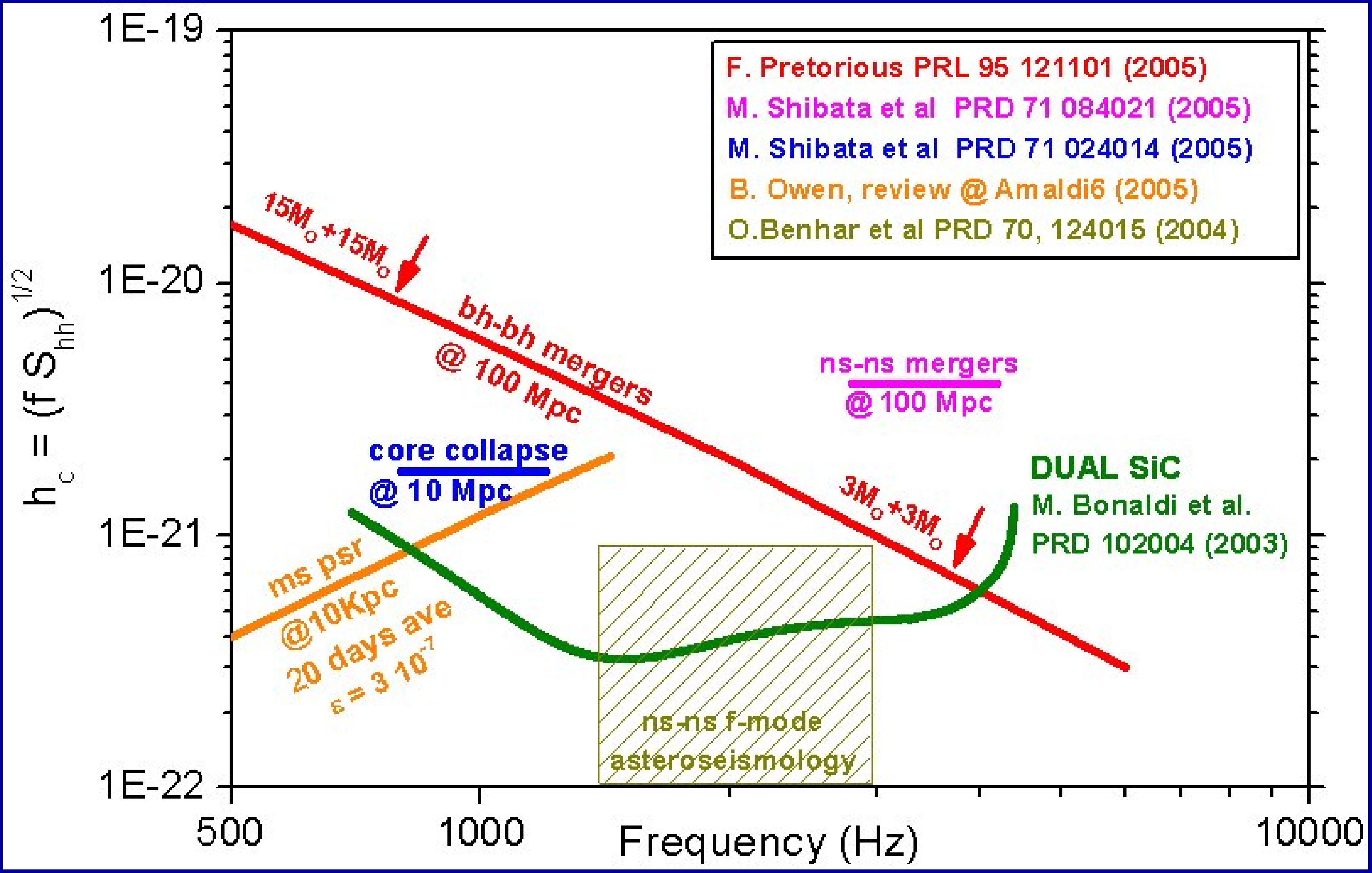
GW detectors optimized for the kHz range

Francesco MARIN for the DUAL-RD collaboration

LIGO-G060328-00-Z

Motivation ...



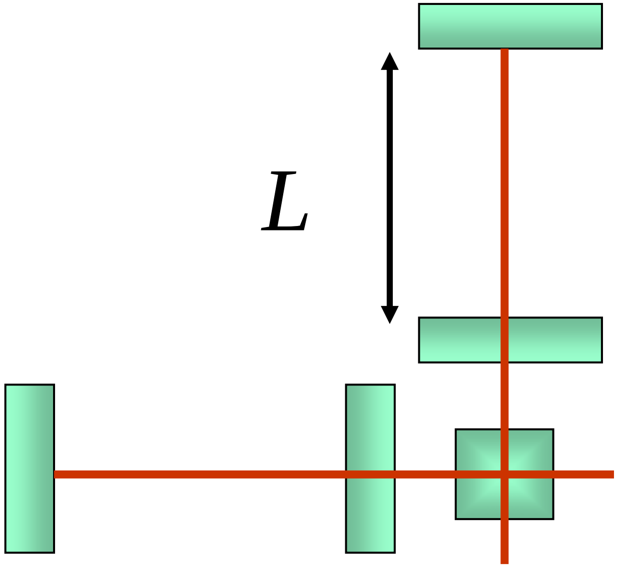
1E-19
1E-20
1E-21
1E-22
500
1000
10000
Frequency (Hz)
$h_c = (f S_{hh})^{1/2}$
F. Pretorious PRL 95 121101 (2005)
M. Shibata et al PRD 71 084021 (2005)
M. Shibata et al PRD 71 024014 (2005)
B. Owen, review @ Amaldi6 (2005)
O.Benhar et al PRD 70, 124015 (2004)
$15M_o+15M_o$
bh-bh mergers
@ 100 Mpc
$3M_o+3M_o$
ns-ns mergers
@ 100 Mpc
core collapse
@ 10 Mpc
ms psr
@10Kpc
20 days ave
$\varepsilon = 3\ 10^{-7}$
DUAL SiC
M. Bonaldi et al.
PRD 102004 (2003)
ns-ns f-mode
asteroseismology

Try to design a detector design devoted to the kHz range. Hopefully

- compact
- sensitive
- cheap...

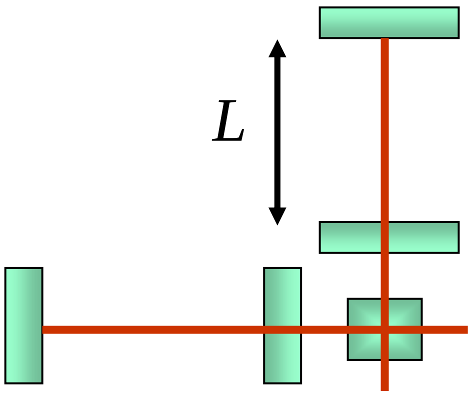
... Back to the beginning

Usual detection strategies…

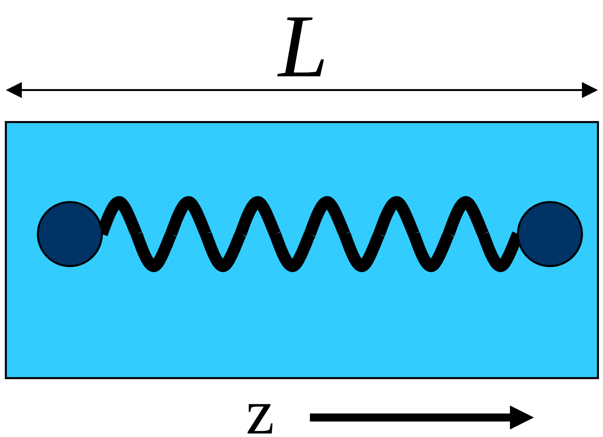


$$\delta L = \frac{h}{2} L$$

Usual detection strategies...



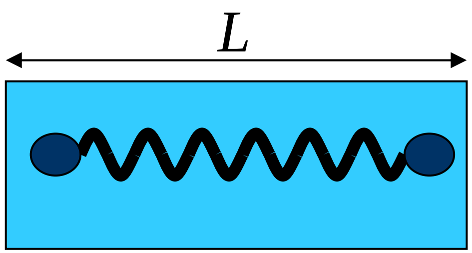
$$\delta L = \frac{h}{2} L$$



$$F = \frac{h}{2} \varpi^2 \rho z$$

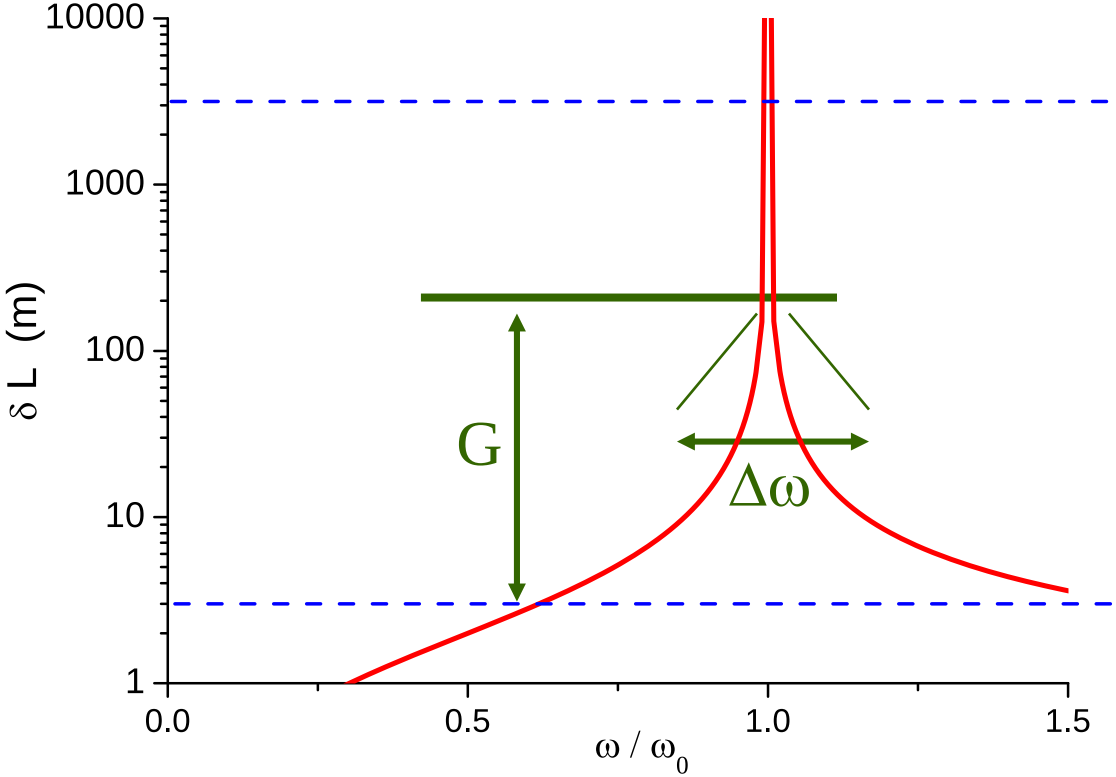
$$\delta L = \frac{h}{2} L \frac{\varpi^2}{-(\varpi_0^2 - \varpi^2 + i \Phi \varpi_0^2)}$$

Usual detection strategies...



