} \end{cases}$$

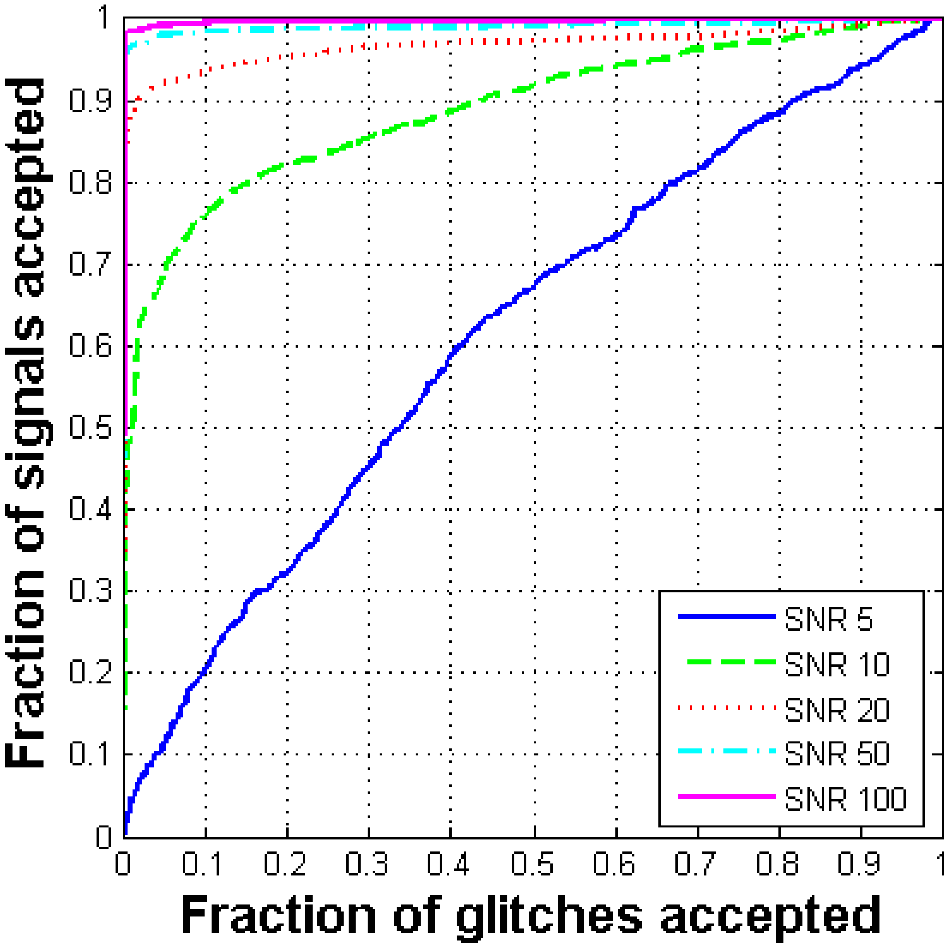
# Example: 5000 GWBs vs. 5000 Glitches

- One point from each simulation.
  - sky position giving strongest cancellation
- GWB and glitch populations clearly distinguished for SNR > 10-20.
  - Similar to detection threshold in LIGO.



# ROC: Distinguishing GWBs from Glitches

- Good discrimination for SNR > 10-20.
  - Similar to detection threshold in LIGO.



# Maximum Entropy Waveform Estimation

- *This section: work by Summerscales, Ph.D. Thesis, Pennsylvania State University (2006).*
- Another way to regularize waveform reconstruction and minimize fitting to noise.
- Add entropy prior P(h) to maximum-likelihood formulation:

I: model

$$P(\mathbf{h} \mid \mathbf{d}, I) \propto \underbrace{P(\mathbf{d} \mid \mathbf{h}, I)}_{\text{standard likelihood}} \underbrace{P(\mathbf{h} \mid I)}_{\text{prior on GWB waveform}}$$

