Non linear effects in pulsations of compact stars

Andrea Passamonti
Aristotle University of Thessaloniki

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Outline

- Motivations
- Nonlinear perturbation theory
- Coupling Radial/Nonradial oscillations
- Numerical results
- Conclusions
- Future extensions

Motivations

Astrophysical systems where compact stars can undergo oscillating phases:

- In a core collapse of a rapidly rotating massive star, after the bounce, a) the proto-neutron can store $E_{pul} \sim 10^6 - 10^8 M_\odot c^2$.
  b) fall-back accretion of surrounding material onto the protoneutron star.
- Accretion-induced collapse of a WD.
- Binary systems due to tidal forces.

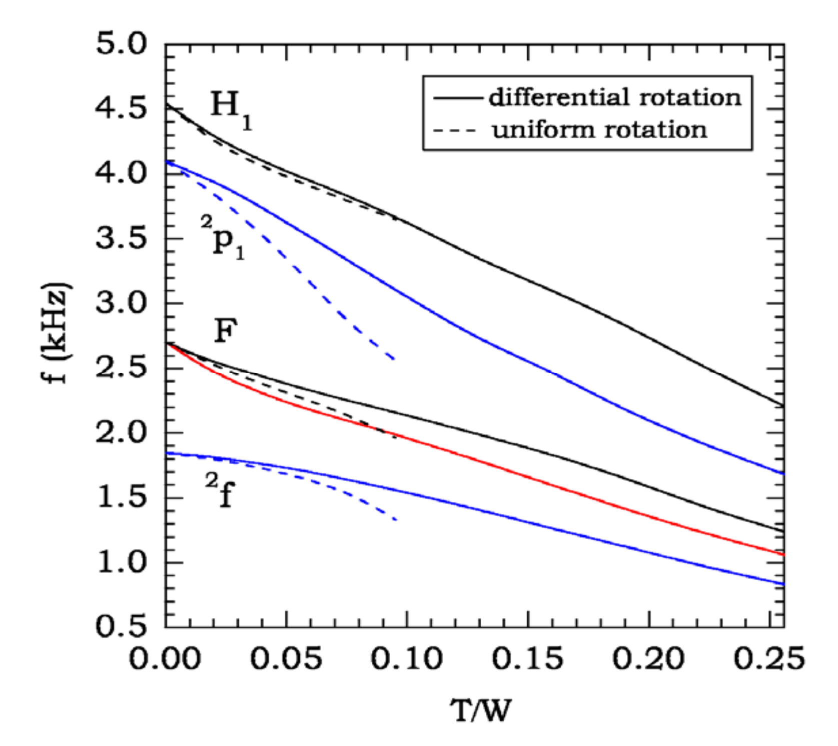
Excitation of Quasi Normal Modes (QNM) $\omega = \nu + i/\tau$

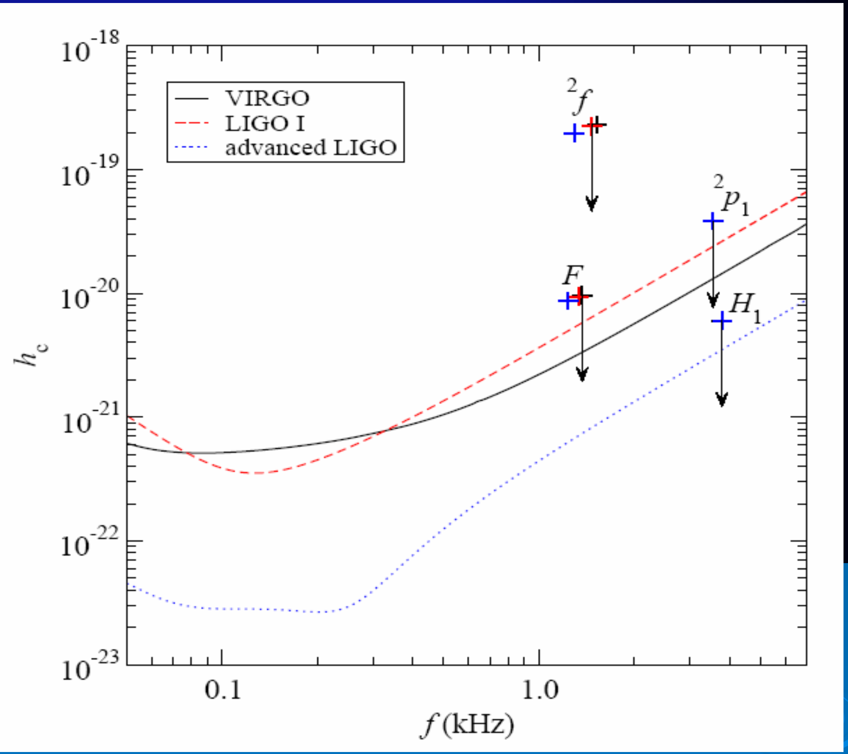
Spectral properties → constrain NS EoS → physics at supranuclear densities

High frequency band ν > 500-600 Hz, where Earth-based interferometer detectors are dominated by the shot noise of the laser.

