Stability and Dynamics in Fabry-Perot cavities due to combined photothermal and radiation-pressure effects

Francesco Marino[1,4], Maurizio De Rosa[2], Francesco Marin[1,3,4]

[1] Istituto Nazionale di Fisica Nucleare, Firenze
[2] CNR-INOA, Sezione di Napoli
[3] Dip. di Fisica, Università di Firenze
[4] European Laboratory for Non-linear Spectroscopy (LENS), Firenze

LIGO-G060310-00-Z

Radiation-Pressure and Thermal expansion

Kerr cavity: The intracavity power modifies the refraction index (then the optical path) leading to changes in the intracavity power

Radiation-pressure driven cavity: The radiation pressure modifies the cavity length $\Rightarrow$ the intracavity power changes $\Rightarrow$ the radiation-pressure force varies

Photo-thermal expansion: Thermal expansion of the mirrors modifies the cavity length $\Rightarrow$ the intracavity power changes $\Rightarrow$ the thermal expansion varies

Nonlinear dependence of the intracavity path on the optical power

$\Rightarrow$ *Multi-stability: cohexistence of stationary solutions*

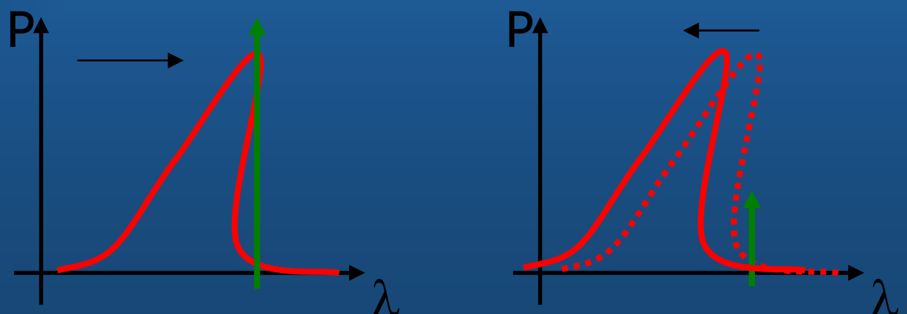
Physical Mechanism

Interplay between radiation pressure and photothermal effect

Optical injection on the long-wavelength side of the cavity resonance

1. The radiation pressure tends to increase the cavity lenght respect to the cold cavity value$\Rightarrow$ the intracavity optical power increases

2. The increased intracavity power *slowly* varies the temperature of mirrors $\Rightarrow$ heating induces a decrease of the cavity length through thermal expansion



Physical Model

We write the cavity lenght variations as $L(t) = L_{rp}(t) + L_{th}(t)$

Radiation Pressure Effect

Limit of small displacements⇒

Damped oscillator forced by the intracavity optical power

$$\ddot{L}_{rp} + \frac{\Omega}{Q}\dot{L}_{rp} + \Omega^2 L_{rp} = \frac{2P_c}{mc}$$

$$\ddot{\phi} + \frac{1}{Q}\dot{\phi} + \phi = \tilde{\alpha} P_c$$

$$\tilde{\alpha} = 2/mc\Omega^2\Delta$$

Photothermal Effect

Single-pole approximation⇒

The temperature relaxes towards equilibrium at a rate ε and $L_{th} \propto T$

$$\dot{T} = -\varepsilon\left(T - T_0 - \frac{dT}{dP_c}P_c\right)$$

$$\dot{L}_{th} = -\varepsilon\left(L_{th} + \left|\frac{dL_{th}}{dP_c}\right| P_c\right)$$

$$\dot{\theta} = -\varepsilon(\theta + \tilde{\beta} P_c)$$

Intracavity optical power

Simple case: Adiabatic approximation

⇒ The optical field instantaneously follows the cavity length variations

$$P_c = \frac{AP_{in}}{1 + \left\{\frac{4F}{\lambda}[L(t) + L_0]\right\}^2}$$

Physical model

$$\dot{\phi} = v,$$

$$\dot{v} + \frac{1}{Q}v = -\phi + \frac{\alpha}{1 + (\delta_0 + \phi + \theta)^2}$$

$$\dot{\theta} = -\varepsilon\left[\theta + \frac{\beta}{1 + (\delta_0 + \phi + \theta)^2}\right].$$

$\delta_0 = L_0/\Delta,\ \alpha = \tilde{\alpha}AP_{in},\ \beta = \tilde{\beta}AP_{in}$

