Lock Acquisition

- What is it? What is it required to do?

Control the random motion of the six test masses, bringing the relative displacements of the test masses into integer multiples of the source frequency, within a short time, and within the limits of the actuators and sensors available, without exciting long time-scale degrees of freedom ,and then hold these lengths stably against noise until the detection mode controls can be switched in.

- How does one go about this?

Lock acquisition is primarily concerned with longitudinal degrees of freedom; alignment degrees primarily only affect the overall gain of the plant (power lost into higher modes doesn't change the transfer function of the TEM00 plant, except for overall gain).

As a modification of the steady-state detection mode plant and controller, one can study the behavior of the plant in the acquisition states and design controllers accordingly.

- How does one study the plant and transitions?
  - SMAC
    - Transfer functions (of the plant only!)
    - Time evolution
- One True Path to Lock Acquisition

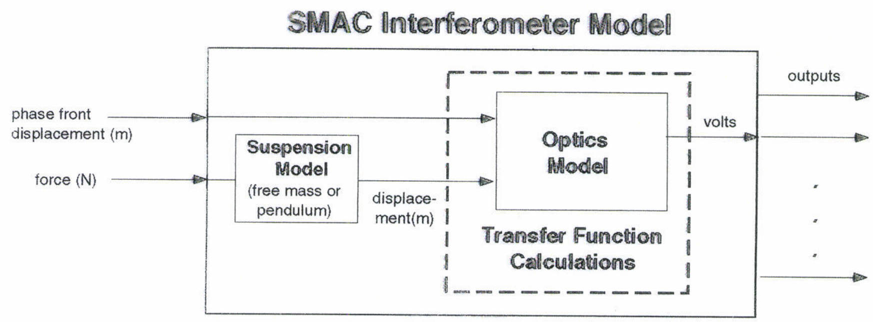
Design Considerations/ Constraints

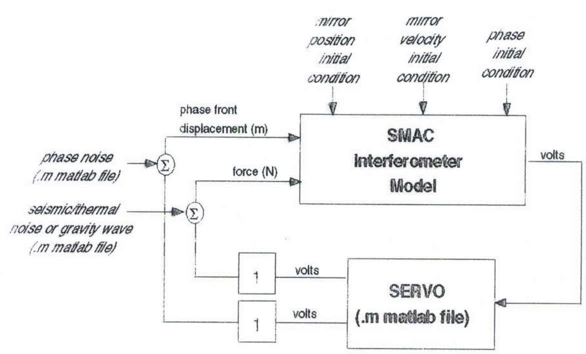
- Acquisition time
- Plant changes with state transitions
- Stability
- Ground motion
- Sensor/actuator limits
- Internal TM resonances
- Similarity to detection mode controls

Acquisition Time

- L- threshold velocity of 10 lambda/s achieved
- l- threshold velocity of 0.5 lambda/s achieved
- MTTL of seconds implied and simulated

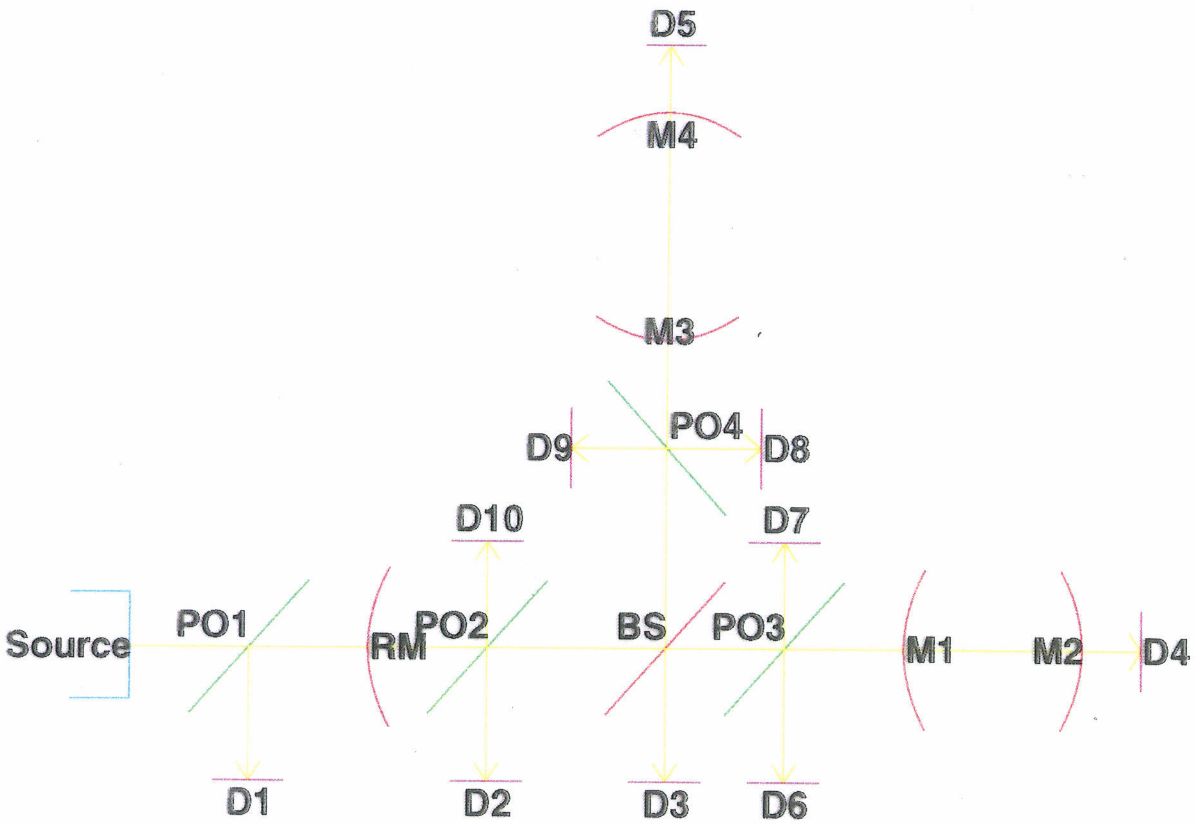
SMAC Model





LIGO

SMAC Configuration



Lengths

$$L_+ = \frac{L_1 + L_2}{2} \qquad l_+ = \frac{l_1 + l_2}{2}$$

$$L_- = \frac{L_1 - L_2}{2} \qquad l_- = \frac{l_1 - l_2}{2}$$

$$\delta\Phi_1 = \frac{1}{2}(\Phi_+ + \Phi_-) \qquad \delta\phi_1 = \frac{1}{2}(\phi_+ + \phi_-)$$

$$\delta\Phi_2 = \frac{1}{2}(\Phi_+ - \Phi_-) \qquad \delta\phi_2 = \frac{1}{2}(\phi_+ - \phi_-)$$

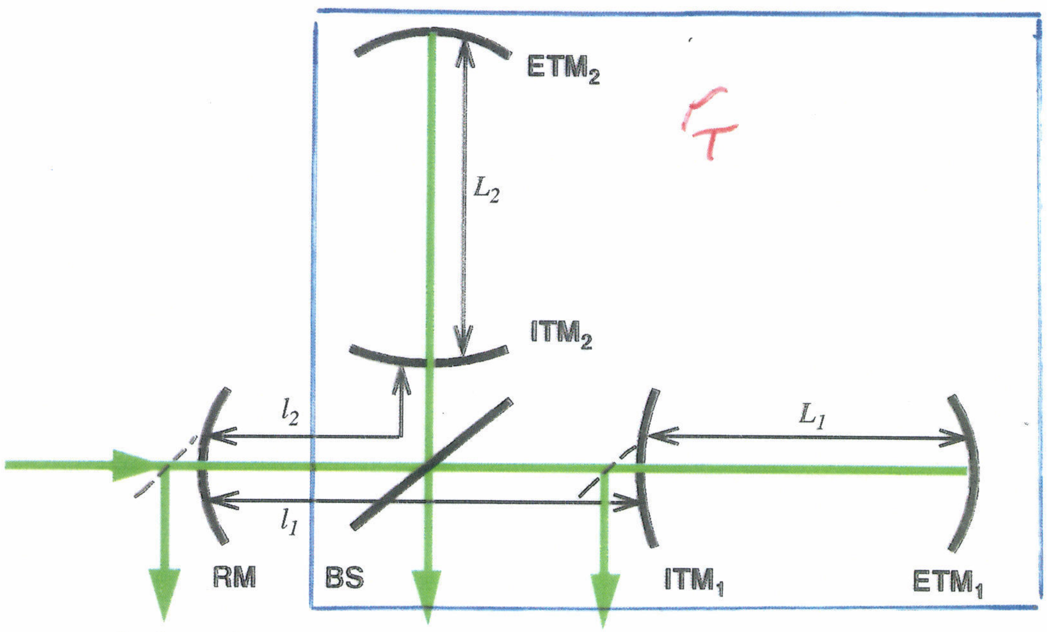


Figure 1: Definition of servo lengths in the system.

$$\frac{E_{\text{refl}}}{E_{\text{inc}}} = r_i + \frac{t_i^2 r_e e^{-i\phi}}{1 + r_i r_e e^{-i\phi}} \equiv r_c(\phi)$$

