Gravitational Wave Sources near 1 Hz

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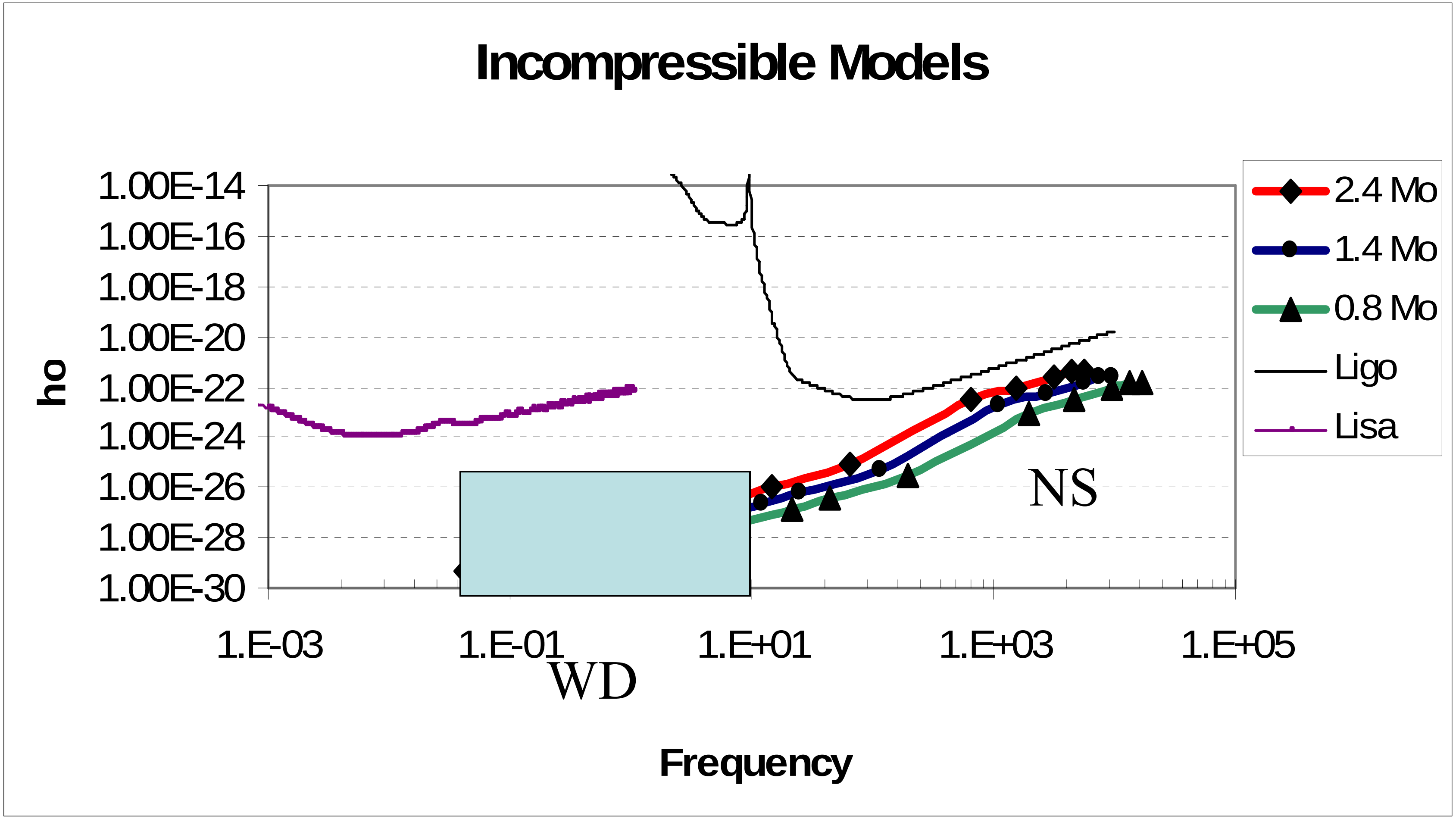
Outline

- White dwarf frequencies
- Gravitational radiation mechanisms
- Stochastic background level near 1 hz

Why White Dwarfs?