The stability domains and dynamics depends on the type of steady states bifurcations

Stationary solutions

$v_s = 0$ $\quad -\phi_s[1 + (\delta_0 + \phi_s + \theta_s)^2] + \alpha = 0$ $\quad \theta_s = -(\beta/\alpha)\phi_s$

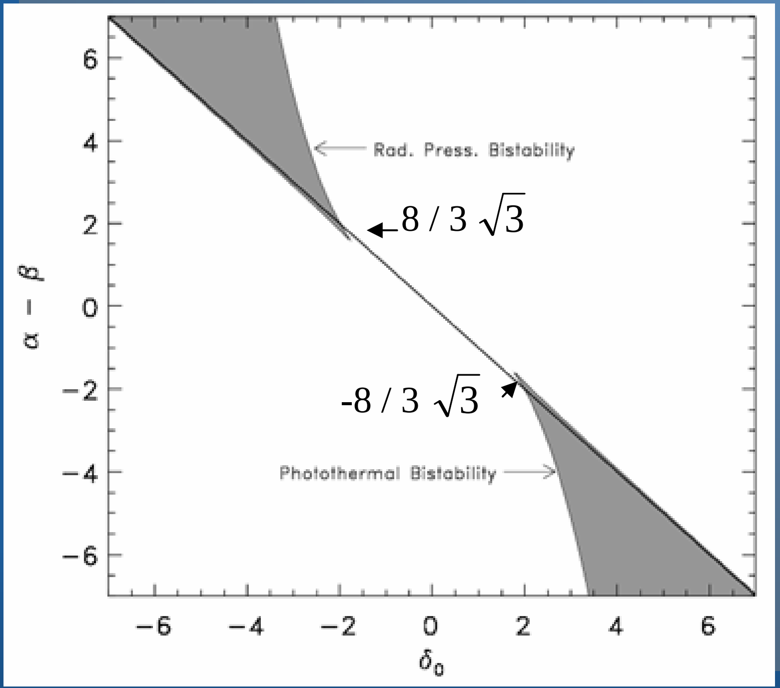
Depending on the parameters the system can have either one or three fixed points

Bistability

Analyzing the cubic equation for ϕ $\Rightarrow$

$$\left(\frac{\delta_0^2-3}{9}\right)^3=\left[\frac{(\alpha-\beta)}{2}+\frac{1}{3}\delta_0\left(1+\frac{\delta_0^2}{9}\right)\right]^2$$