# Maximum Entropy Cont.

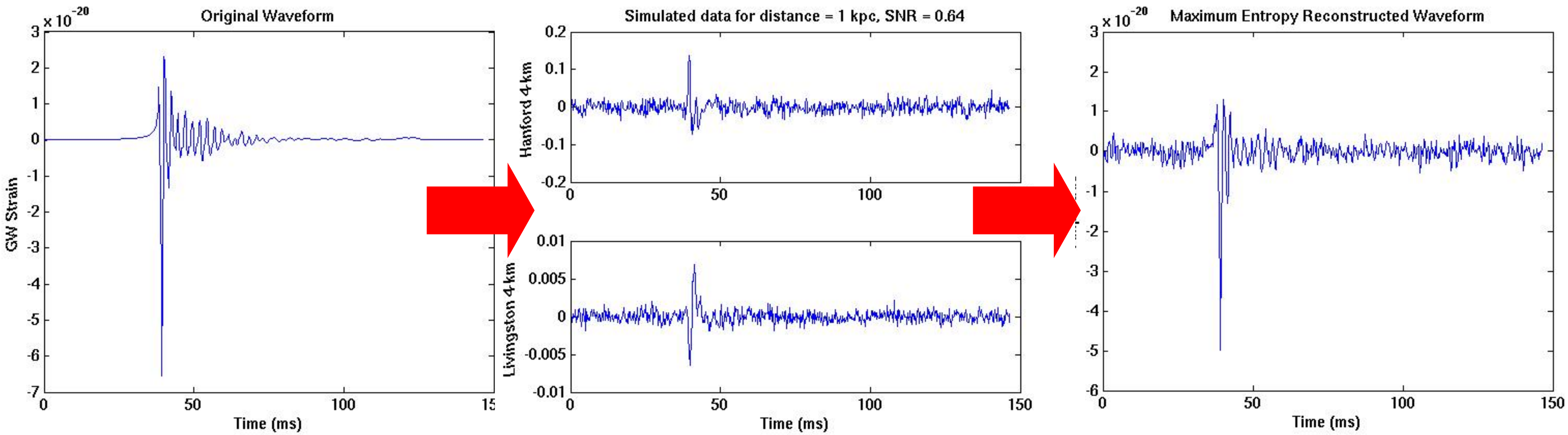
- Choice of prior:

$$P(\mathbf{h}\,|\,I) = \exp[\alpha S(\mathbf{h},\mathbf{m})]$$

$$S(\mathbf{h},\mathbf{m}) = \sum_{time\ i} \left(4{m_i}^2 + {h_i}^2\right)^{1/2} - 2m_i - h_i \log \frac{\left(4{m_i}^2 + {h_i}^2\right)^{1/2} + h_i}{2m_i}$$

  - S: Related to Shannon Information Entropy (or number of ways quanta of energy can be distributed in time to form the waveform).
    - Not quite usual $\rho \ln \rho$ form of entropy because h can be negative.
    - Hobson and Lasenby MNRAS **298** 905 (1998).
  - Model $m_i$: Mean number of “positive” or “negative” quanta per time bin *i*.
    - Determined from data **d** using Bayesian analysis.
  - $\alpha$ is a Lagrange parameter that balances being faithful to the signal (minimizing $\chi^2$) and avoiding overfitting (maximizing entropy)

# Maximum Entropy Performance, Weak Signal



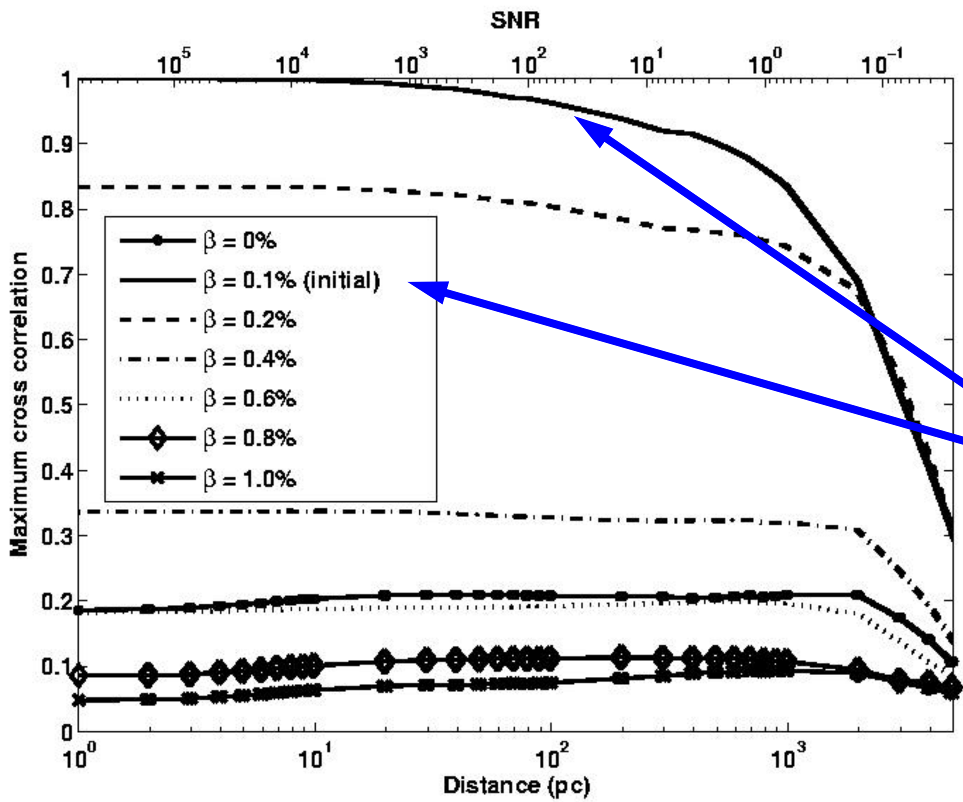

input GWB signal

noisy data from two detectors

recovered GWB signal

Summerscales, Finn, Ott, & Burrows (in preparation): study ability to recover supernova waveform parameters (rotational kinetic energy, degree of differential rotation, equation of state polytropic index).

