WhiskyMHD: a New Numerical Code for General Relativistic Magnetohydrodynamics

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Plan of the talk

- The need of a better (MHD) Whisky
- Outline of the code
- Testing the code
- Computation of initial models
- Summary and conclusions

Importance of Magnetic Fields

- GRB progenitors
- Jet formation from BH
- Gravitational Waves from collapse and NS mergers

To study all these phenomena we need to evolve both Einstein and MHD equations

Previous works

GRMHD code with a non-evolutionary metric:

- Komissarov 1999, MNRAS 303, 343
- Gammie, McKinney and Toth 2003, ApJ 589,444
- De Villiers and Hawley 2003, ApJ 589, 458
- Nishikawa, Richardson, Koide, Shibata, Kudoh, Hardee and Fishman 2005, ApJ 625, 60
- Anton, Zanotti et al 2006, ApJ 637, 296

Full GRMHD codes:

- Duez, Liu, Shapiro and Stephens 2005, PRD 72, 024028

From `Whisky` to `WhiskyMHD`

With the `Whisky` code we were already able to study several different astrophysical scenario with attention to the emission of gws:

- The collapse of uniformly rotating NS
- The collapse of differentially rotating NS
- Head-on collision of two NSs
- Head-on collision of a Ns with a BH

Our goal is to extend all the work done so far to the case in which magnetic fields are present.

The WhiskyMHD code

- Full GR Magneto-Hydro-Dynamical Code
  - Based on the Cactus toolkit (www.cactuscode.org)
  - Solves the MHD equations on dynamical curved background
  - Uses HRSC (High Resolution Shock Capturing) methods
  - Implements a Constrained Transport scheme to enforce div·B=0 during the evolution
  - Can activate domain Excision
  - Implements the Method of Line
  - Adopts fixed and Progressive Mesh Refinement techniques (Carpet)

GRMHD equations

The metric is expressed in the usual 3+1 formalism

$$ds^2 = -\left(\alpha^2 - \beta_i \beta^i\right) dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j$$

Where α is the lapse, β^i is the shift vector, γ_{ij} is the spatial metric

The evolution of the magnetic field obeys Maxwell's equations:

$$F_{[\mu\nu,\lambda]} = 0$$

that give the divergence free condition:

$$\vec{\nabla} \cdot \left(\sqrt{\gamma}\vec{B}\right) = 0$$

and the equations for the evolution of the magnetic field:

$$\frac{\partial}{\partial t}\left(\sqrt{\gamma}\vec{B}\right) = \vec{\nabla} \times \left[\left(\alpha\vec{v} - \vec{\beta}\right) \times \left(\sqrt{\gamma}\vec{B}\right)\right]$$

Matter Field evolution

- The evolution equations of the matter are given as usual by the conservation of the baryon number and energy-momentum:

$$\nabla_\mu J^\mu = 0$$

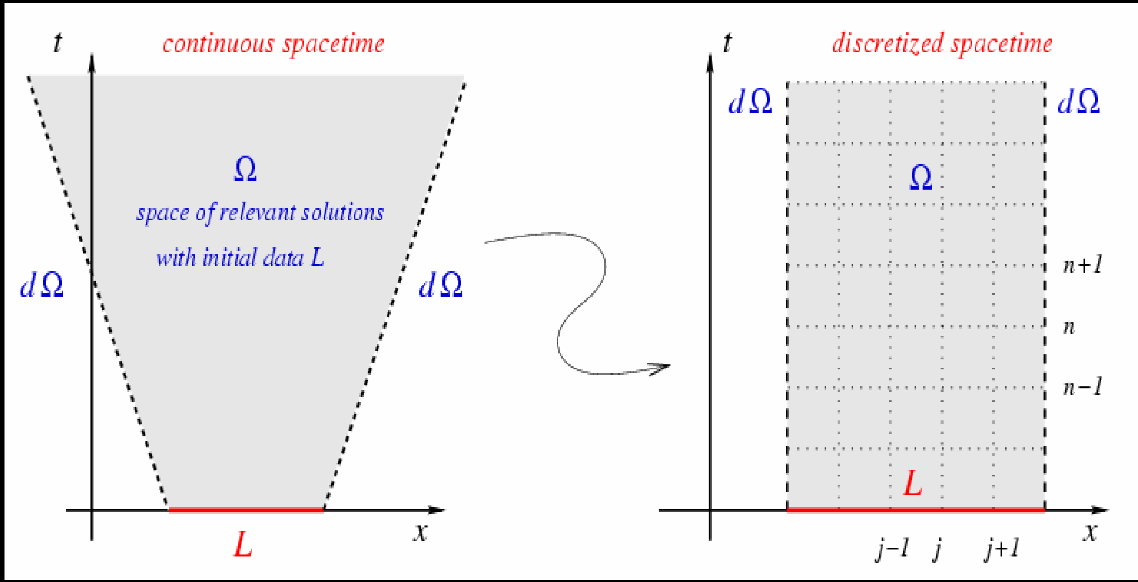
$$\nabla_\nu T^{\mu\nu} = 0$$

$$J^\mu \equiv \rho u^\mu$$

$$T^{\mu\nu} = (\rho + \rho\varepsilon + p + b^2) u^\mu u^\nu + \left(p + \frac{1}{2} b^2 \right) g^{\mu\nu} - b^\mu b^\nu$$

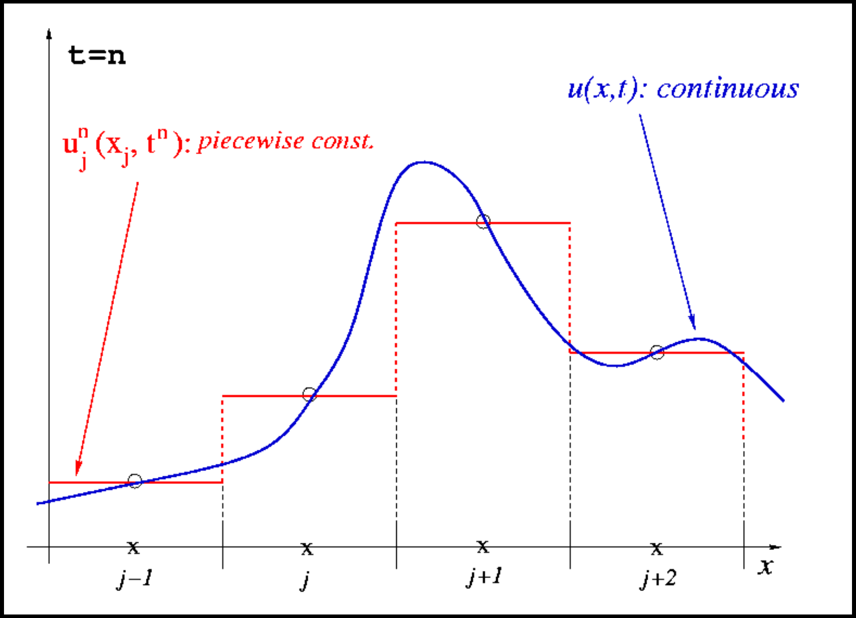
plus an Equation of State $P = P(\rho, \varepsilon)$

HRSC in a slide



A generic problem arises when a Cauchy problem described by a set of *continuous* PDEs is solved in a *discretized form:* the numerical solution is, at least, ***piecewise constant.***

This can be a serious problem in compressible fluids that *generically* produce *nonlinear waves* even from smooth initial data



In HRSC methods, each discretization-produced discontinuity is considered a local **Riemann problem**: *a nuisance is turned into a useful property*