Nonlinear mode calculations

Recent progress on the mode calculations (radial, non-radial) for uniform or differential rotating stars



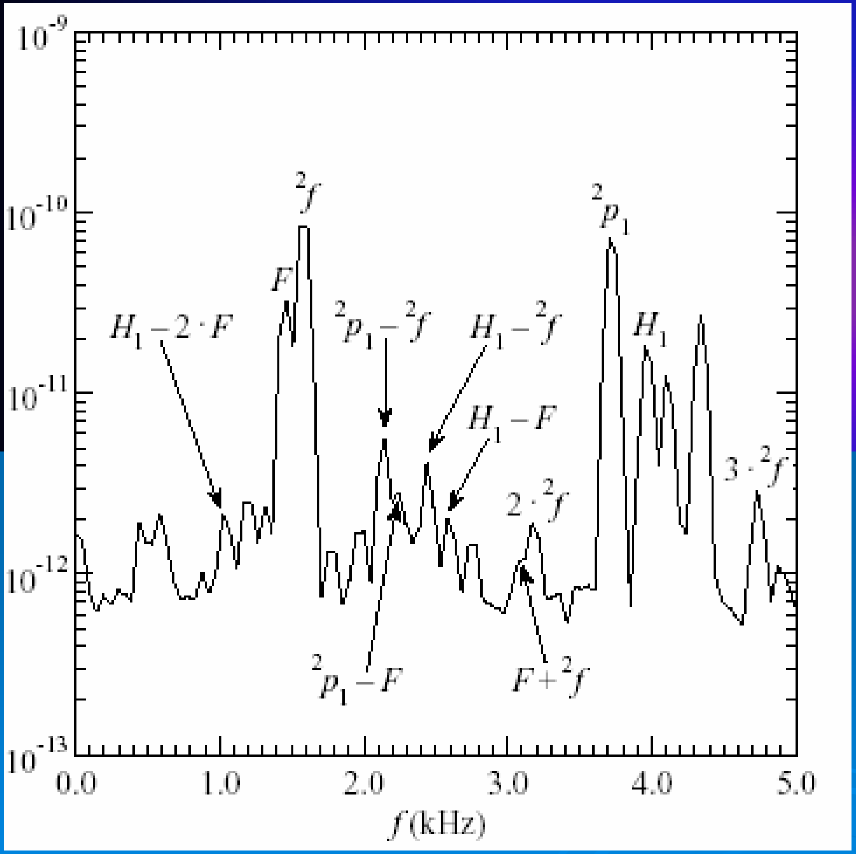


For density variations of the order of 1-5%, neutron star oscillations, can be detected by LIGO and VIRGO from anywhere inside the Milky Way.

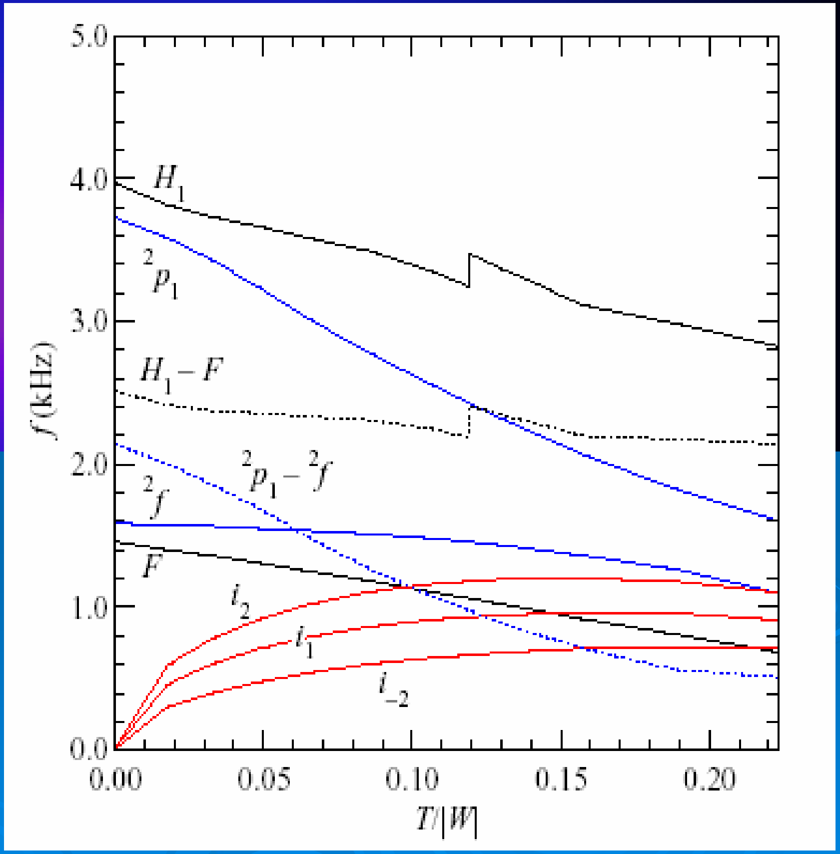
Font, Stergioulas & KK `01, Stergioulas, Apostolatos & Font `04, Dimmelmeier, Stergioulas & Font `05, Stergioulas, KK & Hawke `05

Non-linear mode calculations

Nonlinear Harmonics



Possible Resonances



H. Dimmelmeier, N. Stergioulas and J.A. Font (astro-ph/0511394)

Nonlinear effects

- ***resonances***,   ***energy transfer between different modes***, ***parameter amplifications.***
- Possible ***amplifications of GW signal***.
- ***Spectra***: provide a complete classification of non-linear harmonics, i.e. composition frequencies and sub-harmonics.
- ***Wave forms***: determine more accurate templates for GW detection in the high frequency band.

Nonlinear perturbation theory can be a complementary approach for investigating the above effects, estimating the accuracy of linear perturbation analyses and the results of numerical relativity.