# Extracting Rotational Information




- Cross correlations between reconstructed signal and waveforms from models that differ only by rotation parameter $\beta$ (rotational kinetic energy).
- Reconstructed signal most closely resembles waveforms from models with the same rotational parameters

# Summary

- Coherent analysis is a powerful technique for studying GWBs.
  - Matched-filter-like analysis with no *a priori* knowledge of waveform!
- The past year has seen rapid advances in coherent analysis techniques:
  - Regularization of data inversion
    - Improved detection efficiency, can apply to 2-detector networks
  - Exploration of priors on GWB waveforms (e.g. entropy)
    - Tests of ability extract physics from GWBs (supernovae)
  - Improved tests for discriminating GWBs from background noise
  - Much more work remains to be done (e.g., source localization)
- The first application of fully coherent techniques to real data is in progress
  - Constraint likelihood applied to LIGO S4 data from 2005 (stay tuned!).

# Supplemental Slides

# The Global Network

Several km-scale detectors, bars now in operation

Network gives:

- Detection confidence
- Direction by triangulation
- Waveform extraction

# Wavefrom Estimation by Least-Squares

For trial sky position $\Omega$, compute $\mathbf{F}(\Omega)$ and find best-fit waveform $\mathbf{h}$ that minimizes residual $(\mathbf{d}-\mathbf{F}\mathbf{h})^2$. Simple linear problem!

$$\mathbf{0} = \left.\frac{\partial(\mathbf{d}-\mathbf{F}\mathbf{h})^*(\mathbf{d}-\mathbf{F}\mathbf{h})}{\partial \mathbf{h}^*}\right|_{\mathbf{h}=\hat{\mathbf{h}}} = \mathbf{F}^*(\mathbf{d}-\mathbf{F}\hat{\mathbf{h}})$$

* = conjugate transpose

$$\hat{\mathbf{h}} = (\mathbf{F}^*\mathbf{F})^{-1}\mathbf{F}^*\mathbf{d}$$

linear best-fit solution for $h_+$, $h_x$

"Moore-Penrose inverse" (2xD matrix) :

$$\mathbf{F}_{\mathbf{MP}}^{-1} := (\mathbf{F}^*\mathbf{F})^{-1}\mathbf{F}^* \qquad \mathbf{F}_{\mathbf{MP}}^{-1}\mathbf{F} = \mathbf{I}$$

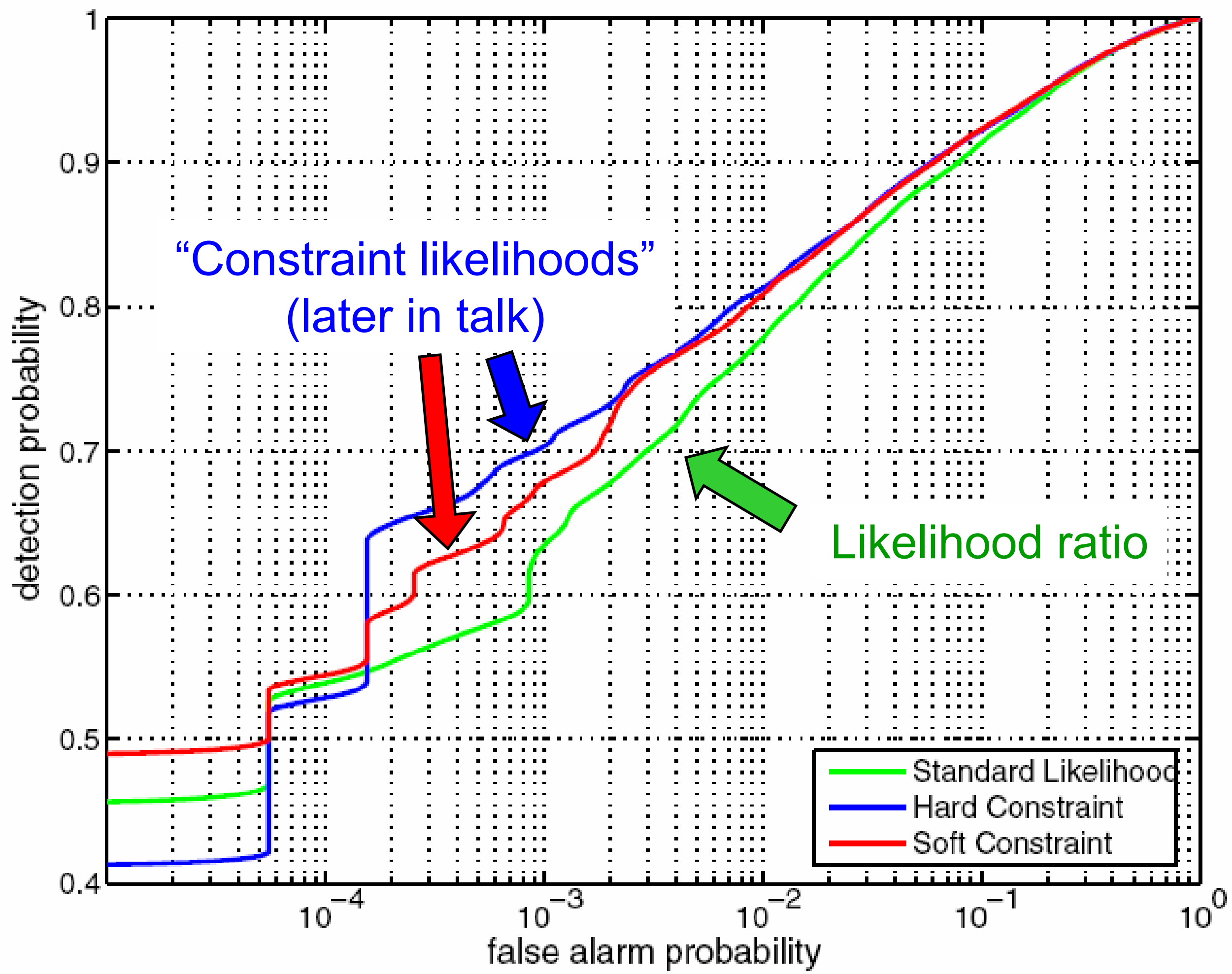
$$\hat{\mathbf{h}} = \mathbf{F}_{\mathbf{MP}}^{-1}(\Omega)\,\mathbf{d}$$