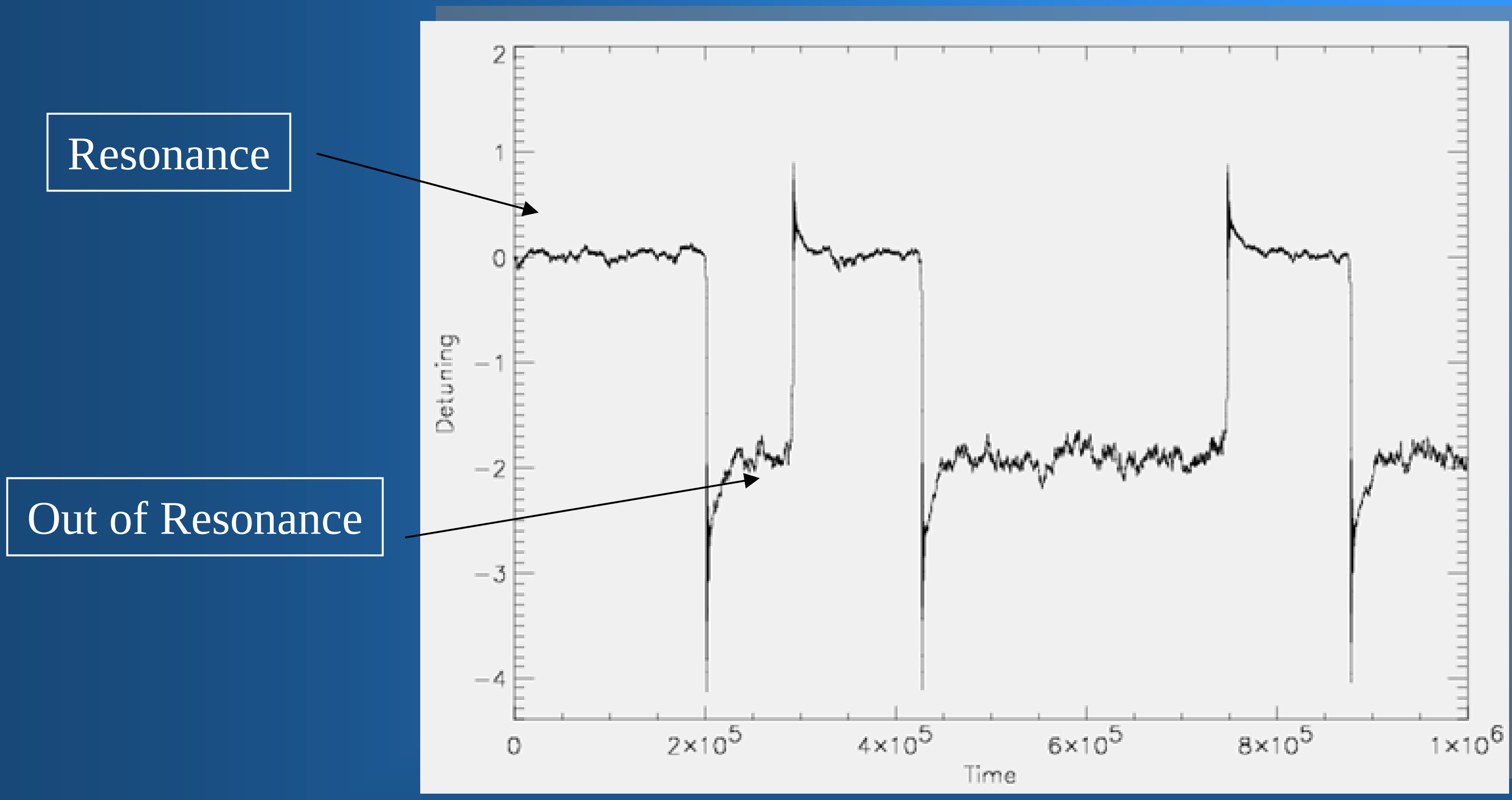
On this curve two new fixed points (one stable and the other unstable) are born in a saddle-node bifurcation



Changes in the control parameters can produce abrupt jumps between the stable states

Bistability: Noise Effects

Noise Effects → Mode-hopping between the two stable states



Local stability region width $\propto 1 / P_{in}$: critical for high power

Single solution: Hopf Bifurcation

Single steady state solution: Stability Analysis

$$\dot{\delta\phi} = \delta v ,$$

$$\dot{\delta v} = -\frac{1}{Q}\delta v - (1 + C\phi_s)\delta\phi - C\phi_s\delta\theta$$

$$\dot{\delta\theta} = -\varepsilon[C\theta_s\delta\phi + (1 + C\theta_s)\delta\theta],$$

$$C = 2(\delta_0 + \phi_s + \theta_s)/[1 + (\delta_0 + \phi_s + \theta_s)^2]$$

They admit nontrivial solutions $\mathbf{K}\, e^{\Lambda t}$ for eigenvalues Λ given by

$$\Lambda^3 + a_1\Lambda^2 + a_2\Lambda + a_3 = 0$$

$$a_1 = \varepsilon(C\theta_s + 1) + \frac{1}{Q} \qquad a_2 = 1 + C\phi_s + \frac{\varepsilon}{Q}(C\theta_s + 1) \qquad a_3 = \varepsilon(C\theta_s + C\phi_s + 1)$$

$$Re\,\Lambda = 0 \quad ; \quad Im\,\Lambda = i\,\nu$$

Hopf Bifurcation: The steady state solution loses stability in correspondence of a critical value of δ_0 (other parameter are fixed) and a finite frequency limit cycle starts to grow