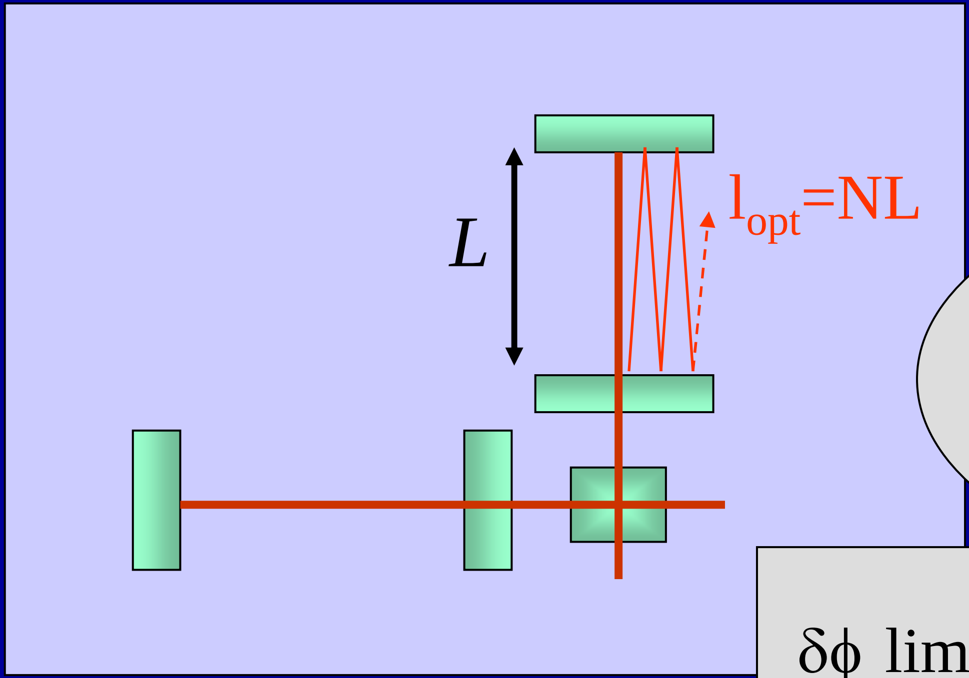
$$F = \frac{h}{2}\varpi^{2}\rho\, z$$

$$\delta L = \frac{h}{2} L \frac{\varpi^{2}}{-(\varpi_0^{2} - \varpi^{2} + i\Phi\,\varpi_0^{2})}$$



$$G \cdot \Delta\omega = \omega_0$$

... Back to interferometers ...



$$\delta\phi = 2\pi\, \delta l_{opt}/\lambda$$
$$= 2\pi\, h\, l_{opt}/\lambda$$

$\delta\phi$ limited by quantum noise (SQL):

$$(\delta\phi)^2 = h\nu_{laser}/2P_{in}$$

Seems better to get the longest possible l_{opt}. However...

… the useful measurement time is just $\sim T/2$

(T is the g.w. period: $T = 2\pi/\omega$)

$$l_{opt} = c\,T/2$$

$$\delta\phi = 2\pi\,\nu_{laser}\,h\,T/2$$

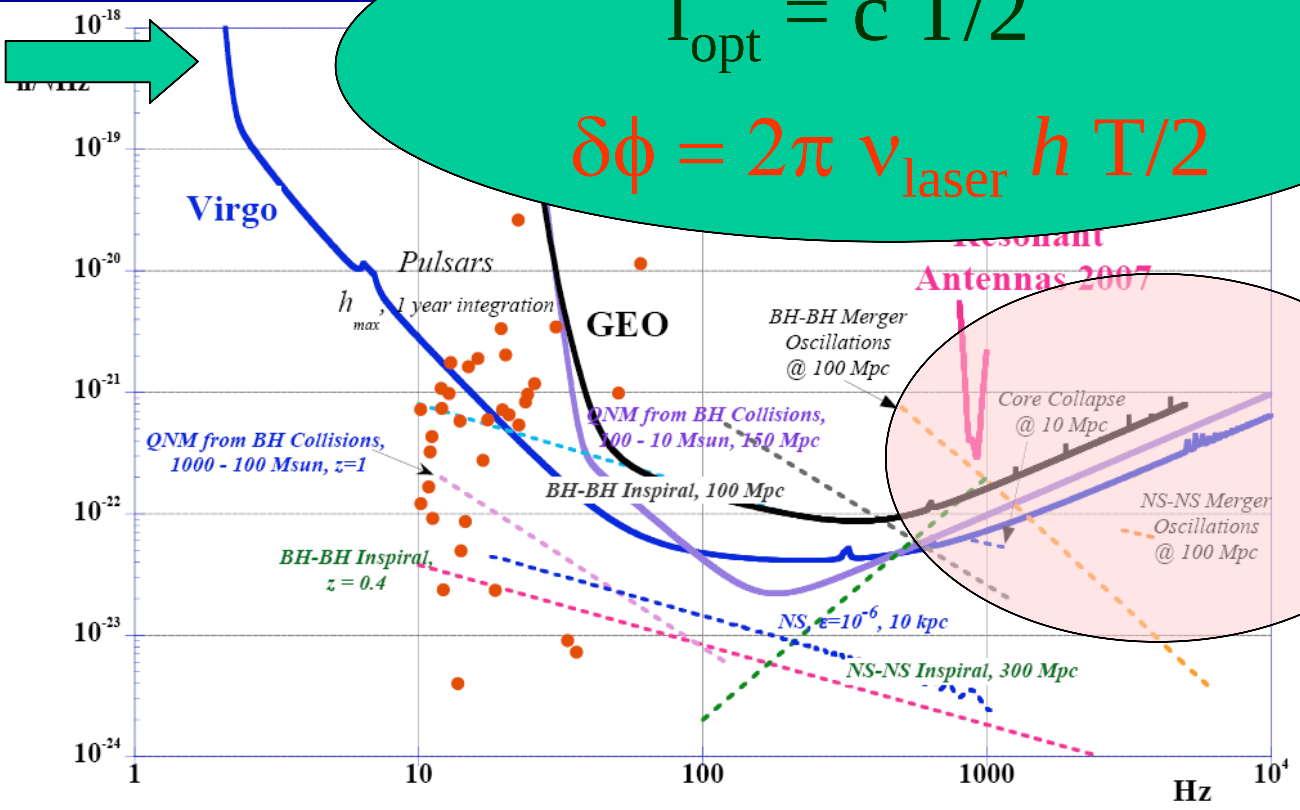


Figure 2: Sensitivity for GEO – LIGO - Virgo – Explorer - Nautilus – Auriga, with estimated levels of some possible sources. The Pulsars points show SNR with respect to background noise of Virgo after one year of integration, for maximum amplitude of the emitted GW.

$$h_{min} \propto \omega$$

$$l_{opt} = c\,T/2$$

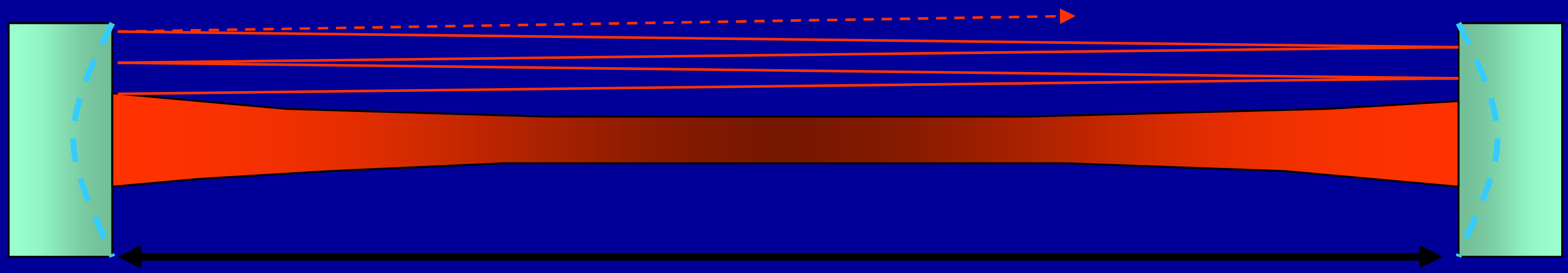
$$\delta\phi = 2\pi\, \nu_{laser}\, h\, T/2$$

No more dependence on L if we can keep an high N

(we just have to keep ‘light on flight’ for a time T/2)

Ex.: $\omega/2\pi \approx 3$ kHz $\Rightarrow$ $l_{opt} \approx 50$ km

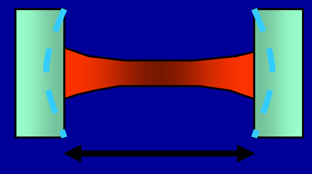
$N \approx 17$



L = 3 km

OR… reduce L, increase N.

Present limit from mirror losses (~ 1 ppm): $N \sim 10^6$



L = 5 cm

In favour of long interferometers: less sensitive to ‘local’ noise (effect $\propto 1/L$) such as

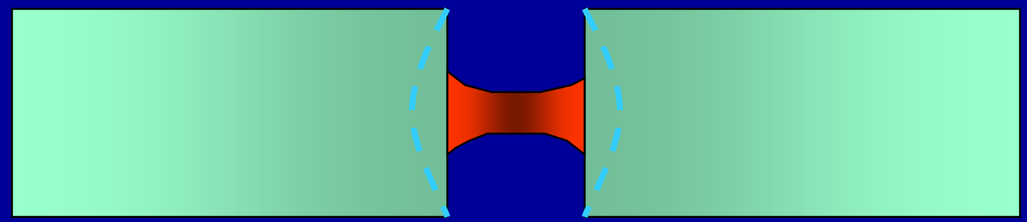
- Thermal noise ($\rightarrow$ go to cryogenic)
- Radiation pressure ($\rightarrow$ go to larger masses)