State Equations (cont)

$$r_T(\Phi,\phi) \equiv \frac{1}{2}\left[r_c(\Phi_1)\, e^{-i\phi_1} + r_c(\Phi_2)\, e^{-i\phi_2}\right]$$

$$r_T\Big|_4 = \frac{1}{2}(r_c^{\pi} + r_c^{\pi}) = -0.98984$$

$$r_T\Big|_2 = \frac{1}{2}(r_c(0) + r_c(0)) = 0.99996$$

$$r_T\Big|_3 = (r_c^0 + r_c^{\pi})/2 = 5.06 \times 10^{-3} \simeq 1/198$$

$$g_{cr} = \frac{t_r}{1 + r_r r_T} \qquad g_{sb} = \frac{t_r}{1 - r_r r_m}$$

root of recycling gain

$$r_{cr} = \frac{r_r + r_T}{1 + r_r r_T} \qquad r_{sb} = \frac{r_r - r_m}{1 - r_r r_m}$$

amplitude of reflected fields

$$t_{cr} = 0 \qquad t_{sb} = \frac{t_r\sqrt{1 - r_m^2}}{1 - r_r r_m}$$

amplitude of fields transmitted to asymmetric port

$$s_c = \frac{i\omega_a}{\omega_c} \qquad \omega_c = \frac{c}{2L}\left(\frac{1 - r_i r_e}{\sqrt{r_i r_e}}\right) \qquad f_c = 91 \text{ Hz}$$

cavity pole, all states

$$s_{cc} = \frac{i\omega_a}{\omega_{cc}} \qquad \omega_{cc} = \left(\frac{1 + r_r r_T}{1 + r_r}\right)\omega_c \qquad f_{cc} = 91,\ 46,\ 1.16 \text{ Hz}$$

double cavity pole

$$s_r = \frac{i\omega_a}{\omega_r} \qquad \omega_r = \left(1 + \frac{g_{cr}^2 r_{sb} r_T}{g_{sb}^2 r_{cr} r_m}\right)\omega_{cc} \qquad f_r = 91,\ 46,\ 6.0 \text{ Hz}$$

reflection zero

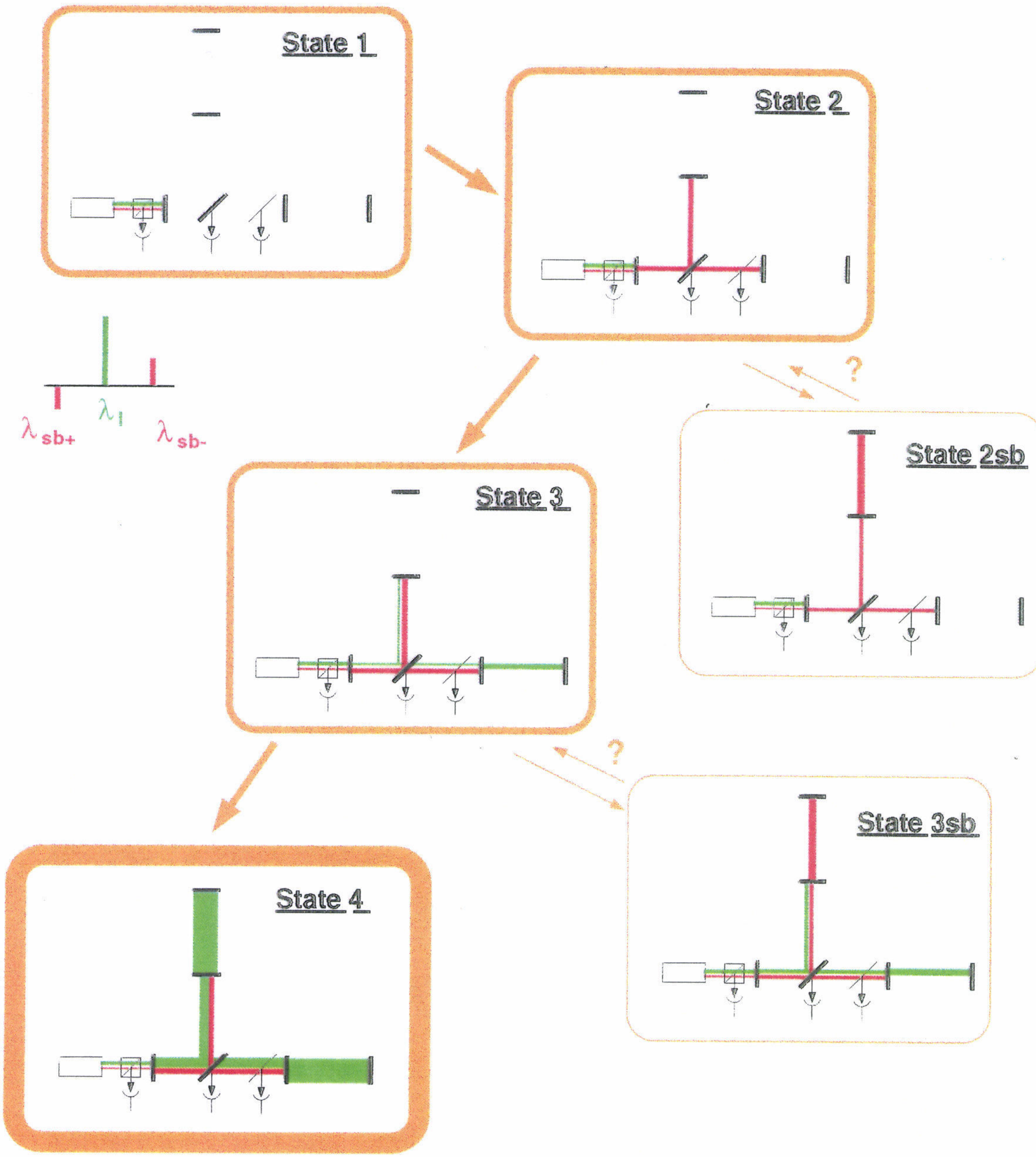
$$s_p = \frac{i\omega_a}{\omega_p} \qquad \omega_p = \left(1 - \frac{g_{cr}}{g_{sb}}\right)\omega_{cc} \qquad f_p = 91,\ 46,\ -0.74 \text{ Hz}$$

negative recycling cavity zero

State Equations

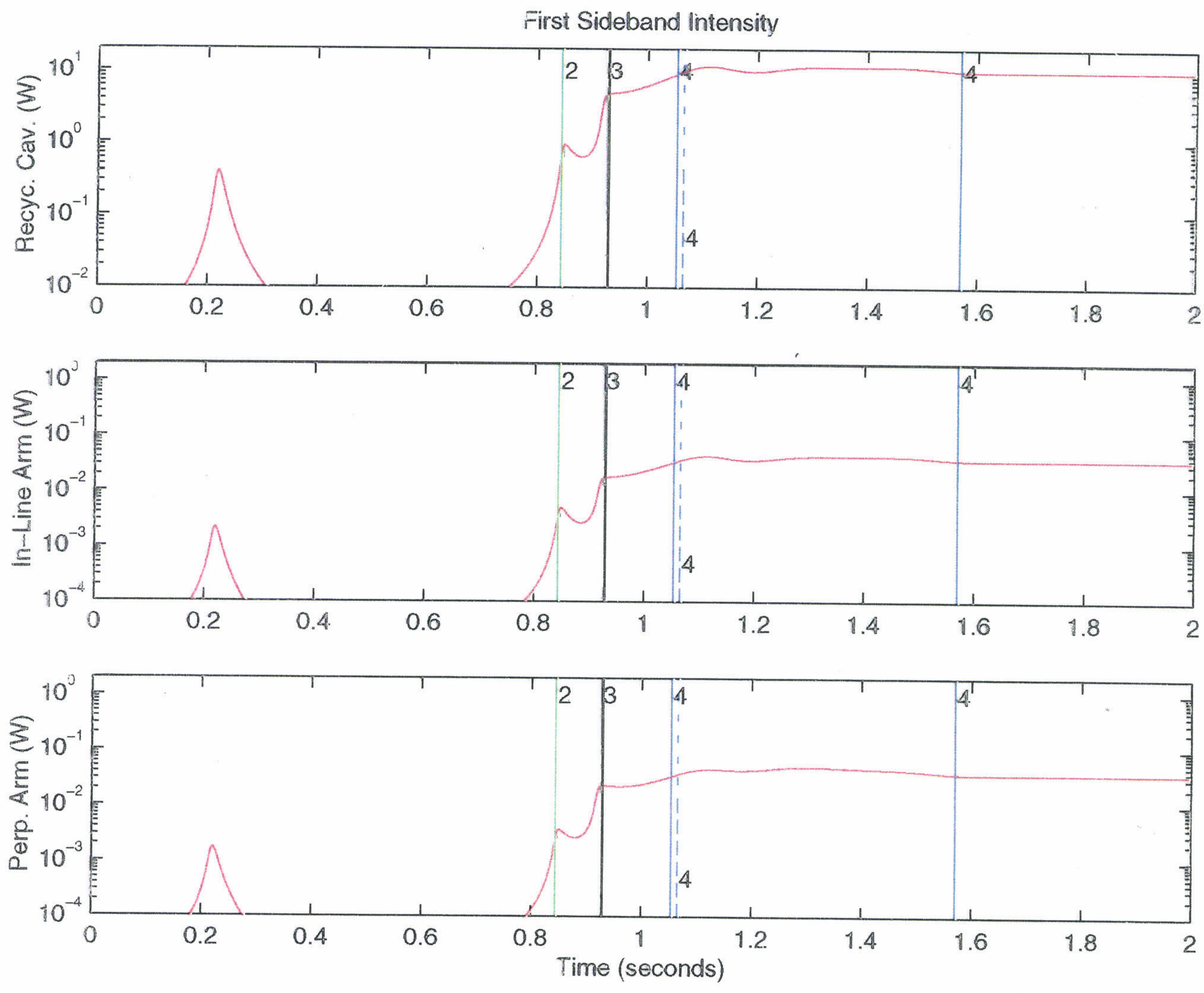
$$
\begin{aligned}
S_A &= 4Sk\, g_{cr} t_{sb} \frac{1}{1+s_c} \left[r_c\, \boldsymbol{\delta l}_- - r'_c\, \boldsymbol{\delta L}_- \right] \sin \omega_m t \\
S_R &= -\,4Sk\, g_{sb} t_{sb} r_{cr} \left[r'_m\, \boldsymbol{\delta L}_- + \boldsymbol{\delta l}_- \right] \sin \omega_m t \\
&+\; 4Sk\, \frac{1}{1+s_{cc}} \left[g_{cr}^2 r_{sb} r'_c\, \boldsymbol{\delta L}_+ - (g_{cr}^2 r_{sb} r_c + g_{sb}^2 r_{cr} r_m)(1+s_r)\, \boldsymbol{\delta l}_+ \right] \cos \omega_m t \\
S_P &= 4Sk\, \frac{g_{cr} g_{sb} t_{sb}}{t_r} \left[r'_m\, \boldsymbol{\delta L}_- - \boldsymbol{\delta l}_- \right] \sin \omega_m t \\
&+\; 4Sk\, \frac{g_{cr} g_{sb} r_m}{t_r} \frac{1}{1+s_{cc}} \left[r_c (g_{cr} - g_{sb})(1+s_p)\, \boldsymbol{\delta l}_+ - g_{cr} r'_c\, \boldsymbol{\delta L}_+ \right] \cos \omega_m t \\
S_P^\nu &= S g_{cr} g_{sb} r_m\, \frac{1-r_c}{1+r_r}\, \frac{i s_{cc}}{1+s_{cc}}\, \boldsymbol{\nu}\, \cos \omega_m t \\
S_R^\nu &= -\, S r_{sb}\, (1 - r_{cr})\, \frac{i s_{cc}}{1+s_{cc}}\, \boldsymbol{\nu}\, \cos \omega_m t
\end{aligned}
$$