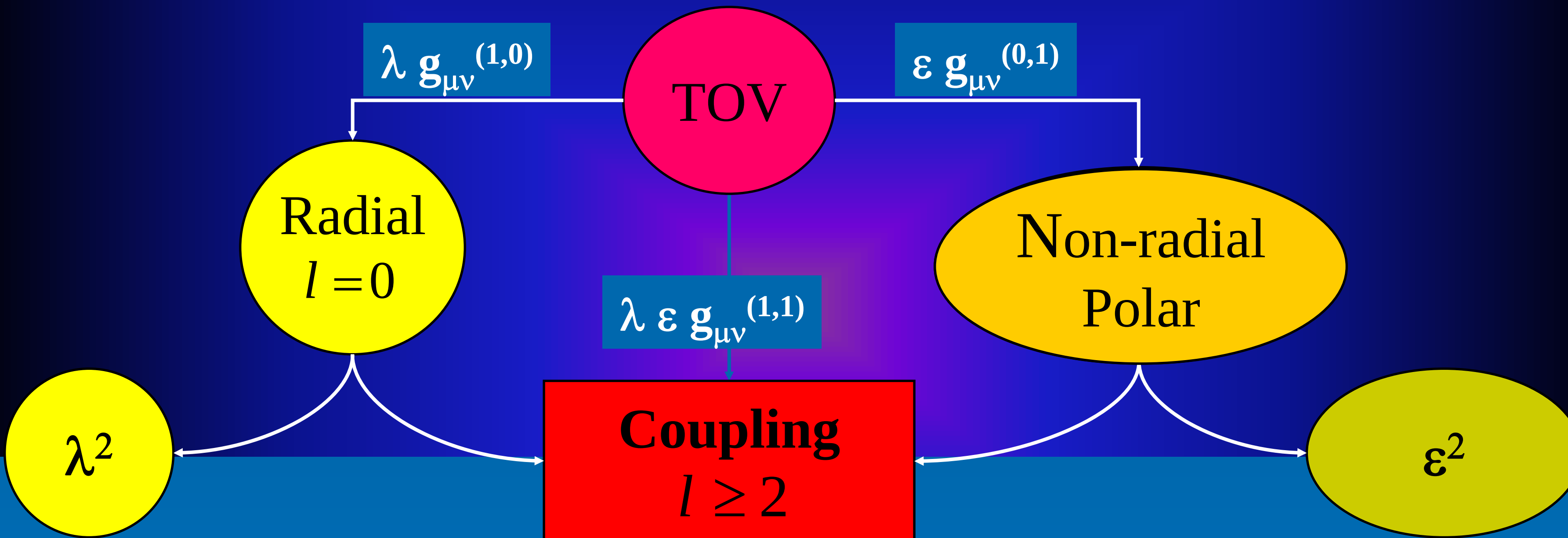
Coupling radial/non-radial oscillations of a spherical star:

- Radial pulsations can drive nonradial oscillations and loss energy in GW through the coupling.

Perturbative framework

Two-parameter second order perturbation theory

C.Sopuerta, M.Bruni, L.Gualtieri (2003)



$$g_{\mu\nu} = g_{\mu\nu}^{(0,0)} + \lambda g_{\mu\nu}^{(1,0)} + \varepsilon g_{\mu\nu}^{(0,1)} + \lambda\varepsilon g_{\mu\nu}^{(1,1)} + O(\lambda,\varepsilon)$$

$$L^{NR}\left[g^{(1,1)} \right] = S\left(g^{(1,0)} \otimes g^{(0,1)} \right)$$

Axial coupling

A. Passamonti, M. Bruni, L. Gualtieri, A. Nagar and C.F. Sopuerta
Phys. Rev. D **73**, 084010 (2006)

- Perturbative equations:

- Gravitational wave equation for axial master metric function Ψ .
  In the exterior spacetime, it reduces to the Regge-Wheeler wave equation.

- Conservation equation for δu_ϕ which is stationary at first order and describes a differentially fluid axial flow.

- Initial data for Linear radial pulsations: selected radial normal modes

$$\delta \mathrm{u}_r = A\,\delta u_n^{00}(r) Y^{00}(\theta,\phi) \qquad E_k = 10^{-6} E_{sun}$$

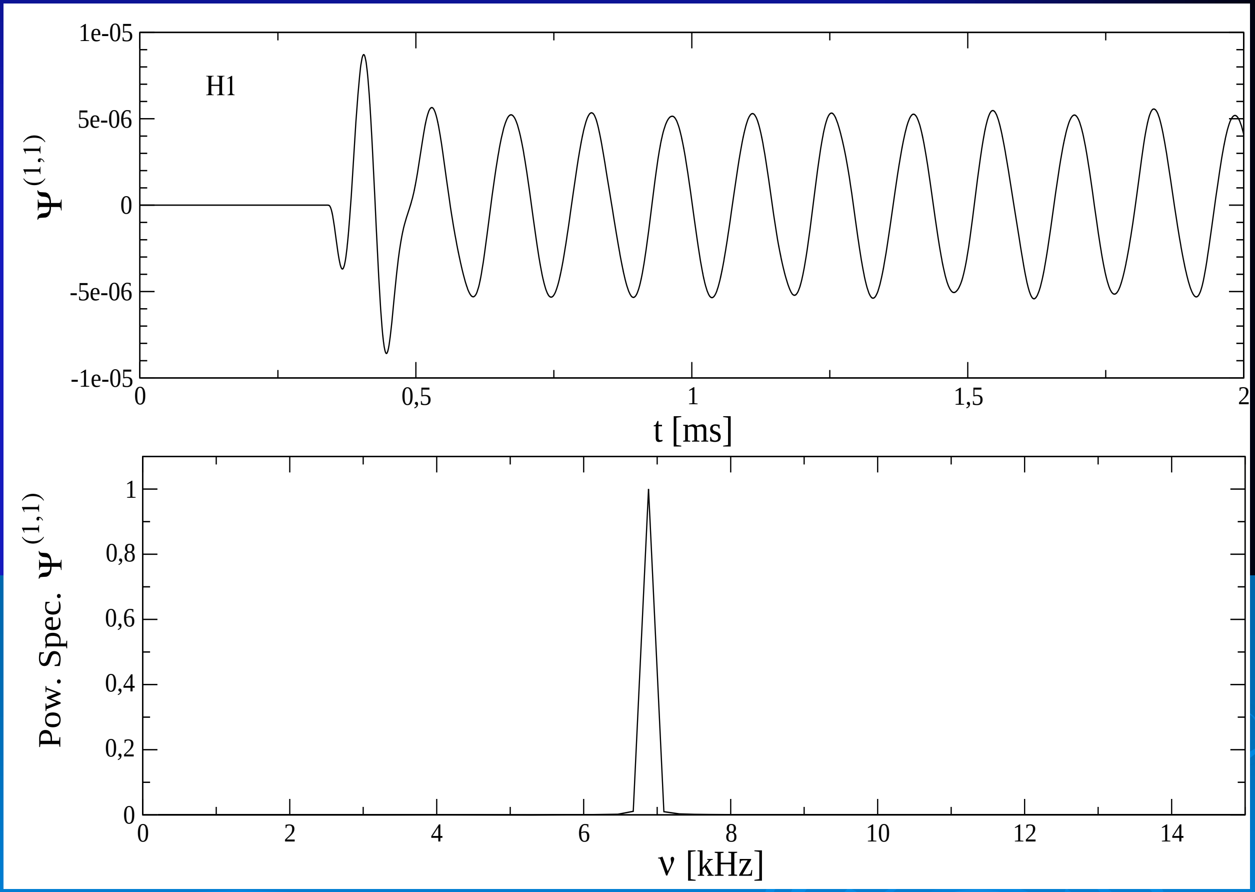
- Initial data for Linear axial nonradial perturbations: l=3 harmonic component of a J-constant differential rotation law → dragging of inertial frames.