Why White Dwarfs?

- White Dwarfs(WD) are stellar configurations with central densities ~10^6-10^9 g/ cm^3
  -they are on the border between normal stars and relativistic configurations
- Quadrupole moment of WDs is Q~10^{48}g cm^2
  - several orders higher then Neutron Star's
  Quadrupole moment

Why White Dwarfs ?

- White Dwarfs(WD) are the most close potential sources of GWs
  - there are White Dwarfs at 8 pc distance.

- WD Population is estimated about $\sim 10^8$ in the Galaxy
  -WDs are the largest population among potential astrophysical sources of GWs

Strain Amplitudes

- Oblate shape due to rotation
- Oscillation is *self-similar* and is described by:

$$x_\alpha = x_\alpha^0\left(1+\eta\sin(\omega t)\right)$$

- Quadrupole moment

$$Q_{ij} = \int \rho\left(x_i x_j - \tfrac{1}{3}x^2\delta_{ij}\right)dV = Q_{ij}^0\left(1+\eta\sin(\omega t)\right)$$

Choose *z*-axis along rotation axis: $Q^0{}_{zz}=-2Q^0{}_{xx}=-2Q^0{}_{yy}=-2Q^0$

Polarizations

In TT gauge with *z*-axis along the wave vector:

$$h_+ = h_{xx} - h_{yy} = \frac{4GQ^0\eta\omega^2}{c^4 r}\sin^2\theta\sin(\omega t)$$

$$h_\times = 2h_{xy} = 0$$

where θ is the angle between the wave vector and the white dwarf axis of rotation

Gravitational Radiation Intensity

$$J = \frac{6G}{5c^5}\eta^2\omega^6\left|Q_{zz}^0\right|^2\cos^2\omega t' = J_0\cos^2\omega t'$$

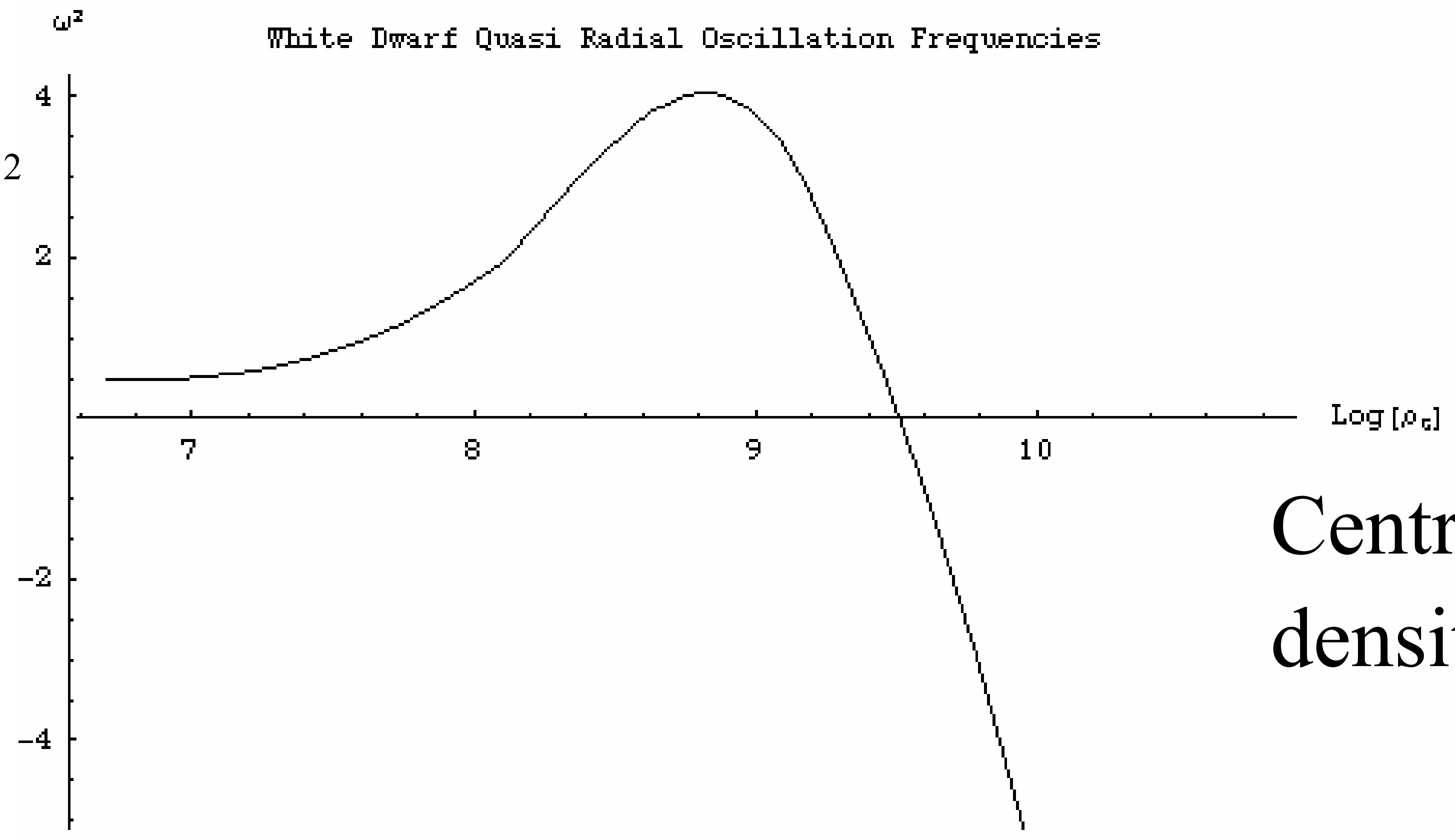
$$J_0 << \omega^6\left|Q^0{}_{zz}\right|^2\frac{6}{5}\frac{G}{c^5}$$