State Transitions

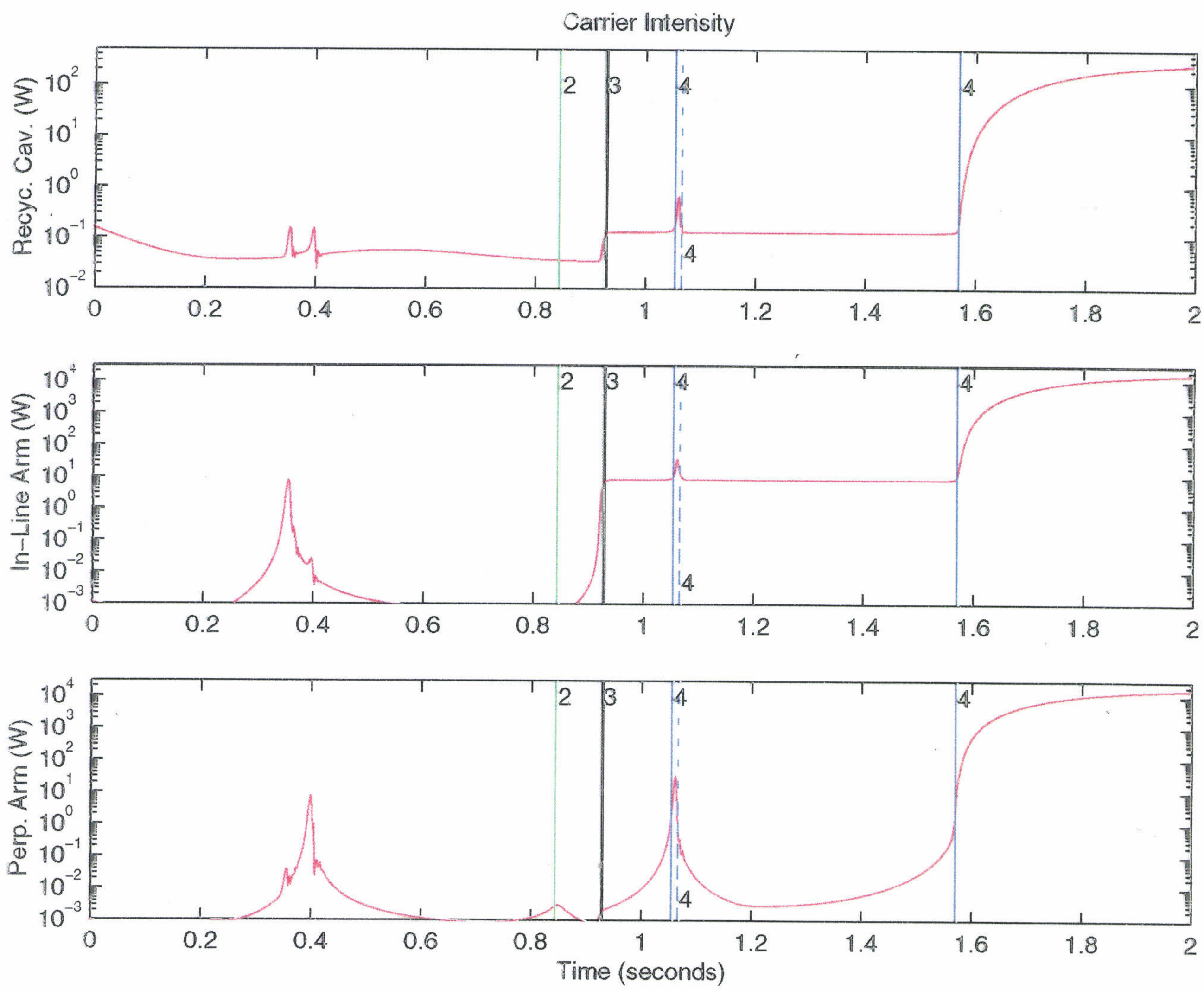


LIGO

State Transitions



State Transitions



quantity	**State**	**Description**
		Recycling Mirror
r_r	$\sqrt{0.97} = 0.985$	reflectivity
l_r	$\sqrt{30 \times 10^{-6}}$	absorption loss
t_r	$\sqrt{1 - r^2 - l^2} = \sqrt{0.02997} = 0.173$	transmission
		Input Test Mass
l_i	$\sqrt{75 \times 10^{-6}} = 0.008660$	absorption loss
r_{AR}	$\sqrt{300 \times 10^{-6}}$	AR loss
t_i	$\sqrt{0.03} = 0.173$	transmission
r_i	$\sqrt{1 - t^2 - l^2 = 0.970} = 0.985$	reflectivity
		End Test Mass
t_e	$\sqrt{10 \times 10^{-6}} = 0.003162$	transmission
l_e	$\sqrt{70 \times 10^{-6}} = 0.008367$	loss
r_e	0.99996	reflectivity
		Beam Splitter
l_{bs}	$\sqrt{30 \times 10^{-6}}$	absorption loss
r_{AR}	$\sqrt{300 \times 10^{-6}}$	AR loss
r_{bs}	$\sqrt{1/2}$	reflectivity
t_{bs}	$\sqrt{1 - r^2 - l^2 = 0.4997} = 0.707$	transmission
		Misc.
l_{asym}	0.23 m	Schnupp asymmetry
$l_{\pm}$	9.38 m	recycling cavity average length
ω_m	$2\pi\,(23.97)$ MHz	modulation frequency
λ	1.064 μm	laser wavelength

Fabry-Perot derived quantities

	antiresonant	**resonant**	
r_c	0.99996	−0.98984	carrier reflectivity of FP cavity (r_c^0, r_c^π)
r_m		0.97342	sideband reflectivity of FP cavity
r'_c		130.31	dr_c/dL (resonant)
r'_m		0.007634	dr_m/dL
f_c	91 Hz		Fabry-Perot cavity pole

IFO derived quantities

	1	**2**	**3**	**4**	
g_{cr}		0.0872	0.173	$\sqrt{46} = 6.74$	recycling carrier gain
g_{sb}		$\sqrt{17} = 4.135$			recycling sideband gain
r_T		0.99996	0.005	−0.98984	reflected Thevenin equivalent
r_2		0.002	−0.0017	−6.74	carrier field at PO2
r_3		0.001	−0.061	−4.77	carrier field at PO3
r_P		0.12	0.122	4.76	carrier field reflected from ITM
r_{cr}		1.000	0.958	−0.1971	reflected carrier field
r_{sb}		0.30128			reflected sideband field
t_{cr}				0	fields transmitted to asymmetric port
t_{sb}		0.94452			fields transmitted to asymmetric port
f_{cc}		f_c	$f_c/2$	1.16 Hz	double cavity pole (recycing + FP)
f_r		f_c	$f_c/2$	6.0 Hz	reflection zero
f_p		f_c	$f_c/2$	−0.74 Hz	recycling cavity zero

Table 1: Interferometer parameters.

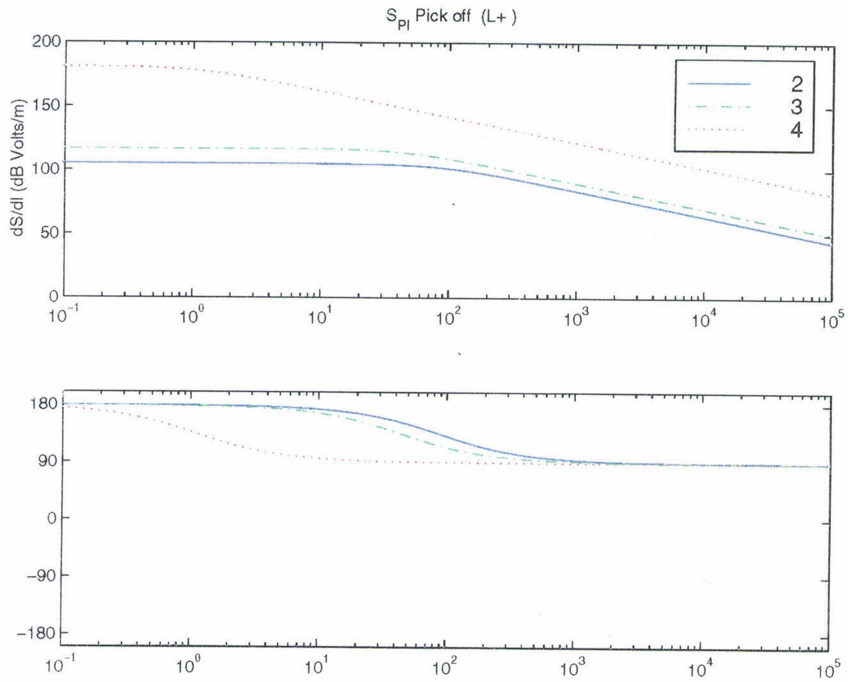


Figure 5: Derived common mode transfer function (L_+), all states.

