Estimating Instrumental Correlations between Collocated Gravitational-Wave Interferometers

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Stochastic gravitational-wave background

- Type of gravitational radiation produced by a very large number of independent and unresolved sources
- Energy density $\rho_{\mathrm{GW}} = \frac{c^2}{32\pi G} \left\langle \dot{h}_{ab}\dot{h}^{ab} \right\rangle$
- Characterized by log-frequency spectrum

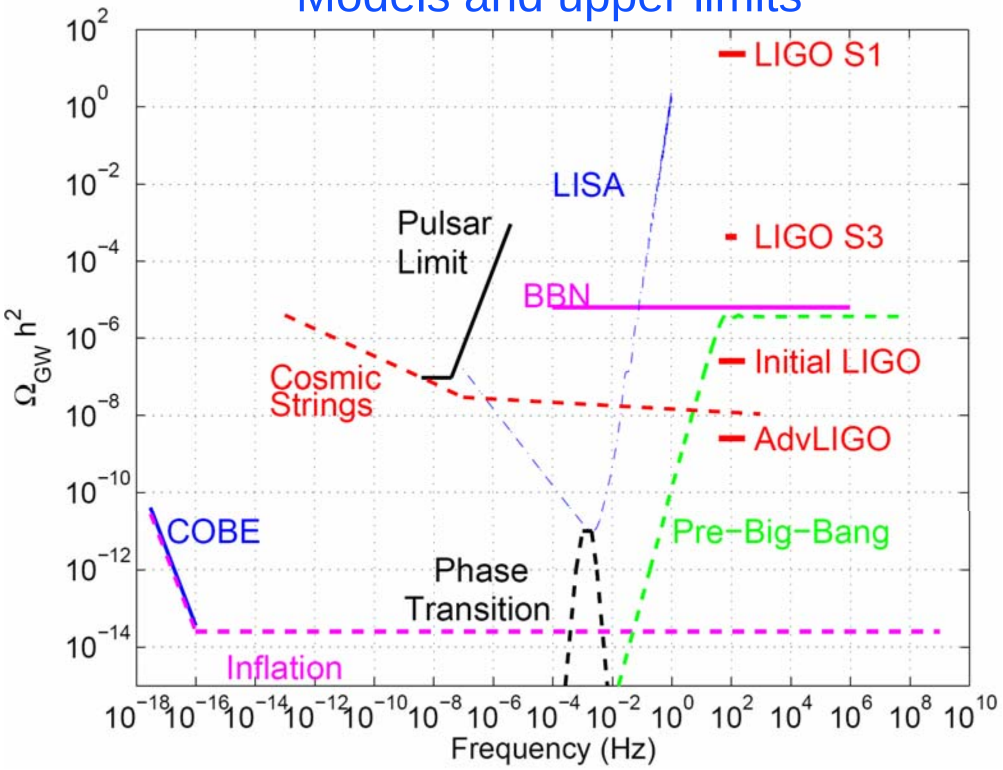
$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\mathrm{GW}}(f)}{d\ln f} \qquad \left(\rho_c = \frac{3c^2 H_0^2}{8\pi G}\right)$$

Various models

- Cosmological origin:
  - » Inflation
  - » Phase transitions
  - » Cosmic strings
  - » Pre-Big-Bang models
- Astrophysical origin:
  - » Low mass X-ray binaries
  - » Rotating neutron stars
- Most models predict power-law spectrum in LIGO frequency range

$$\Omega_{GW}(f) = \Omega_0 \left(\frac{f}{f_0}\right)^{\alpha}$$

Models and upper limits



Cross-correlation search

- Search performed for a template spectrum $\Omega_t(f) = \Omega_\alpha \left(\frac{f}{100\text{Hz}}\right)^\alpha$
- Cross-correlation estimator (in frequency domain)

$$Y = \int_{-\infty}^{+\infty} df\, Y(f) = \int_{-\infty}^{+\infty} df\, \tilde{s}_1^*(f)\tilde{s}_2(f)\tilde{Q}(f)$$