$$\delta \mathrm{u}_\phi \propto \Omega(r,\vartheta) \propto \sum_{l,m} \delta u_\phi^{lm}(r)\, S_\phi^{lm}(\theta,\phi)$$

$T_c = 10$ ms

Axial Coupling: H1 wave form and spectrum

w-mode excitation
is due to the constraint violation on the initial Cauchy surface.



Periodic signal driven by radial pulsations through the sources.

$$S \propto J^{R} \otimes G^{NR} \propto e^{i\omega_R t}$$

Axial Coupling: Resonance

Radial Initial Data:

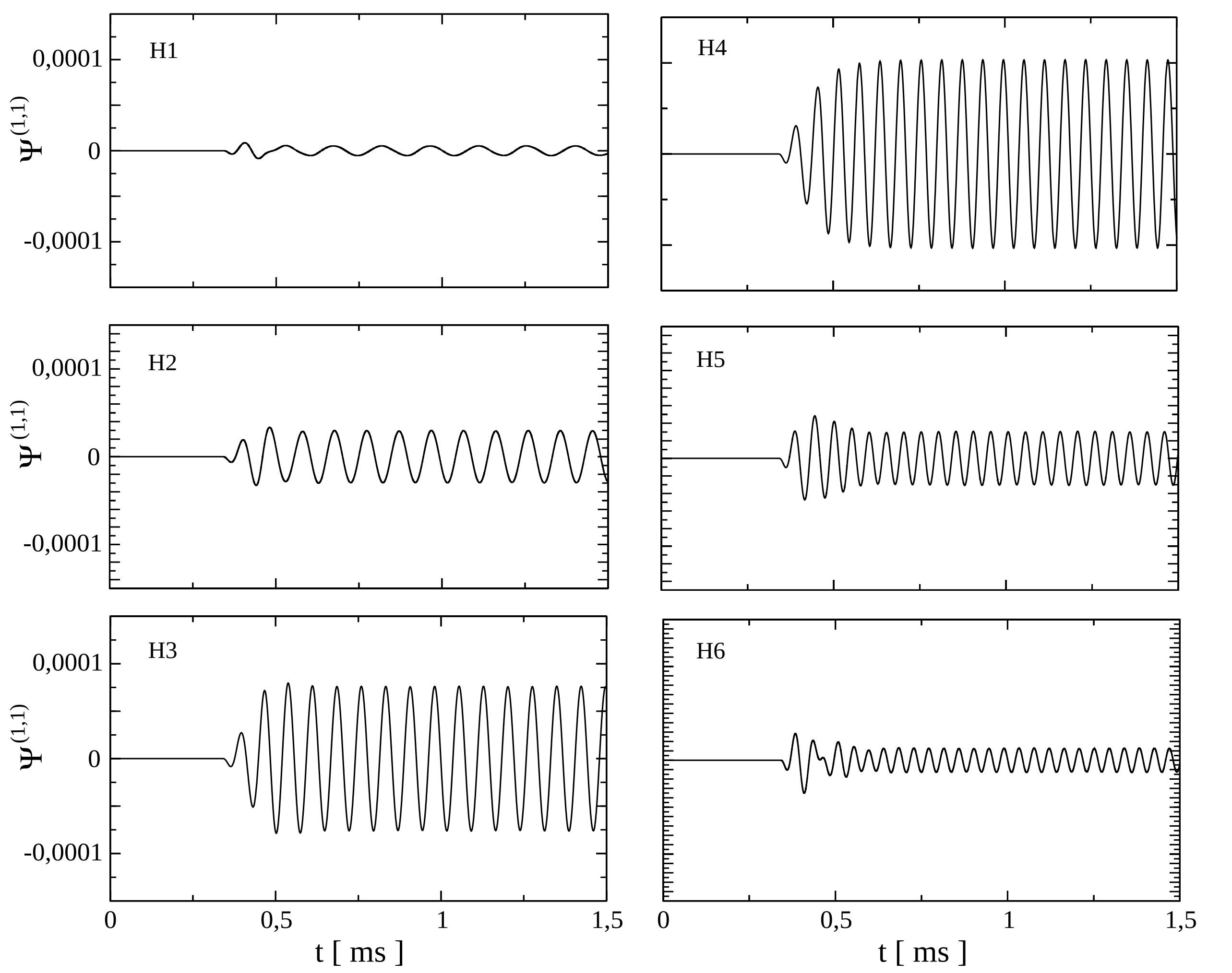
Single radial modes

ν_{H3} = 13.5 kHz

ν_{ω} = 16.09 kHz

ν_{H4} = 16.70 kHz

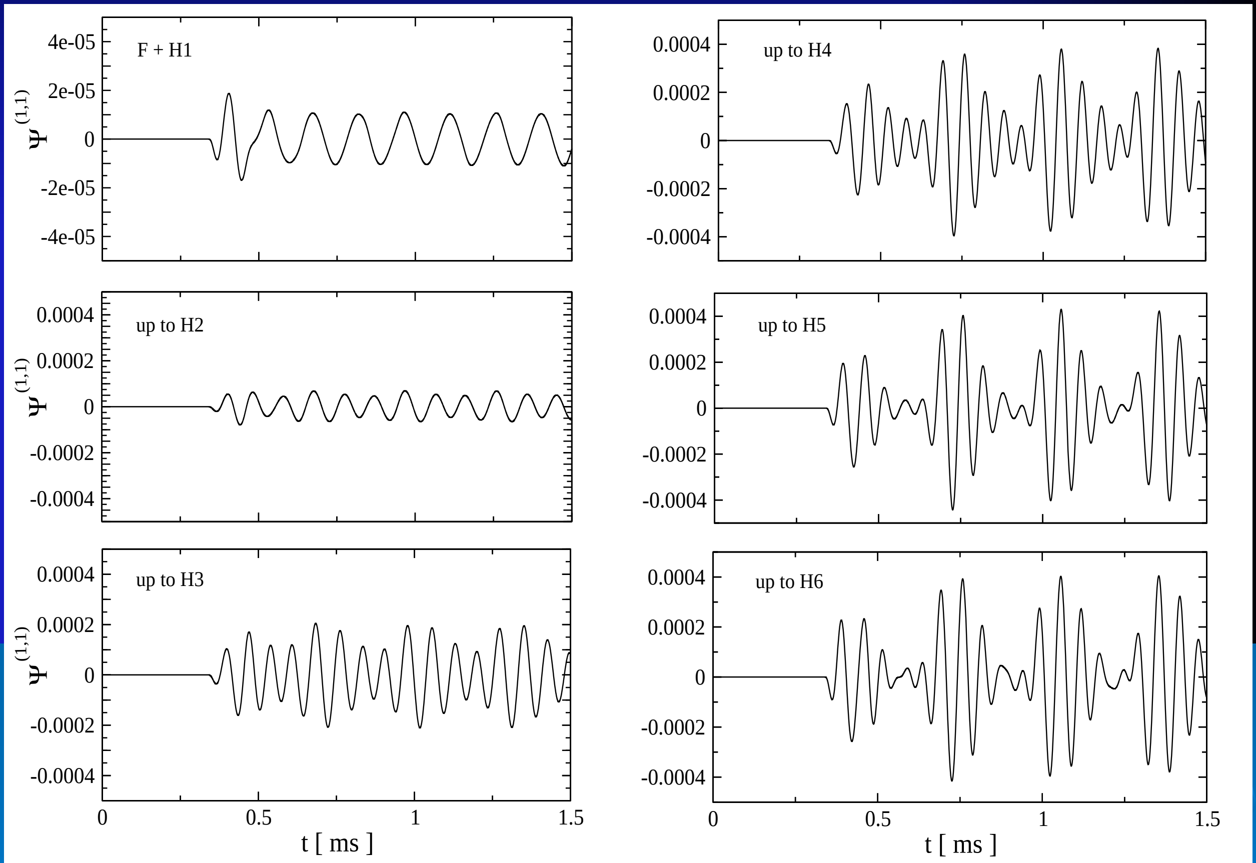
$\Psi + V\Psi = S$



Axial Coupling: Resonance

Radial Initial Data:

Linear combination of normal modes.



$$h_{\times}^{30} = 3.63\times10^{-17}\,\Psi^{30}\,\frac{10kpc}{r}\cos\theta\sin^2\theta$$

$$h_{\times}^{30}\Big|_{peak} = 1.45\times10^{-20}\,\frac{10kpc}{r}\cos\theta\sin^2\theta$$