Hopf Bifurcation Boundary

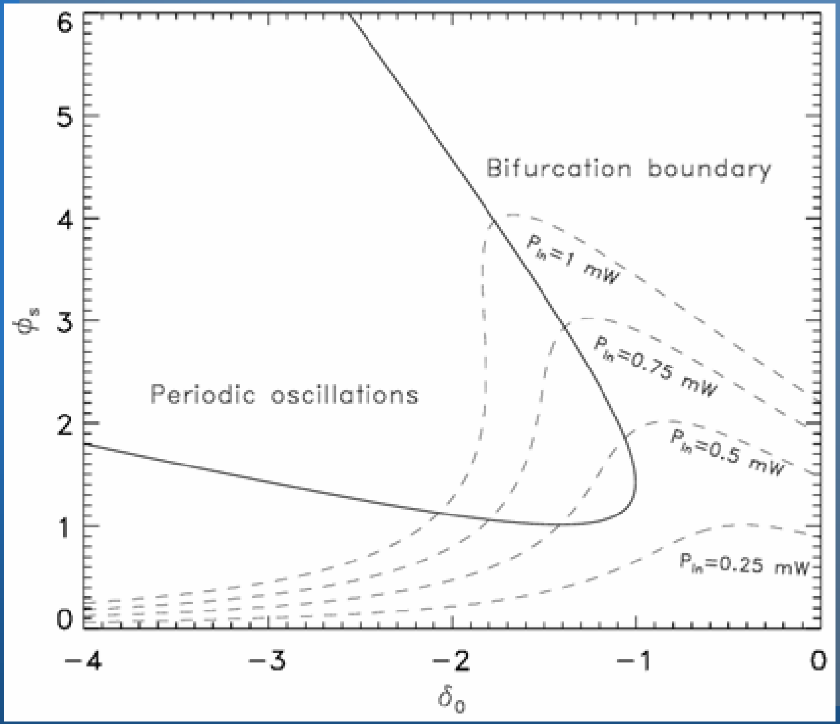
$$\nu^2 = a_2,$$ Frequency of the limit cycle

$$a_2 a_1 - a_3 = 0$$ Boundary of the bifurcation

Q = 1, ε = 0.01, $\alpha = 4 P_{in}$, $\beta = 2.4 P_{in}$

For sufficiently high power the steady state solution loses stability in correspondence of a critical value of δ_0

Further incresing of δ_0 leads to the “inverse” bifurcation



Linear stability analysis is valid in the vicinity of the bifurcations
Far from the bifurcation **?**

Relaxation oscillations

ε small ⇒ separation of the system evolution in two time scales: $O(1)$ and $O(\varepsilon)$

We consider $\varepsilon = 0$ and $1/Q \gg \varepsilon$ ⇒ θ is constant

Fast Evolution

$$\dot{\phi} = v,$$

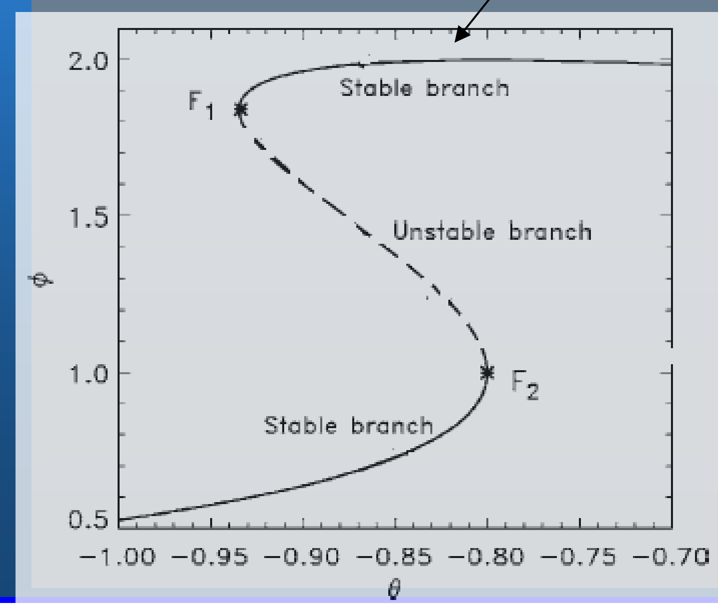
$$\dot{v} + \frac{1}{Q} v = -\phi + \frac{\alpha}{1 + (\delta_0 + \phi + \theta)^2}$$

$$\dot{\theta} = -\varepsilon\left[\theta + \frac{\beta}{1 + (\delta_0 + \phi + \theta)^2}\right].$$

Fixed Points

$$-\phi + \frac{\alpha}{1 + (\delta_0 + \phi + \theta)^2} = F(\phi, \theta) = 0$$

⇒ $\phi(\theta)$



By linearization we find that the stability boundaries ($F_{1,2}$) are given by $C\phi = -1$

Relaxation oscillations

Slow Evolution $\Rightarrow$ θ is slowly varying (ϕ instantaneously follows θ variations)