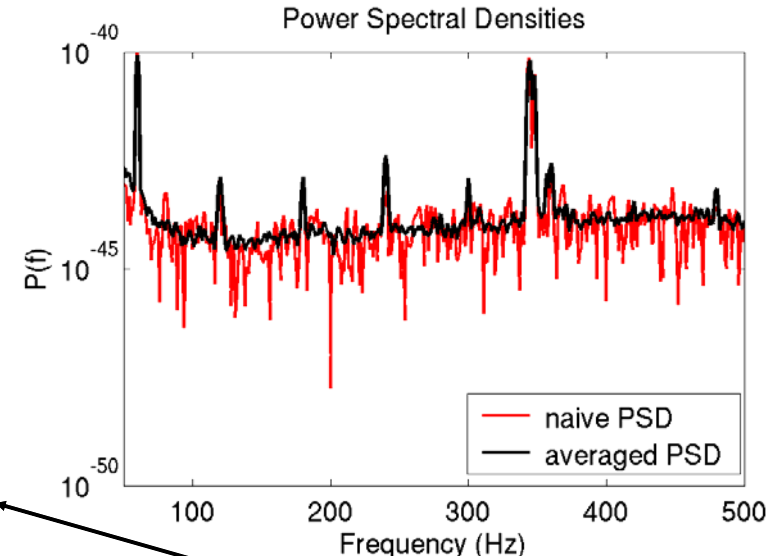
- Theoretical variance $\sigma_Y^2 = \int_0^{+\infty} df\, \sigma_Y^2(f) = \int_0^{+\infty} df\, \frac{T}{2} P_1(f) P_2(f) |\tilde{Q}(f)|^2$
- $\tilde{Q}(f)$ is the optimal filter: $\tilde{Q}(f) = \frac{1}{N} \frac{\gamma(|f|)\Omega_t(|f|)}{|f|^3 P_1(|f|) P_2(|f|)}$
- Normalization constant N determined by $\langle Y \rangle = \Omega_\alpha T$

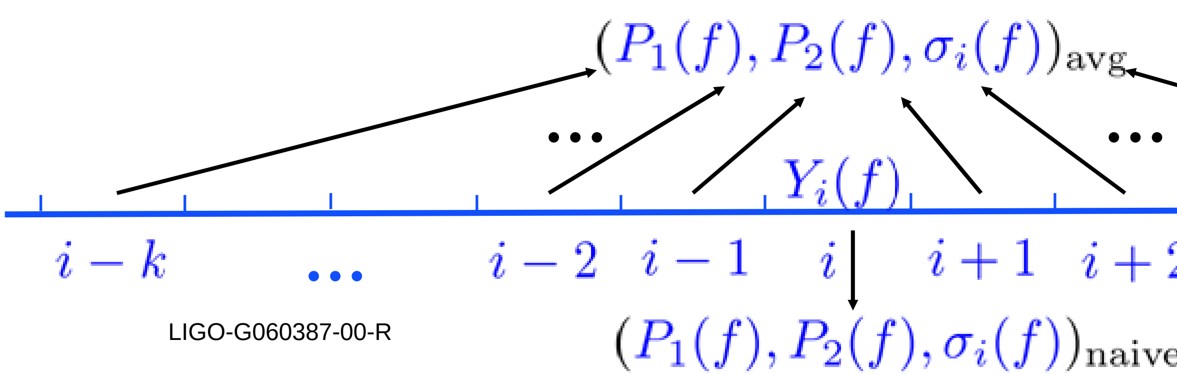
Analysis details

- Data divided into segments:
  - » $Y_i(f)$ and $\sigma_i(f)$ calculated for each segment i
  - » Weighted average performed
- Sliding point estimate:
  - » Produce values of $P_1(f), P_2(f), \sigma_i(f)$ which are less sensitive to noise transients and more reliable