# Example: ROC for Detecting Black-Hole Mergers

From Klimenko et al., PRD **72** 122002 (2005)

Injected signal:
"Lazarus" black-hole merger, SNR=6.9 [Baker et al., PRD **65** 124012 (2002)]

Network:
H1-L1-GEO (white noise approximation)



# (Brief) History of Coherent Techniques for GWBs

# Ancient History

- Y. Gursel & M. Tinto PRD **40** 3884 (1989)
  - *"Near Optimal Solution to the Inverse Problem for GWBs"*.

- First solution of inverse problem for GWBs.
  - Source location, waveform extraction.
  - For detectors at 3 sites.

- Procedure:
  - Use 2 detectors to estimate GWB waveforms at each point on the sky.
  - Check estimated waveform for consistency with data from 3rd detector ($\chi^2$ test).
  - Symmetrize $\chi^2$ expression over the 3 detectors.
  - Used timing estimates to restrict region of sky to be scanned, find minimum of $\chi^2(\Omega)$.

# Medieval Times

- E.E. Flanagan & S.A. Hughes, PRD 57 4566 (1998)
  - *"Measuring gravitational waves from binary black hole coalescences: II. the waves' information and its extraction, with and without templates"*
  - Appendix A (!)

- Discovered maximum-likelihood formulation of detection & inverse problems.
  - Generalized to 3+ detectors, colored noise.
  - Equivalent to Gursel-Tinto for 3 detectors.

# One Example: Constraint Likelihood

- Klimenko, Mohanty, Rakhmanov, & Mitselmakher: PRD **72** 122002 (2005), J. Phys. Conf. Ser. **32** 12 (2006).
- In dominant polarization frame:

$$L_{max} = L_{max,1} + L_{max,2} \qquad F_{MP}^{-1} = \frac{1}{g}\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\varepsilon} \end{pmatrix} F^*$$

$$L_{max,1} = \frac{1}{2g}\left(\left[F_1/\sigma\right]^* d\right)^2 \approx \frac{1}{2}\left(g\, h_1^2 + n^2\right)$$

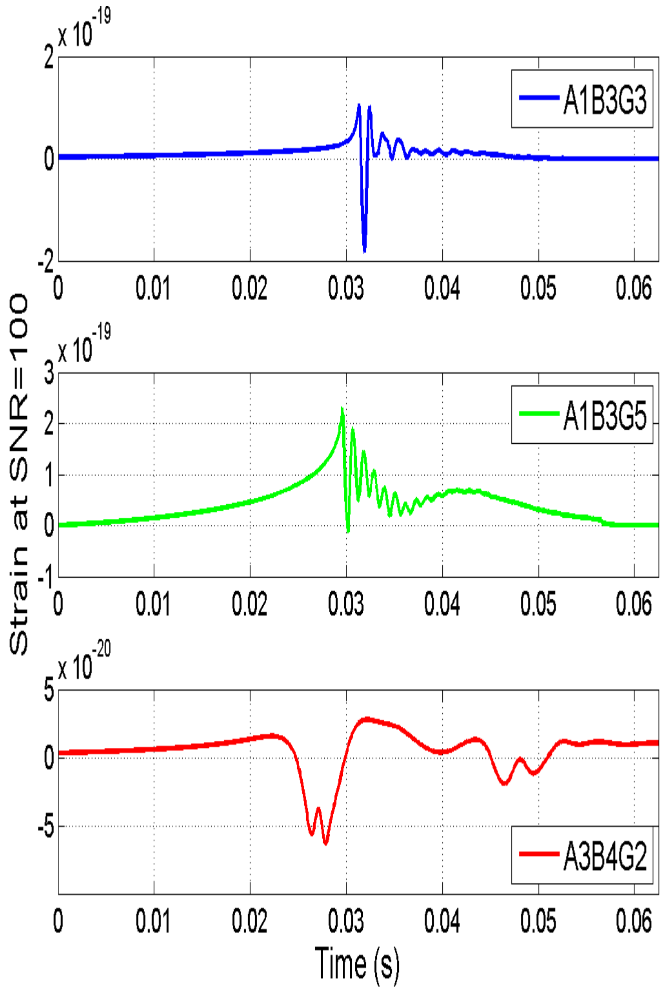
$$L_{max,2} = \frac{1}{\varepsilon}\frac{1}{2g}\left(\left[F_2/\sigma\right]^* d\right)^2 \approx \frac{1}{2}\left(\varepsilon\, g\, h_2^2 + n^2\right)$$