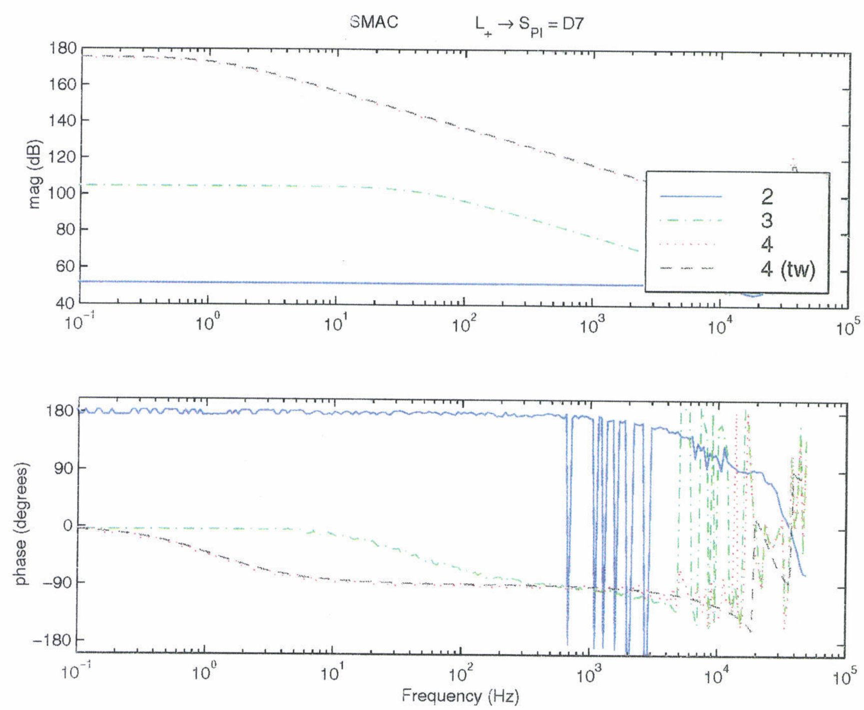


Figure 6: Common mode transfer function (L_+) at D7, SMAC and Twiddle, all states.

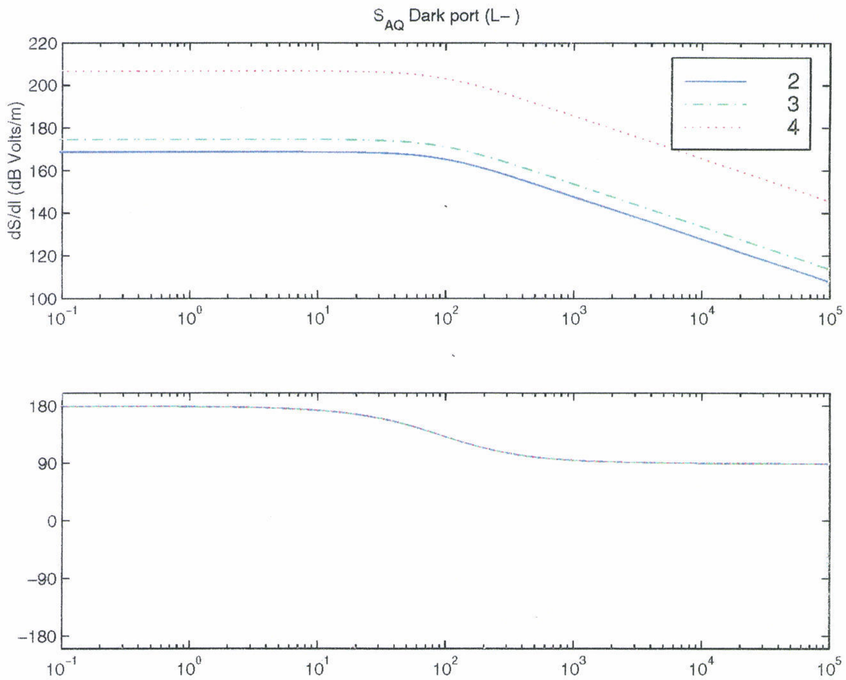


Figure 7: Derived differential mode transfer fúnction (L_-), all states.

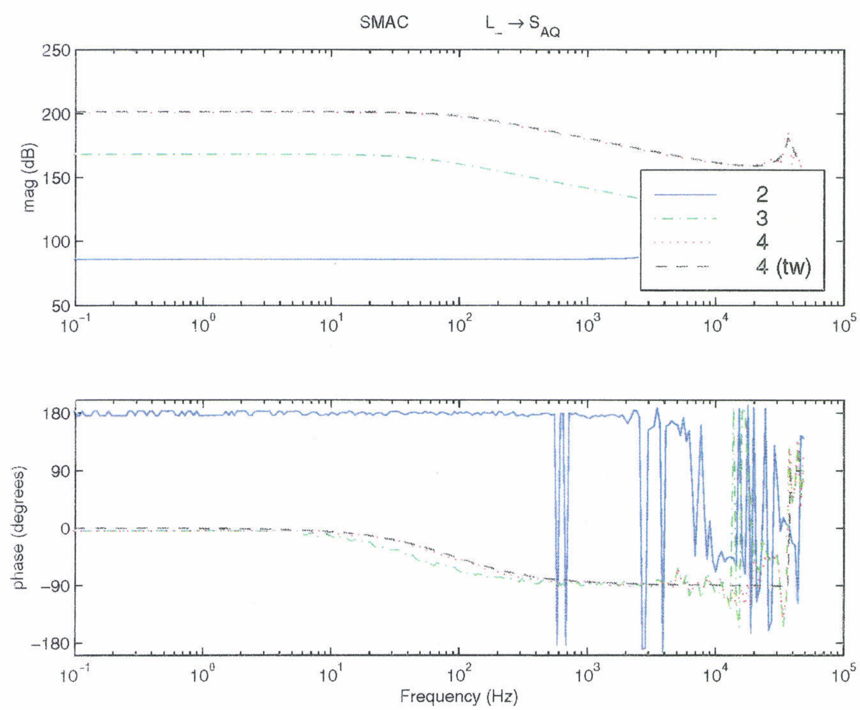


Figure 8: Michelson differential mode transfer function (L_-), SMAC and Twiddle, all states.

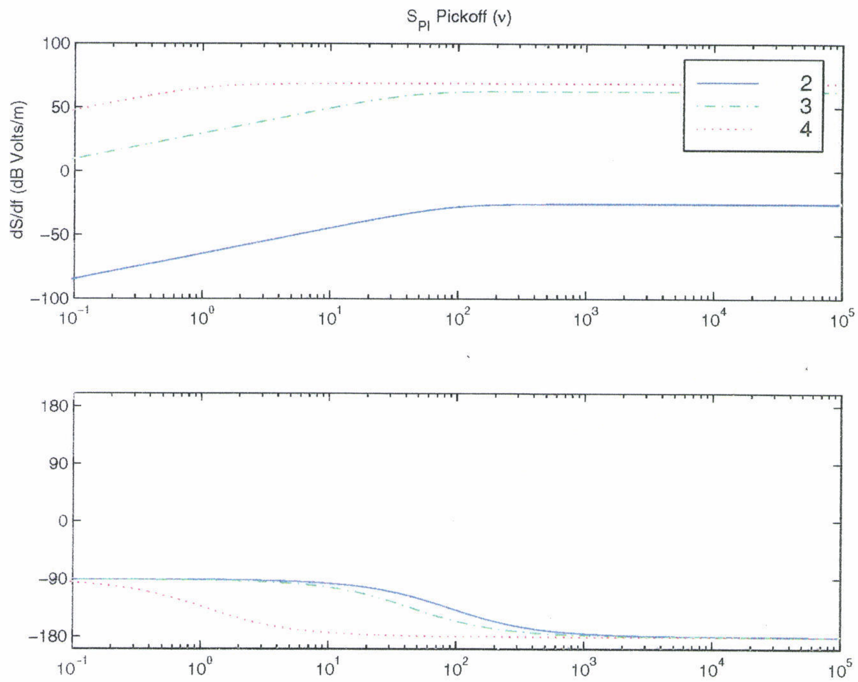


Figure 9: Derived source transfer functión (ν), all states.

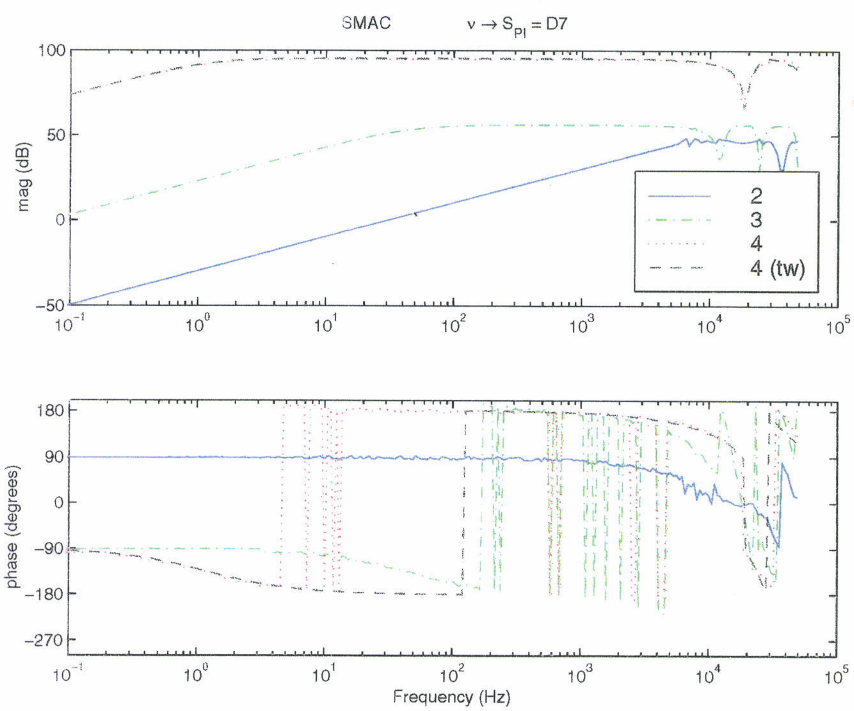


Figure 10: Source transfer function (ν) at D7, SMAC and Twiddle, all states.