By means of the time-scale change $\tau = \varepsilon t$ and putting $\varepsilon = 0$

$$\dot{\theta} = -\left[\theta + \frac{\beta}{1+(\delta_0+\phi+\theta)^2}\right] \equiv G(\phi,\theta)$$

$$-\phi + \frac{\alpha}{1+(\delta_0+\phi+\theta)^2} \equiv F(\phi,\theta) = 0$$

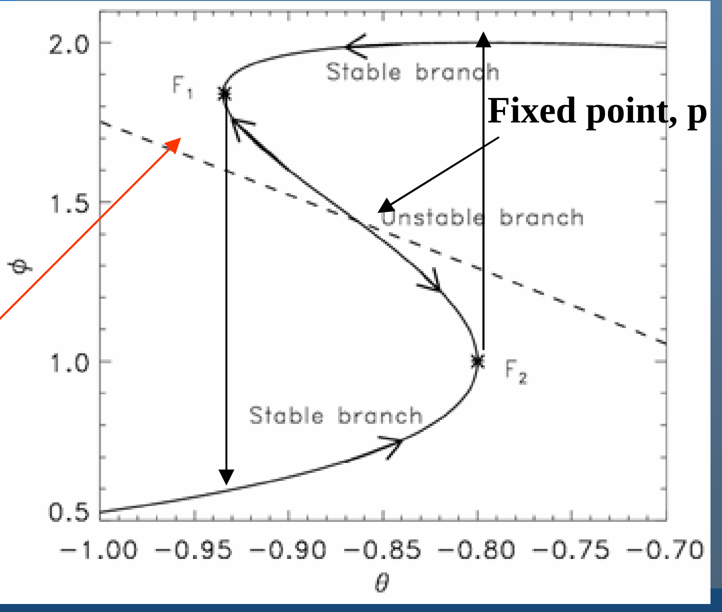
$$\dot{\theta} = -\left[\theta + \frac{\beta}{\alpha}\phi(\theta)\right]$$

Defines the branches of slow motion (Slow manifold)

