DETECTING THE `MAGNETIC' COMPONENT OF THE GRAVITATIONAL-WAVE FORCE

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Motion of a charge in the field of an electromagnetic wave

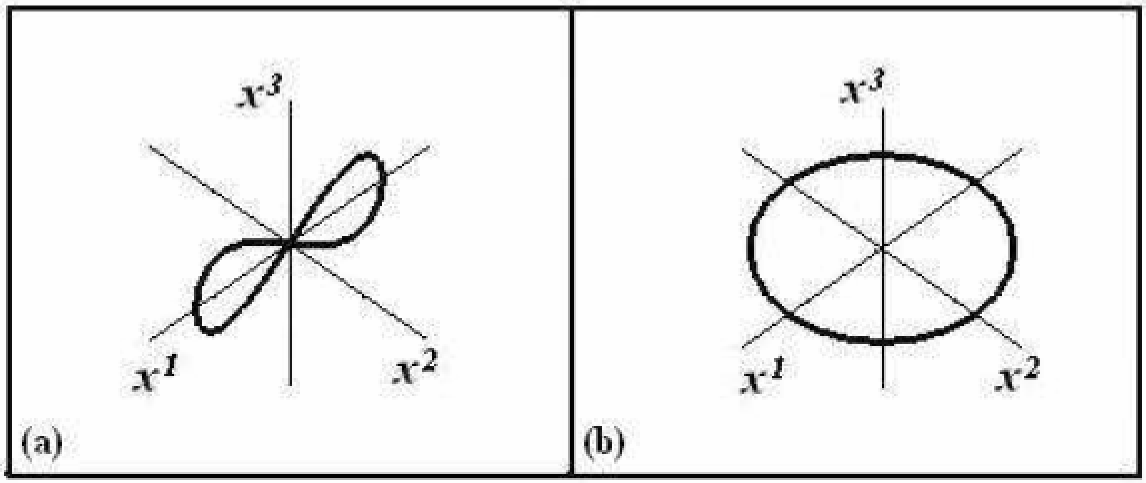

Figure 1. The figure a) on the left shows the trajectory of a charged particle in the field of a linearly polarized electromagnetic wave, whereas the figure b) on the right shows the trajectory in the field of a circularly polarized wave.

Electromagnetic (Lorentz) force:

$$m\frac{d^2\mathbf{x}}{dt^2} = e\mathbf{E} + \frac{e}{c}\left(\frac{d\mathbf{x}}{dt} \times \mathbf{H}\right).$$

A weak gravitational wave:

$$ds^2 = c^2 dt^2 - [\delta_{ij} + h_{ij}]\, dx^i dx^j.$$

$$h_{ij} = \overset{1}{p}_{ij} a + \overset{2}{p}_{ij} b,$$

$$\overset{1}{p}_{ij} = m_i m_j - n_j n_i, \quad \overset{2}{p}_{ij} = -m_i n_j - m_j n_i,$$

$$a = h_+ \sin\left(k(x^0 + x^3) + \psi_+\right), \quad b = h_\times \sin\left(k(x^0 + x^3) + \psi_\times\right),$$

In the frame based on principal axes:

$$a = h_+ \sin\left(k(x^0 + x^3) + \psi\right), \quad b = h_\times \cos\left(k(x^0 + x^3) + \psi\right),$$

$$ds^2 = c^2 dt^2 - (1 + a) {dx^1}^2 - (1 - a) {dx^2}^2 + 2b\; dx^1 dx^2 - {dx^3}^2,$$

Coordinate transformation to a local inertial coordinate system (the closest thing to a global Lorentzian coordinate system):

$$\bar{x}^0 = x^0 + \tfrac{1}{4}\dot{a}\,\left({x^1}^2 - {x^2}^2\right) - \tfrac{1}{2}\dot{b}\,x^1x^2,$$

$$\bar{x}^1 = x^1 + \tfrac{1}{2}a\,x^1 - \tfrac{1}{2}b\,x^2 + \tfrac{1}{2}\dot{a}\,x^3x^1 - \tfrac{1}{2}\dot{b}\,x^3x^2,$$

$$\bar{x}^2 = x^2 - \tfrac{1}{2}a\,x^2 - \tfrac{1}{2}b\,x^1 - \tfrac{1}{2}\dot{a}\,x^3x^2 - \tfrac{1}{2}\dot{b}\,x^3x^1,$$

$$\bar{x}^3 = x^3 - \tfrac{1}{4}\dot{a}\,\left({x^1}^2 - {x^2}^2\right) + \tfrac{1}{2}\dot{b}\,x^1x^2.$$