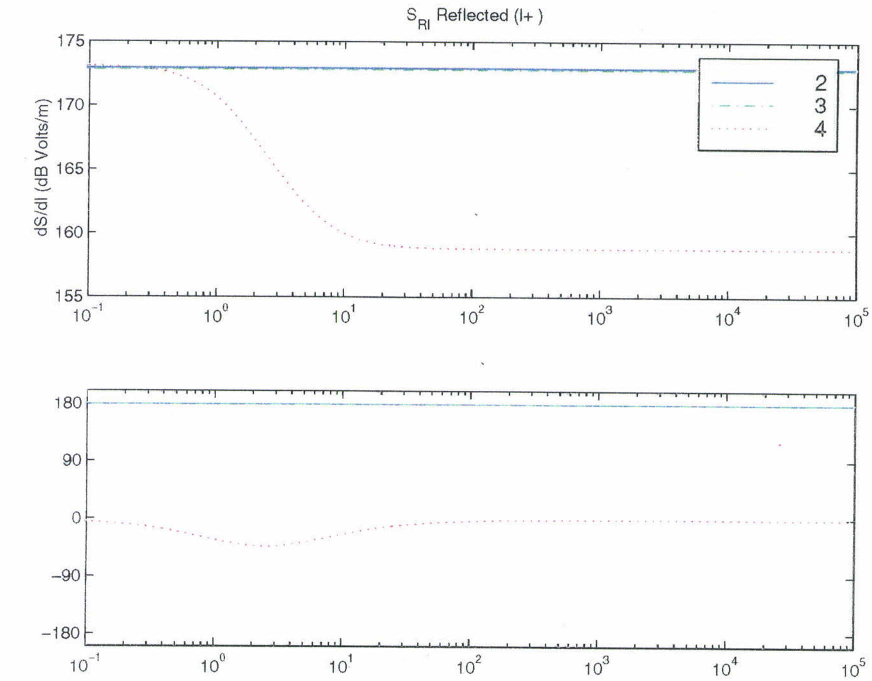


Figure 3: Derived Michelson common mode transfer function (l_+), all states.

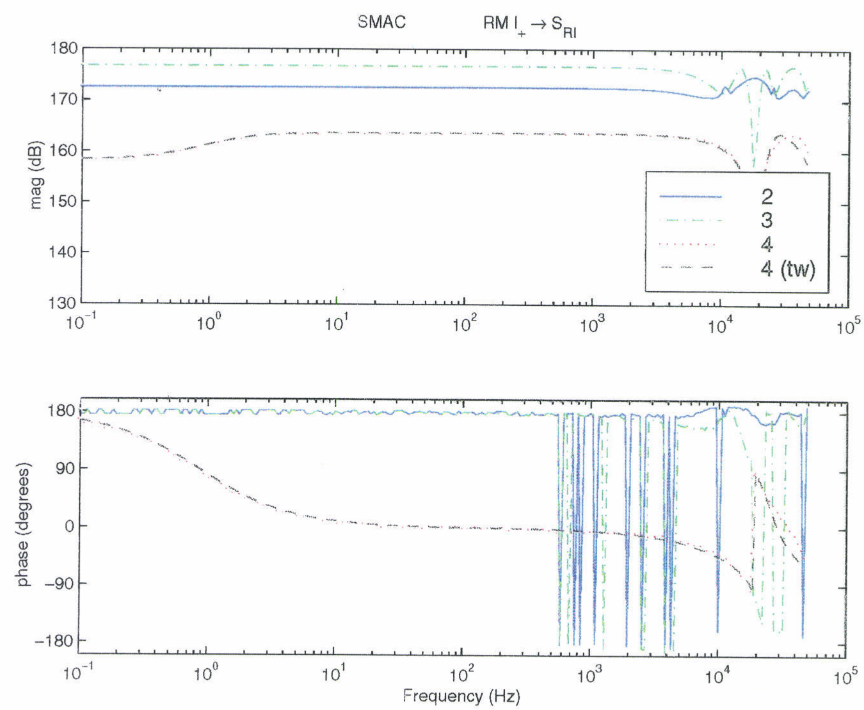


Figure 4: Michelson common mode transfer function (recycling mirror-driven l_+), SMAC and Twiddle (+120 dB), all states.

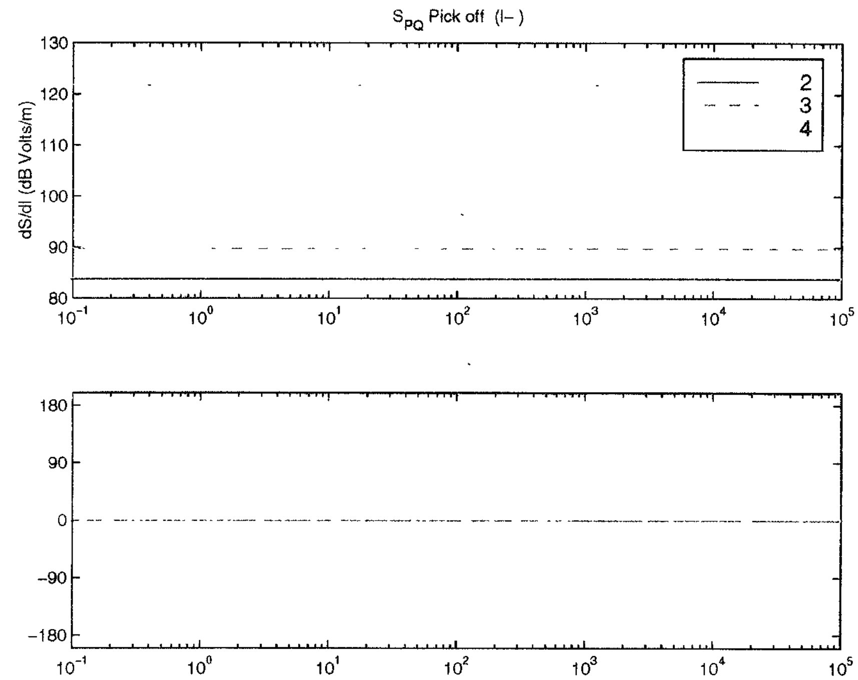


Figure 1: Derived Michelson differential mode transfer function (l_{-}), all states.