If $\phi(\theta) > p \Rightarrow d_t\theta < 0$

If $\phi(\theta) < p \Rightarrow d_t\theta > 0$

G=0



At the critical points $F_{1,2}$ the system istantaneously jumps

Numerical Results

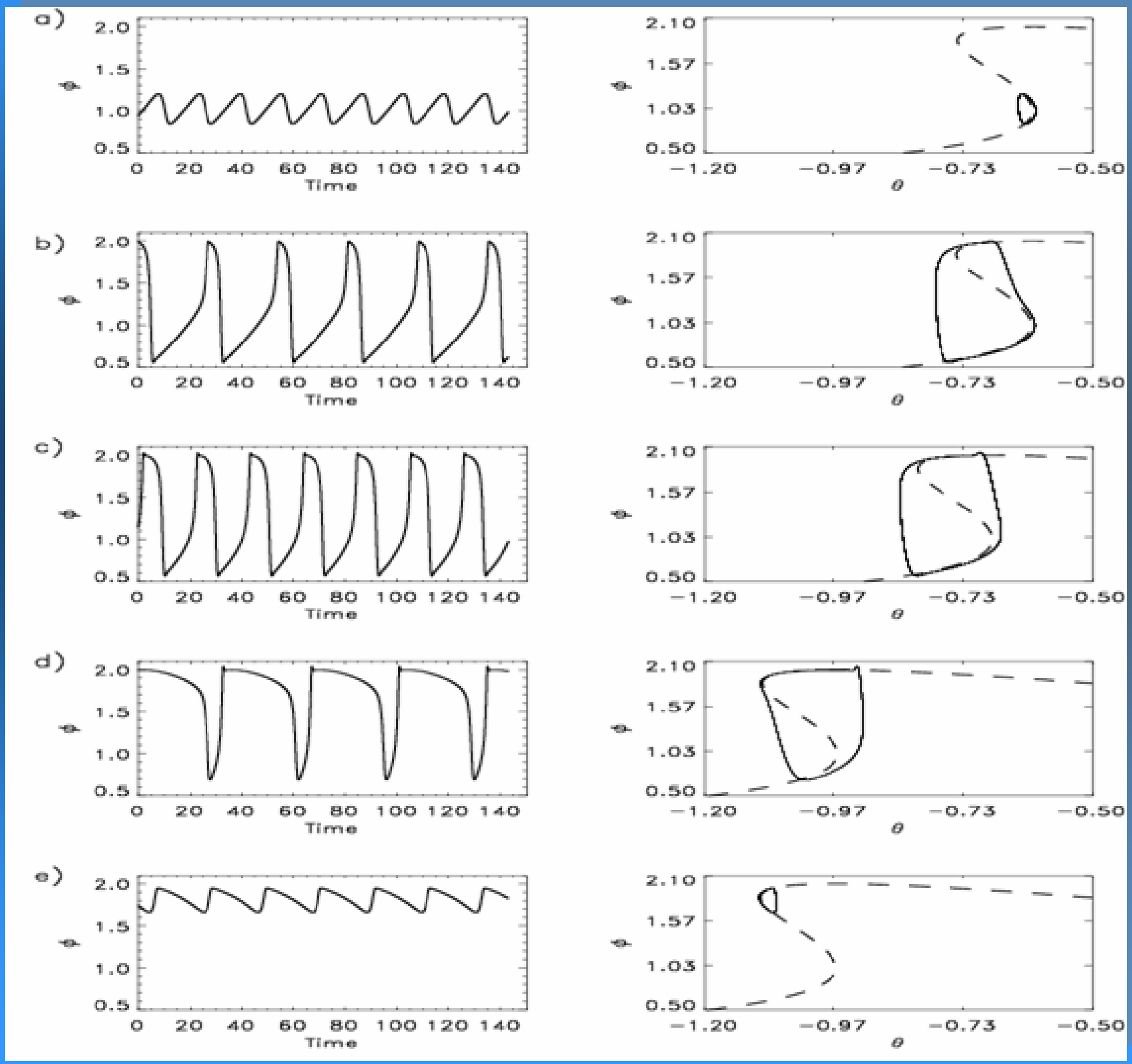
Q = 1

Temporal evolution of the ϕ variable and corresponding phase-portrait as δ_0 is varied

Far from resonance:
Stationary behaviour

In correspondence of δ_0^c:
Quasi-harmonic Hopf limit cycle

Further change of δ_0:
Relaxation oscillations, reverse sequence and a new stable steady state is reached