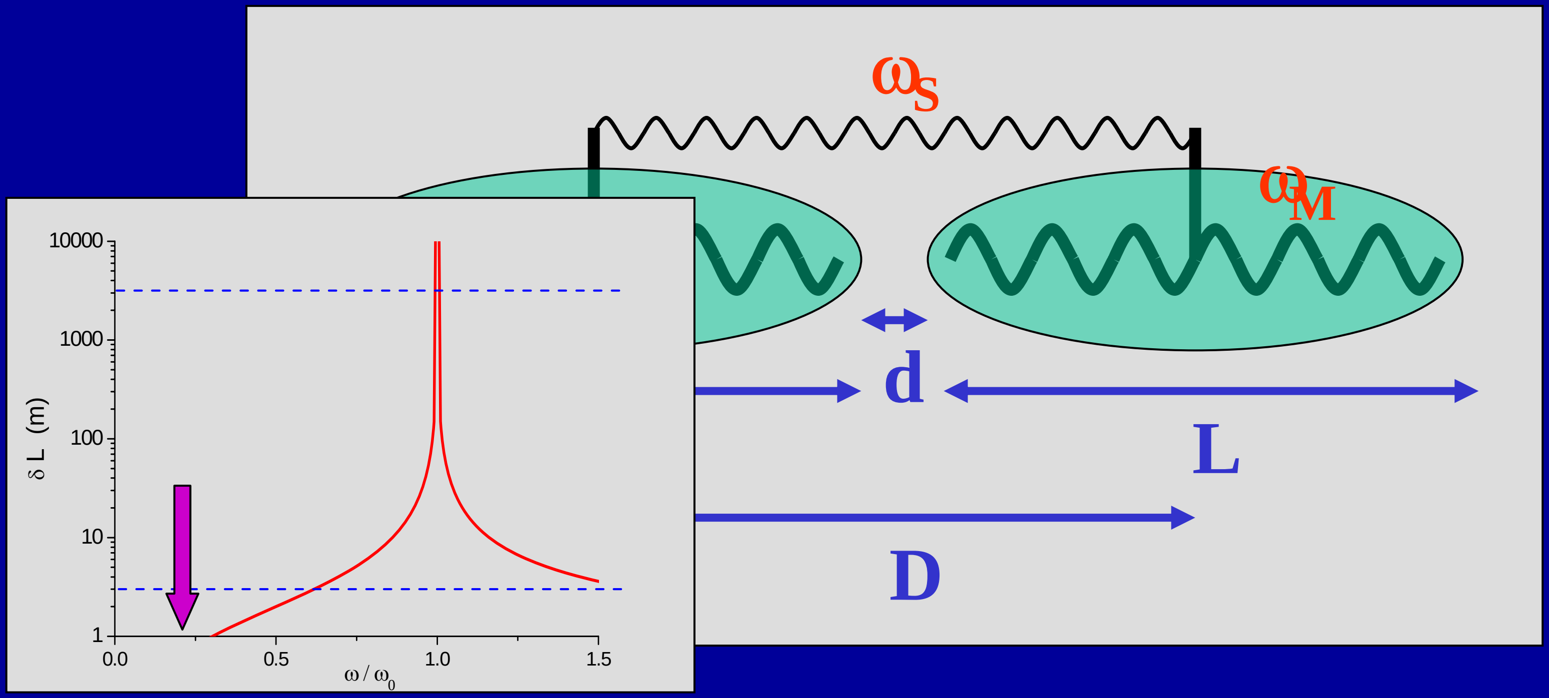
However, present (and future) interferometers are not limited by such effects in the > kHz range.

$\rightarrow$ room for shortening

Moreover...

For close mirrors, again some (new) useful effects from elastic forces



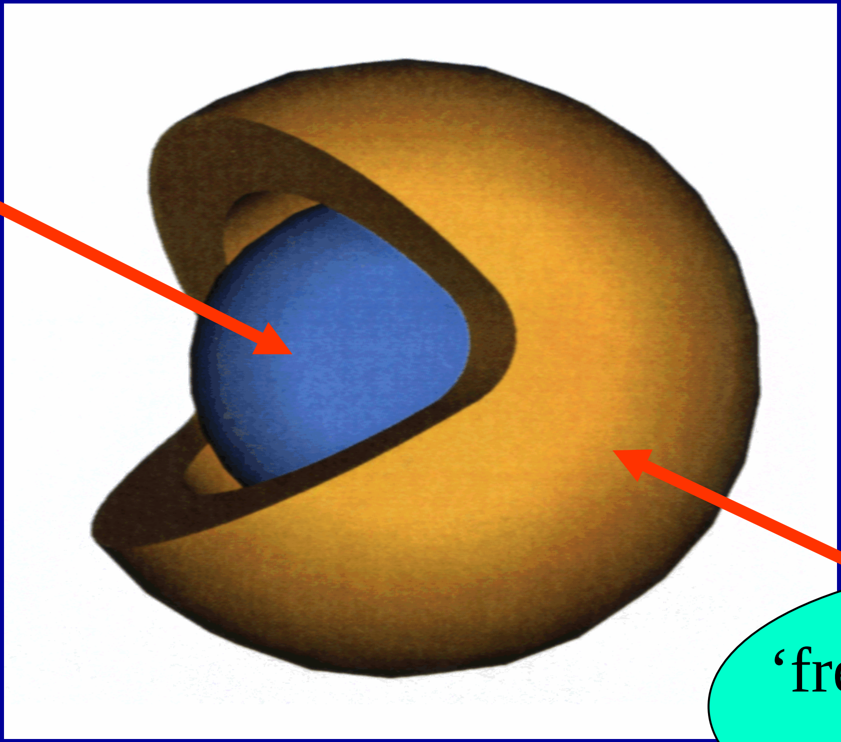


For $\omega_S \ll \omega \ll \omega_M$ we have ‘free’, ‘rigid’ masses

$\Rightarrow$ $\delta d \cong h/2\ D$ ($\gg h/2\ d$) $\Rightarrow$ ‘Gain’ D/d (~ 100)

First (??? at least recently) proposed system of this kind:

the DUAL SPHERE (2001)



‘rigid’ sphere

‘free moving’ sphere

$\delta R_{int} \approx h\, R_{int}$

M. Cerdonio et al. Phys. Rev. Lett. 87 031101 (2001)

Constraints

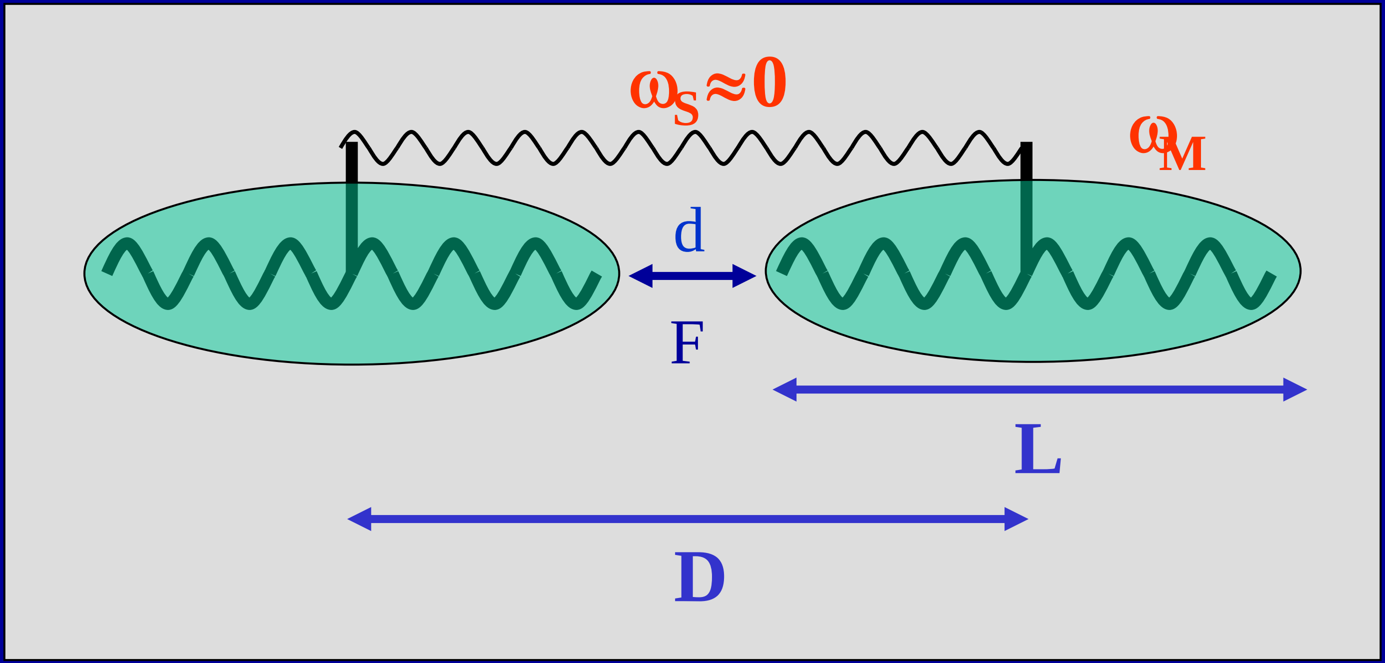
- If L is too large, ω_M is low

$\Rightarrow$ - limit to L

- better high sound velocity

- Limit to L $\rightarrow$ limit to the mass M $\rightarrow$ effect of back-action (radiation pressure) cannot be lowered indefinitely