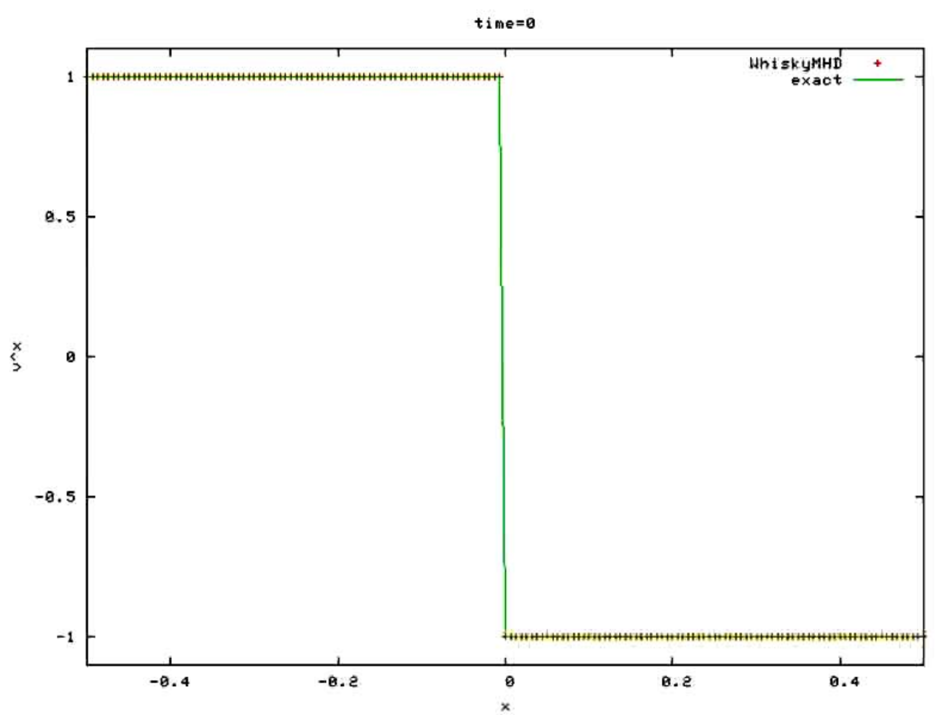
Testing the code

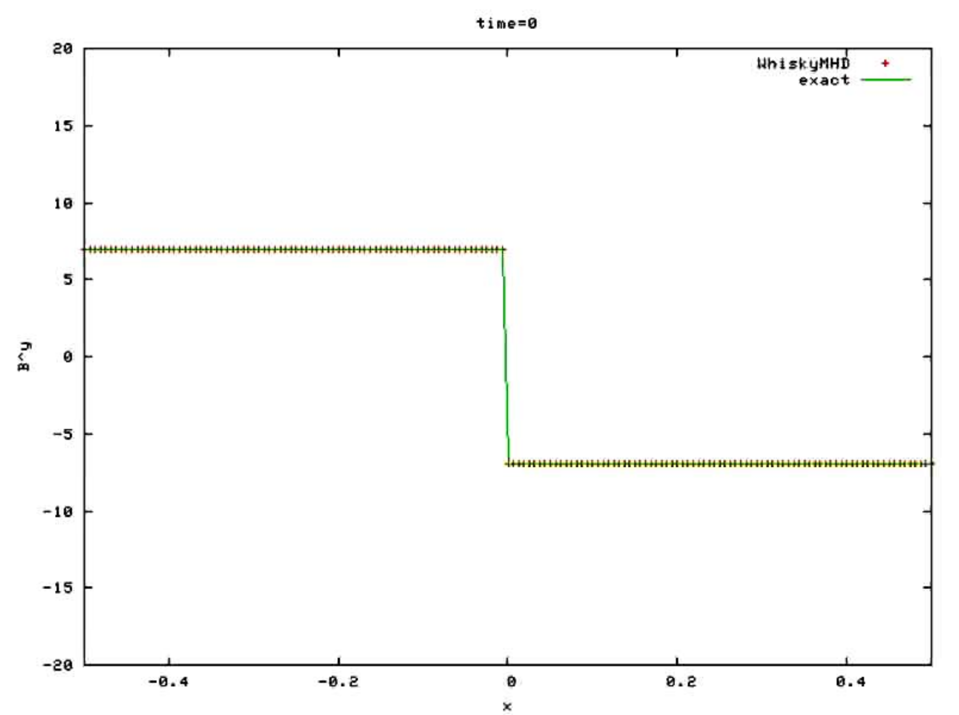
- Riemann Problems:
  - All HRSC methods are based on the solution of Riemann problems, so the numerical code has been tested using also this type of problems
  - All the results were compared with the exact solution computed by BG, Rezzolla (J. Fluid Mech., 06)
- Excision Tests
  - Shock-Tube tests with an excised region
- Curved background tests
  - Spherical accretion on a Schwarzschild Black Hole
  - Evolution of a stable magnetized NS

Testing the code

Test number 4 of Balsara (collision test)

Comparing the numerical solution (points) with the exact one (solid line)