White Dwarf Properties and Resonant Frequencies

ρ_c (g/cm^3)	M_0 ($M_\odot$)	M ($M_\odot$)	Ω_{max}	Q^0_{max} (10^{48}g cm^2)	$N_{(57)}$	ω
$1.76 \quad 10^6$	0.498	0.572	0.196	20.48	0.4997	0.757
$1.54 \quad 10^7$	0.867	0.976	0.476	14.27	0.8398	0.766
$1.28 \quad 10^8$	1.145	1.254	1.063	4.766	1.0695	1.399
$7.036 \; 10^8$	1.245	1.34	2.042	1.554	1.1340	2.001
$2.09 \quad 10^9$	1.257	1.339	3.105	0.673	1.1261	1.299

Frequency Range of WD Oscillations

$$\omega^2 = (2\pi\nu)^2$$



Central density

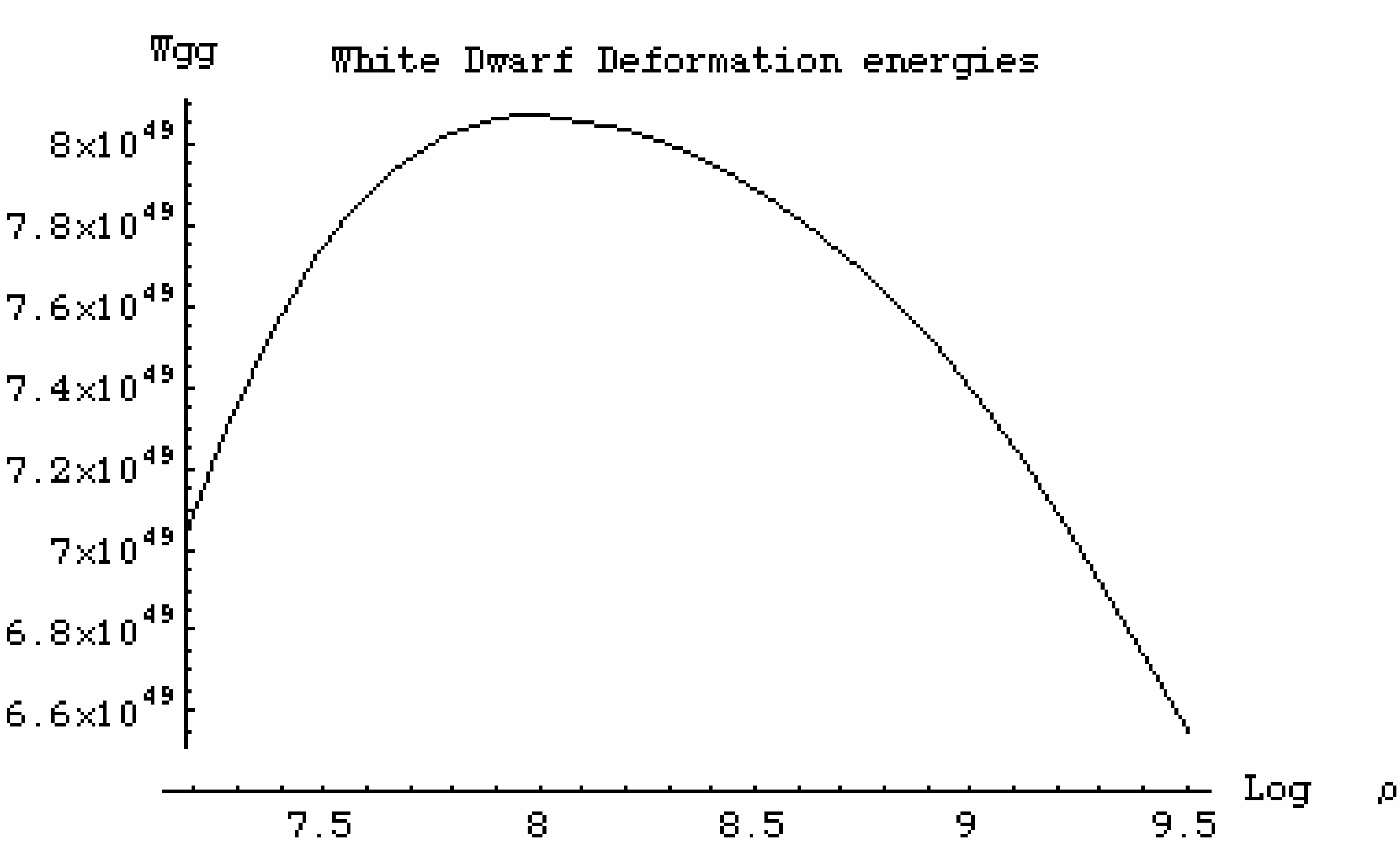
Deformation Energy

$$W_{def}(\Omega) = (M - M_0)c^2 - W_r(\Omega)$$

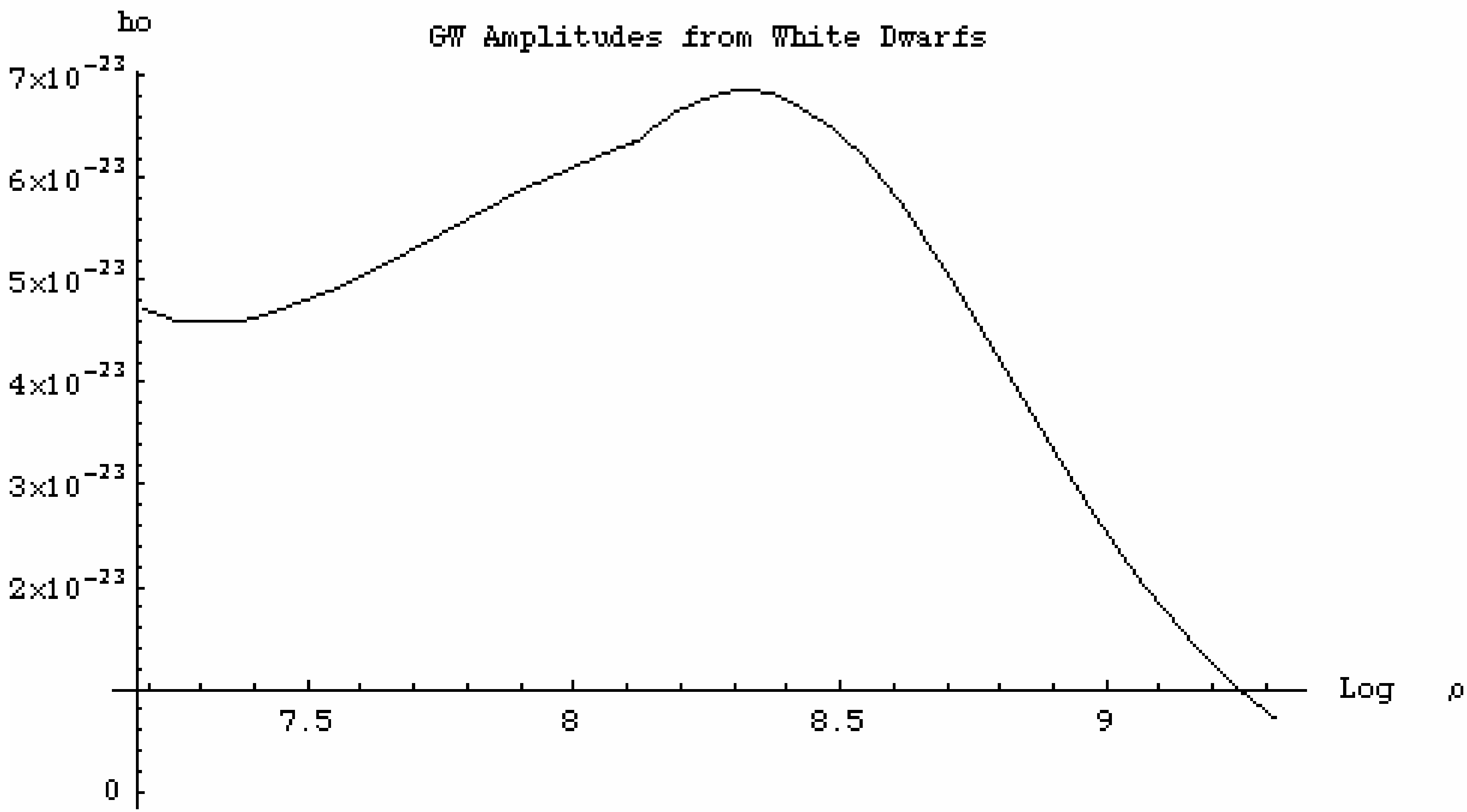
- M and M_o are mass of rotating and non-rotating configurations with same complete number of baryons N