small GWB contribution

full noise contribution

# Testing the method

- GWBs:
  - 3 core-collapse supernova waveforms.
    - Dimmelmeier, Font, & Müller, A&A **393** 523-542 (2002).
  - Pick one DFM and add to each detector data stream.
- Glitches:
  - Inject a different supernova waveform into each detector
  - Use same time delays, amplitudes as a GWB. Pathological glitches!
- Detector Network:
  - LIGO-Virgo network @ design sensitivity



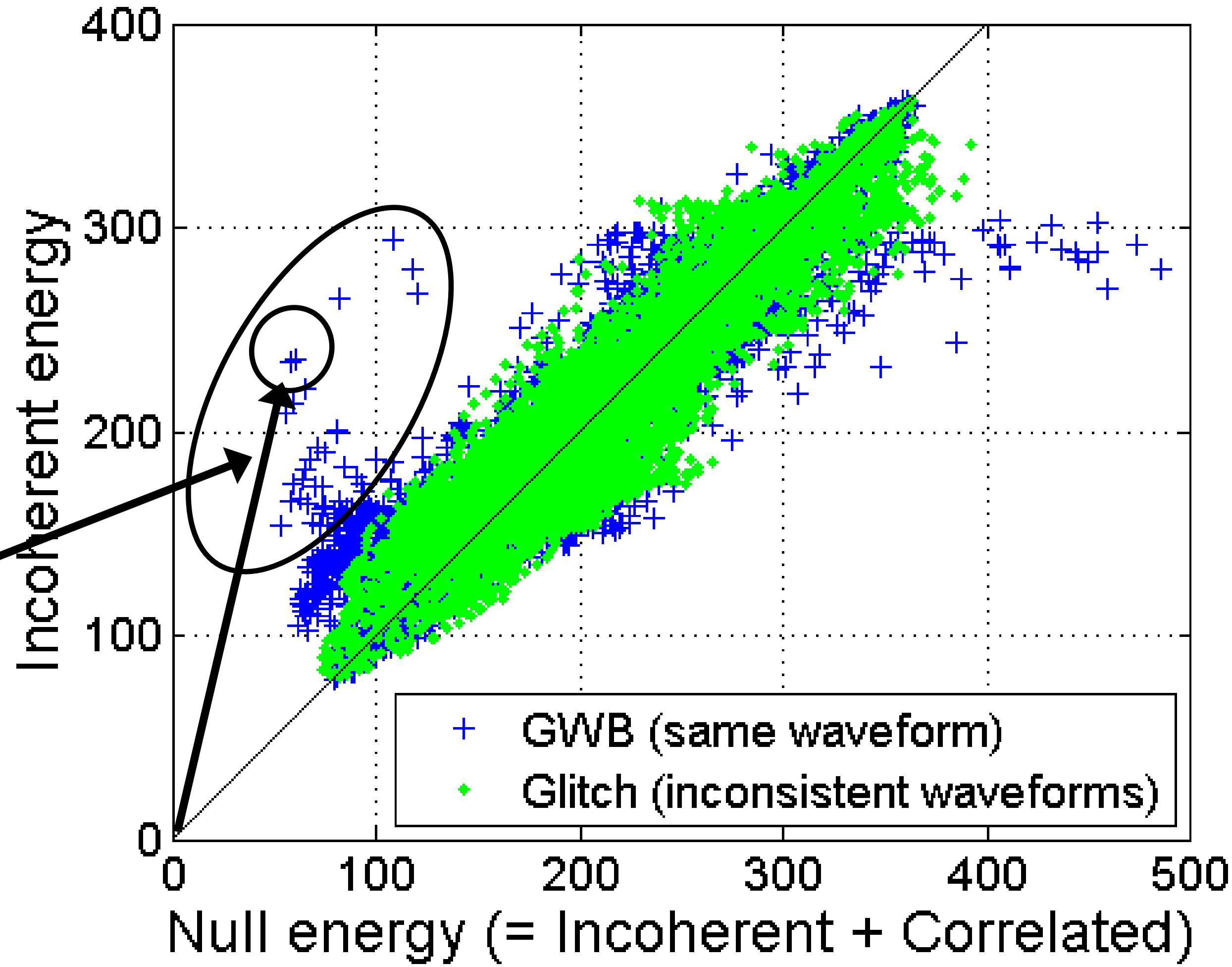
# Example: 1 GWB vs. 1 Glitch

One point for each sky position tested ($10^4$ total).