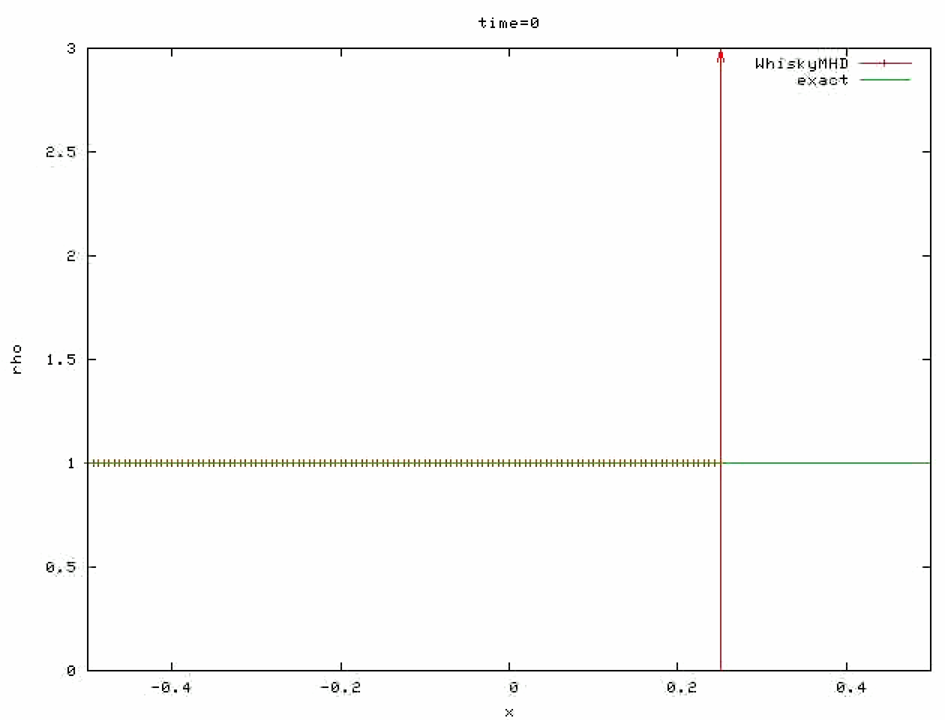


v^x B^y

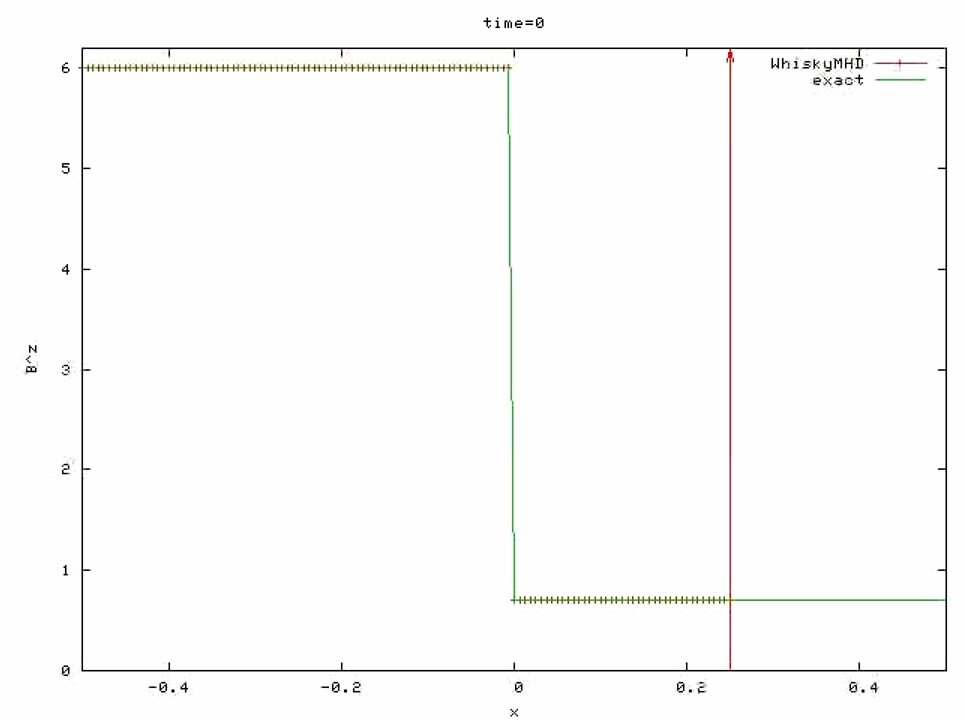
Testing the code

Test number 2 of Balsara with an excised region

Comparing the numerical solution (points) with the exact one (solid line)