Back-action reduction

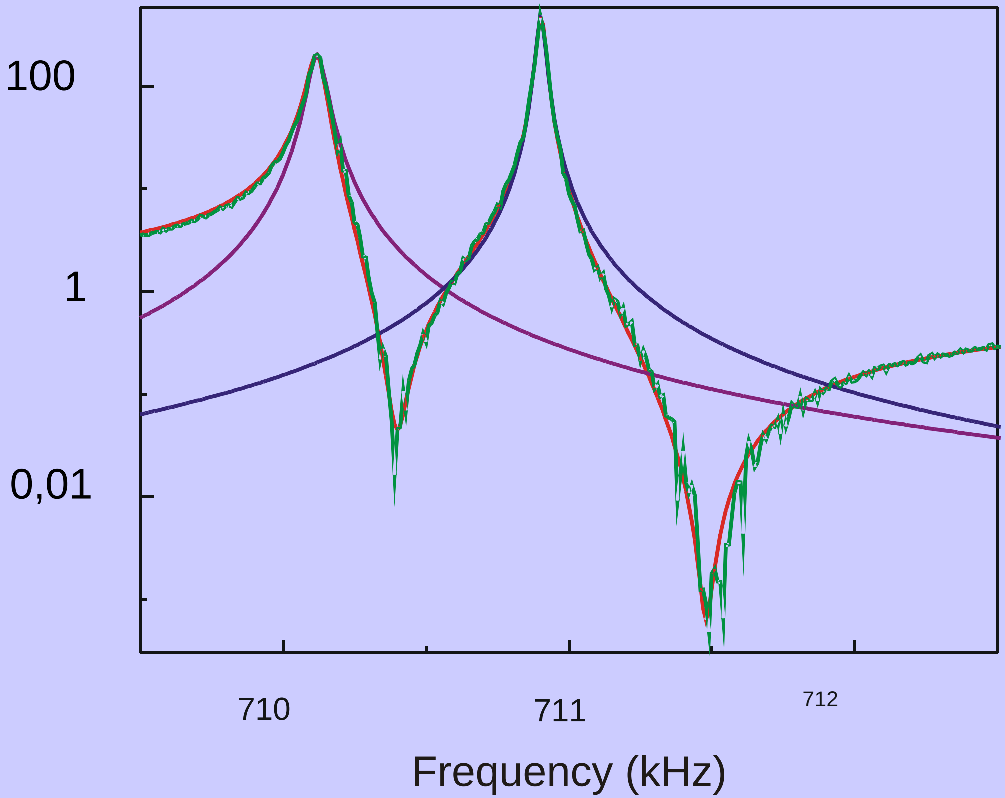


$\delta D = - F/M\omega^2$ $\delta L = F/M(\omega_M^2 - \omega^2)$

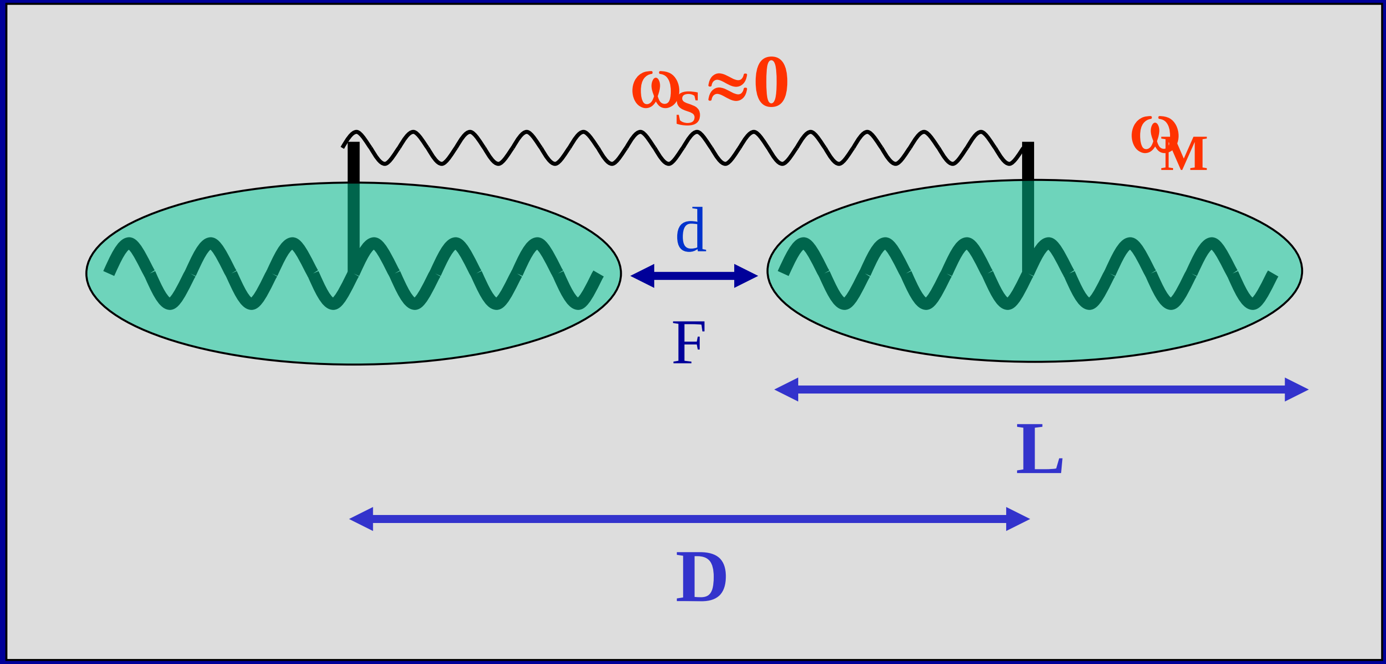
$\delta d = (F/M)\ [-1/\omega^2 + 1/\ (\omega_M^2 - \omega^2)]$

$\delta d = 0$ for $\omega = \omega_M/\sqrt{2}$

Back-action reduction



From T. Briant, talk given last Sunday



$$S_{hh} = \frac{1}{|D/2 - 2T|^2}\left[S_{xx} + S_T + \left|\frac{2}{M\omega^2} - \chi\right|^2 S_{FF}\right]$$

$$\bar{T} = T/L \qquad v_s = \sqrt{Y/\rho} \qquad \Omega = v_s/L \qquad \bar{\chi} = M\Omega^2\chi \qquad x = \omega/\Omega$$

$$S_{hh} = \frac{1}{|0.5 - 2\bar{T}|^2}\left[\frac{S_{xx} + S_T}{L^2} + \left(\frac{L}{Mv_s^2}\right)^2 \left|\frac{2}{x^2} - \bar{\chi}\right|^2 S_{FF}\right]$$

If S_{xx} dominates, the best material has the largest v_s (allowing the largest L)

$$S_{xx}S_{FF} = N^2\hbar^2$$

SQL: $N = 1$

$$\frac{S_{xx}}{L^2} = \eta^2 \left(\frac{L}{Mv_s^2}\right)^2 S_{FF}$$

$\eta \cong 1$ optimized to get a ‘flat’ S/N

$$S_{hh} = \frac{S_T}{L^2|0.5 - 2\bar{T}|^2} + N S_{opt} \frac{\eta + \frac{1}{\eta}\left|\frac{2}{x^2} - \bar{\chi}\right|^2}{|0.5 - 2\bar{T}|^2}$$

Thermal noise

$f(x) \approx 1$

$$S_{opt} = \hbar \frac{\rho^{3/2}}{Y^{5/2}} (2\pi\nu_0)^3 \left(\frac{1}{x_0^3} \frac{L^3}{V}\right)$$

Test mass volume

ω_M/Ω

Form factor

- S_{opt} scales as ν_0^3
- The best material has the lower $\rho^{1.5}/Y^{2.5}$

	Mo	Si	SiC
Density ρ	10.3	2.3	3.2
Young (GPa)	330	150	450
$\rho^{1.5}/Y^{2.5}$	1.7 E-23	1.3 E-23	1.3 E-24
Sound velocity (m/s)	5600	8000	11900
Diameter (m) for ν_M = 4 kHz	0.6	1.0	1.5
$\sqrt{S_{opt}}$ (cylinder 3 m)	1.9 E-23	2.0 E-23	7.6 E-24

Thermal noise

$$S_{thermal} = \frac{4kT}{\omega} Im\chi\,.$$

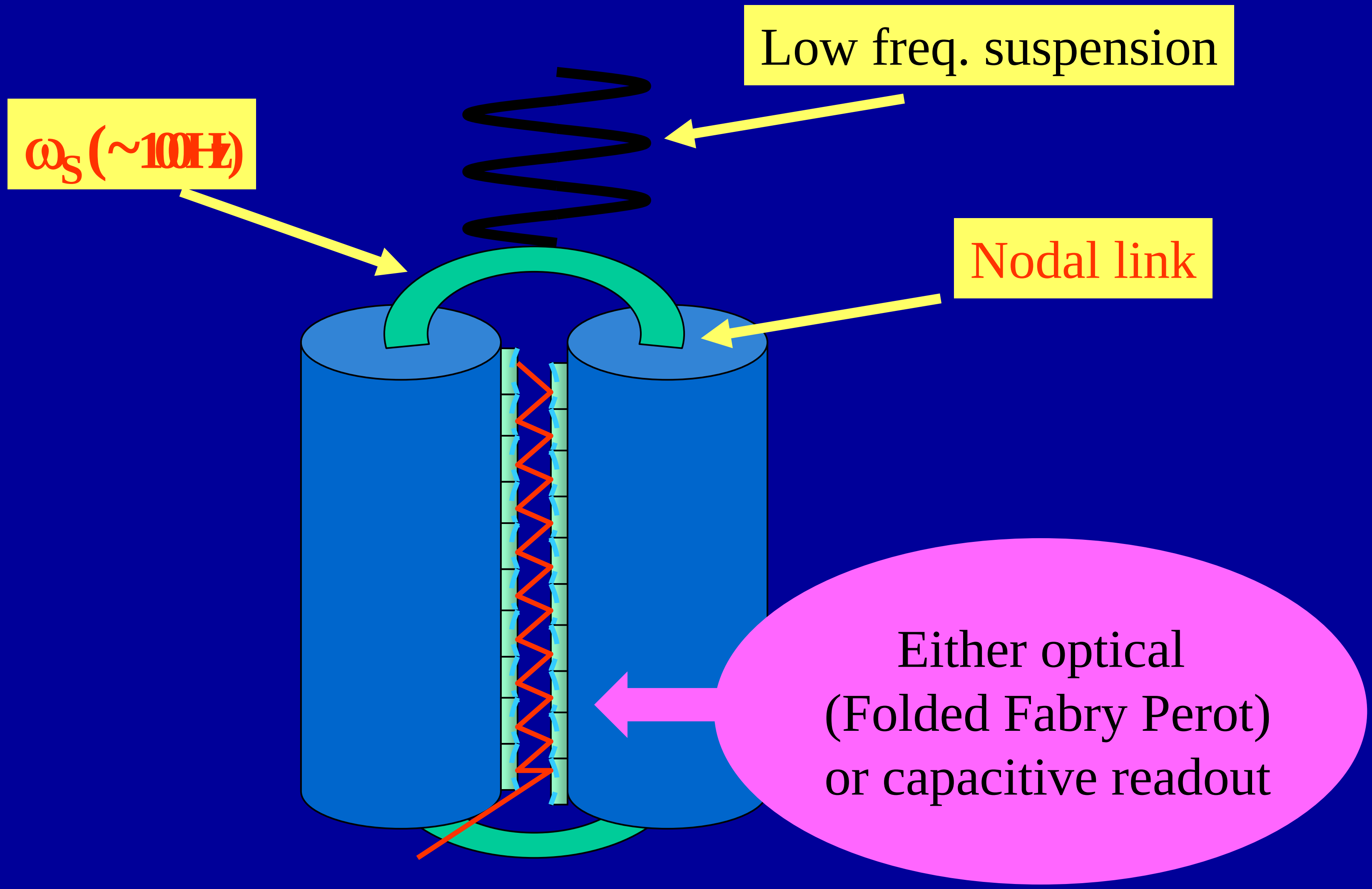
$$S_T = \frac{8kT}{M\Omega^3}\left(\frac{\phi_S x_S^2}{x^5} + \frac{\phi}{\mu}\frac{x_0^2}{x(x_0^2 - x^2)}\right)$$

$$\frac{8kT}{\hbar\omega_M}\phi \ll N$$

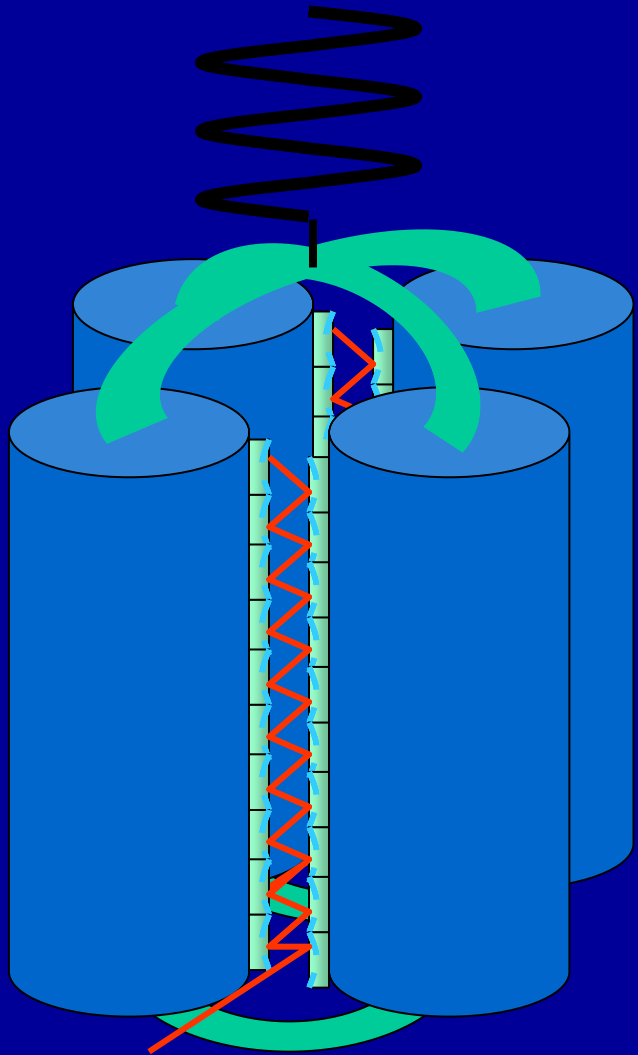
For ν_0 = 5 kHz:

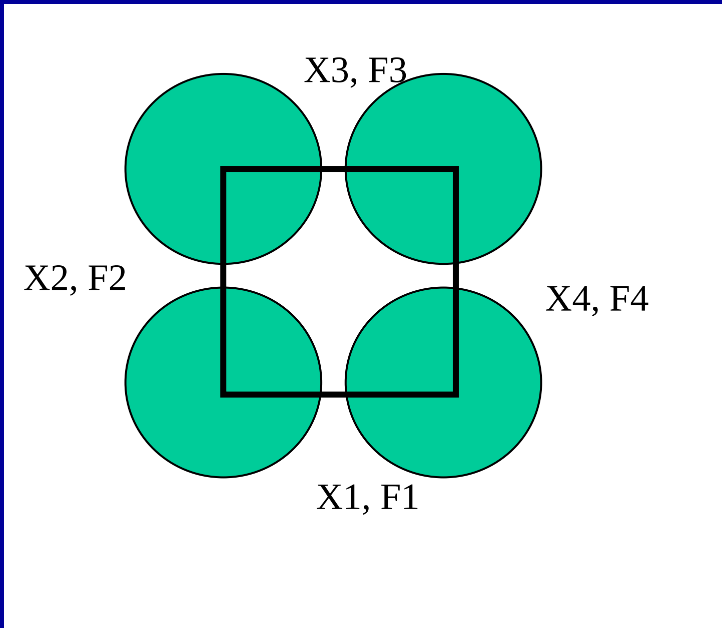
$$\frac{N}{T\phi} \gg 3 \times 10^7$$

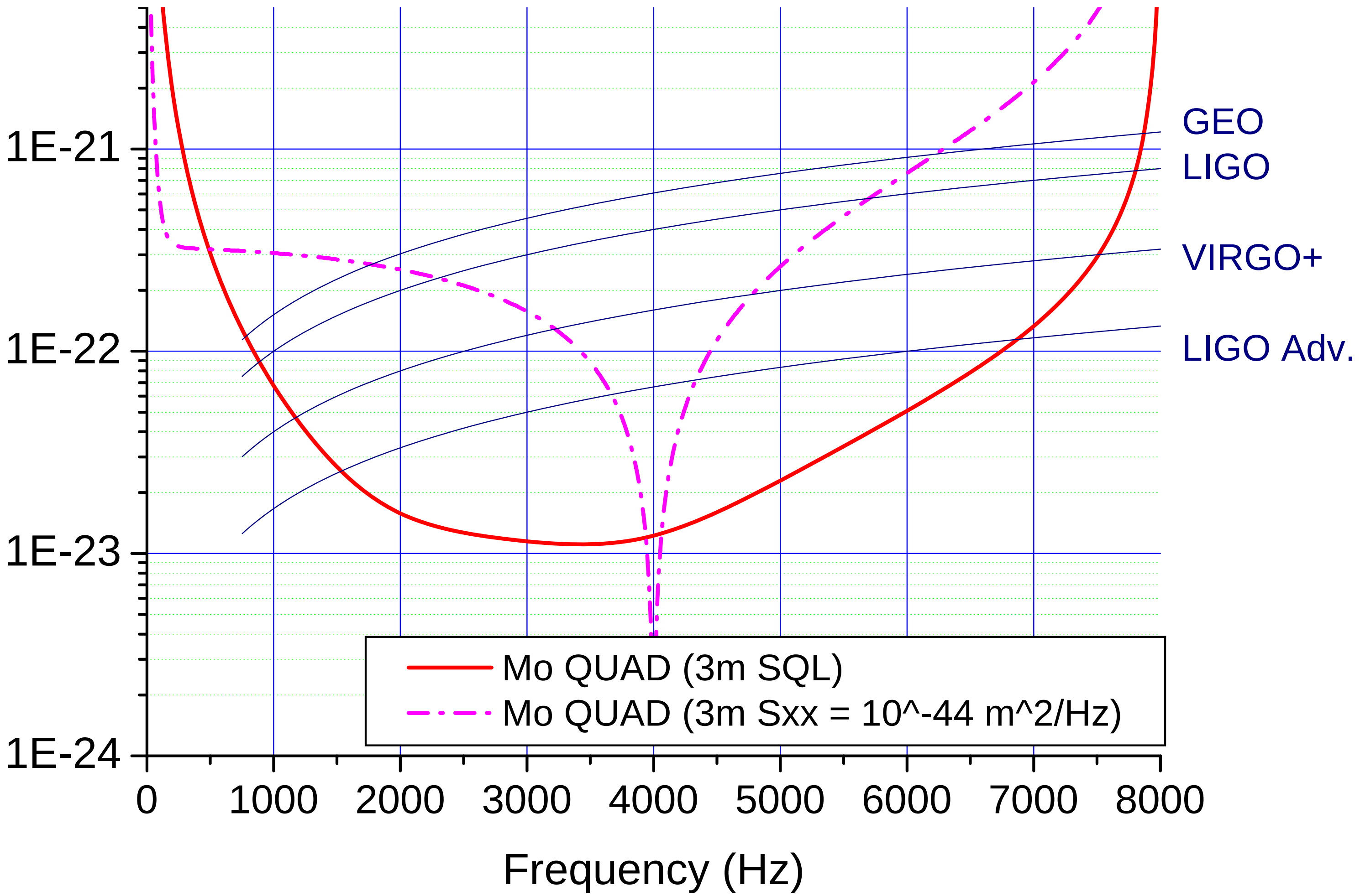
Possible implementation



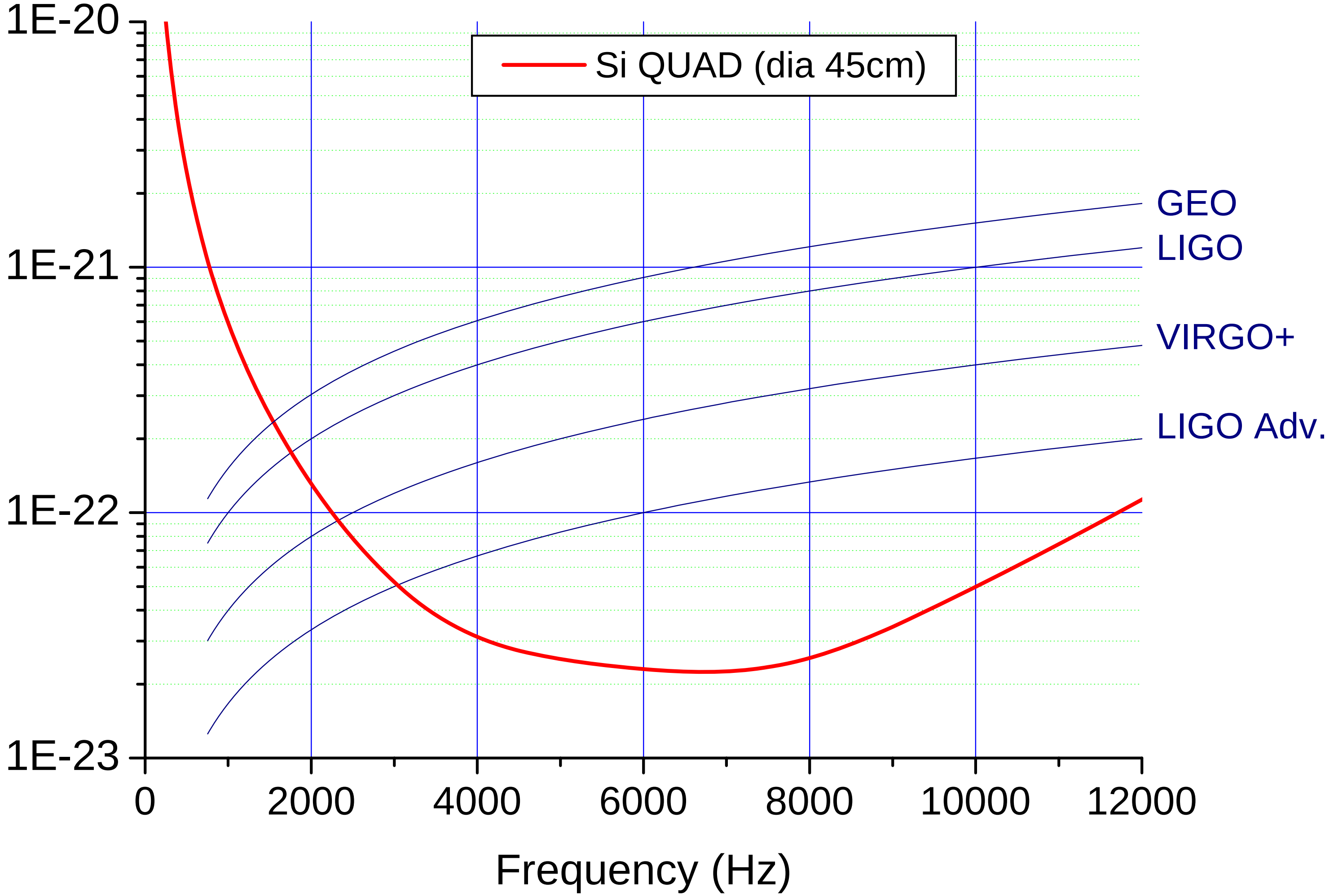
'QUAD' detector



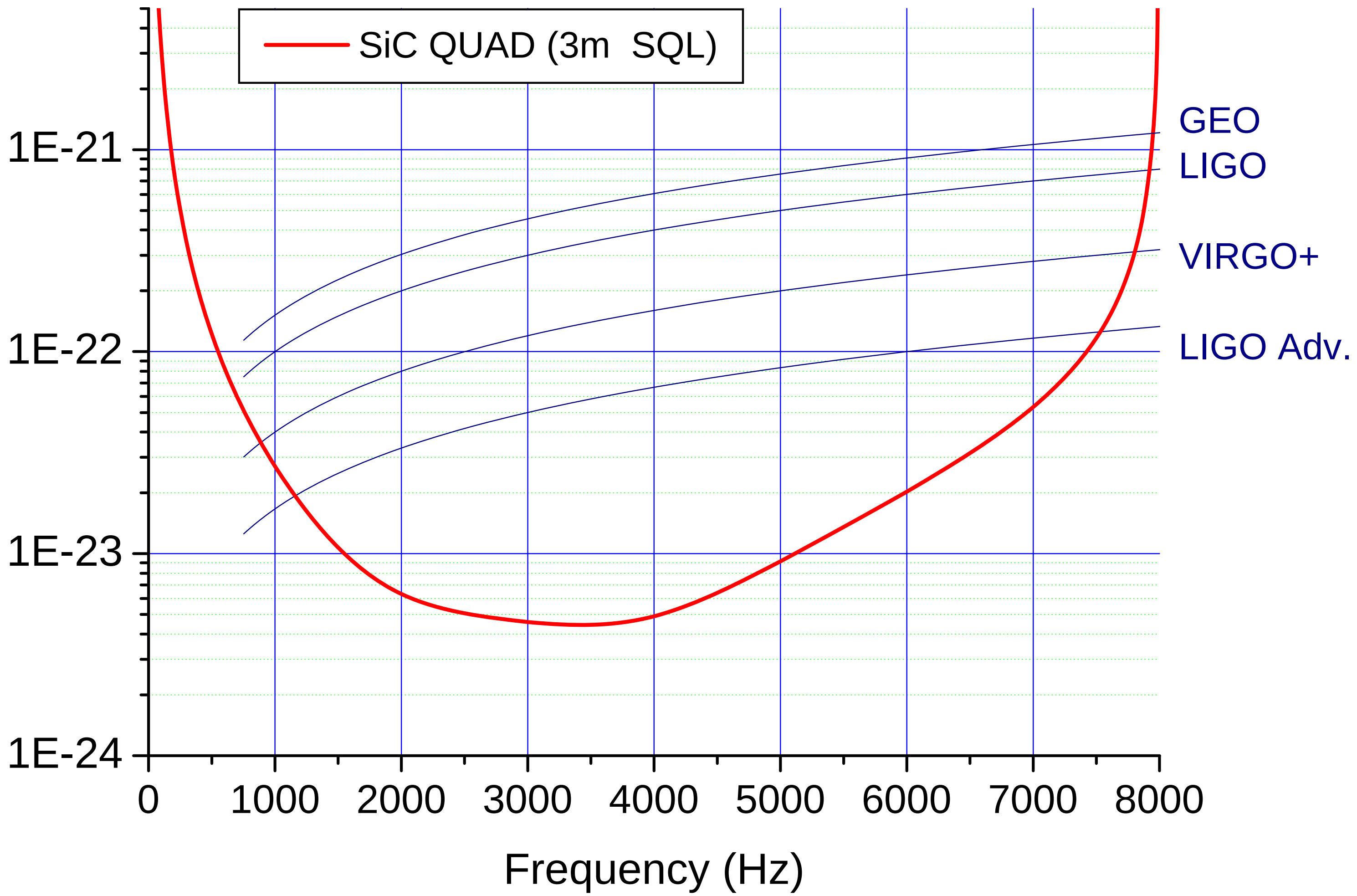




1E-21
1E-22
1E-23
1E-24
0
1000
2000
3000
4000
5000
6000
7000
8000
Frequency (Hz)
GEO
LIGO
VIRGO+
LIGO Adv.
Mo QUAD (3m SQL)