Glitch lies on diagonal $\mathbf{E_{null} \sim E_{inc}}$ (low correlation)

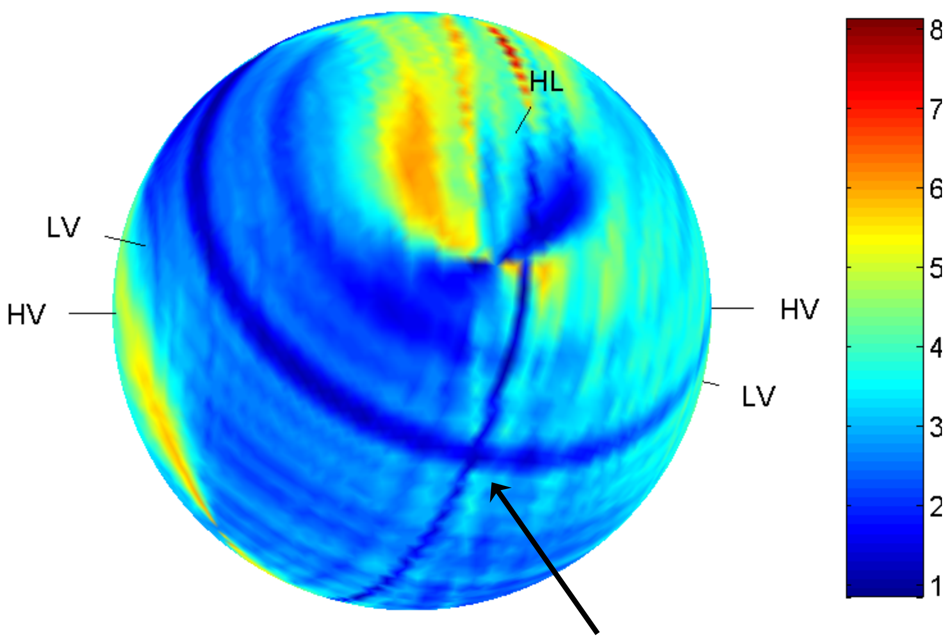
GWB has off-diagonal points $\mathbf{E_{null} \sim E_{inc}}$ (high correlation)

Strongest cancellation (lowest $\mathbf{E_{null}/E_{inc}}$)



# Source Localization: Not Good!

$E_{null}$ across the sky for 1 GWB
(Hanford-Livingston-Virgo network)



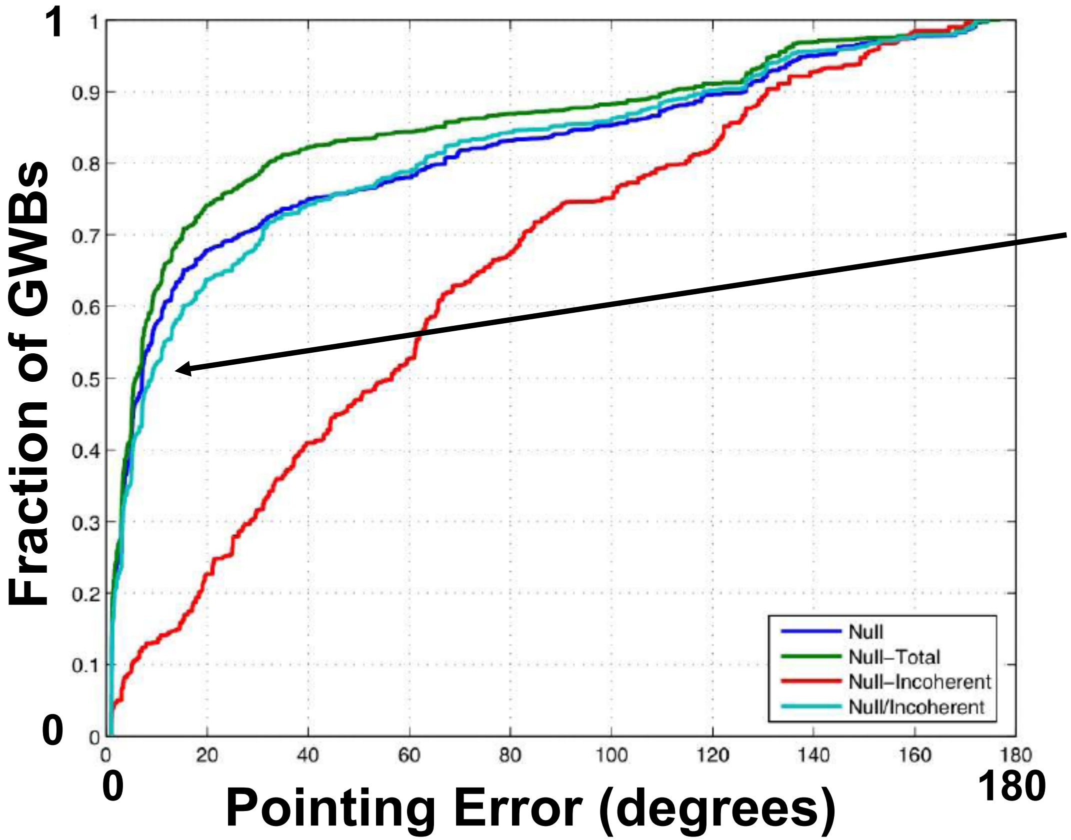

source
location

Null energy $E_{null}$ varies slowly along rings of constant time delay with respect to any detector pair.