Numerical Results

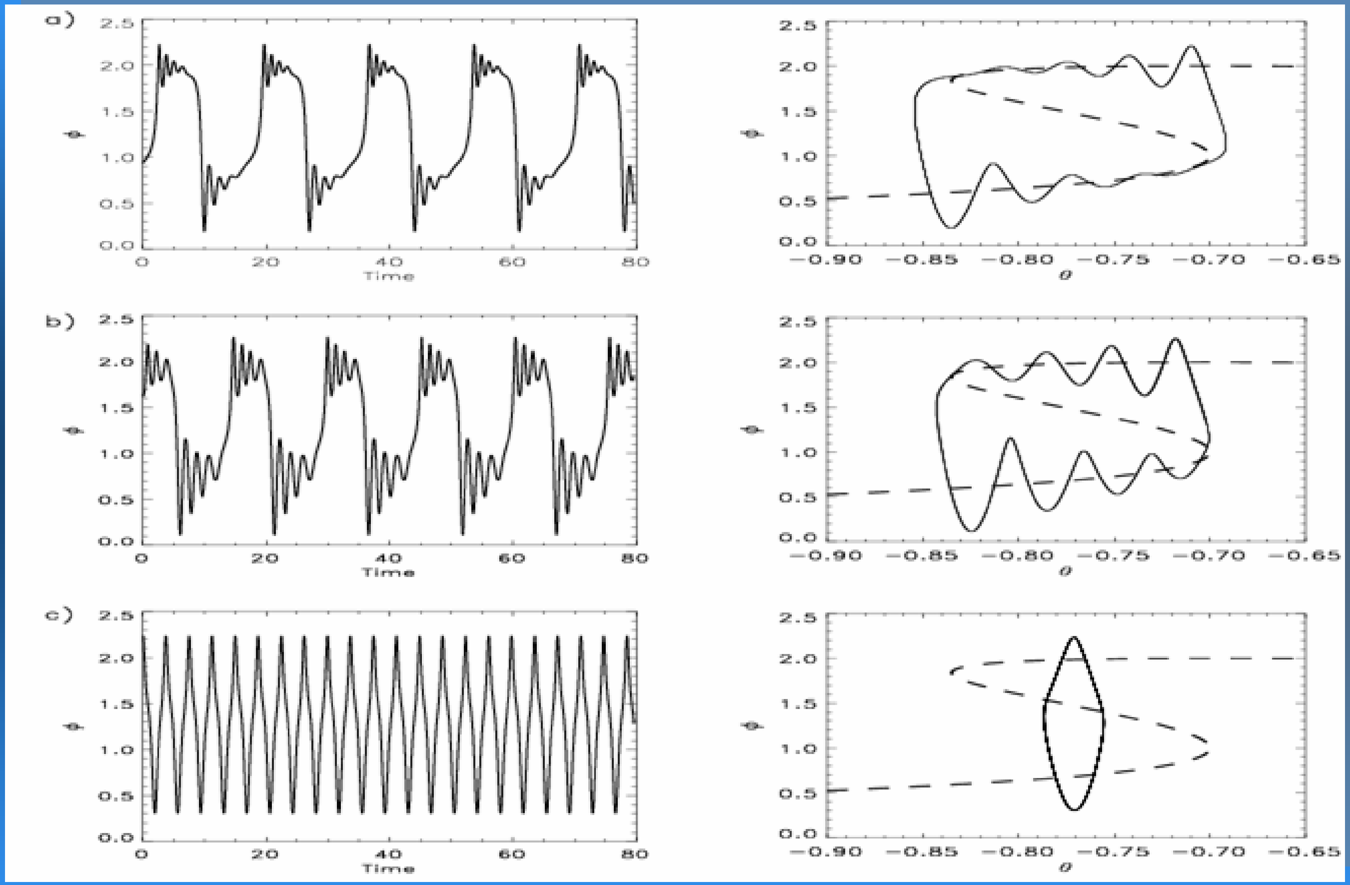
Q > 1

Temporal evolution of the ϕ variable (in the relaxation oscillations regime) and corresponding phase-portrait as Q is incresed

Q= 5

Q=10

Q=20



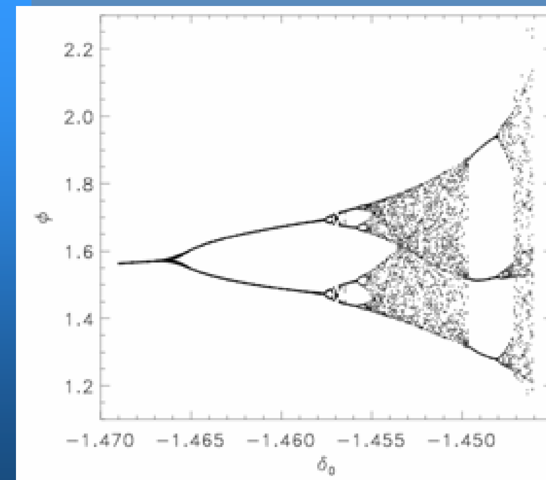
Relaxation oscillations with damped oscillations when jumps between the stable branches of the slow manifold occurs

Numerical Results

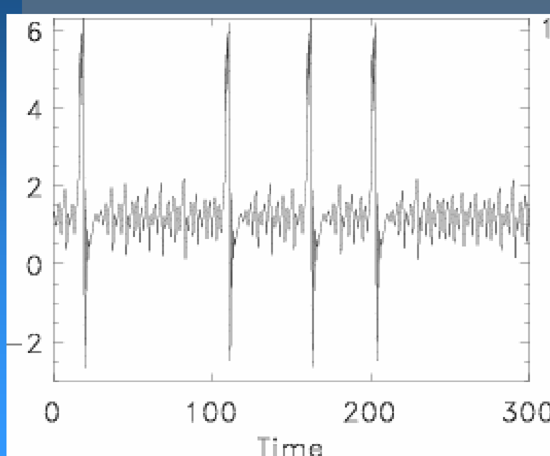
Q > 1 Three interacting time scales:

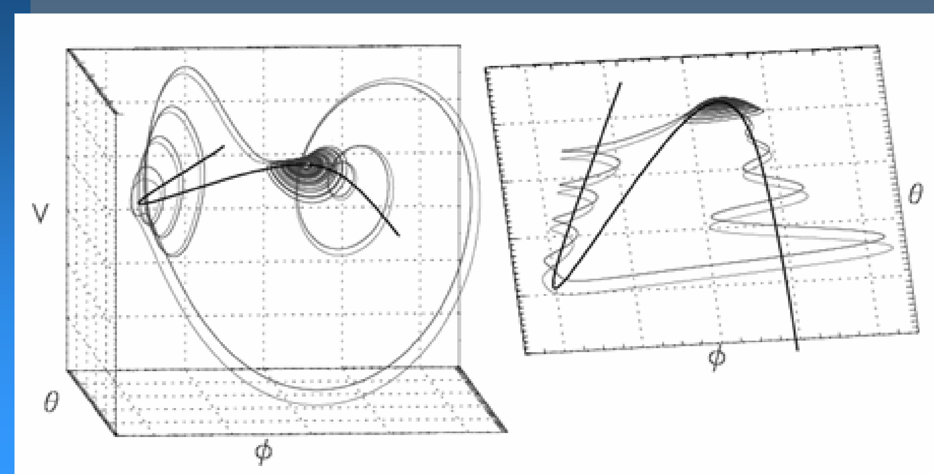
O(1), O(1/Q), O(ε)