Mo QUAD (3m Sxx = 10^-44 m^2/Hz)

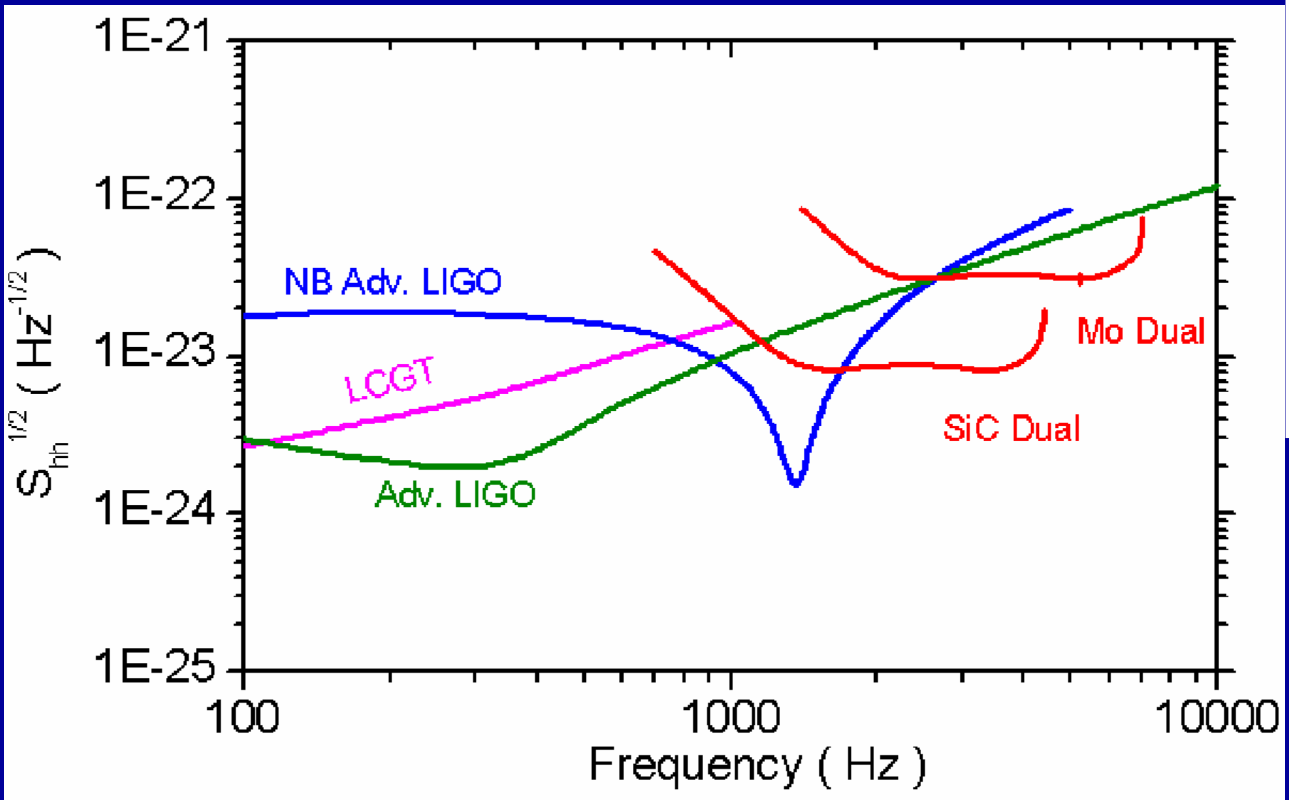


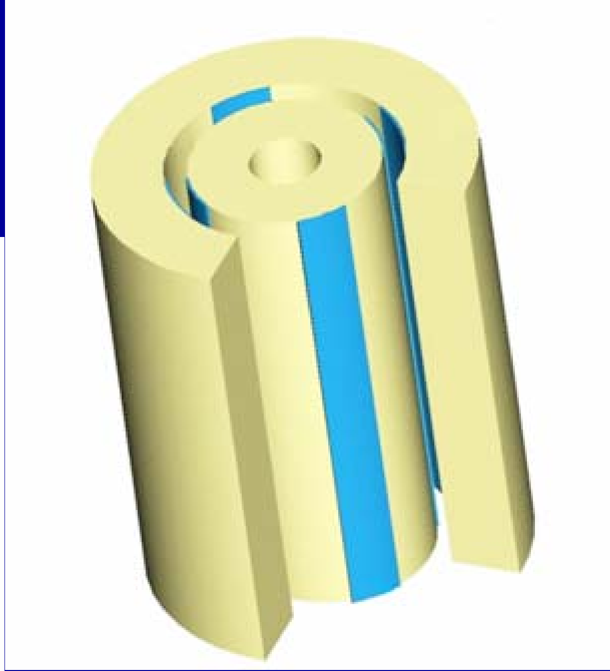
Si QUAD (dia 45cm)
GEO
LIGO
VIRGO+
LIGO Adv.
1E-20
1E-21
1E-22
1E-23
0
2000
4000
6000
8000
10000
12000
Frequency (Hz)



SiC QUAD (3m SQL)
1E-21
1E-22
1E-23
1E-24
0
1000
2000
3000
4000
5000
6000
7000
8000
Frequency (Hz)
GEO
LIGO
VIRGO+
LIGO Adv.