Rest mass density

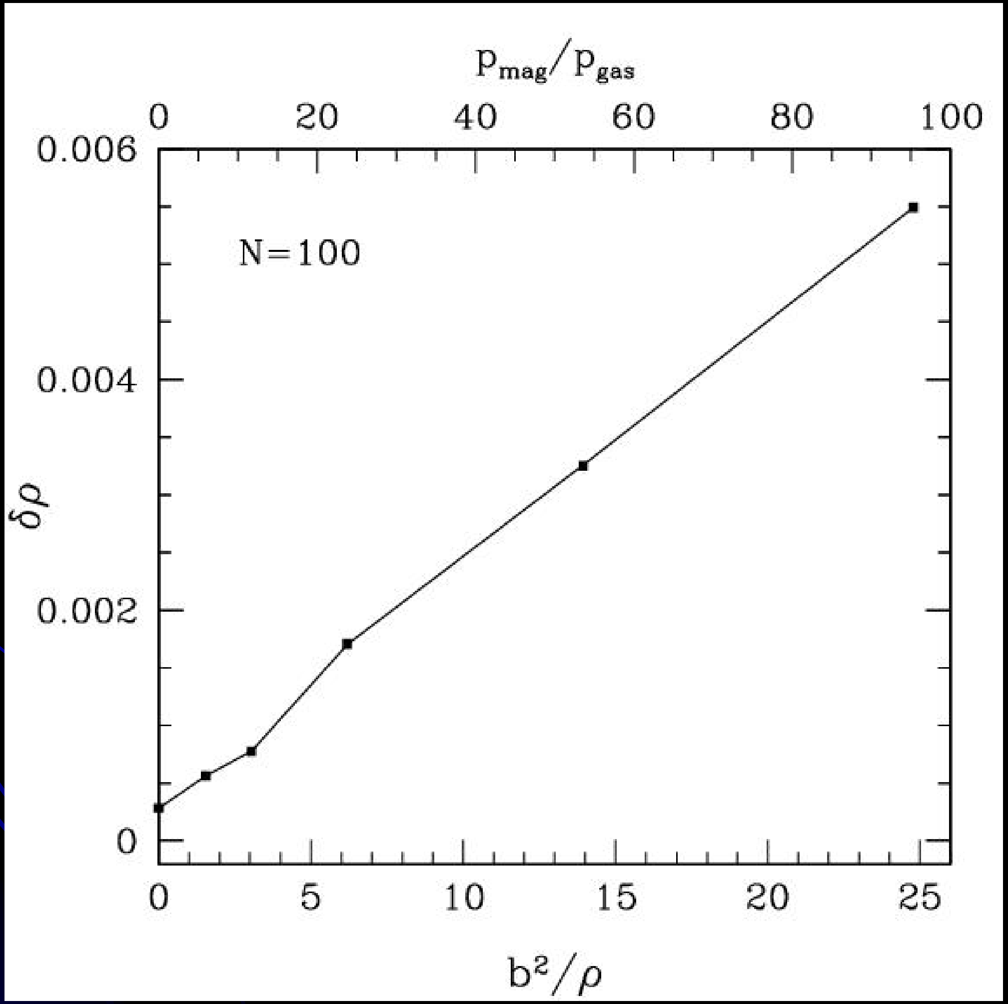


B^z

Testing the code

Spherical Accretion onto a Schwarzschild BH with a radial magnetic field