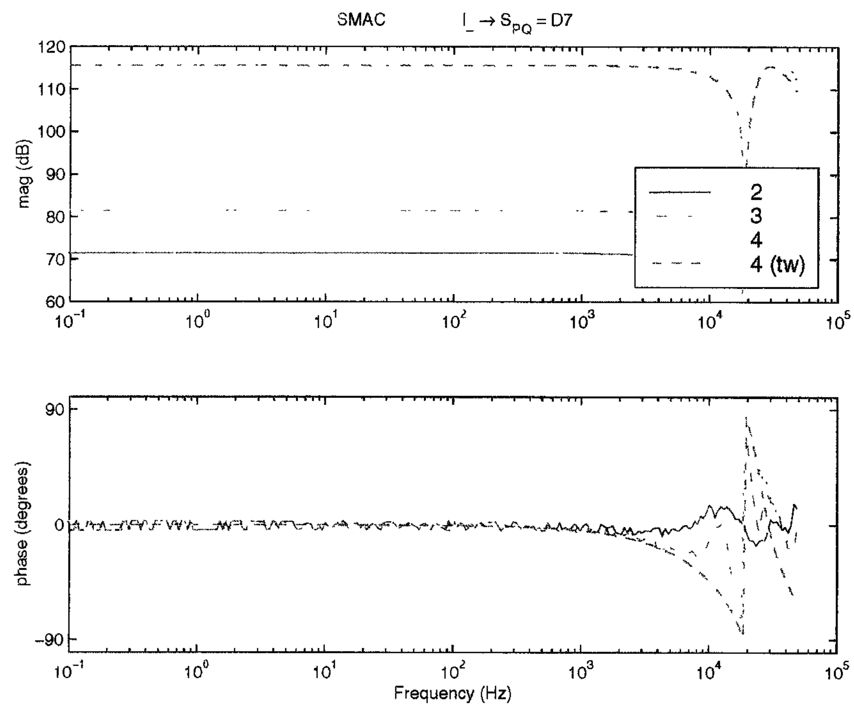
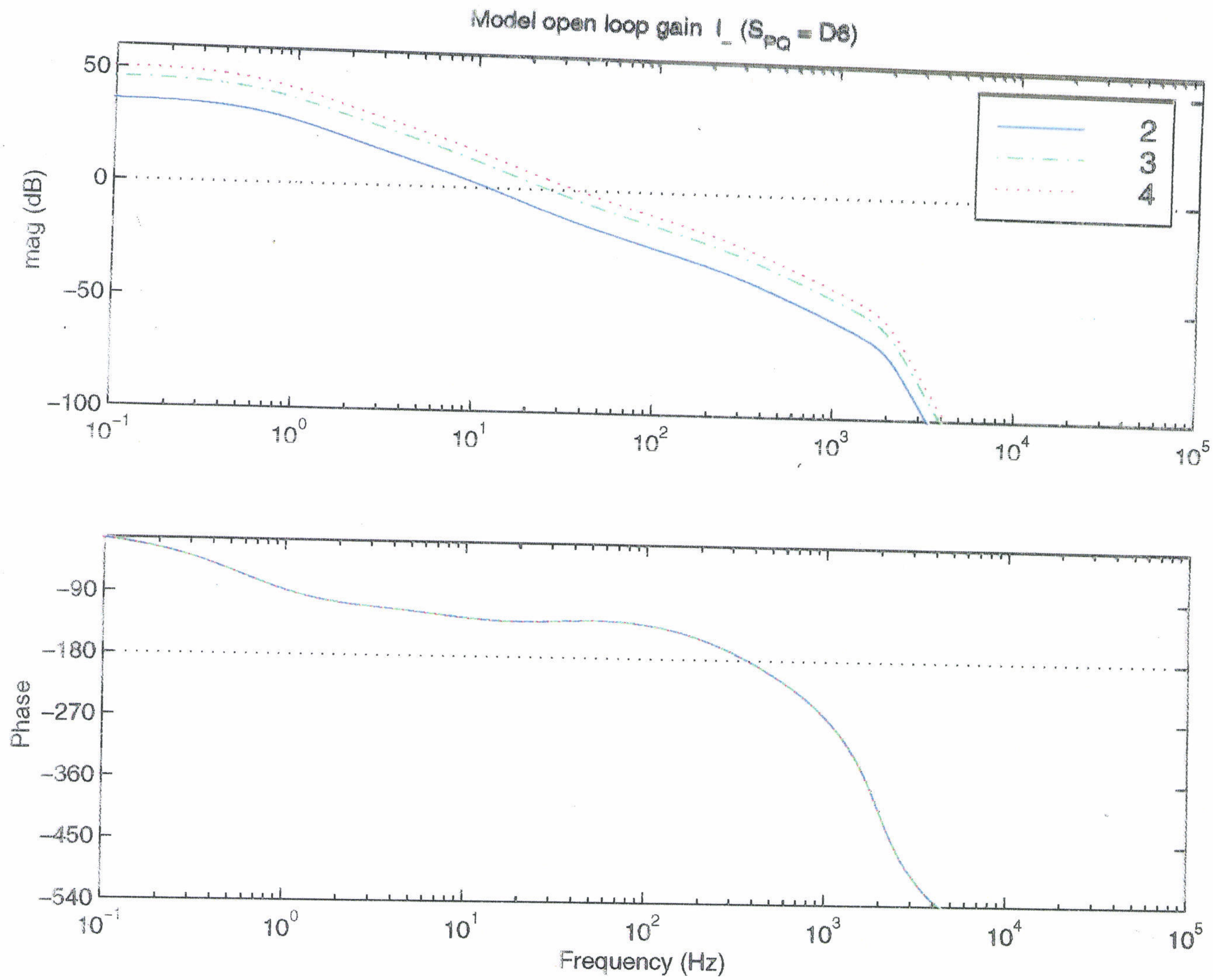
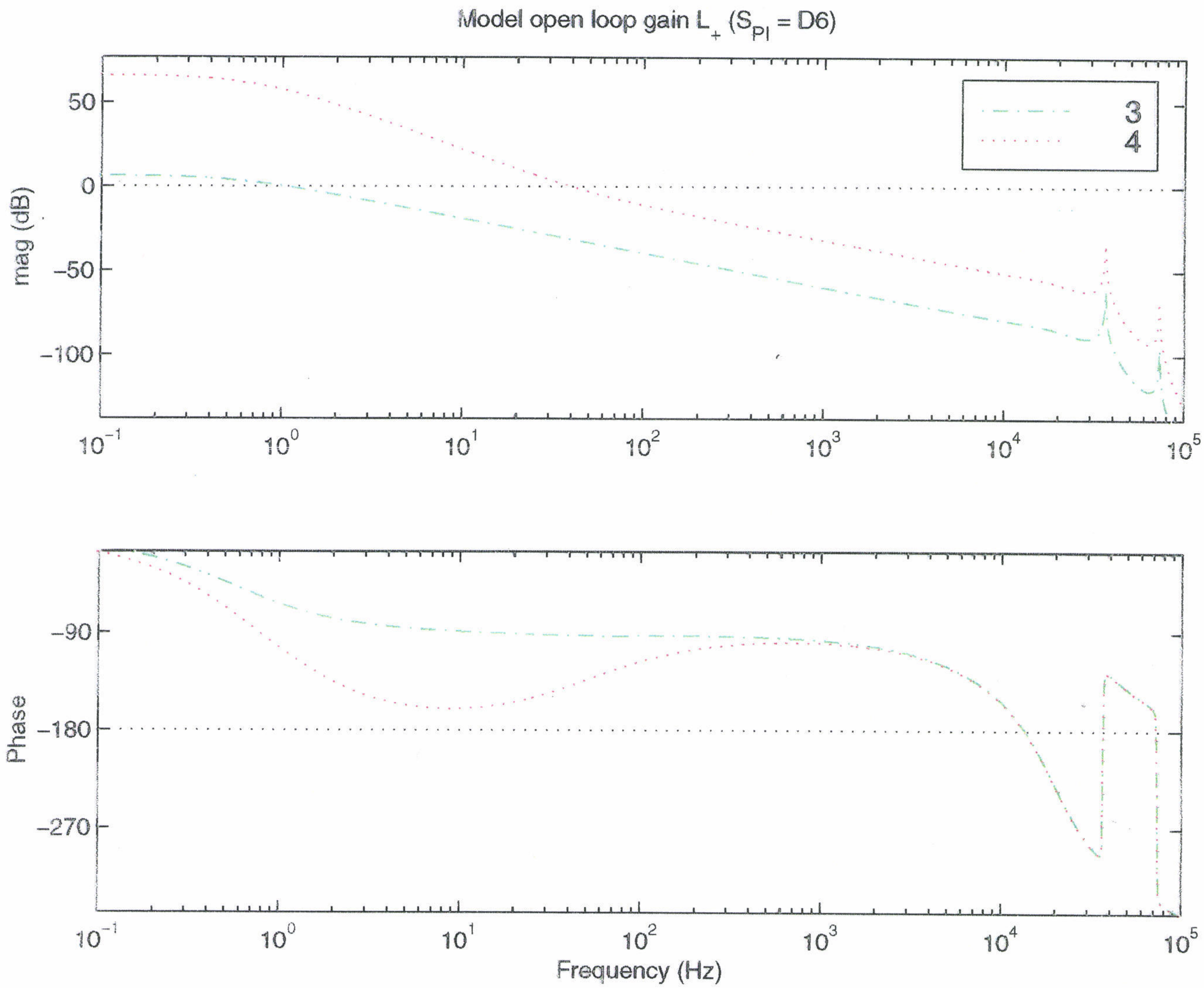


Figure 2: Michelson differential mode transfer function $(-(M_1 + M_2) + (M_3 + M_4))$ at D7, SMAC and Twiddle, all states.