Damping of radial pulsations

- *Power emitted in Gravitational waves*

$$P = \frac{dE}{dt} = \frac{1}{16\pi}\sum_{l,m}\frac{l(l+1)}{(l-1)(l+2)}\dot{\Psi}_{lm}^{2}$$

$$\tau = \frac{E^{(1,0)}}{P^{(1,1)}}$$

$$\tau \sim \varepsilon^{-2}$$

	F	H1	H2	H3	H4	H5	H6
N_{osc} ~	10^{10}	10^{5}	900	50	12	73	240

Polar coupling

A. Passamonti, M. Bruni, L. Gualtieri and C.F. Sopuerta
Phys. Rev. D **71**, 024022 (2005)

- Perturbative equations: System of three partial differential equations for 2 metric and and 1 fluid variable (Enthalpy). (Nagar et al. 2004)

- Initial data: analytic function that simulates the Enthalpy eigenfunctions

H = r/R sin(N p/2 r/R)

- Gravitational wave signal: construnction of Zerilli function Z

- Gravitational Wave energy at infinity:
(Cunnigham, Price and Moncrief 1978)

$$\frac{dE}{dt} = \frac{1}{64\pi} \sum_{l,m} \frac{(l+2)!}{(l-2)!} |\dot{Z}_{lm}|^2$$