$$W_r = I\Omega^2/2 \qquad \Delta M = (\alpha_0 - \alpha)mN$$

White Dwarfs Maximal deformation Energy versus Central density



GW Amplitudes from WDs rotating with Keplerian angular velocities



Mechanisms of GW Radiation

1. GWs from Magnetized WDs:
   -deformation energy is feeding oscillations
   -magnetodipol radiation torque is breaking rotation

2. GWs from differentially rotating WDs

3. GWs from triaxial WDs

Types of Models of WDs

- Model 1.a is calculated by requiring that the largest Doppler broadening of spectral lines due to pulsations be less than thermal Doppler broadening
- Model 1.m is based on assumption that all non-dissipated part of deformation energy is going to oscillations, it is maximal possible model to that sense.

GWs from Magnetized WDs 1.a

WD Name	**r (pc)**	**B (MG)**	h_o	**F**	**t (Gy)**	η
PG 1031+234	142	500	$6.0\ 10^{-29}$	$6.1\ 10^{-17}$	11.0	$1.02\ 10^{-02}$
EUVE J0317-855	35	450	$1.0\ 10^{-27}$	$6.7\ 10^{-15}$	1.7	$4.03\ 10^{-03}$
PG 1015+015	66	90	$9.3\ 10^{-30}$	$1.1\ 10^{-18}$	571.9	$7.09\ 10^{-04}$
Feige 7	49	35	$1.6\ 10^{-28}$	$4.9\ 10^{-17}$	125.1	$5.18\ 10^{-04}$
G99-47	8	25	$3.5\ 10^{-27}$	$5.9\ 10^{-16}$	50.6	$3.70\ 10^{-04}$
KPD 0253+5052	81	17	$2.9\ 10^{-30}$	$4.6\ 10^{-20}$	11852	$3.46\ 10^{-04}$
PG 1312+098	----	10	$1.5\ 10^{-30}$	$3.8\ 10^{-21}$	70313.	$2.04\ 10^{-04}$
G217-037	11	0.2	$9.0\ 10^{-31}$	$8.2\ 10^{-23}$	$2\ 10^{8}$	$4.08\ 10^{-06}$