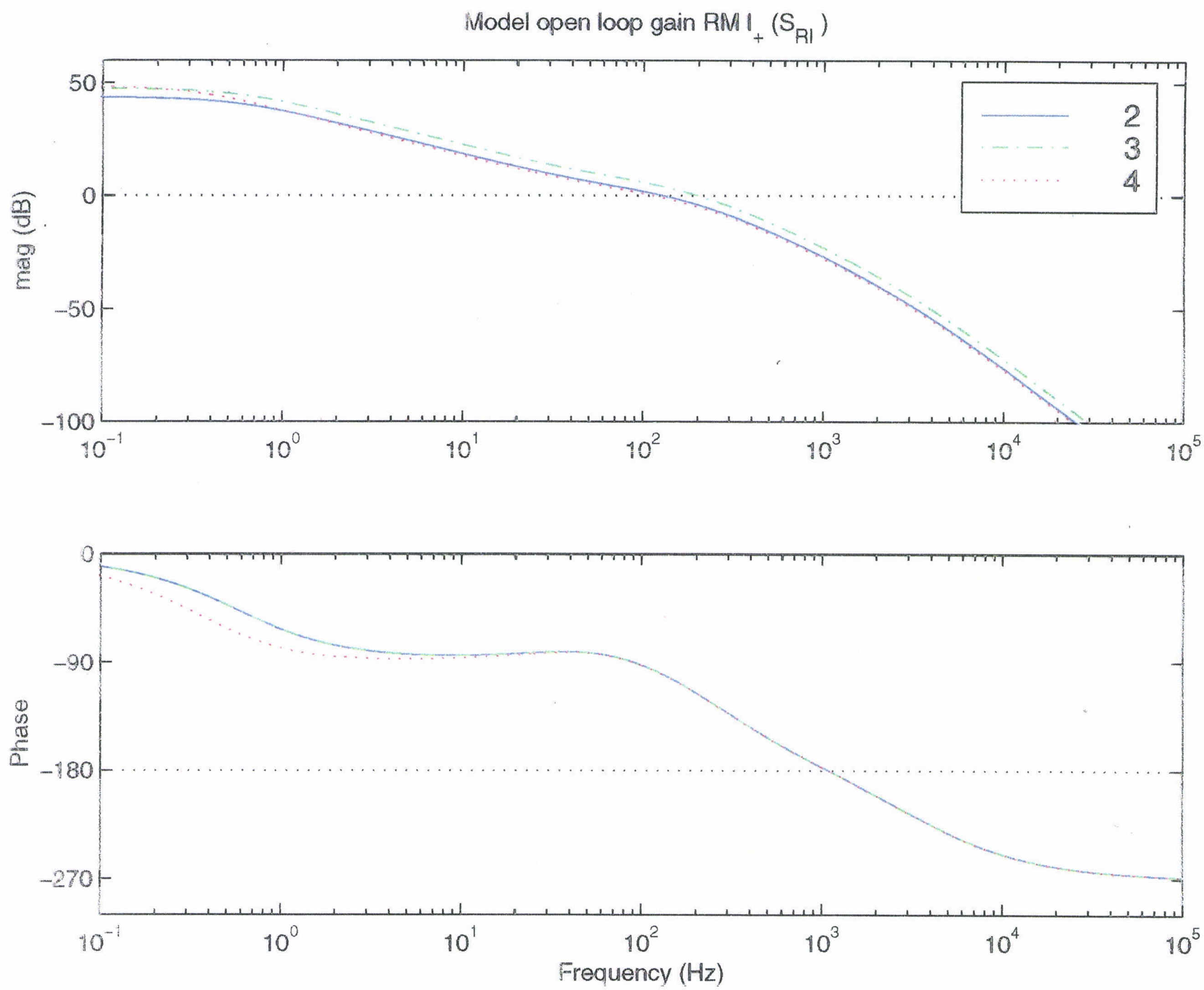
I- Open Loop Gain



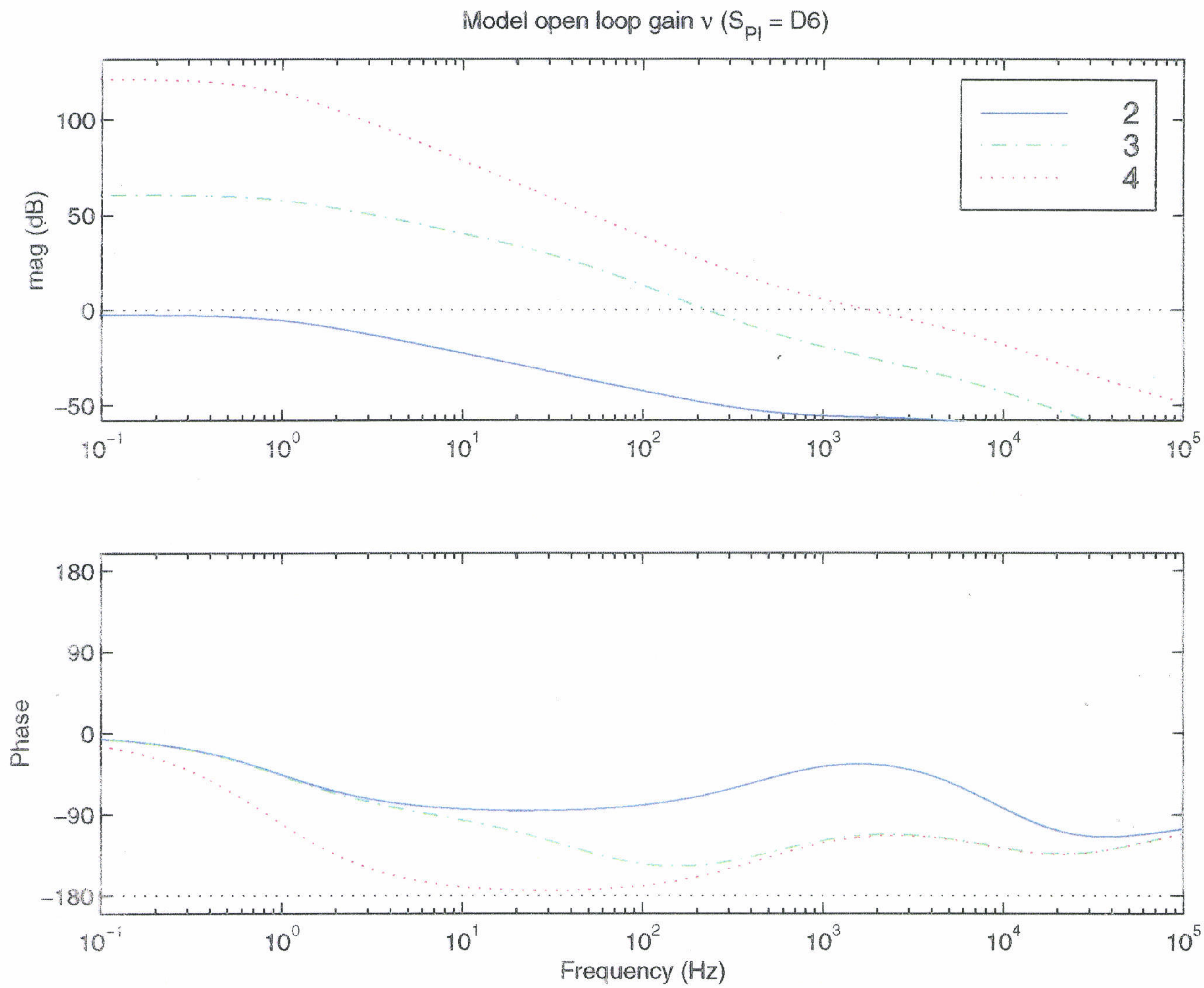
L+ Open Loop Gain



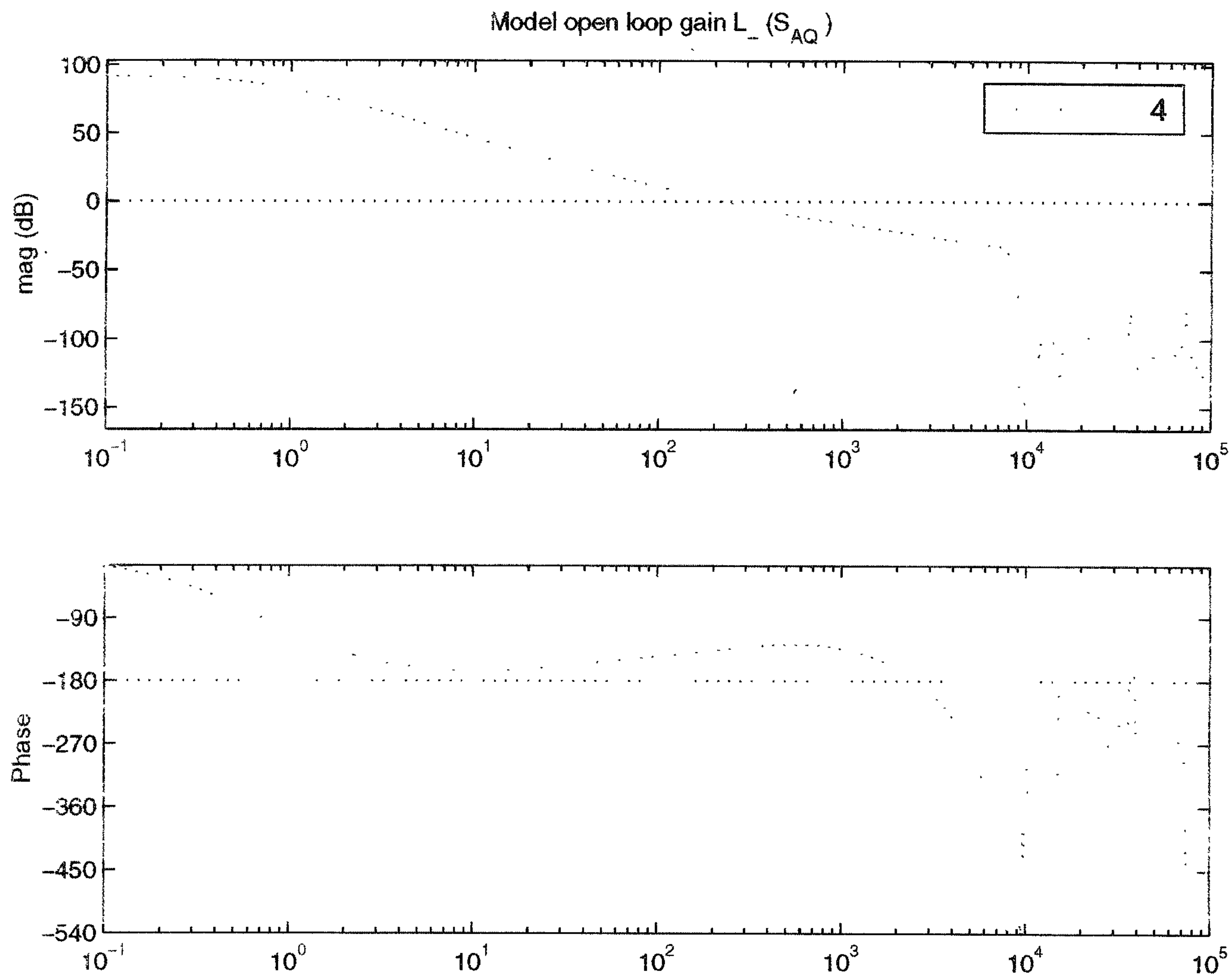
I+ Open Loop Gain



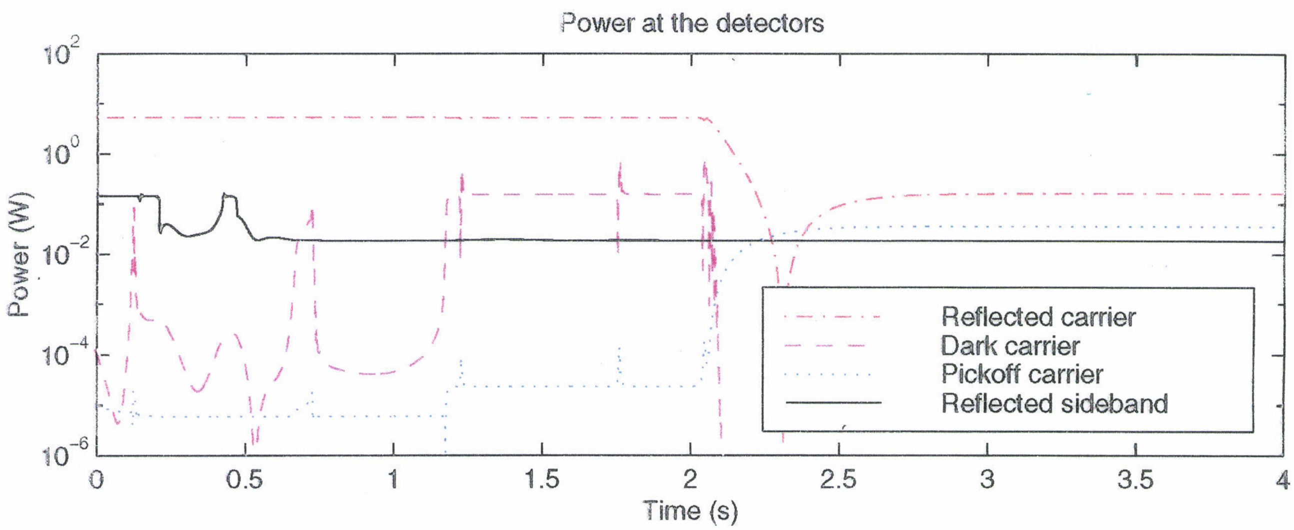
Laser Frequency Open Loop Gain

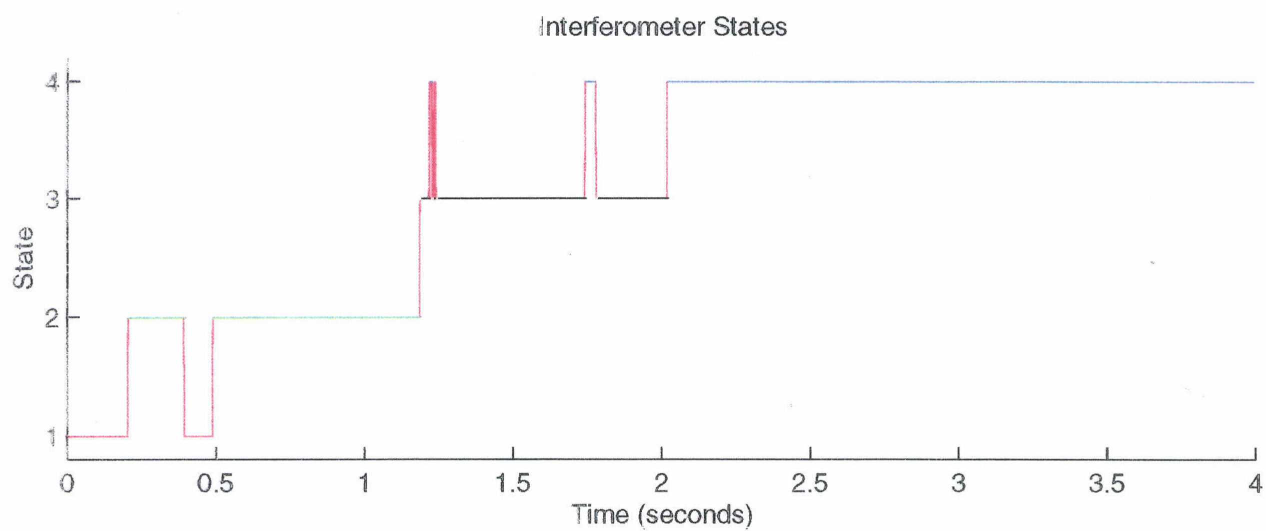


L- Open Loop Gain



Interferometer States





Triggers and controller switching

State Transition	Loop	Trigger	Effect
$1 \rightarrow 2$	$l_{\pm}$	$P_R^{sb} < 0.04$ W	State 2 acquired, enable L_+
$2 \rightarrow 3$	ϕ, L_+ L_-	$P_A^c > 0.15$ W	State 3 acquired, –30 dB l_- $\pm$ l_- actuator sign enable L_- loop
$3 \rightarrow 4$ (acq)	all	$P_R^c < 5.25$ & $P_A^c < 0.15$ W	
4 (acq) $\rightarrow$ 4 (det)	all	$P_{tr\parallel}^c$ & $P_{tr\perp}^c > 0.305$ W	switch to detection mode

Table 1: Transition through the acquisition states, showing triggers and controller enabling. c carrier, sb – sideband, tr – transmitted, a – asymmetric port, r – reflected port.

CONTROLLER	STATE 2		STATE 3		STATE 4	
	Gain (dB)	Phase (deg.)	Gain (dB)	Phase (deg.)	Gain (dB)	Phase (deg.)
$S_{PQ} \to l_-$	40 @ 380 Hz	55 @ 12 Hz	30 @ 380 Hz	80 @ 135 Hz	26 @ 380 Hz	55 @ 39 Hz
$S_{RI} \to l_+$	28 @ 1.12 kHz	80 @ 135 Hz	16 @ 1.12 kHz	64 @ 210 Hz	29 @ 1.17 kHz	83 @ 121 Hz
$S_{PI} \to L_+$			81 @ 14 kHz	116 @ 1 Hz	53 @ 14 kHz	42 @ 43 Hz
$S_{AQ} \to L_-$			@	@	24 @ 2.67 kHz	46 @ 234 Hz
$S_{PI} \to \nu$			∞ @ –	40 @ 300 Hz	∞ @ –	45 @ 2 kHz

Table 1: Gain and phase margins of controllers

Controllers

$$S_{PQ} \to L_- = -21.6 \frac{(s + 2\pi\ 2)(s + 2\pi\ 30)}{(s + 2\pi\ 10)(s + 2\pi\ 300)}$$