- Gravitational wave strain:

$$h_+^{20} = 9.63 \times 10^{-18} Z^{20} \frac{10kpc}{r} \sin^2\theta$$

Polar coupling - Enthalpy

Initial data:

- Radial:

  Lin. Comb. up to H5

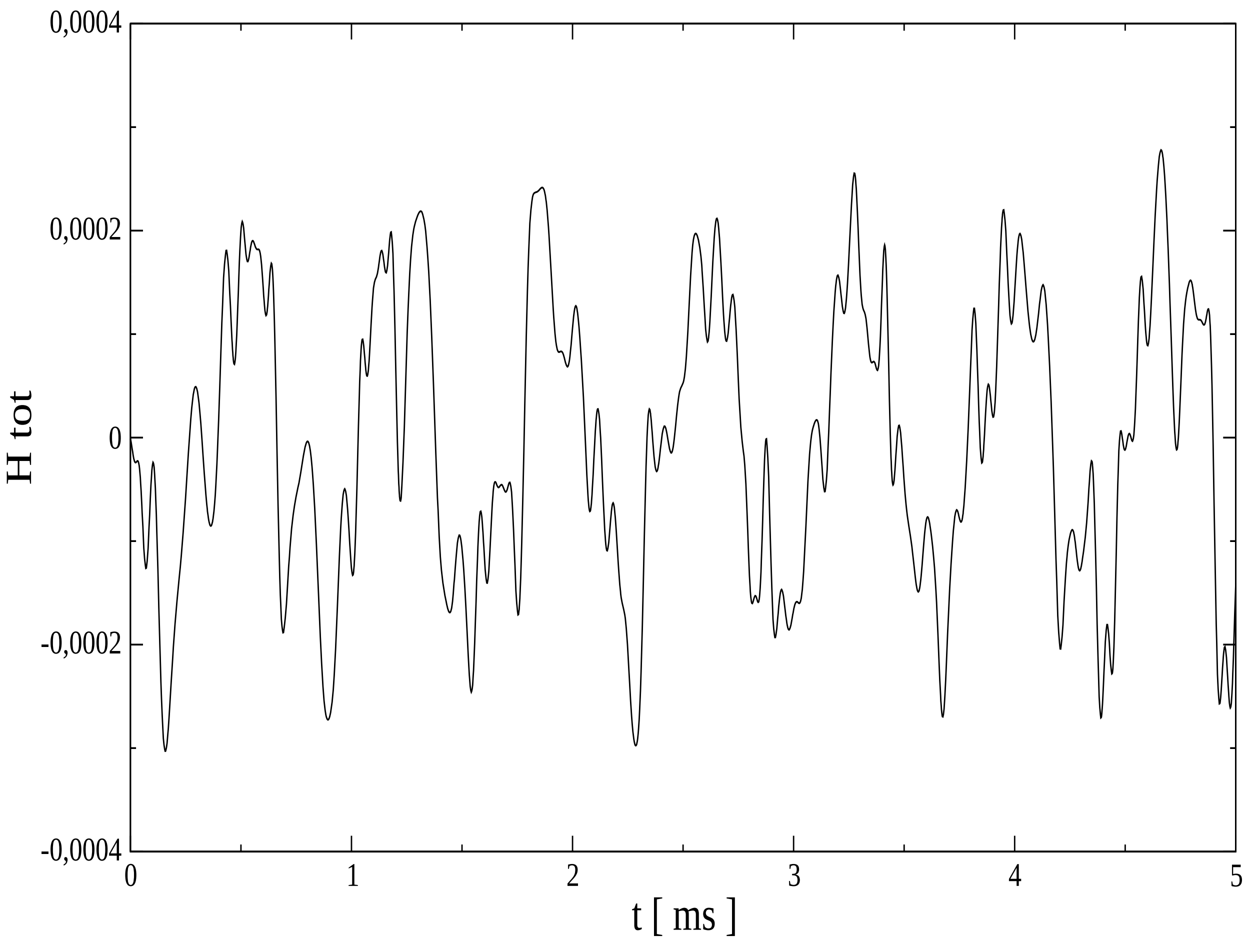
  Erad = $10^6\ M_\odot\ c^2$

- Nonradial:

  Enr ~ Erad /10

  Eigenfunction

  Node = 2.