GWs from Magnetized WDs 1.m

WD Name	r (pc)	B (MG)	h_o	F	t (Gy)	η
PG 1031+234	142	500	$2.58\cdot10^{-28}$	$1.13\cdot10^{-15}$	11.0	$4.7\cdot10^{-2}$
EUVE J0317-855	35	450	$9.69\cdot10^{-26}$	$6.04\cdot10^{-11}$	1.7	$3.8\cdot10^{-1}$
PG 1015+015	66	90	$3.81\cdot10^{-28}$	$1.93\cdot10^{-15}$	571.9	$2.9\cdot10^{-2}$
Feige 7	49	35	$1.47\cdot10^{-26}$	$3.96\cdot10^{-13}$	125.1	$4.7\cdot10^{-2}$
G99-47	8	25	$3.45\cdot10^{-25}$	$5.84\cdot10^{-12}$	50.6	$3.7\cdot10^{-2}$
KPD 0253+5052	81	17	$2.06\cdot10^{-28}$	$2.33\cdot10^{-16}$	11852.8	$2.5\cdot10^{-2}$
PG 1312+098	----	10	$9.38\cdot10^{-29}$	$1.56\cdot10^{-17}$	70313.8	$1.3\cdot10^{-2}$
G217-037	11	0.2	$8.97\cdot10^{-29}$	$8.19\cdot10^{-19}$	$2.4\cdot10^{7}$	$4.1\cdot10^{-4}$

Differentially Rotating WDs model 2.1

	Edifrot I	Ediss I	LifeTime (Gyr)	Jo I	ho	η Etta	F Flux
PG 1031+234	8.7411E+42	1.2574E+26	2,2	1.26E+25	1.39E-27	6.54E-01	3.3E-14
EUVE J0317-855	4.0005E+44	8.5162E+28	0,1	8.52E+27	5.86E-26	2.19E-01	2.2E-11
PG 1015+015	1.4919E+43	9.8724E+26	0,5	9.87E+25	4.39E-27	6.78E-01	2.6E-13
Feige 7	3.4674E+43	9.3271E+25	11,8	9.33E+24	3.63E-27	1.72E-01	2.4E-14
τ G99-47	1.6782E+44	4.5143E+26	11,8	4.51E+25	4.89E-26	7.83E-02	1.2E-13
KPD 0253+5052	7.0347E+42	1.012E+26	2,2	1.01E+25	2.18E-27	7.29E-01	2.6E-14
PG 1312+098	3.4271E+42	4.93E+25	2,2	4.93E+24	2.68E-27	1.04E+00	1.3E-14
G217-037	2.5262E+43	3.634E+26	2,2	3.63E+25	3.05E-26	3.85E-01	9.4E-14
Average					1.9E-26		

Differentially Rotating WDs model 2.2

	Edifrot II	Ediss II	LifeTime (Gyr)	Jo II	ho	η Etta	Flux
PG 1031+234	3.638E+42	8.4434E+25	1,4	8.44E+24	1.14E-27	5.36E-01	2.2E-14
EUVE J0317-855	9.4765E+43	5.5308E+28	0,1	5.53E+27	4.72E-26	1.77E-01	1.4E-11
PG 1015+015	4.3709E+42	6.4883E+26	0,2	6.49E+25	3.56E-27	5.50E-01	1.7E-13
Feige 7	1.8693E+43	6.3832E+25	9,3	6.38E+24	3.00E-27	1.42E-01	1.7E-14
G99-47	9.0472E+43	3.0895E+26	9,3	3.09E+25	4.04E-26	6.47E-02	8.0E-14
KPD 0253+5052	2.9278E+42	6.7951E+25	1,4	6.80E+24	1.79E-27	5.98E-01	1.8E-14
PG 1312+098	1.4263E+42	3.3104E+25	1,4	3.31E+24	2.20E-27	8.56E-01	8.6E-15
G217-037	1.0514E+43	2.4402E+26	1,4	2.44E+25	2.50E-26	3.15E-01	6.3E-14