Trajectories of the nearby free particles, including `magnetic' oscillations back and forth in the direction of the wave propagation, the z-direction.

$$\begin{aligned}\bar{x}^1(t) &= l_1 + \tfrac{1}{2}\left[h_+ l_1 \sin(\omega t + \psi) - h_\times l_2 \cos(\omega t + \psi)\right] \\ &\quad + \tfrac{1}{2}k\left[h_+ l_3 l_1 \cos(\omega t + \psi) + h_\times l_3 l_2 \sin(\omega t + \psi)\right],\end{aligned}$$

$$\begin{aligned}\bar{x}^2(t) &= l_2 - \tfrac{1}{2}\left[h_+ l_2 \sin(\omega t + \psi) + h_\times l_1 \cos(\omega t + \psi)\right] \\ &\quad - \tfrac{1}{2}k\left[h_+ l_3 l_2 \cos(\omega t + \psi) - h_\times l_3 l_1 \sin(\omega t + \psi)\right],\end{aligned}$$

$$\bar{x}^3(t) = l_3 - \tfrac{1}{4}k\left[h_+\left({l_1}^2 - {l_2}^2\right)\cos(\omega t + \psi) + 2h_\times l_1 l_2 \sin(\omega t + \psi)\right].$$

Familiar picture of deformations of a disk consisting of free particles. Zero-order approximation in the wavenumber k; `magnetic' contribution ignored:

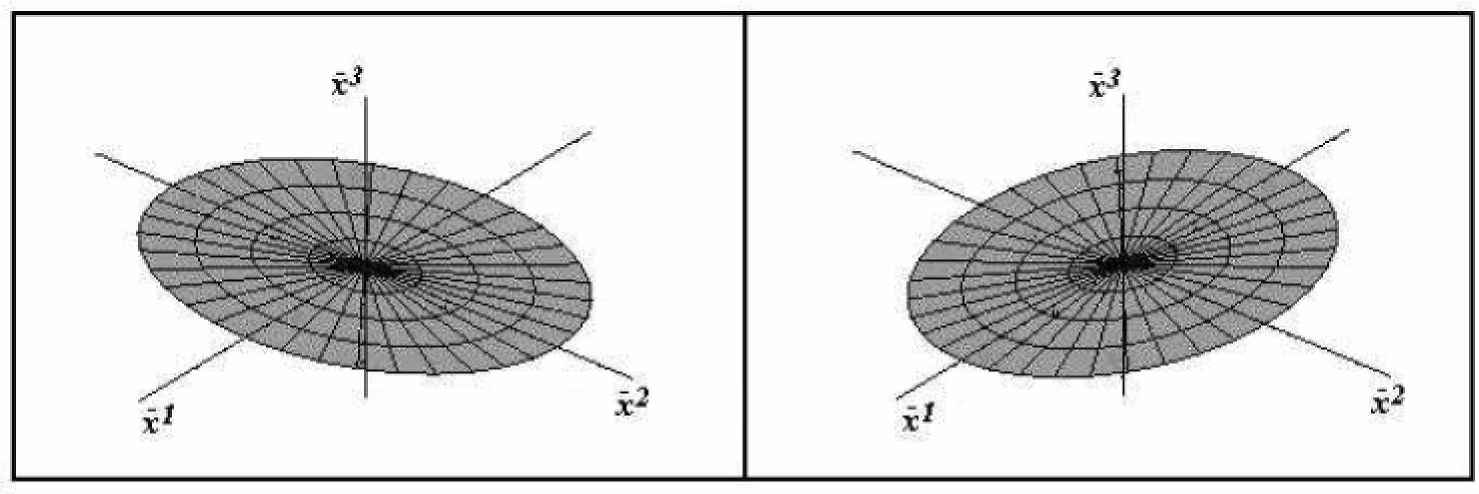

Figure 3. Deformation of a disk of free test particles in the field of a linearly polarized ($h_+ \neq 0, h_\times = 0$) g.w. in the limit of $k = 0$. The two figures show the displacements at the moments of time separated by a half period.