Noise fluctuations often move minimum away from source location (pointing error)

# Example: Lazarus Black Hole Mergers, SNR = 100 (!) (H-L-V network)

**Pointing Error for $10^4$ Lazarus GWBs**




L. Stein B.Sc. Thesis, Caltech, 2006.

Median pointing error O(10) degrees.

Must use additional information for accurate source localization!

- E.g.: timing or global ring structure rather than local energy
- More research required!

# Maximum Entropy Cont.

- Maximizing P(**h**|**d**,I) equivalent to minimizing

$$F(\mathbf{h} \,|\, \mathbf{d}, \mathbf{R}, \mathbf{N}, \mathbf{m}) = \chi^2(\mathbf{R}, \mathbf{h}, \mathbf{d}, \mathbf{N}) - 2\alpha S(\mathbf{h}, \mathbf{m})$$

  – $\alpha$ is a Lagrange parameter that balances being faithful to the signal (minimizing $\chi^2$) and avoiding overfitting (maximizing entropy)
  – $\alpha$ associated with constraint which can be formally established. In summary: half the data contain information about the signal

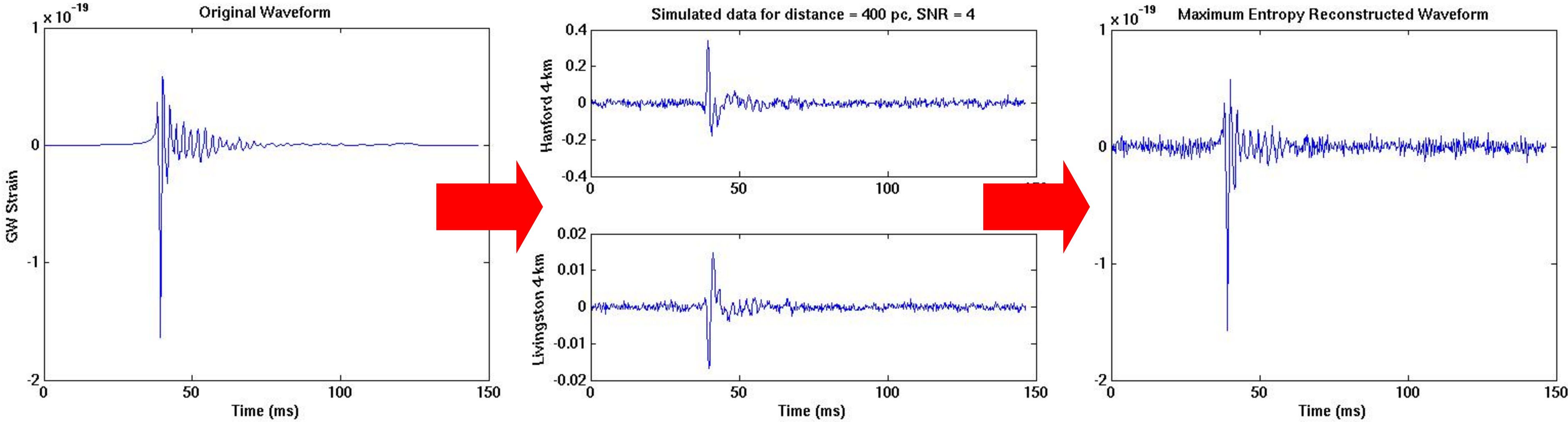
# Maximum Entropy Cont.

- Choosing **m**
  - Pick a simple model where all elements $m_i = m$
  - Model m related to the variance of the signal which is unknown
  - Using Bayes' Theorem: $P(m|\mathbf{d}) \propto P(\mathbf{d}|m)P(m)$
  - Assuming no prior preference, the best m maximizes $P(\mathbf{d}|m)$
  - Bayes again: $P(\mathbf{h}|\mathbf{d},m)P(\mathbf{d}|m) = P(\mathbf{d}|\mathbf{h},m)P(\mathbf{h}|m)$
  - Integrate over **h**: $P(\mathbf{d}|m) = \int D\mathbf{h}\, P(\mathbf{d}|\mathbf{h},m)P(\mathbf{h}|m)$ where

$$P(\mathbf{d}\,|\,\mathbf{h},\mathbf{m}) = \frac{\exp(-\chi^2/2)}{\int D\mathbf{d}\exp(-\chi^2/2)} \qquad P(\mathbf{h}\,|\,\mathbf{m}) = \frac{\exp(\alpha S)}{\int D\mathbf{h}\exp(\alpha S)}$$