Relative error on the rest mass density vs magnetic field intensity

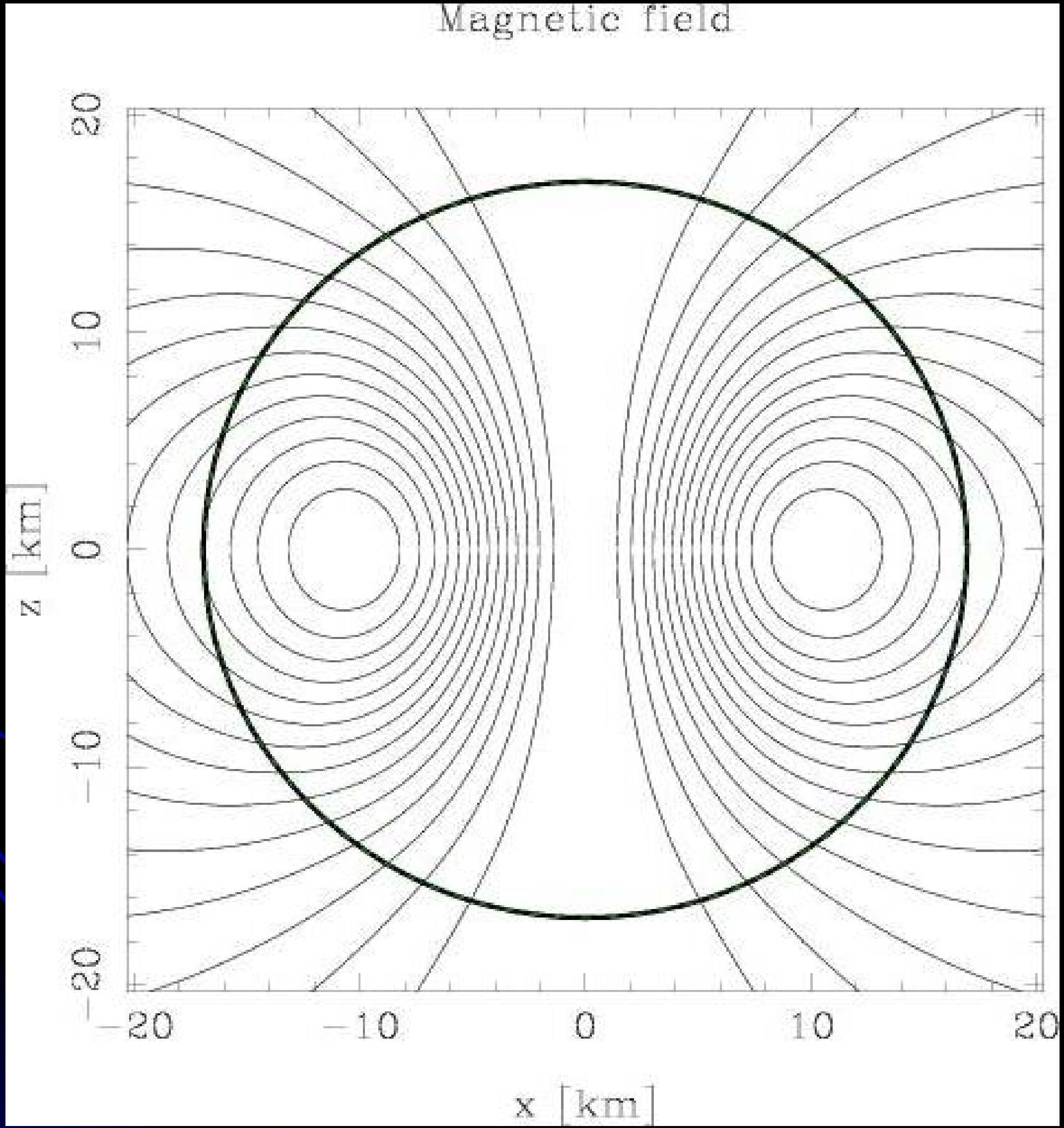


Computation of initial Models (Lorene code)