Deformations of the disk with the `magnetic' contribution included:

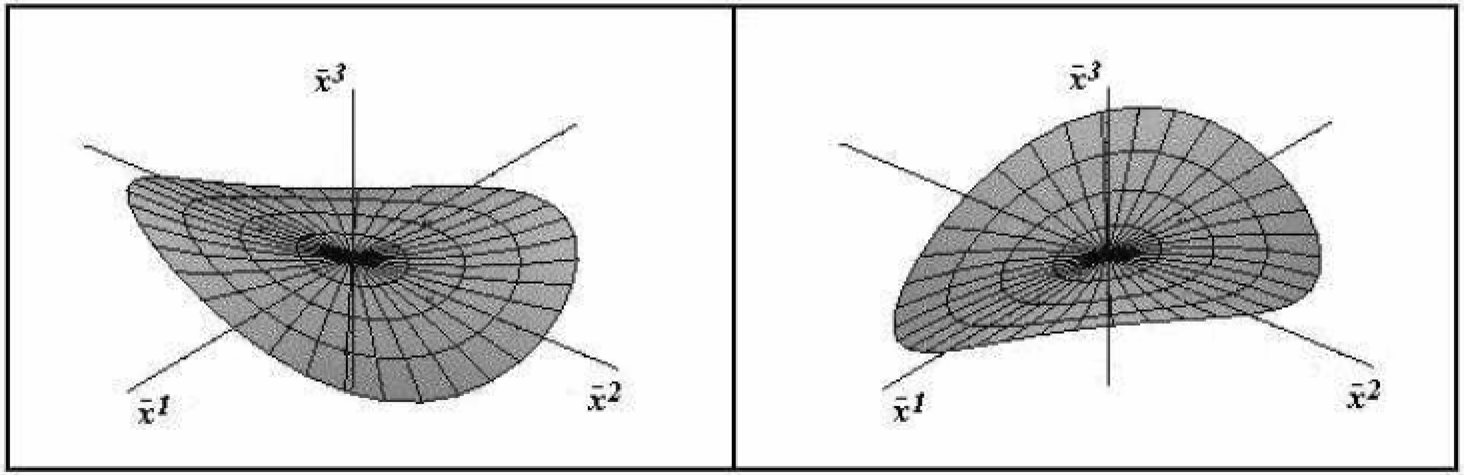

Figure 5. The figure shows the deformations of a circular disk of free test particles under the action of a linearly polarized g.w. $(h_+ \neq 0, h_\times = 0)$. The "magnetic" contribution is responsible for the displacements along the $\bar{x}^3$ axis. The two pictures show the configurations at the moments of time separated by a half of period.

Gravitational-wave force from the geodesic deviation equation:

$$\frac{d^2\xi^i}{dt^2} = \frac{1}{2}\omega^2 l^j \left[\overset{1}{p}{}^i{}_j h_+ \sin(\omega t + \psi) + \overset{2}{p}{}^i{}_j h_\times \cos(\omega t + \psi)\right]$$
$$- \frac{1}{2}\omega^2 l^k l^l \left[k_l \delta^{ij} + \frac{1}{2}k^i \delta^j_l\right] \left[\overset{1}{p}{}_{kj} h_+ \cos(\omega t + \psi) - \overset{2}{p}{}_{kj} h_\times \sin(\omega t + \psi)\right].$$

The second term is the `magnetic' force, proportional to the particle's velocity:

$$m\frac{d^2\xi^i}{dt^2} = F^i_{(e)} + F^i_{(m)}.$$

Variation of the distance between the central particle (corner mirror of interferometer) and a nearby particle (end mirror of interferometer). `Magnetic' contribution (terms proportional to the wavenumber k) is included:

$$d(t) = l + \frac{1}{2l}\left[h_+({l_1}^2 - {l_2}^2)\sin(\omega t + \psi) - 2h_\times l_1 l_2 \cos(\omega t + \psi)\right]$$

$$+ \frac{1}{4l} k l_3 \left[h_+({l_1}^2 - {l_2}^2)\cos(\omega t + \psi) + 2h_\times l_1 l_2 \sin(\omega t + \psi)\right] + O\left(hl(l/\lambda)^2\right).$$