$$Y_{\rm opt} = \frac{\sum_i \sigma_i^{-2} Y_i}{\sum_i \sigma_i^{-2}}$$

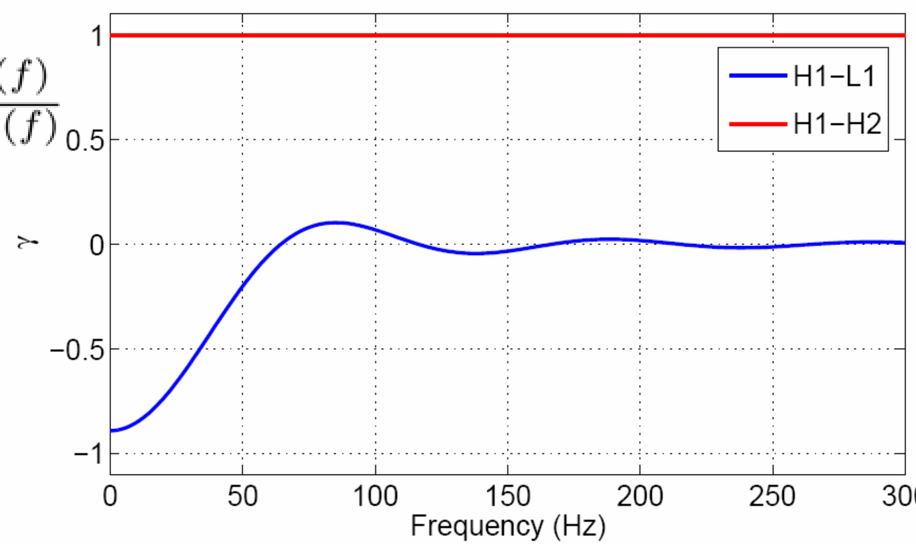
$$\sigma_{\rm opt}^{-2} = \sum_i \sigma_i^{-2}$$





H1-H2: advantages and problems

Overlap Reduction Function $\gamma(f)$



- Signal-to-noise ratio

$$\left(\frac{Y}{\sigma_Y}\right)^2 = \frac{9H_0^4}{50\pi^4} T \int_0^\infty \mathrm{d}f \, \frac{\gamma^2(f)\Omega_{\mathrm{gw}}^2(f)}{f^6 P_1(f) P_2(f)}$$

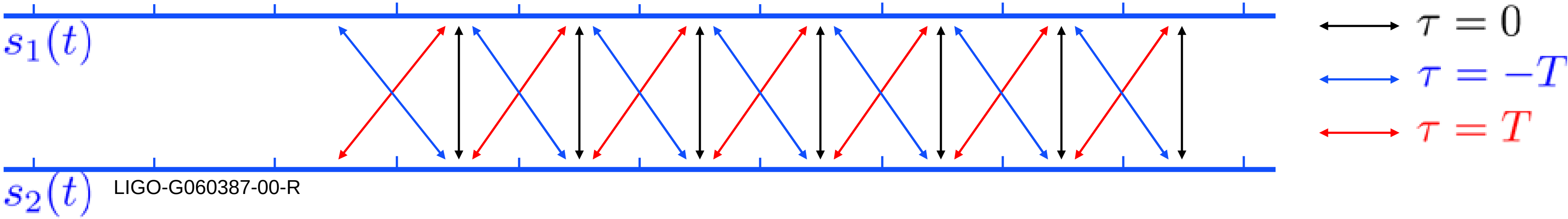
- The H1-H2 pair is about 10 times more sensitive to the stochastic background than the H1-L1 pair
- The H1-H2 cross-correlation is susceptible to instrumental correlations
  - » Common instrumental noise sources
  - » Shared environment

Time shifts

- Gravitational-wave correlations are not expected to be coherent over more than ~10ms
- Instrumental correlations could be coherent over longer time-scales
- Instead of cross-correlating $s_1(t)$ and $s_2(t)$, cross-correlate $s_1(t+\tau)$ and $s_2(t)$, for some value of τ
- Separate the long-lasting correlations from the short-lasting correlations

Appropriate choice of time shifts and segment lengths

- If segment length is T, and the time shift is τ, we need $\tau \geq T$, in order to get independent realizations of noise for different time shifts
- We take $\tau = \pm T, \pm 2T$
- We wish to detect broad features, not expected to be correlated over time-scales longer than $\sim 1\text{s}$
- We wish good frequency resolution ($T < 1\text{s}$ would give frequency resolution $\Delta f = 1/T > 1\text{Hz}$)
- Compromise: $T \sim 1\text{s}$