M. Bonaldi et al. Phys. Rev. D 68 102004 (2003)



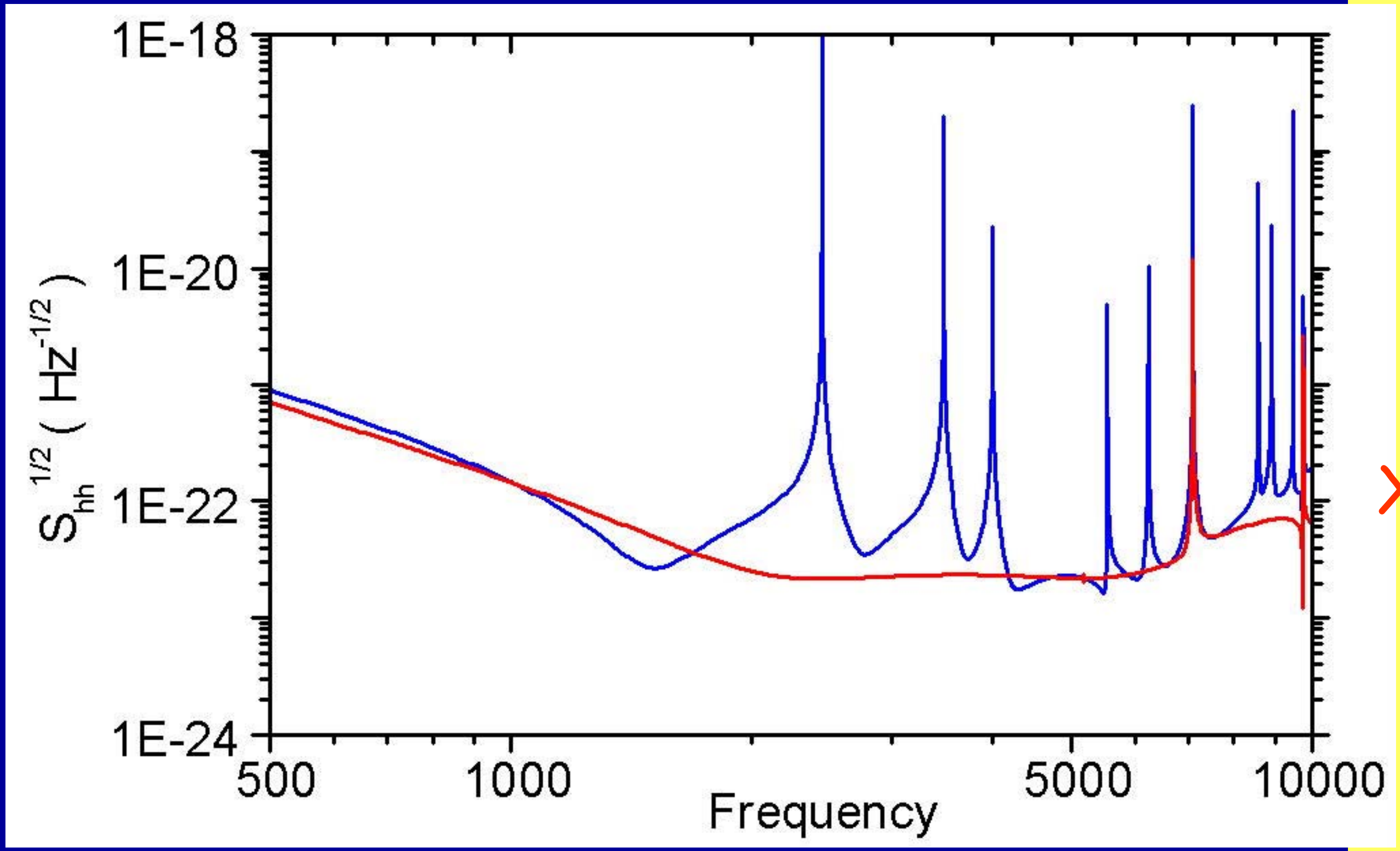


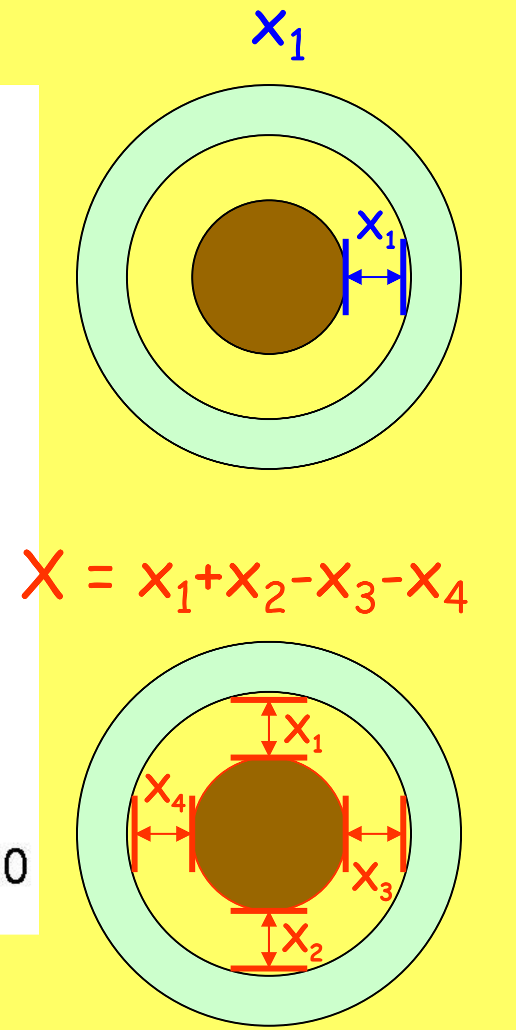
Mo nested cylinders 16.4 ton height 2.3m 0.94m

SiC nested cylinders 62.2 ton height 3.0m 2.9m

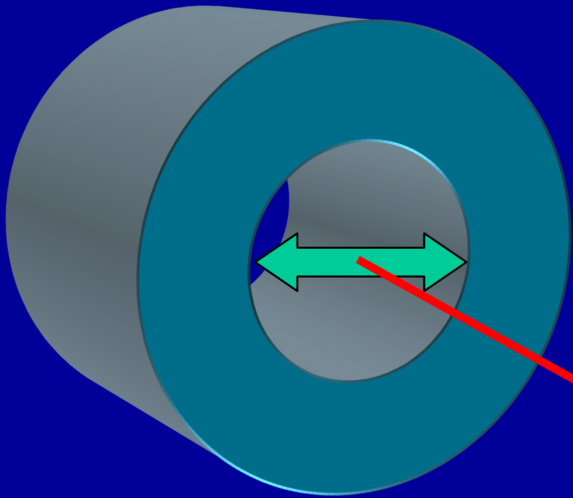
$Q/T = 2 \times 10^8\ K^{-1}$

Example of Thermal and BA noise reduction using selective readout





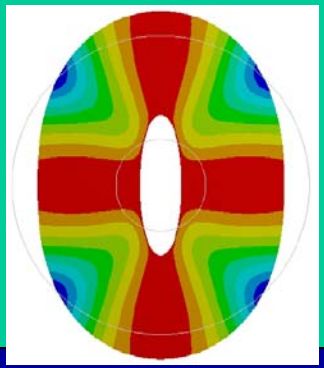
Single mass DUAL Detector

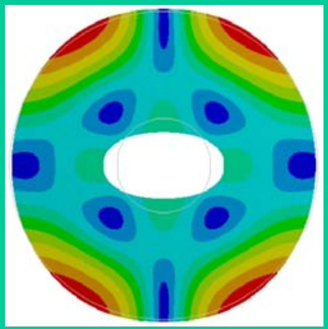


An hollow cylinder can work as a DUAL (mode) detector

The DUAL concept which works between two modes of two different bodies can work also between two modes of the SAME body

the internal diameter is the length to be measured for the detection

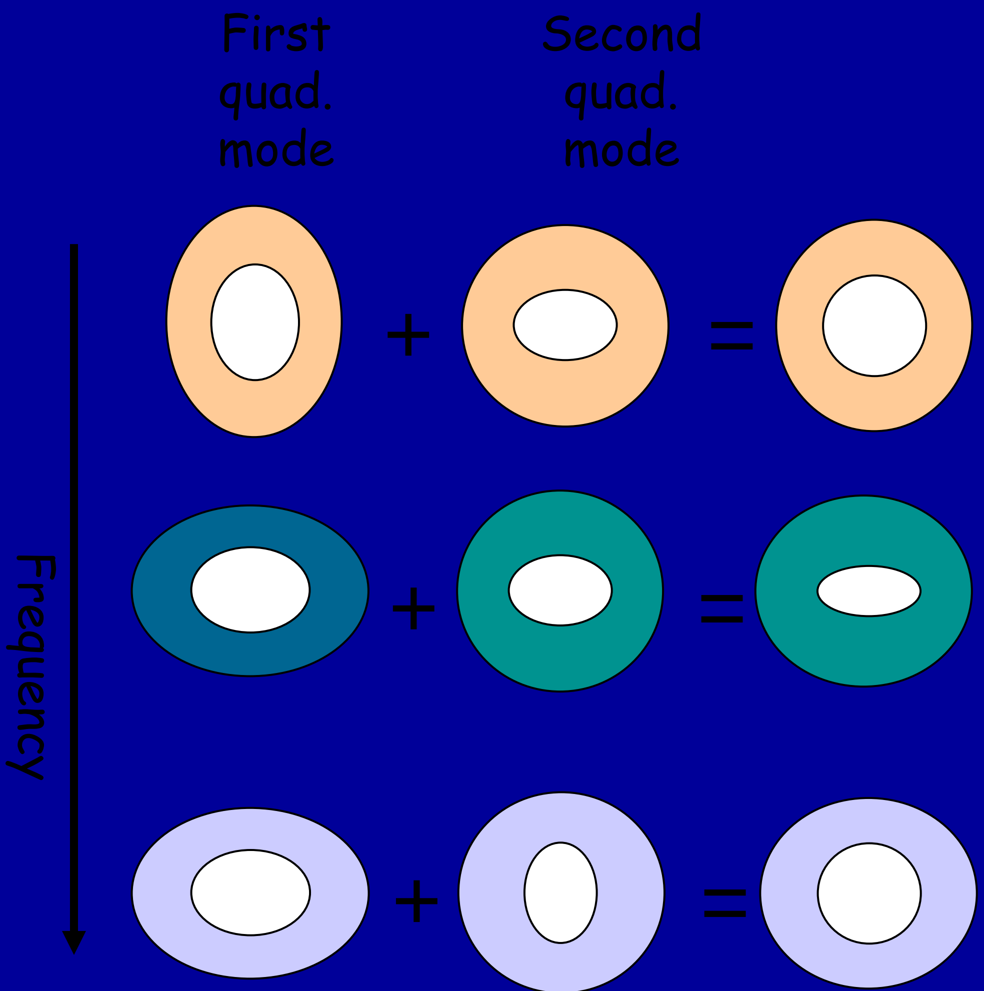


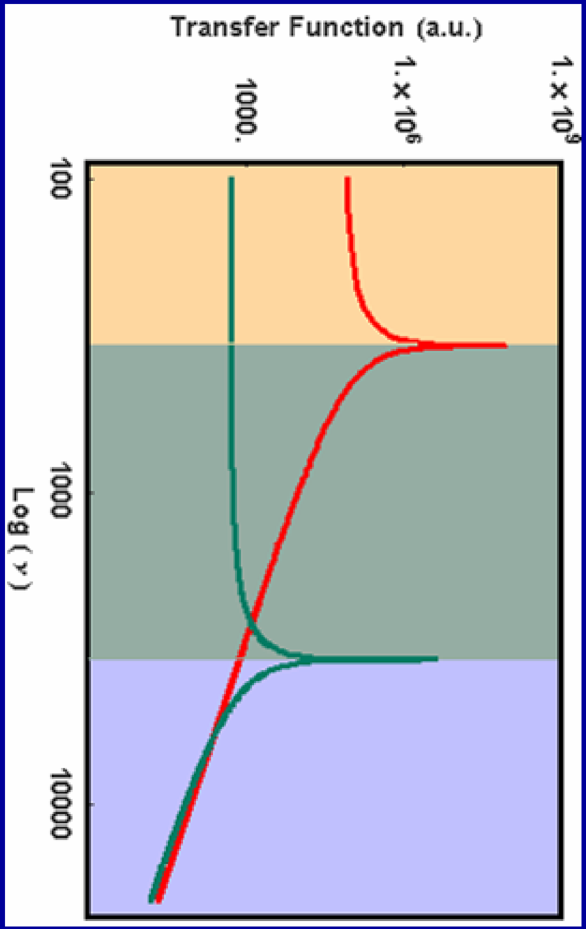


the deformation of the inner surface has “opposite sign” for the first and the second quadrupolar mode