Polar coupling – Enthalpy Spectrum

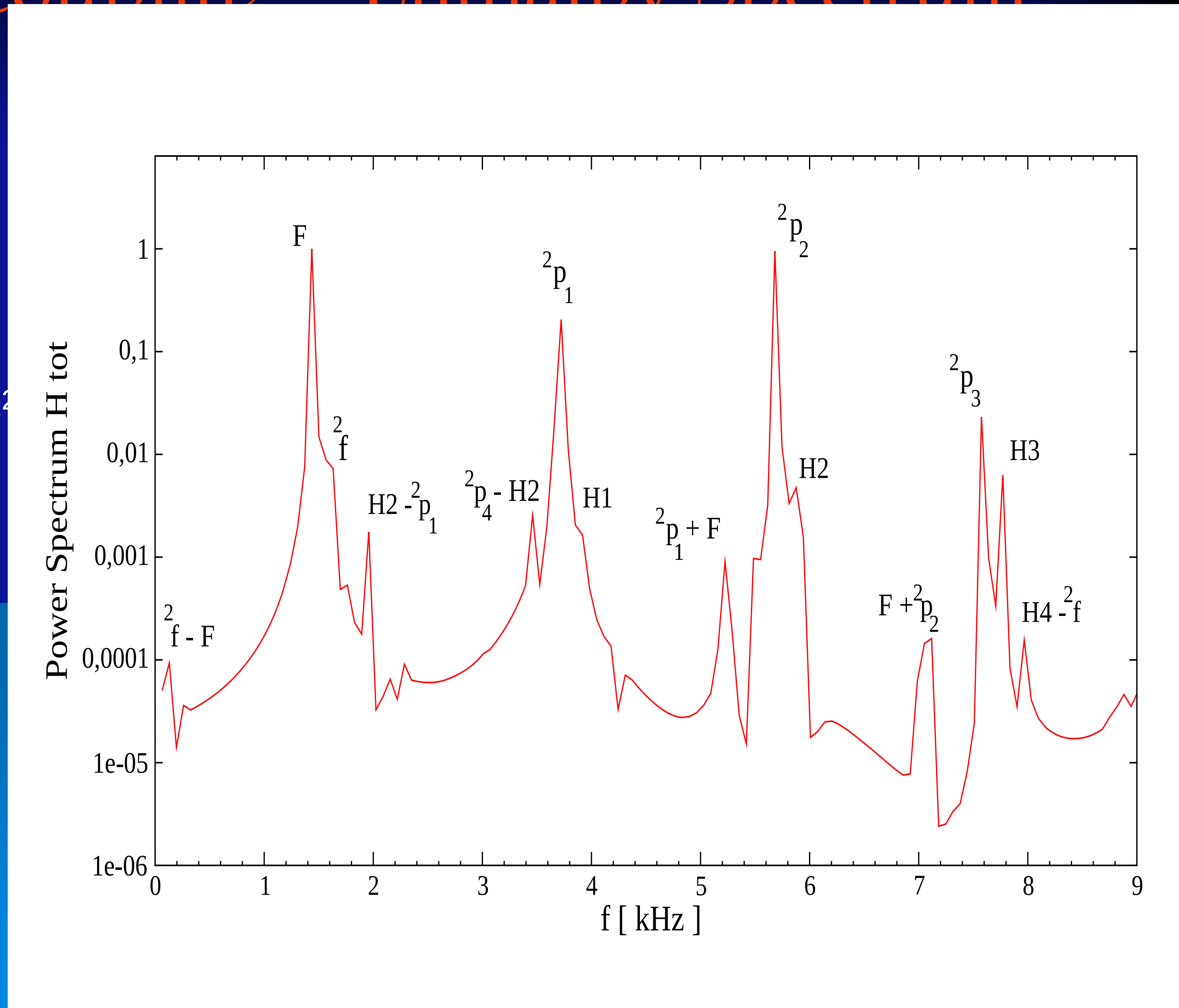
Initial data:

- Radial:

  Erad = $10^6 M_\odot c^2$

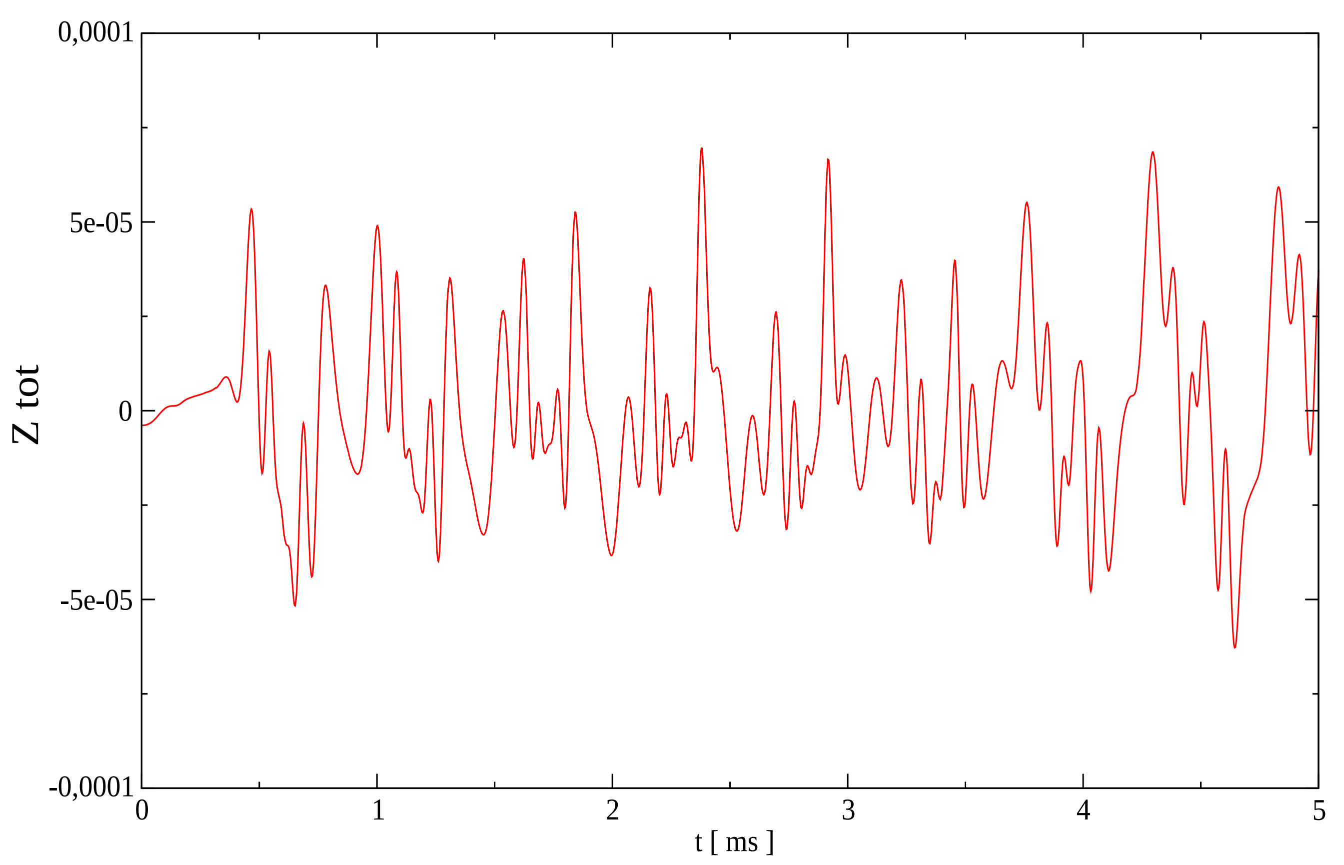
- Nonradial:

  Enr ~ Erad /10.