Triaxsial WDs model 3.r

- Rotating triaxsial ellipsoid

$\rho_c \times 10^6$, g/cm^3	M/M_Θ	$R_e \times 10^8$	$I_3 \times 10^{48}$ g.cm^2	Ω_{max}	H, km	$\varepsilon \times 10^{-5}$	$J_0 \times 10^{29}$ erg/sec	h_0	$\tau_0 \times 10^2$ Gyear
2.403	0.5946	10.93	128	0.196	0.699	6.4	0.667	$0.69\ 10^{-24}$	12.25
19.38	0.9993	7,342	88.6	0.476	0.187	2.56	10.5	$1.13\ 10^{-24}$	3.19
157.7	1.2731	4.625	39.5	1.063	0.058	1.26	62.1	$1.23\ 10^{-24}$	1.19
866.1	1.3502	3.044	15.9	2.04	0.024	0.784	197	$1.14\ 10^{-24}$	0.56
2586	1.3412	2.287	8.17	3.11	0.014	0.059	373	$1.03\ 10^{-24}$	0.35

Triaxsial WDs model 3.n

- Non Rotating, oscillating triaxsial ellipsoid

$\rho_c \times 10^6$ g/cм^3	M_0/M_Θ	$R \times 10^8$ cm	$I_0 \times 10^{50}$ g.cм^2	ω, s^{-1}	H, km	$\varepsilon \times 10^{-5}$	h_o	$\tau \times 10^3$ Gyear
2.403	0.5087	8.873	4.81	0.758	0.539	6.1	$2.1\ 10^{-26}$	0.35
19.38	0.8854	5.903	3.70	0.794	0.137	2.3	$3.4\ 10^{-26}$	2.59
157.7	1.1612	3.747	1.96	1.51	0.042	1.1	$3.7\ 10^{-26}$	1.60
866.1	1.2538	2.492	0.934	1.99	0.017	0.69	$3.4\ 10^{-26}$	2.92
2586	1.2582	1.888	0.538	0.967	0.010	0.52	$3.1\ 10^{-26}$	160

Stochastic background level

- Background is not isotropic: Assuming a galactic distribution of white dwarfs to follow the disk population, we assign a density distribution of WDs:

$$\rho = \rho_0 e^{-r/R_0} e^{-z/h}$$

in galacto-centric cylindrical coordinates, with R_0=2.5kpc and h=200pc

Conclusions

- Gravitational radiation spectrum near 1 hz is inhabited by Isolated White dwarfs
- Model 1.a $h_{av+}= 8.35 \; 10^{-27}$
- Model 1.m $h_{av+}= 7.94 \; 10^{-25}$
- Model 2.1 $h_{av+}= 2.01 \; 10^{-25}$
- Model 2.2 $h_{av+}= 1.62 \; 10^{-25}$

- Standard inflation gives $h \sim 10^{-27}$–10^{-29} in this frequency range.

Equation of State for White Dwarfs

$$P = \frac{4}{3}\left(\frac{m_e}{m_n}\right)^4 K_n \left[x\left(2x^3 - 3\right)\sqrt{1+x^2} + 3\ln\left(x + \sqrt{1+x^2}\right)\right]$$

$$\rho = \frac{32}{3}\left(\frac{m_e}{m_n}\right)^3 \frac{K_n}{c^2}\frac{A}{Z}x^3$$

- where

$$K_n = m_n{}^4 c^5 / 32\pi^2\hbar^3, \quad x = p_e / m_e c$$

$$\frac{A}{Z} = 2 + 1.255\cdot 10^{-2}x + 1.755\cdot 10^{-5}x^2 + 1.376\cdot 10^{-6}x^3$$