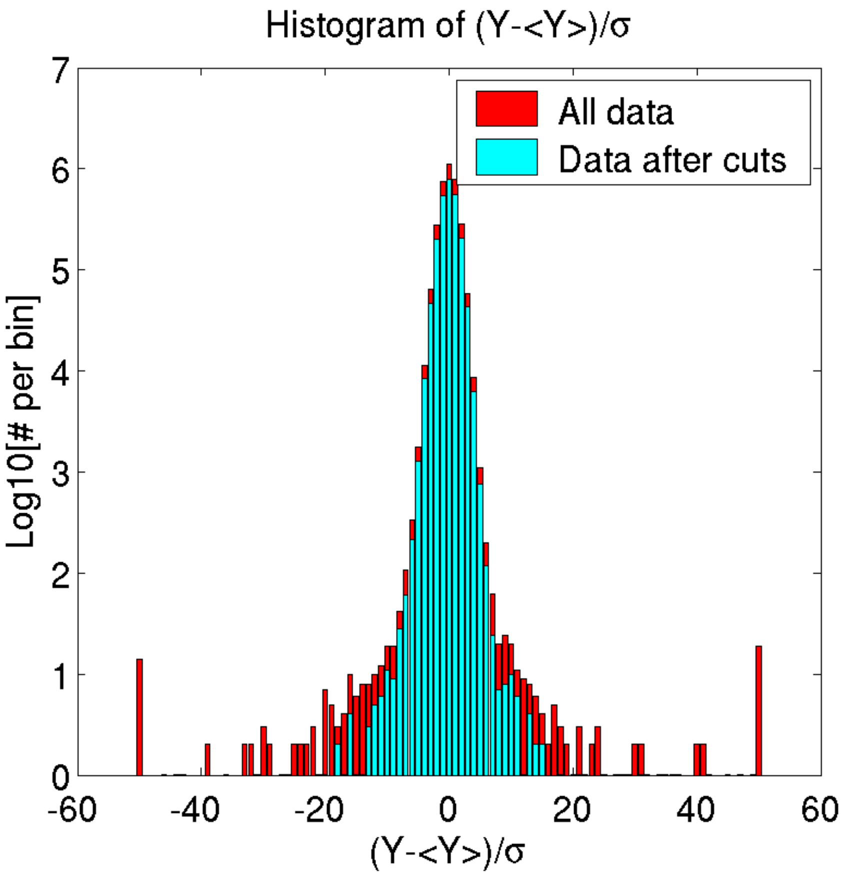
Data quality cuts

S4 data, $T = 1\mathrm{s}$

- Segments of data containing glitches (obtained from the burst group) are rejected
- "Large sigma cut":

  $\sigma_{\mathrm{avg}} < \mathrm{threshold}$
- ΔPSD-cut:

  $$\frac{\int |P_{\mathrm{naive}}(f) - P_{\mathrm{avg}}(f)|\, df}{\int |P_{\mathrm{avg}}(f)|\, df} < \mathrm{threshold}$$
- About 20% of the data is rejected
- Residuals $\frac{Y_i - \langle Y \rangle}{\sigma_i}$



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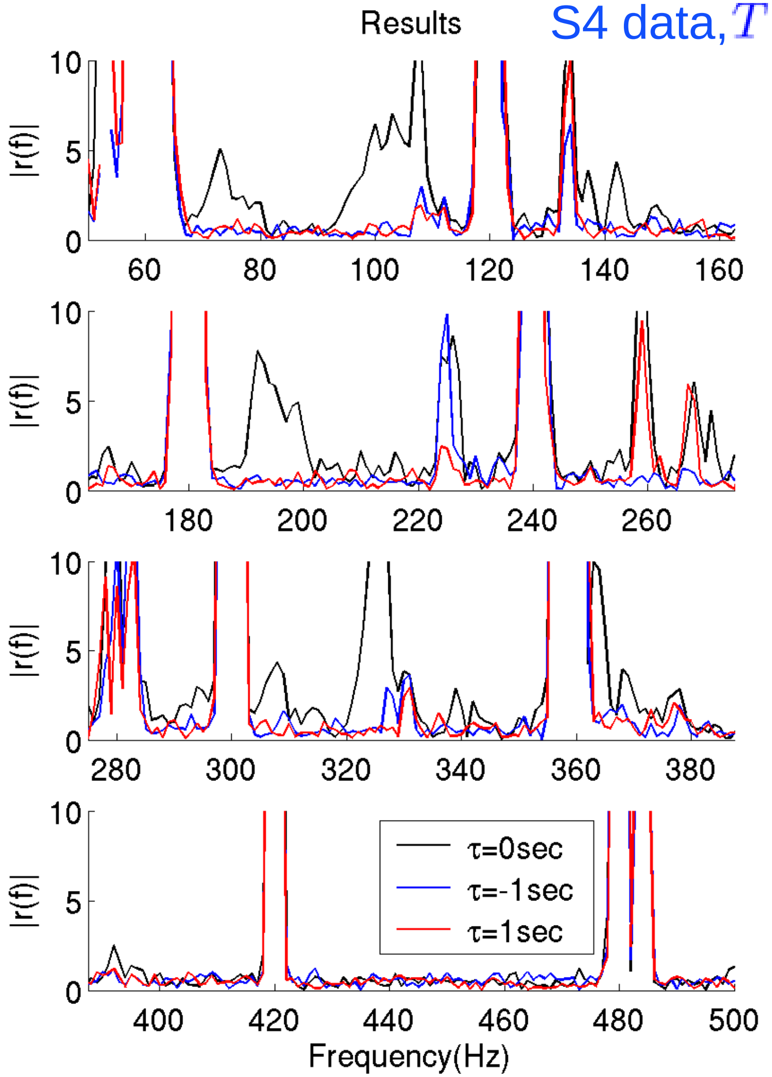
Results