The competition between Hopf-frequency and damping frequency leads to a period-doubling route to chaos



Between the self-oscillation regime and the stable state ⇒ Chaotic spiking





Stability Domains: Oscillatory behaviour

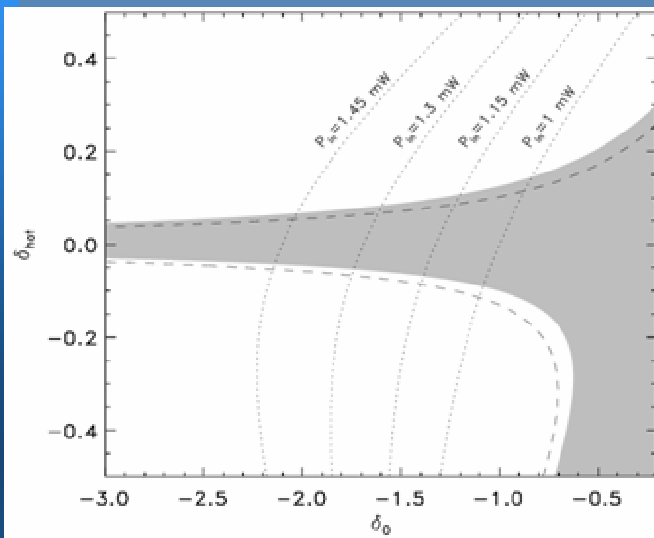
Stability domain in presence of the Hopf bifurcation

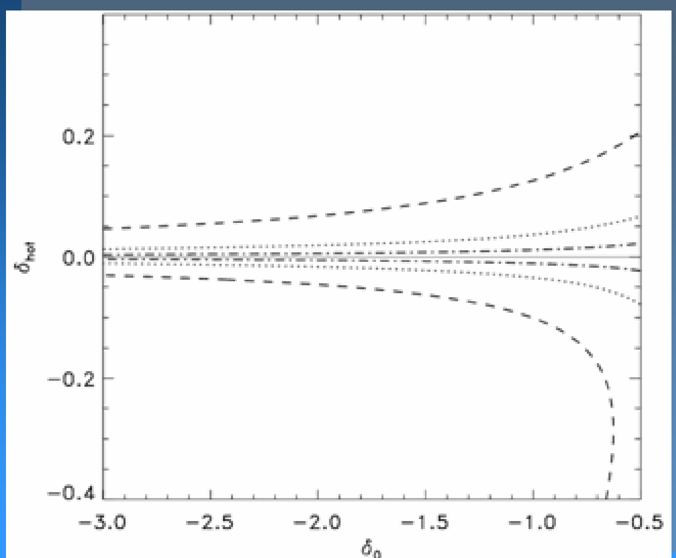
$$a_1 = \varepsilon(C\theta_s + 1) + \frac{1}{Q}\,,$$
$$a_2 = 1 + C\phi_s + \frac{\varepsilon}{Q}(C\theta_s + 1)\,,$$
$$a_3 = \varepsilon(C\theta_s + C\phi_s + 1)\,.$$

$$\nu^2 = a_2$$
$$a_2 a_1 - a_3 = 0\,.$$

Q » 1

$$|\delta_h| = \frac{1}{2}\sqrt{\frac{(1+\varepsilon^2)}{\varepsilon\alpha\beta Q}}$$





For high Q is critical

Stability domains for Q=1000, 10000,100000

Future Perspectives

- Experiment on the interaction between radiation pressure and photothermal effect

- Time Delay Effects (the time taken for the field to adjust to its equilibrium value)
  Model of servo-loop control
  $\Rightarrow$ Extend the model to the case of gravitational wave interferometers