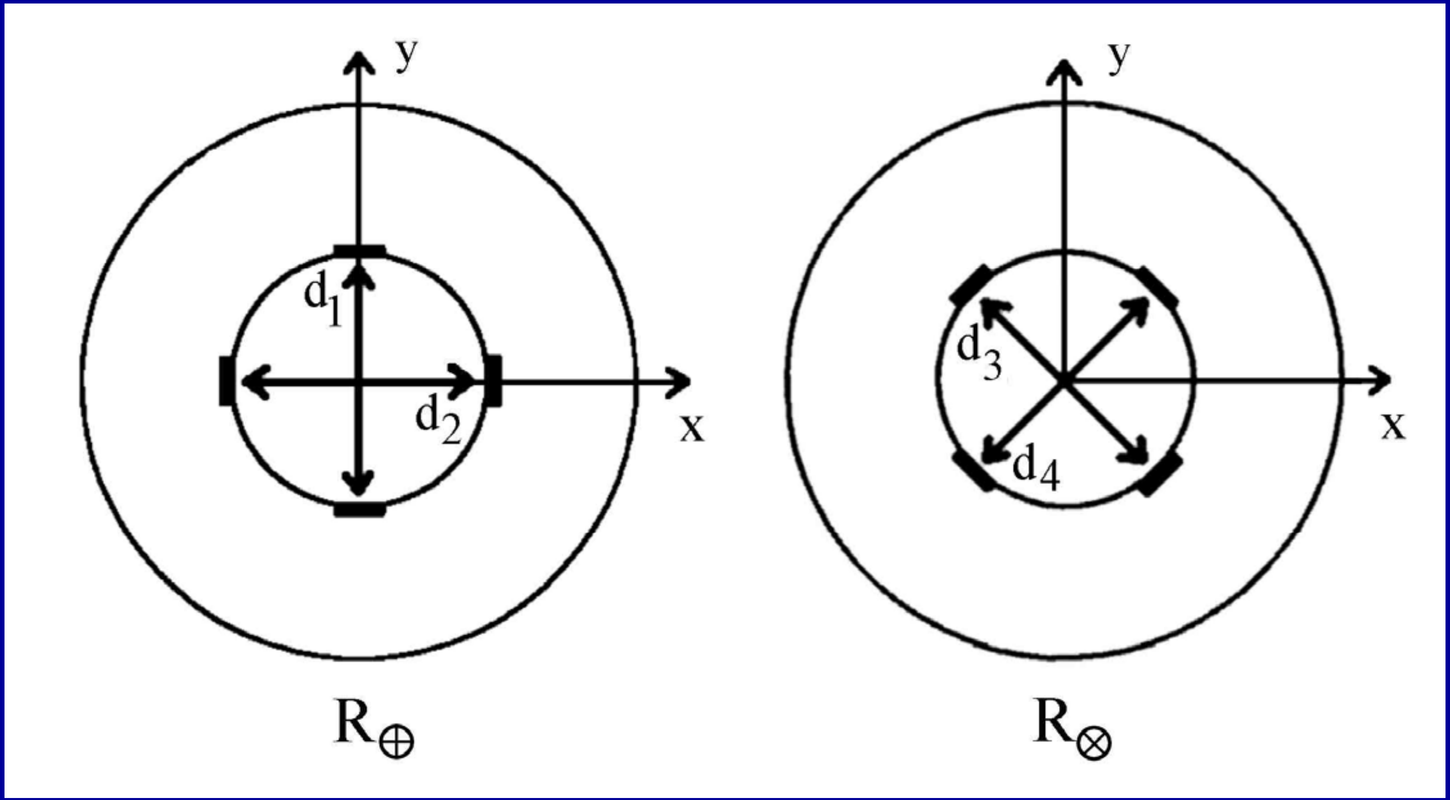
M. Bonaldi et al. Phys. Rev. D submitted. Also on gr-qc/0605004

The π phase difference concept still holds



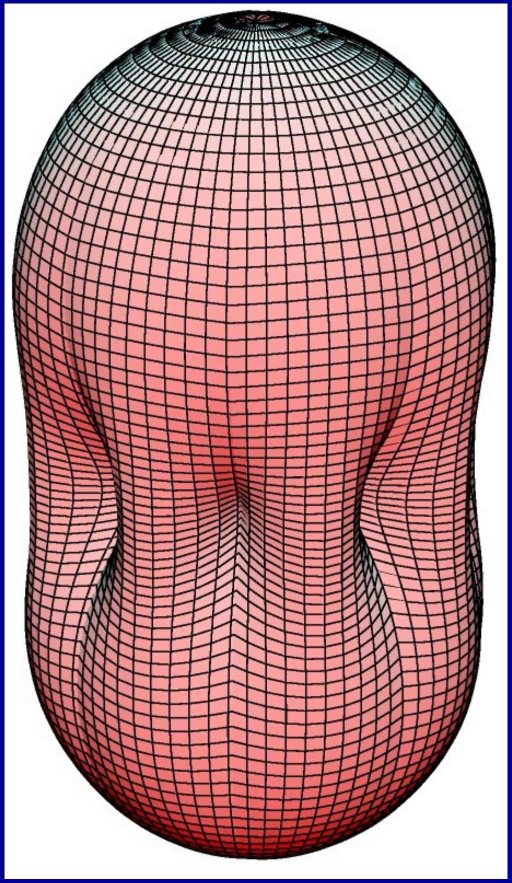


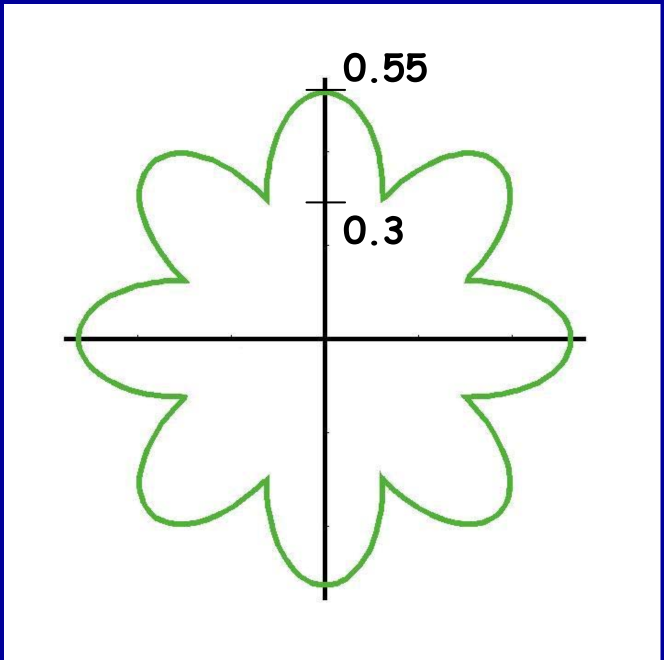
Double channel readout



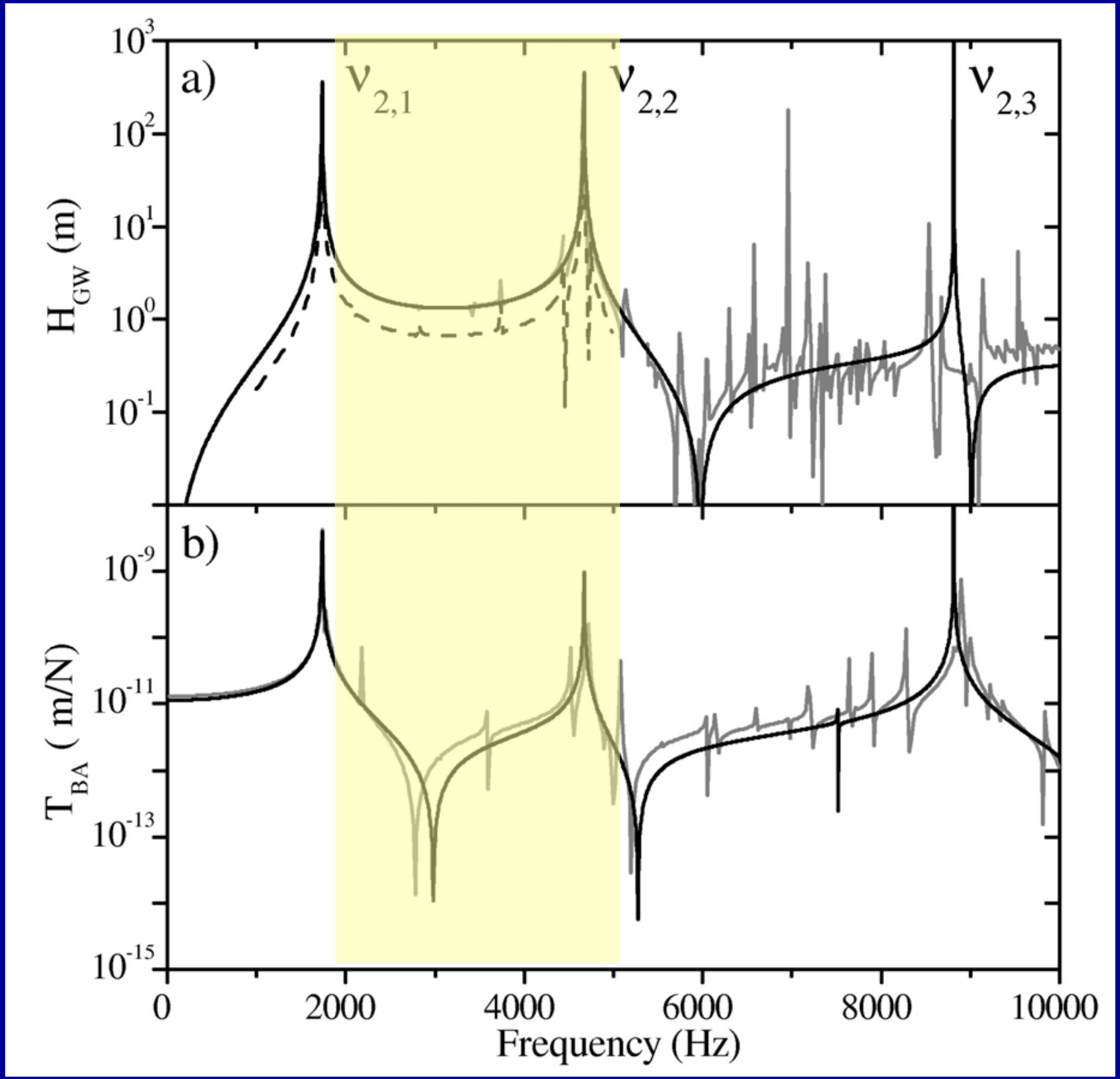
Antenna pattern: like 2 IFO at 45°

$$MAX\{\sqrt{X^2_{T_1,og+}+X^2_{T_1,ogx}},\sqrt{X^2_{T_2,og+}+X^2_{T_2,ogx}}\}$$





Complete transfer function



Molibdenum

R_{ext} = 0.5m

R_{int} = 0.15m

L = 3m

CONCLUSIONS: we are just beginning

Problems:

- SQL readout
- availability of large masses (depends on materials)
- cosmic rays (underground?)
- links between test masses
- cryogenic possibilities vs dissipated power
- ...

❖Strong R&D necessary (2-3 yrs ?)

❖ Help and ideas welcome