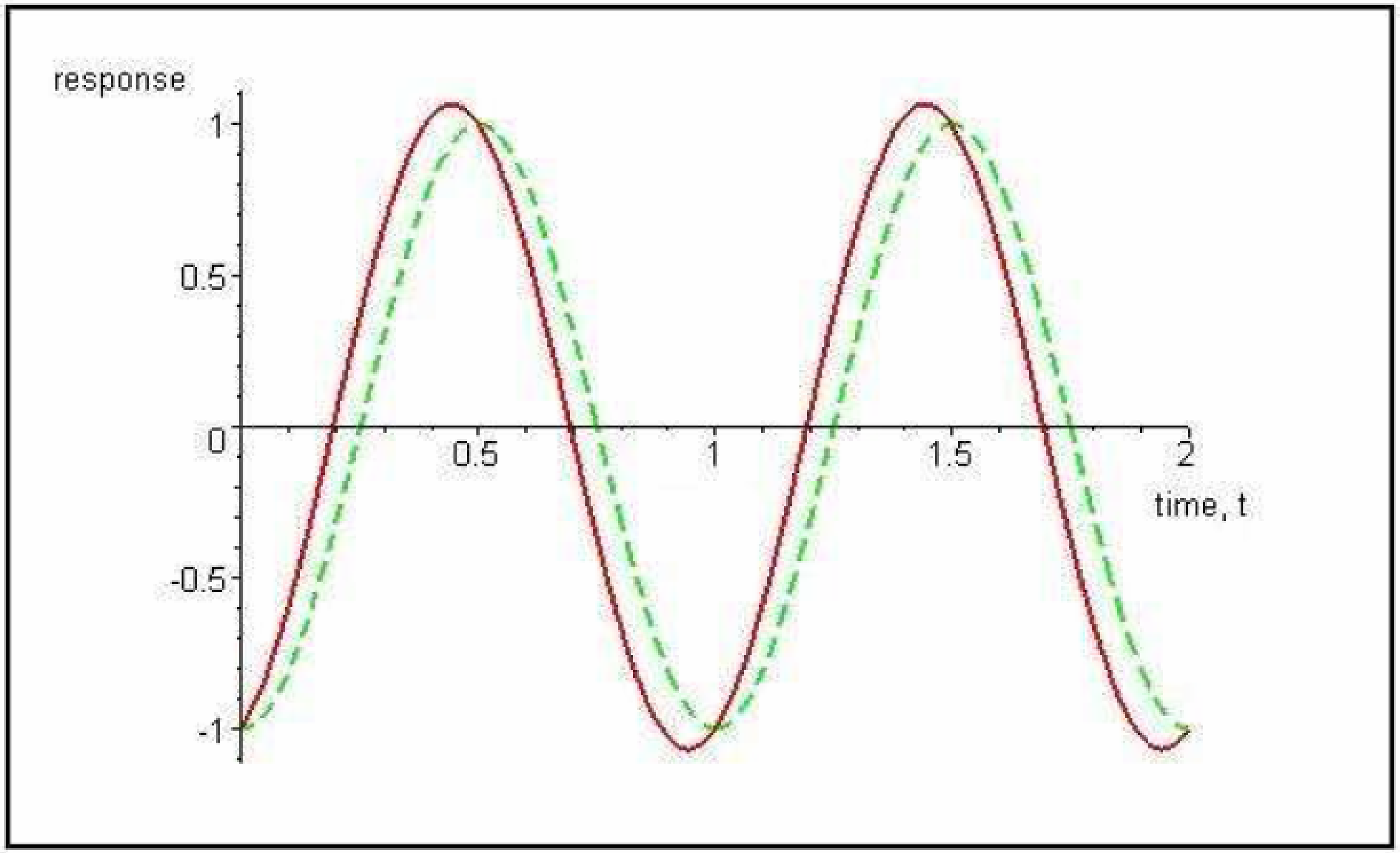

Figure 8. A typical response of an interferometer, as a function of time, to the monochromatic circularly polarized gravitational wave coming from a fixed direction on the sky. The solid line shows the total response, while the dashed line is purely "electric" part.