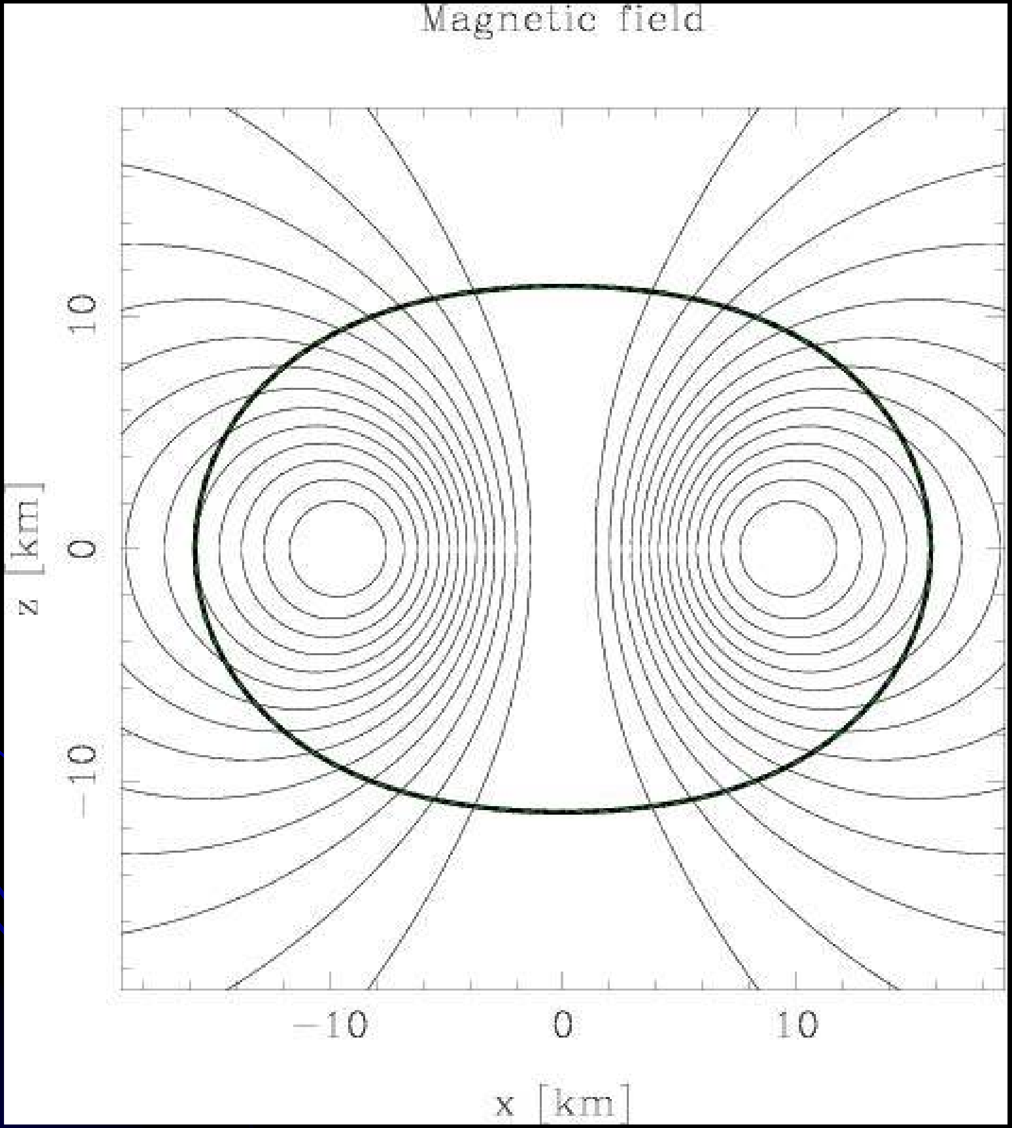
Bocquet et al. 1995, A&A 301, 757

- To build initial models we are using the code `Magstar` included in `Lorene` and developed by J. Novak (Meudon) and others
- This code uses spectral methods to compute initial conditions describing both uniformly rotating and non rotating NS with magnetic fields:
  - it uses spherical coordinates and different equations of state
  - model are built after specifying: central enthalpy, angular velocity, mf distribution
- `WhiskyMHD` imports data from `Magstar` onto a cartesian grid