  - Evaluate $P(\mathbf{d}|m)$ with m ranging over several orders of magnitude and pick the m for which it is highest

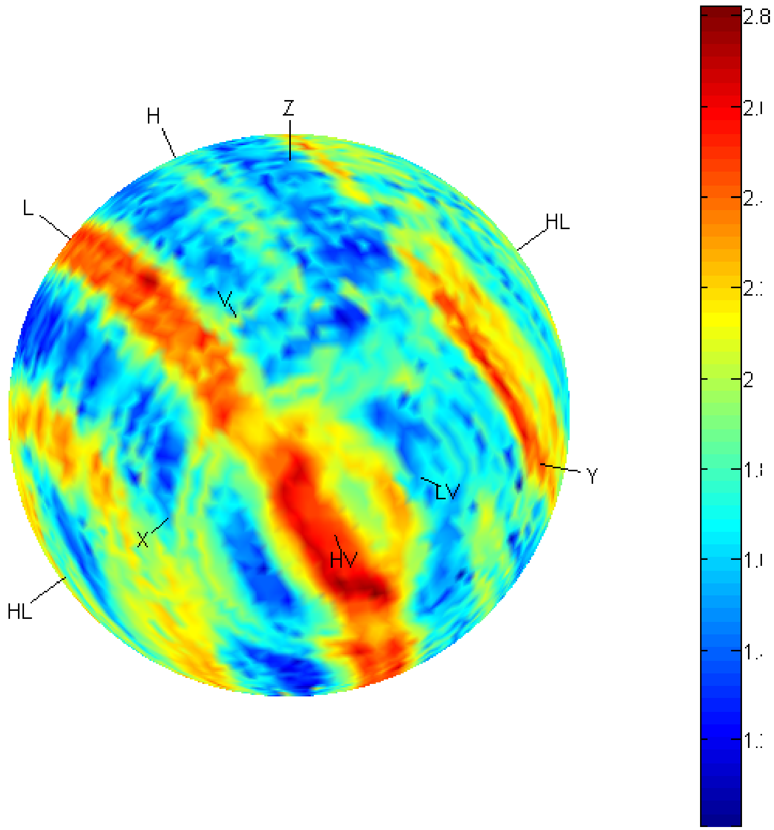
# Maximum Entropy Performance, Strong Signal

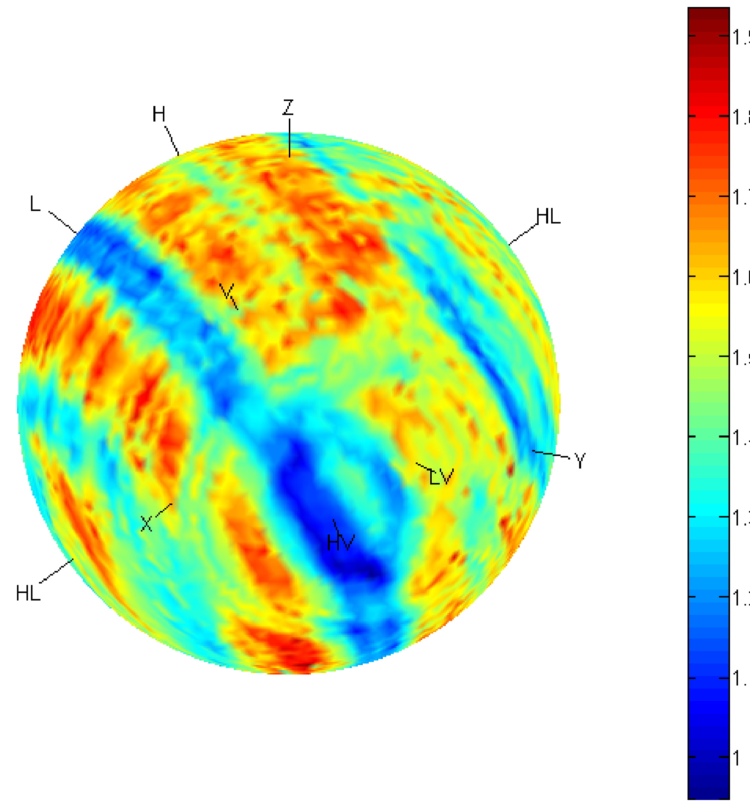



- Maximum entropy recovers waveform with only a small amount of noise added

# Sky Maps: Null Energy / DOF

**GWB: Insert a Lazarus null energy map, Tot-Null map**




- $\chi^2 \sim 1$ somewhere for both GWB an

# Summary

- Gravitational-wave bursts are an interesting class of GW signals
  - Probes of physics of supernovae, black-hole mergers, gamma-ray burst engines, …
- Coherent data analysis for GWBs using the global network of GW detectors is a potentially powerful tool for
  - detection
  - source localization
  - waveform extraction
  - consistency testing