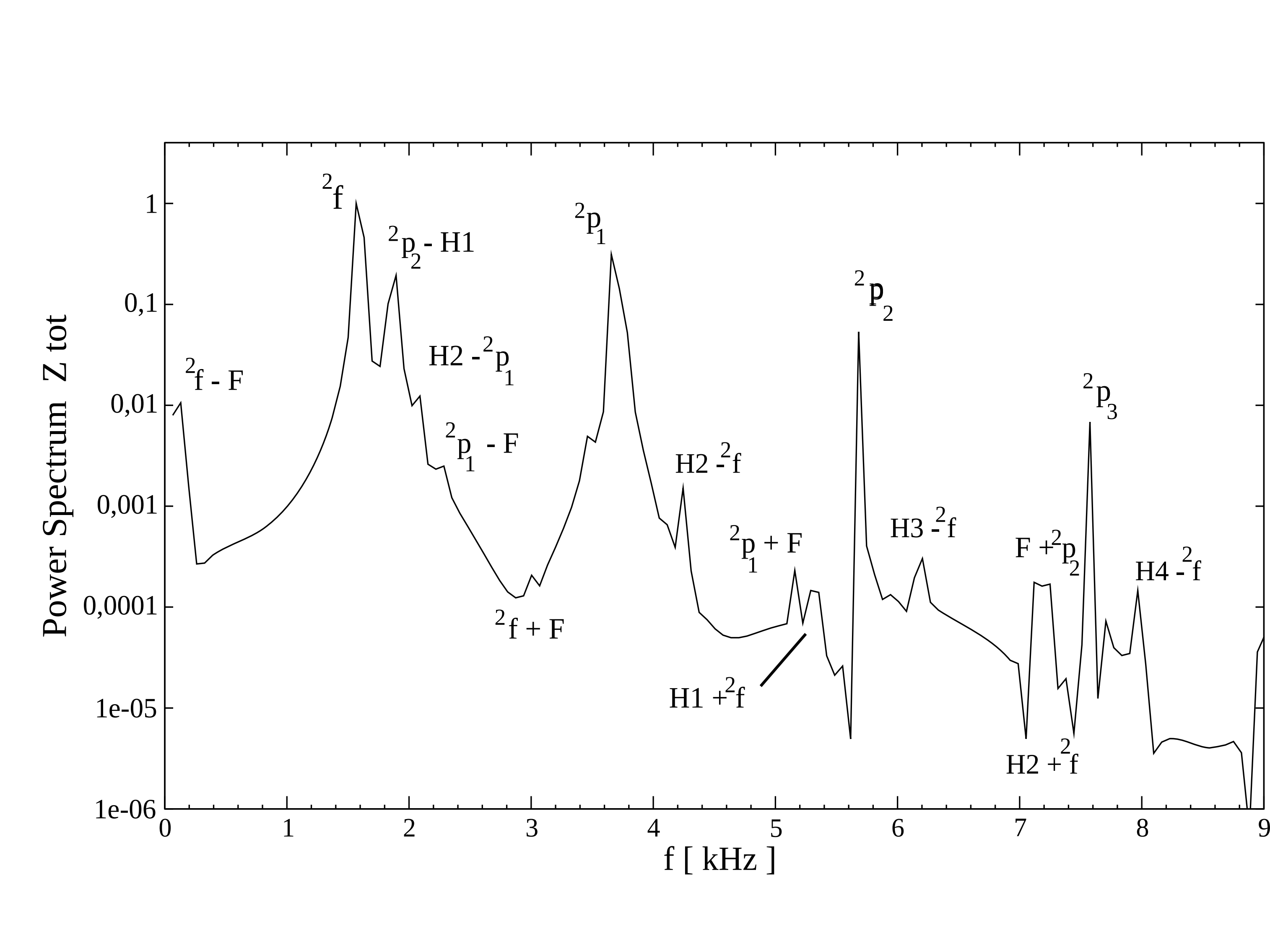
Polar coupling – Zerilli function Wave Form

Observer: 100 km



Polar coupling – Zerilli function

- Combination frequencies
- Sub-harmonics



Conclusions

- Nonlinear effects in stellar oscillations can provide new interesting features in the spectral properties and wave forms of gravitational waves.
- We have implemented a gauge invariant formalism and a numerical code to study in the time domain the coupling between radial and nonradial oscillations.
- Axial coupling shows an interesting ***new gravitational signal at first order in differential rotation, where the radial frequencies are precisely mirrored at coupling order.***
- This signal clearly exhibits a resonance when the radial pulsations frequencies are close to the axial w-mode.
- Polar coupling: first results show the presence of nonlinear harmonics and spectrum pattern similar to full nonlinear results.

Future Works

- ✶Investigate the efficiency of the coupling for more realistic EOS.
- ✶ Rotating stars: Study non-linear axi/nonaxi-symmetric oscillations with perturbation theory and with the 3-D non-linear code Cactus-Whisky.
- ✶Set up initial rotating configurations for studying resonances and parameter amplifications.