$$\times \text{ (2 kHz 5 pole Butterworth)}$$

$$S_{RI} \to L_+ = -1080 \frac{(s + 2\pi\ 1)(s + 2\pi\ 50)}{(s + 2\pi\ 100)(s + 2\pi\ 300)(s + 2\pi\ 3.5\ \text{kHz})}$$

$$S_{DI} \to L_+ = (s + 2\pi\ 1)(s + 2\pi\ 50)$$

$$\times \text{ (6 pole, 0.1\% ripple, 60 dB stopband, 7.5 kHz elliptic)}$$

$$\times \text{ (8 pole 80 dB elliptic notch 9.1 kHz-10.1 kHz)}$$

$$S_{PI} \to \phi = -\frac{\text{[illegible]}}{30} \frac{(s + 2\pi\ 500)(s + 2\pi\ 50\ \text{kHz})}{s(s + 2\pi\ 1)(s + 2\pi\ 10\ \text{kHz})}$$

$$S_{IQ} \to L_- = \text{[illegible] 108 dB)}\ (s + 2\pi\ 50)(s + 2\pi\ 250)$$

$$\times \text{ (6 pole, 0.1\% ripple, 60 dB stopband, 7.5 kHz elliptic)}$$

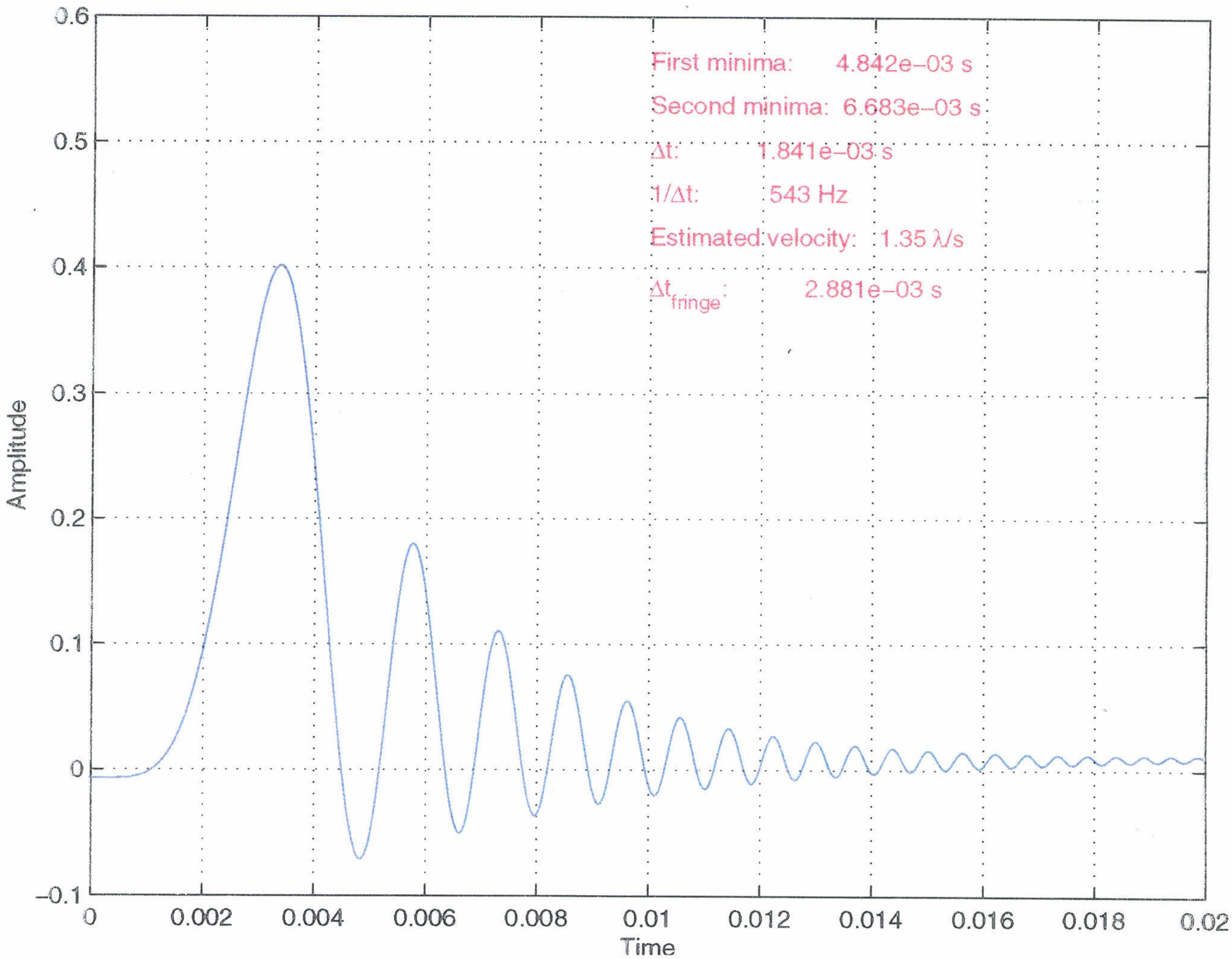
$$\times \text{ (8 pole 80 dB elliptic notch 9.1 kHz-10.1 kHz)}$$

Results and Refinements

- Designed a stable, robust lock acquisition system requiring minimal switching, with predicted short MTTL.
- Better understanding of plant in divers states.
- Frequency crossovers in L+, frequency loops to MC, PSL, IFO
- Simplify controllers further to minimize switching between acquisition and detection modes
- Evolve SMAC as diagnostic tool for LIGO turn-on
- Use STM code to further study alignment effects
- Study transitions between states (short time scale effects), improve triggering
- Implement LA in digital/analog control system

LIGO

Fringe



First minima: 4.842e−03 s
Second minima: 6.683e−03 s
Δt: 1.841e−03 s
1/Δt: 543 Hz
Estimated velocity: 1.35 λ/s
Δt fringe: 2.881e−03 s
Amplitude
Time
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02

Ground Motion