- The quotient $r(f) = Y(f)/\sigma(f)$ is a measure of the significance of the signal at a given frequency
- The functions $r(f)$ obtained for time shifts of $\tau = \pm 1\mathrm{s}$ contain most narrow features present in the function $r(f)$ for $\tau = 0\mathrm{s}$; broad structures elude detection with this choice of parameters
- Analysis is under way with shorter data segments and smaller time shifts

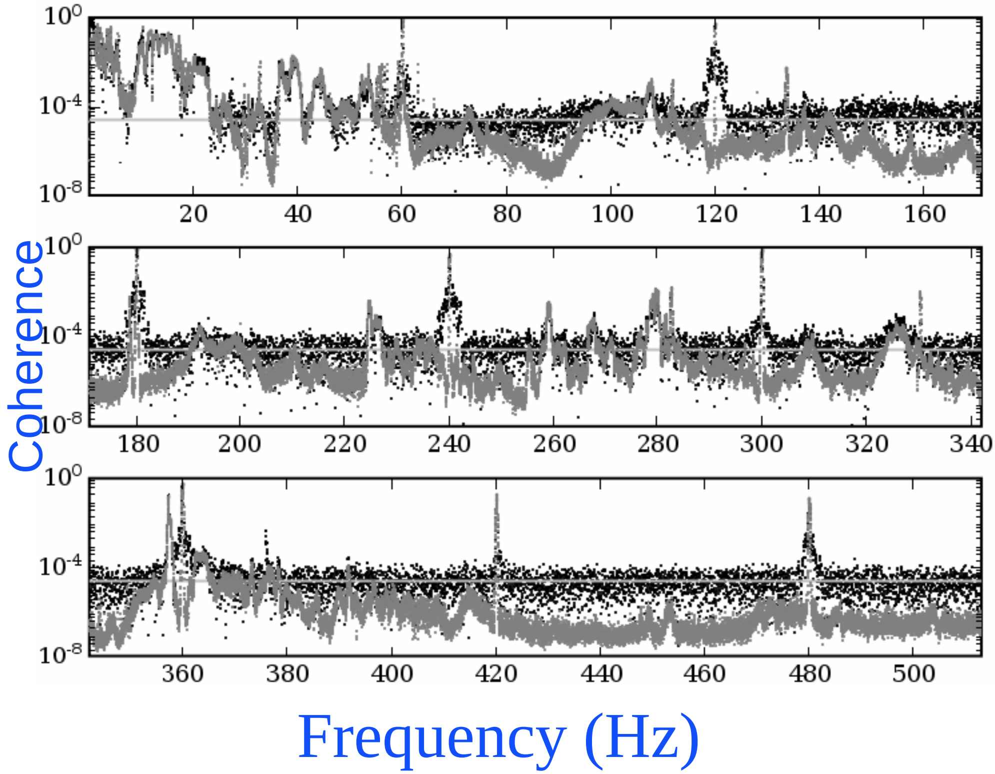
S4 data, $T = 1\mathrm{s}$



Coherence in environmental channels

- LIGO sites are equipped with numerous environmental monitors, monitoring various environmental noise sources
- Nickolas Fotopoulos (MIT): Coherences between strain channels and environmental monitor channels used to estimate H1-H2 coherences originating from shared environment

$$\text{Coherence} = \frac{|s_1^*(f)s_2(f)|^2}{P_1(f)P_2(f)}$$



Conclusions

- By performing the time-shift analysis and the environmental channel analysis, we expect to be able to identify the frequencies at which instrumental correlations occur
- After identifying the “corrupted” frequencies, we can notch them out of the analysis, or we may be able to subtract the environmental contribution
- That would allow us to use the H1-H2 pair for setting upper limits (or detections!) in future science runs
  - » Potentially 10 times more sensitive

Acknowledgements

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- Caltech