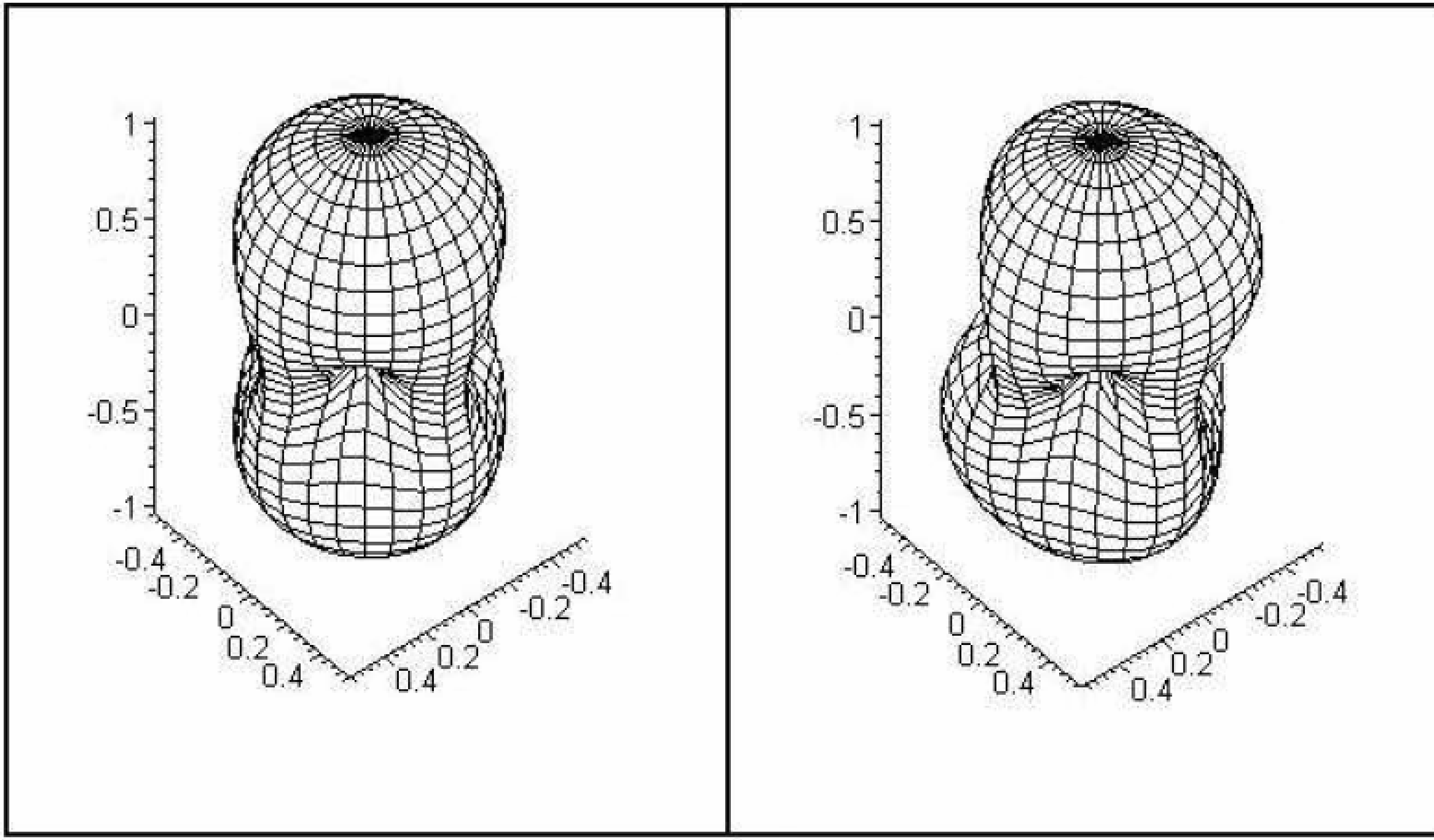

Figure 9. The amplitude of the interferometer's response to circularly polarized waves. The graphs are normalized in such a way that the amplitude is equal 1 for $\Theta = 0$. The left figure ignores the "magnetic" effect, whereas the right figure shows the total response.

Astrophysical example: a pair of stars on a circular orbit in a plane orthogonal to the line of sight.

$$h_+ = h_\times = h_R = \frac{32\pi^2 G}{Rc^4} M r^2 \omega^2,$$

Correct response of the interferometer, including its `magnetic' part:

$$\Delta d(t) = l h_R \left[\left\{ \cos 2\Phi \left(\frac{1+\cos^2\Theta}{2} \right) - \frac{1}{4}\frac{\omega l}{c} \sin 2\Phi \left(\cos\Phi + \sin\Phi \right) \cos\Theta \sin\Theta \right\} \right.$$
$$\times \sin(\omega t + \psi)$$
$$- \left\{ \sin 2\Phi \cos\Theta - \frac{1}{4}\frac{\omega l}{c} \left(\cos^2\Theta + \sin 2\Phi \left(\frac{1+\cos^2\Theta}{2} \right) \right) \left(\cos\Phi - \sin\Phi \right) \sin\Theta \right\}$$
$$\left. \times \cos(\omega t + \psi) \right].$$

Response based on the `electric' contribution only (incorrect):

$$\Delta d(t) = l\ h_R \left[\cos 2\Phi \left(\frac{1+\cos^2\Theta}{2} \right) \sin(\omega t + \psi) - \sin 2\Phi \cos\Theta \cos(\omega t + \psi) \right].$$

Conclusions

In the LIGO interferometers, the `magnetic' component of the g.w. force, proportional to (kl), provides a correction to the interferometer's response at the level of 5 percent in the frequency band of 600 Hz, and up to 10 percent in the frequency band of 1200 Hz.

Data analysis based on the `electric' contribution only can significantly affect the determination of the parameters of the g.w. source.

`Magnetic' contribution is measurable and must be taken into account in the accurate observations of periodic and quasi-periodic astrophysical sources by advanced interferometers.