Non rotating NS with $B_c = 3.1 \cdot 10^{16}$G

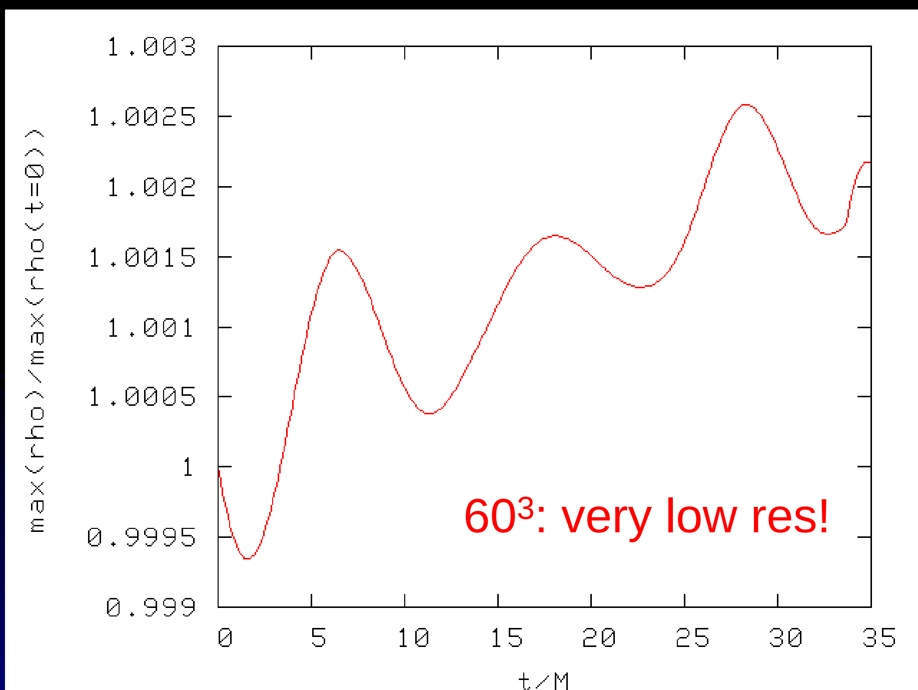


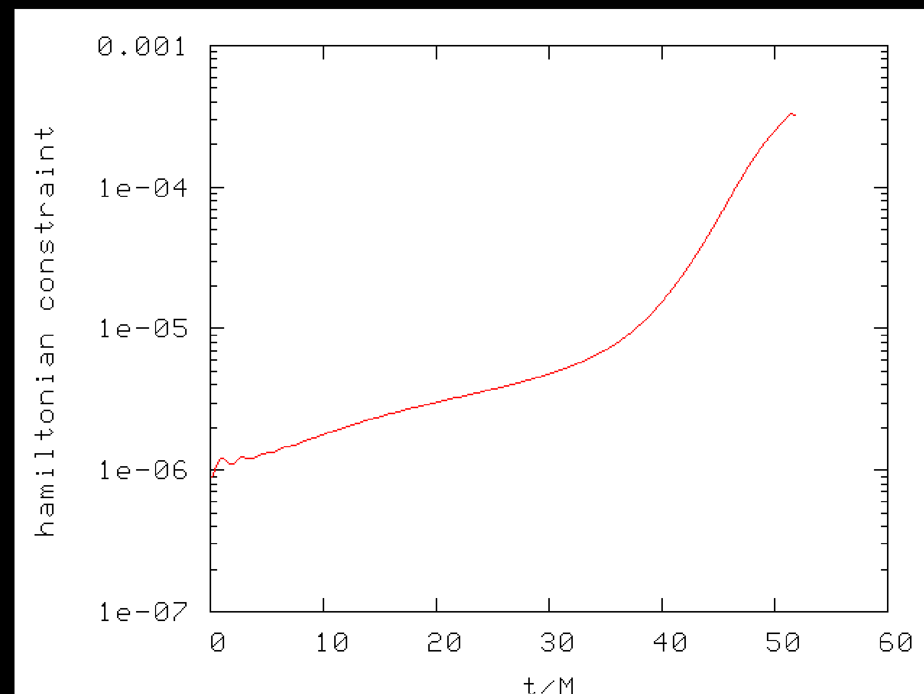
Non rotating NS with $B_c=1.1\cdot 10^{18}$G



Testing the code (work in progress!)

Evolution of a stable non rotating NS with $B_c=2.3\cdot 10^{14}$G





These are the first calculations of an oscillating magnetized star. This is a whole new area which has not been investigated not even perturbatively.

Conclusions

- We have implemented the GRMHD eqs `Whisky` extending considerably its range of applications
- A long list of projects awaits to be tackled starting from the collapse of both uniformly rotating and non-rotating NSs with magnetic fields
- The differences in the gw signal will provide important information on the NSs which cannot be extracted from EM signals
- You will know more on this at the next GWADW meeting!…